disha
Publication Inc

71 JEE Main MATHEMATICS

Chapterwise & Topicwise Solved Papers

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Contents

JEE Main Score Vs Rank 2020

Estimated JEE Main Score Vs Rank range for 2020 Jan. session:

Mark/300	Rank
285–300	Top 100
275 - 284	100 - 200
260 - 274	200 - 500
250 - 259	500 - 1000
240 - 249	1000 - 1500
220 - 239	1500 - 3500
200 - 219	3500 - 6000
180 - 199	6000 - 9500
150 -180	9500 - 15000
120 - 149	15000 - 35000
<120	More than 35000

JEE Main Percentile vs Ranks

It is also important to understand which rank will be awarded to a candidate on a certain percentile. It is as important as understanding the JEE Main marks vs percentile. Candidates can refer the table below for the same:

Percentile Scores (NTA Score)	Expected Rank (Approximate)
100	1
99	8750
98	17,500
97	26,200
96	35,000
95	43,700
94	52,400
93	61,200
92	70,000
91	78,700
90	87,450

JEE Main 2019 Marks vs Rank

It should be noted that JEE Main 2019 was of 360 marks where as JEE Main 2020 was of 300 Marks. So Please make adjustments why comparing the marks and the rank.

Score Range	Rank range
310 marks to 360 marks	1 to 100
290 marks to 309 marks	101 to 200
270 marks to 289 marks	201 to 500
255 marks to 269 marks	501 to 1000
247 marks to 254 marks	1001 to 1500
240 marks to 246 marks	1501 to 2000
232 marks to 239 marks	2001 to 2500
225 marks to 231 marks	2501 to 3000
217 marks to 224 marks	3001 to 3500
210 marks to 216 marks	3501 to 4000
207 marks to 209 marks	4001 to 4500
204 marks to 206 marks	4501 to 5000
200 marks to 203 marks	5001 to 5500
197 marks to 199 marks	5501 to 6000
195 marks to 196 marks	6001 to 6500
192 marks to 194 marks	6501 to 7000
185 marks to 189 marks	7501 to 8000
182 marks to 184 marks	8001 to 8500
179 marks to 181 marks	8501 to 9000
177 marks to 178 marks	9001 to 9500
175 marks to 176 marks	9501 to 10000
165 marks to 174 marks	10001 to 20000
152 marks to 164 marks	20001 to 35000
140 marks to 151 marks	35001 to 50000
130 marks to 139 marks	50001 to 75000
125 marks to 129 marks	75001 to 98000
117 marks to 124 marks	98001 to 118000
109 marks to 116 marks	118001 to 139400
102 marks to 108 marks	139401 to 182200
94 marks to 101 marks	160801 to 182200
Less than 93 marks	More than 182201

Seats offered by NITs, IIITs, and CFTIs

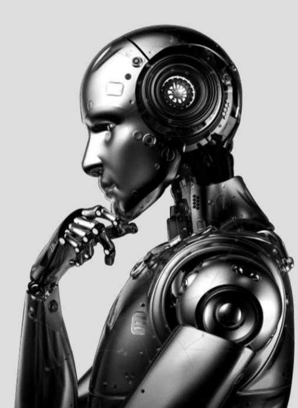
Institutes	Participating Institutes	OBC NCL	ST	Open	SC	Total Seats
IIITs	23	1089	310	2078	609	4023
NITs	31	4858	1736	9264	2762	17967
CFTIs	23	776	391	2878	658	4683

JEE Main Cutoff Trends (2013-2019)

Tabulated below is the category wise JEE Main cut off marks for the previous years:

Year	ST	SC	OBC NCL	General
2013	45 Marks	50 Marks	70 Marks	113 Marks
2014	47 Marks	53 Marks	74 Marks	115 Marks
2015	44 Marks	50 Marks	70 Marks	105 Marks
2016	48 Marks	52 Marks	70 Marks	100 Marks
2017	27 Marks	32 Marks	49 Marks	81 Marks
2018	24	29	45	74
2019	44.33	54.01	74.3	89.7

1



Sets

TOPIC 1 Sets, Types of Sets, Disjoint Sets, Complementary Sets, Subsets, Power Set, Cardinal Number of Sets, Operations on Sets

1. Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is ______. **[Sep. 06, 2020 (I)]**

2. Let $S = \{1, 2, 3, \ldots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is : **[Jan. 12, 2019 (I)]**

(a) $2^{100} - 1$ (b) $2^{50}\left(2^{50} - 1\right)$

(c) $2^{50} - 1$ (d) $2^{50} + 1$

3. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$. Then S : **[2018]**

(a) contains exactly one element.
(b) contains exactly two elements.
(c) contains exactly four elements.
(d) is an empty set.

4. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S: **[2016]**

(a) contains exactly two elements.
(b) contains more than two elements.
(c) is an empty set.
(d) contains exactly one element.

5. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then: **[Online April 10, 2016]**

(a) $P \subset Q$ and $Q - P \neq \phi$
(b) $Q \not\subset P$
(c) $P = Q$
(d) $P \not\subset Q$

6. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is: **[Online April 12, 2014]**

(a) 32 (b) 16 (c) 8 (d) 64

7. Let $X = \{1,2,3,4,5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is : **[2012]**

(a) 5^2 (b) 3^5 (c) 2^5 (d) 5^3

8. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[2009]**

(a) $A = C$ (b) $B = C$

(c) $A \cap B = \phi$ (d) $A = B$

TOPIC 2 Venn Diagrams, Algebraic Operations on Sets, De Morgan's Law, Number of Elements in Different Sets

9. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be : **[Sep. 05, 2020 (I)]**

(a) 63 (b) 36 (c) 54 (d) 38

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be : **[Sep. 04, 2020 (I)]**

(a) 29 (b) 37 (c) 65 (d) 55

11. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to **[Sep. 04, 2020 (II)]**

(a) 15 (b) 50 (c) 45 (d) 30

12. Let $X = \{n \in N: 1 \le n \le 50\}$. If
$A = \{n \in X: n \text{ is } a \text{ multiple of } 2\}$ and
$B = \{n \in X: n \text{ is } a \text{ multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is __________. **[Jan. 7, 2020 (II)]**

13. Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is : **[Jan. 12, 2019 (II)]**
(a) 2^{15} (b) 2^{18} (c) 2^{12} (d) 2^{10}

14. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is: **[Jan. 10, 2019 (II)]**
(a) 102 (b) 42 (c) 1 (d) 38

15. Let A, B and C be sets such that $\phi \ne A \cap B \subseteq C$. Then which of the following statements is not true ? **[April 12, 2019 (II)]**
(a) $B \cap C \ne \phi$
(b) If $(A - B) \subseteq C$, then $A \subseteq C$
(c) $(C \cup A) \cap (C \cup B) = C$
(d) If $(A - C) \subseteq B$, then $A \subseteq B$

16. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is: **[April. 09, 2019 (II)]**
(a) 13.9 (b) 12.8 (c) 13 (d) 13.5

17. In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements : **[Online April 10, 2015]**
(A) 5% families own both a car and a phone
(B) 35% families own either a car or a phone
(C) 40,000 families live in the town
Then,
(a) Only (A) and (C) are correct.
(b) Only (B) and (C) are correct.
(c) All (A), (B) and (C) are correct.
(d) Only (A) and (B) are correct.

Hints & Solutions

1. **(28)** $2^m = 112 + 2^n \Rightarrow 2^m - 2^n = 112$

$\Rightarrow 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$

$\therefore m = 7,\ n = 4 \Rightarrow mn = 28$

2. **(b)** $\because$ Product of two even number is always even and product of two odd numbers is always odd.

$\therefore$ Number of required subsets

= Total number of subsets – Total number of subsets having only odd numbers

$= 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

3. **(b)** **Case-I:** $x \in [0,9]$

$2(3-\sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2$

$\Rightarrow x = 16, 4$

Since $x \in [0,9]$

$\therefore \quad x = 4$

Case-II: $x \in [9, \infty]$

$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

Since $x \in [9, \infty]$

$\therefore x = 16$

Hence, $x = 4$ & 16

4. **(a)** $f(x) + 2f\left(\frac{1}{x}\right) = 3x \quad ....(1)$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \quad ....(2)$

Adding (1) and (2)

$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = x + \frac{1}{x} \quad ...(3)$

Substracting (1) from (2)

$\Rightarrow f(x) - f\left(\frac{1}{x}\right) = \frac{3}{x} - 3x \quad ...(4)$

On adding (3) and (4)

$\Rightarrow f(x) = \frac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x \Rightarrow x = \frac{2}{x}$

$x^2 = 2 \quad$ or $\quad x = \sqrt{2}, -\sqrt{2}$

5. **(c)** $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \cos\theta + \sqrt{2}\cos\theta$

$= (\sqrt{2}+1)\cos\theta = \left(\frac{2-1}{\sqrt{2}-1}\right)\cos\theta$

$\Rightarrow (\sqrt{2}-1)\sin\theta = \cos\theta$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\therefore \quad P = Q$

6. **(b)** $A = \{x : |x| < 3,\ x \in Z\}$

$A = \{-2, -1, 0, 1, 2\}$

$R = \{(x, y) : y = |x|,\ x \neq -1\}$

$R = \{(-2, 2), (0, 0), (1, 1), (2, 2)\}$

R has four elements

Number of elements in the power set of R

$= 2^4 = 16$

7. **(b)** Let $X = \{1, 2, 3, 4, 5\}$

$n(x) = 5$

Each element of x has 3 options. Either in set Y or set Z or none. ($\because Y \cap Z = \phi$)

So, number of ordered pairs $= 3^5$

8. **(b)** $\because \quad B = (B \cap A) \cup B$

$= (A \cap C) \cup B$

$= (A \cup B) \cap (C \cup B)$

$= (A \cup C) \cap (B \cup C)$

$= (A \cap B) \cup C$

$= (A \cap C) \cup C$

$= C$

9. **(b)** Given, $n(C) = 73,\ n(T) = 65,\ n(C \cap T) = x$

$\therefore 65 \geq n(C \cap T) \geq 65 + 73 - 100$

$\Rightarrow 65 \geq x \geq 38 \Rightarrow x \neq 36.$

10. **(d)** Let $n(U) = 100$, then $n(A) = 63, n(B) = 76$

$n(A \cap B) = x$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) \leq 100$

$= 63 + 76 - x \leq 100$

$\Rightarrow x \geq 139 - 100 \Rightarrow x \geq 39$

$\because n(A \cap B) \le n(A)$

$\Rightarrow x \le 63$

$\therefore 39 \le x \le 63$

11. **(d)** $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$

$\because \quad n(X_i) = 10,\ n(Y_i) = 5$

So, $\bigcup_{i=1}^{50} X_i = 500,\ \bigcup_{i=1}^{n} Y_i = 5n$

$\Rightarrow \frac{500}{20} = \frac{5n}{6} \Rightarrow n = 30$

12. **(29)** From the given conditions,

$n(A) = 25, n(B) = 7$ and $n(A \cap B) = 3$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3 = 29$

13. **(a)** Let $x \in A$, then

$\because \quad 2^{(x+2)(x^2-5x+6)} = 1 \Rightarrow (x+2)(x-2)(x-3) = 0$

$x = -2, 2, 3$

$A = \{-2, 2, 3\}$

Then, $n(A) = 3$

Let $x \in B$, then

$-3 < 2x - 1 < 9$

$-1 < x < 5$ and $x \in Z$

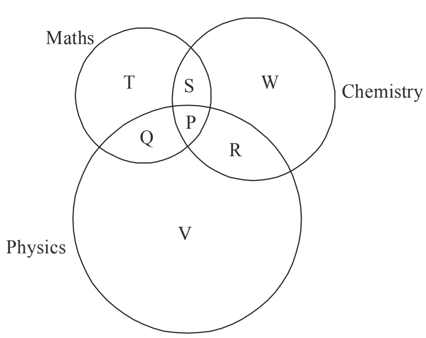
$\therefore \quad B = \{0, 1, 2, 3, 4\}$

$n(B) = 5$

$n(A \times B) = 3 \times 5 = 15$

Hence, Number of subsets of $A \times B = 2^{15}$

14. **(d)**



$P = \{30, 60, 90, 120\}$

$\Rightarrow \quad n(P) = 4$

$Q = \{6n: n \in N, 1 \le n \le 23\} - P$

$\Rightarrow \quad n(Q) = 19$

$R = \{15n: n \in N, 1 \le n \le 9\} - P$

$\Rightarrow \quad n(R) = 5$

$S = \{10n: n \in N, 1 \le n \le 14\} - P$

$\Rightarrow \quad n(S) = 10$

$n(T) = 70 - n(P) - n(Q) - n(S) = 70 - 33 = 37$

$n(V) = 46 - n(P) - n(Q) - n(R) = 46 - 28 = 18$

$n(W) = 28 - n(P) - n(R) - n(S) = 28 - 19 = 9$

$\Rightarrow$ Number of required students

$= 140 - (4 + 19 + 5 + 10 + 37 + 18 + 9)$

$= 140 - 102 = 38$

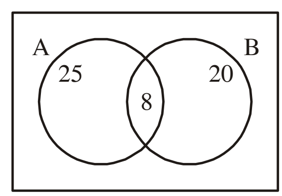
15. **(d)** (1), (2) and (4) are always correct.

In (3) option,

If $A = C$ then $A - C = \phi$

Clearly, $\phi \subseteq B$ but $A \subseteq B$ is not always true.

16. **(a)**



% of people who reads A only = 25 – 8 = 17%

% of people who read B only = 20 – 8 = 12%

% of people from A only who read advertisement

$= 17 \times 0.3 = 5.1\%$

% of people from B only who read advertisement

$= 12 \times 0.4 = 4.8\%$

% of people from A & B both who read advertisement

$= 8 \times 0.5 = 4\%$

$\therefore$ total % of people who read advertisement

$= 5.1 + 4.8 + 4 = 13.9\%$

17. **(c)** $n(P) = 25\%$

$n(C) = 15\%$

$n(P' \cup C') = 65\%$

$\Rightarrow n(P \cup C)' = 65\%$

$n(P \cup C) = 35\%$

$n(P \cap C) = n(P) + n(C) - n(P \cup C)$

$25 + 15 - 35 = 5\%$

$x \times 5\% = 2000$

$x = 40,000$

2

Relations and Functions

TOPIC 1 Relations, Domain, Codomain and Range of a Relation, Functions, Domain, Codomain and Range of a Function

1. Let R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in \mathbf{R}^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in \mathbf{R}^2 : a^2 + b^2 \notin Q\}$, where Q is the set of all rational numbers. Then : **[Sep. 03, 2020 (II)]**

(a) Neither R_1 nor R_2 is transitive.
(b) R_2 is transitive but R_1 is not transitive.
(c) R_1 is transitive but R_2 is not transitive.
(d) R_1 and R_2 are both transitive.

2. The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty]$. Then a is equal to : **[Sep. 02, 2020 (I)]**

(a) $\frac{\sqrt{17}}{2}$ (b) $\frac{\sqrt{17}-1}{2}$ (c) $\frac{1+\sqrt{17}}{2}$ (d) $\frac{\sqrt{17}}{2}+1$

3. If $R = \{(x, y) : x, y \in \mathbf{Z}, x^2 + 3y^2 \le 8\}$ is a relation on the set of integers $\mathbf{Z}$, then the domain of R^{-1} is : **[Sep. 02, 2020 (I)]**

(a) $\{-2, -1, 1, 2\}$ (b) $\{0, 1\}$
(c) $\{-2, -1, 0, 1, 2\}$ (d) $\{-1, 0, 1\}$

4. Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is : **[Jan. 11, 2019 (I)]**

(a) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (b) $R - [-1, 1]$
(c) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $(-1, 1) - \{0\}$

5. The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is: **[April. 09, 2019 (II)]**

(a) $(-1, 0) \cup (1, 2) \cup (3, \infty)$
(b) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$
(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
(d) $(1, 2) \cup (2, \infty)$

6. The range of the function $f(x) = \frac{x}{1+|x|}$, $x \in R$, is **[Online May 7, 2012]**

(a) R (b) $(-1, 1)$ (c) $R - \{0\}$ (d) $[-1, 1]$

7. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is **[2011]**

(a) $(0, \infty)$ (b) $(-\infty, 0)$
(c) $(-\infty, \infty) - \{0\}$ (d) $(-\infty, \infty)$

8. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is **[2003]**

(a) $(-1,0) \cup (1,2) \cup (2,\infty)$ (b) $(a,2)$
(c) $(-1,0) \cup (a,2)$ (d) $(1,2) \cup (2,\infty)$

TOPIC 2 Even and Odd Functions, Explicit and Implicit Functions, Greatest Integer Function, Periodic Functions, Value of a Function, Equal Functions, Algebraic Operations on Functions

9. Let $[t]$ denote the greatest integer $\le t$. Then the equation in x, $[x]^2 + 2[x+2] - 7 = 0$ has : **[Sep. 04, 2020 (I)]**

(a) exactly two solutions
(b) exactly four integral solutions
(c) no integral solution
(d) infinitely many solutions

10. Let $f(x)$ be a quadratic polynomial such that $f(-1)+f(2)=0$. If one of the roots of $f(x)=0$ is 3, then its other root lies in : **[Sep. 02, 2020 (II)]**

(a) $(-1, 0)$ (b) $(1, 3)$ (c) $(-3, -1)$ (d) $(0, 1)$

11. Let $f(1, 3) \to R$ be a function defined by $f(x)=\dfrac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\le x$. Then the range of f is: **[Jan. 8, 2020 (II)]**

(a) $\left(\dfrac{2}{5}, \dfrac{3}{5}\right) \cup \left(\dfrac{3}{4}, \dfrac{4}{5}\right)$ (b) $\left(\dfrac{2}{5}, \dfrac{1}{2}\right) \cup \left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

(c) $\left(\dfrac{2}{5}, \dfrac{4}{5}\right)$ (d) $\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

12. If $f(x)=\log_e\left(\dfrac{1-x}{1+x}\right)$, $|x|<1$, then $f\left(\dfrac{2x}{1+x^2}\right)$ is equal to : **[April 8, 2019 (I)]**

(a) $2f(x)$ (b) $2f(x^2)$ (c) $(f(x))^2$ (d) $-2f(x)$

13. Let $f(x)=a^x$ $(a>0)$ be written as $f(x)=f_1(x)+f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y)+f_1(x-y)$ equals : **[April. 08, 2019 (II)]**

(a) $2f_1(x)f_1(y)$ (b) $2f_1(x+y)f_1(x-y)$

(c) $2f_1(x)f_2(y)$ (d) $2f_1(x+y)f_2(x-y)$

14. Let $f(n)=\left[\dfrac{1}{3}+\dfrac{3n}{100}\right]n$, where [n] denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56} f(n)$ is equal to: **[Online April 19, 2014]**

(a) 56 (b) 689 (c) 1287 (d) 1399

15. Let f be an odd function defined on the set of real numbers such that for $x \ge 0$, $f(x) = 3\sin x + 4\cos x$.

Then f(x) at $x=-\dfrac{11\pi}{6}$ is equal to: **[Online April 11, 2014]**

(a) $\dfrac{3}{2}+2\sqrt{3}$ (b) $-\dfrac{3}{2}+2\sqrt{3}$

(c) $\dfrac{3}{2}-2\sqrt{3}$ (d) $-\dfrac{3}{2}-2\sqrt{3}$

16. A real valued function $f(x)$ satisfies the functional equation $f(x-y)=f(x)f(y)-f(a-x)f(a+y)$ where a is a given constant and $f(0)=0$, $f(2a-x)$ is equal to **[2005]**

(a) $-f(x)$ (b) $f(x)$

(c) $f(a)+f(a-x)$ (d) $f(-x)$

17. The graph of the function $y=f(x)$ is symmetrical about the line $x=2$, then **[2004]**

(a) $f(x)=-f(-x)$ (b) $f(2+x)=f(2-x)$

(c) $f(x)=f(-x)$ (d) $f(x+2)=f(x-2)$

18. If $f: R \to R$ satisfies $f(x+y)=f(x)+f(y)$, for all $x, y \in R$ and $f(1)=7$, then $\sum_{r=1}^{n} f(r)$ is **[2003]**

(a) $\dfrac{7n(n+1)}{2}$ (b) $\dfrac{7n}{2}$

(c) $\dfrac{7(n+1)}{2}$ (d) $7n+(n+1)$

Hints & Solutions

1. **(a)** For R_1 let $a = 1+\sqrt{2},\ b = 1-\sqrt{2},\ c = 8^{1/4}$

$aR_1b \Rightarrow a^2+b^2 = (1+\sqrt{2})^2+(1-\sqrt{2})^2 = 6 \in Q$

$bR_1c \Rightarrow b^2+c^2 = (1-\sqrt{2})^2+(8^{1/4})^2 = 3 \in Q$

$aR_1c \Rightarrow a^2+c^2 = (1+\sqrt{2})^2+(8^{1/4})^2 = 3+4\sqrt{2} \notin Q$

$\therefore R_1$ is not transitive.

For R_2 let $a = 1+\sqrt{2},\ b = \sqrt{2},\ c = 1-\sqrt{2}$

$aR_2b \Rightarrow a^2+b^2 = (1+\sqrt{2})^2+(\sqrt{2})^2 = 5+2\sqrt{2} \notin Q$

$bR_2c \Rightarrow b^2+c^2 = (\sqrt{2})^2+(1-\sqrt{2})^2 = 5-2\sqrt{2} \notin Q$

$aR_2c \Rightarrow a^2+c^2 = (1+\sqrt{2})^2+(1-\sqrt{2})^2 = 6 \in Q$

$\therefore R_2$ is not transitive.

2. **(c)** $\because f(x) = \sin^{-1}\left(\dfrac{|x|+5}{x^2+1}\right)$

$\therefore -1 \le \dfrac{|x|+5}{x^2+1} \le 1$

$\Rightarrow |x|+5 \le x^2+1 \qquad [\because x^2+1 \ne 0]$

$\Rightarrow x^2-|x|-4 \ge 0$

$\Rightarrow \left(|x|-\dfrac{1-\sqrt{17}}{2}\right)\left(|x|-\dfrac{1+\sqrt{17}}{2}\right) \ge 0$

$\Rightarrow x \in \left(-\infty, -\dfrac{1+\sqrt{17}}{2}\right) \cup \left[\dfrac{1+\sqrt{17}}{2}, \infty\right)$

$\therefore a = \dfrac{1+\sqrt{17}}{2}$

3. **(d)** Since, $R = \{(x, y) : x, y \in \mathbf{Z},\ x^2+3y^2 \le 8\}$

$\therefore R = \{(1, 1), (2, 1), (1, -1), (0, 1), (1, 0)\}$

$\Rightarrow D_{R^{-1}} = \{-1, 0, 1\}$

4. **(a)** $f(x) = \dfrac{x}{1+x^2}, x \in R$

Let, $y = \dfrac{x}{1+x^2}$

$\Rightarrow yx^2 - x + y = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1-4y^2}}{2}$

$\Rightarrow 1-4y^2 \ge 0$

$\Rightarrow 1 \ge 4y^2$

$\Rightarrow |y| \le \dfrac{1}{2}$

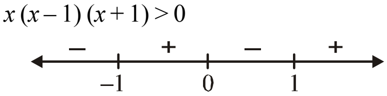
$\Rightarrow -\dfrac{1}{2} \le y \le \dfrac{1}{2}$

$\Rightarrow$ The range of f is $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$.

5. **(c)** To determine domain, denominator $\ne 0$ and $x^3-x>0$

i.e., $4-x^2 \ne 0\ x \ne \pm 2$ ...(1)

and $x(x-1)(x+1) > 0$



$x \in (-1, 0) \cup (1, \infty)$ ...(2)

Hence domain is intersection of (1) & (2).

i.e., $x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

6. **(b)** $f(x) = \dfrac{x}{1+|x|}, x \in R$

If $x > 0, |x| = x \Rightarrow f(x) = \dfrac{x}{1+x}$

which is not defined for $x = -1$

If $x < 0, |x| = -x \Rightarrow f(x) = \dfrac{x}{1-x}$

which is not defined for $x = 1$

Thus $f(x)$ defined for all values of R except 1 and – 1

Hence, range = (– 1, 1).

7. **(b)** $f(x) = \dfrac{1}{\sqrt{|x|-x}}$, $f(x)$ is define if $|x|-x>0$

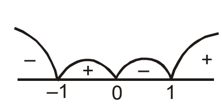
$\Rightarrow |x| > x, \Rightarrow x < 0$

Hence domain of $f(x)$ is $(-\infty, 0)$

8. **(a)** $f(x) = \dfrac{3}{4-x^2} + \log_{10}(x^3-x)$

$4-x^2 \ne 0;\ x^3-x>0;$

$x \ne \pm\sqrt{4}$ and $-1<x<0$ or $1<x<\infty$



$\therefore D = (-1, 0) \cup (1, \infty) - \{\sqrt{4}\}$

$D = (-1, 0) \cup (1, 2) \cup (2, \infty)$.

9. **(d)** The given equation

$[x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x]^2 + 2[x] - 3 = 0$

$\Rightarrow [x]^2 + 3[x] - [x] - 3 = 0$

$\Rightarrow ([x]+3)([x]-1) = 0 \Rightarrow [x] = 1$ or -3

$\Rightarrow x \in [-3, -2) \cup [1, 2)$

$\therefore$ The equation has infinitely many solutions.

10. **(a)** Let $f(x) = ax^2 + bx + c$

Given : $f(-1) + f(2) = 0$

$a - b + c + 4a + 2b + c = 0$

$\Rightarrow 5a + b + 2c = 0$ ...(i)

and $f(3) = 0 \Rightarrow 9a + 3b + c = 0$ ...(ii)

From equations (i) and (ii),

$$\frac{a}{1-6} = \frac{b}{18-5} = \frac{c}{15-9} \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$$

Product of roots, $\alpha\beta = \frac{c}{a} = \frac{-6}{5}$ and $\alpha = 3$

$$\Rightarrow \beta = \frac{-2}{5} \in (-1, 0)$$

11. **(b)** $f(x)\begin{cases} \frac{x}{x^2+1}; & x \in (1,2) \\ \frac{2x}{x^2+1}; & x \in [2,3) \end{cases}$

$$f'(x)\begin{cases} \frac{1-x^2}{1+x^2}; & x \in (1,2) \\ \frac{1-2x^2}{1+x^2}; & x \in [2,3) \end{cases}$$

$\therefore$ $f(x)$ is a decreasing function

$$\therefore \quad y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{6}{10}, \frac{4}{5}\right]$$

$$\Rightarrow \quad y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$$

12. **(a)** $f(x) = \log\left(\frac{1-x}{1+x}\right), |x| < |$

$$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+2x^2}}\right)$$

$$= \log\left(\frac{1+x^2-2x}{1+x^2+2x}\right) = \log\left(\frac{1-x}{1+x}\right)^2$$

$$= 2\log\left(\frac{1-x}{1+x}\right) = 2f(x)$$

13. **(a)** Given function can be written as

$$f(x) = a^x = \left(\frac{a^x + a^{-x}}{2}\right) + \left(\frac{a^x - a^{-x}}{2}\right)$$

where $f_1(x) = \frac{a^x + a^{-x}}{2}$ is even function

$f_2(x) = \frac{a^x - a^{-x}}{2}$ is odd function

$\Rightarrow f_1(x+y) + f_1(x-y)$

$$= \left(\frac{a^{x+y} + a^{-x-y}}{2}\right) + \left(\frac{a^{x-y} + a^{-x+y}}{2}\right)$$

$$= \frac{1}{2}\left[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})\right]$$

$$= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2} = 2f_1(x).f_1(y)$$

14. **(d)** Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right]n$

where $[n]$ is greatest integer function,

$$= \left[0.33 + \frac{3n}{100}\right]n$$

For $n = 1, 2, ..., 22$, we get $f(n) = 0$

and for $n = 23, 24, ..., 55$, we get $f(n) = 1 \times n$

For $n = 56$, $f(n) = 2 \times n$

So, $\sum_{n=1}^{56} f(n) = 1(23) + 1(24) + ... + 1(55) + 2(56)$

$= (23 + 24 + ... + 55) + 112$

$= \frac{33}{2}[46 + 32] + 112$

$= \frac{33}{2}(78) + 112 = 1399.$

15. **(c)** Given f be an odd function

$f(x) = 3\sin x + 4\cos x$

Now,

$$f\left(\frac{-11\pi}{6}\right) = 3\sin\left(\frac{-11\pi}{6}\right) + 4\cos\left(\frac{-11\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right) = 3\sin\left(-2\pi+\frac{\pi}{6}\right)+4\cos\left(-2\pi+\frac{\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right) = 3\sin\left\{-\left(2\pi-\frac{\pi}{6}\right)\right\}+4\cos\left\{-\left(2\pi-\frac{\pi}{6}\right)\right\}$$

$$\left\{\text{For odd functions } \begin{matrix}\sin(-\theta) = -\sin\theta \\ \text{and}\cos(-\theta) = -\cos\theta\end{matrix}\right\}$$

$$\therefore\ f\left(\frac{-11\pi}{6}\right) = -3\sin\left(2\pi-\frac{\pi}{6}\right)-4\cos\left(2\pi-\frac{\pi}{6}\right)$$

$$\Rightarrow f\left(\frac{-11\pi}{6}\right) = +3\sin\left(\frac{\pi}{6}\right)-4\cos\frac{\pi}{6}$$

$$\Rightarrow f\left(\frac{-11\pi}{6}\right) = 3\times\frac{1}{2}-4\times\frac{\sqrt{3}}{2}$$

$$\text{or } f\left(\frac{-11\pi}{6}\right) = \frac{3}{2}-2\sqrt{3}$$

16. **(a)** Given that f(0) = 0 and put $x=0, y=0$,

$f(0) = f^2(0) - f^2(a)$

$\Rightarrow f^2(a) = 0 \Rightarrow f(a) = 0$

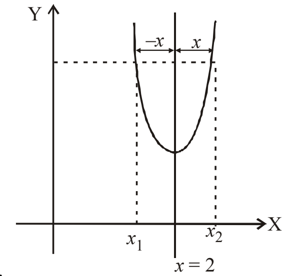
$f(2a-x) = f(a-(x-a))$

$= f(a)f(x-a) - f(0)f(x)$

$= f(a)f(x-a) - f(x) = -f(x)$

$\Rightarrow f(2a-x) = -f(x)$

17. **(b)** Given that a graph symmetrical. with respect to line $x = 2$ as shown in the figure.



From the figure

$f(x_1) = f(x_2)$, where $x_1 = 2-x$ and $x_2 = 2+x$

$\therefore f(2-x) = f(2+x)$

18. **(a)** $f(x+y) = f(x)+f(y)$.

$\because f(1) = 7$

$f(2) = f(1+1) = f(1)+f(1) = 14$

$f(3) = f(1+2) = f(1)+f(2) = 21$

— — — — — — — — — —

— — — — — — — — — —

$$\therefore \sum_{r=1}^{n} f(r) = 7(1+2+3.....+n)$$

$$= \frac{7n(n+1)}{2}$$

3

Trigonometric Functions

TOPIC 1 Circular System, Trigonometric Ratios, Domain and Range of Trigonometric Functions, Trigonometric Ratios of Allied Angles

1. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals: **[Jan. 9, 2019 (I)]**

(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(b) $13 - 4\cos^6\theta$

(c) $13 - 4\cos^2\theta + 6\cos^4\theta$

(d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

2. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$.

Then $f_4(x) - f_6(x)$ equals **[2014]**

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

3. If $2\cos\theta + \sin\theta = 1 \left(\theta \neq \frac{\pi}{2}\right)$,

then $7\cos\theta + 6\sin\theta$ is equal to: **[Online April 11, 2014]**

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{11}{2}$ (d) $\frac{46}{5}$

4. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$

can be written as : **[2013]**

(a) $\sin A \cos A + 1$ (b) $\sec A \operatorname{cosec} A + 1$

(c) $\tan A + \cot A$ (d) $\sec A + \operatorname{cosec} A$

5. The value of $\cos 255° + \sin 195°$ is **[Online May 26, 2012]**

(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}-1}{\sqrt{2}}$

(c) $-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ (d) $\frac{\sqrt{3}+1}{\sqrt{2}}$

6. Let $f(x) = \sin x$, $g(x) = x$.

Statement 1: $f(x) \leq g(x)$ for x in $(0, \infty)$

Statement 2: $f(x) \leq 1$ for x in $(0, \infty)$ but $g(x) \to \infty$ as $x \to \infty$. **[Online May 7, 2012]**

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(d) Statement 1 is false, Statement 2 is true.

7. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is **[2006]**

(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$

(c) $\frac{1}{2}x^2$ (d) πx^2

TOPIC 2 Trigonometric Identities, Conditional Trigonometric Identities, Greatest and Least Value of Trigonometric Expressions

8. The value of $\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$ is **[Jan. 9, 2020 (I)]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

9. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$,

$\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to _____.

[Jan. 8, 2020 (II)]

10. If $L=\sin^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$ and

$M=\cos^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$, then : **[Sep. 05, 2020 (II)]**

(a) $L=-\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$ (b) $L=\frac{1}{4\sqrt{2}}-\frac{1}{4}\cos\frac{\pi}{8}$

(c) $M=\frac{1}{4\sqrt{2}}+\frac{1}{4}\cos\frac{\pi}{8}$ (d) $M=\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

11. The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x+y=1$ is : **[Sep. 02, 2020 (II)]**

(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

(c) $\left(0,\frac{3\pi}{4}\right)$ (d) $\left(0,\frac{\pi}{4}\right)$

12. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is: **[April 12, 2019 (II)]**

(a) $15(3+\sqrt{3})$ (b) $15(5-\sqrt{3})$

(c) $15(3-\sqrt{3})$ (d) $15(1+\sqrt{3})$

13. The value of **[April 9, 2019 (II)]**

$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

(a) $\frac{3}{4}+\cos 20°$ (b) 3/4

(c) $\frac{3}{2}(1+\cos 20°)$ (d) 3/2

14. Two poles standing on a horizontal ground are of heights 5m and 10m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is: **[April. 09, 2019 (II)]**

(a) $5(2+\sqrt{3})$ (b) $5(\sqrt{3}+1)$

(c) $\frac{5}{2}(2+\sqrt{3})$ (d) $10(\sqrt{3}-1)$

15. The value of sin 10° sin 30° sin 50° sin 70° is:

[April. 09, 2019 (II)]

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$

(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

16. If $\cos(\alpha+\beta)=\frac{3}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha, \beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to : **[April 8, 2019 (I)]**

(a) $\frac{63}{52}$ (b) $\frac{63}{16}$

(c) $\frac{21}{16}$ (d) $\frac{33}{52}$

17. If $\sin^4\alpha+4\cos^4\beta+2=4\sqrt{2}\sin\alpha\cos\beta$; $\alpha,\beta\in[0,\pi]$, then $\cos(\alpha+\beta)-\cos(\alpha-\beta)$ is equal to : **[Jan. 12, 2019 (II)]**

(a) 0 (b) -1

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

18. Let $f_k(x)=\frac{1}{k}\left(\sin^k x+\cos^k x\right)$ for k = 1, 2, 3, ... Then for all $x\in R$, the value of $f_4(x)-f_6(x)$ is equal to : **[Jan. 11, 2019 (I)]**

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$

(c) $\frac{-1}{12}$ (d) $\frac{5}{12}$

19. The value of **[Jan. 10, 2019 (II)]**

$\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot....\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$

(a) $\frac{1}{512}$ (b) $\frac{1}{1024}$

(c) $\frac{1}{256}$ (d) $\frac{1}{2}$

20. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is : **[2017]**

(a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

21. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x - 2\cos^4 x, x \in R,$ then M – m is equal to : **[Online April 9, 2016]**

(a) $\frac{9}{4}$ (b) $\frac{15}{4}$

(c) $\frac{7}{4}$ (d) $\frac{1}{4}$

22. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta$ $\frac{1}{2}$ and θ is the the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to : **[Online April 11, 2015]**

(a) $\frac{3}{5}$ (b) $\frac{7}{5}$

(c) $\frac{4}{5}$ (d) $\frac{8}{5}$

23. If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$ $(p \neq q \neq 0)$, then $\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|$ is equal to: **[Online April 9, 2014]**

(a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{q}{p}}$

(c) $\sqrt{pq}$ (d) pq

24. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[2011]**

(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$

(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

25. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ **[2010]**

(a) $\frac{56}{33}$ (b) $\frac{19}{12}$

(c) $\frac{20}{7}$ (d) $\frac{25}{16}$

26. Let **A** and **B** denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then : **[2009]**

(a) **A** is false and **B** is true

(b) both **A** and **B** are true

(c) both **A** and **B** are false

(d) **A** is true and **B** is false

27. If p and q are positive real numbers such that $p^2+q^2=1$, then the maximum value of $(p+q)$ is **[2007]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) 2.

28. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[2006]**

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

29. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by **[2004]**

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$

(c) $(a+b)^2$ (d) $2(a^2+b^2)$

30. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ **[2004]**

(a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$

(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

31. The function $f(x) = \log\left(x+\sqrt{x^2+1}\right)$, is **[2003]**

(a) neither an even nor an odd function

(b) an even function

(c) an odd function

(d) a periodic function.

32. The period of $\sin^2\theta$ is **[2002]**

(a) π^2 (b) π

(c) 2π (d) $\pi/2$

33. Which one is not periodic? **[2002]**

(a) $|\sin 3x|+\sin^2 x$ (b) $\cos\sqrt{x}+\cos^2 x$
(c) $\cos 4x+\tan^2 x$ (d) $\cos 2x+\sin x$

TOPIC 3 Solutions of Trigonometric Equations

34. If the equation $\cos^4\theta+\sin^4\theta+\lambda=0$ has real solutions for θ, then λ lies in the interval : **[Sep. 02, 2020 (II)]**

(a) $\left(-\frac{5}{4},-1\right)$ (b) $\left[-1,-\frac{1}{2}\right]$
(c) $\left(-\frac{1}{2},-\frac{1}{4}\right]$ (d) $\left[-\frac{3}{2},-\frac{5}{4}\right]$

35. The number of distinct solutions of the equation, $\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|$ in the interval $[0,2\pi]$, is _____. **[Jan. 9, 2020 (I)]**

36. The number of solutions of the equation $1+\sin^4 x=\cos^2 3x, x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$ is : **[April 12, 2019 (I)]**

(a) 3 (b) 5
(c) 7 (d) 4

37. Let S be the set of all $\alpha\in R$ such that the equation, $\cos 2x+\alpha\sin x=2\alpha-7$ has a solution. Then S is equal to : **[April 12, 2019 (II)]**

(a) R (b) [1, 4]
(c) [3,7] (d) [2, 6]

38. If $[x]$ denotes the greatest integer $\le x$, then the system of linear equations

$[\sin\theta]\,x+[-\cos\theta]\,y=0$

$[\cot\theta]\,x+y=0$ **[April 12, 2019 (II)]**

(a) have infinitely many solutions if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ and has a unique solution if $\theta\in\left(\pi,\frac{7\pi}{6}\right)$.

(b) has a unique solution if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$.

(c) has a unique solution if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta\in\left(\pi,\frac{7\pi}{6}\right)$.

(d) have infinitely many solutions if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

39. Let $S=\{\theta\in[-2\pi,2\pi]:2\cos^2\theta+3\sin\theta=0\}$. Then the sum of the elements of S is: **[April 9, 2019 (I)]**

(a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$
(c) 2π (d) π

40. If $0\le x<\frac{\pi}{2}$, then the number of values of x for which $\sin x-\sin 2x+\sin 3x=0$, is: **[Jan. 09, 2019 (II)]**

(a) 3 (b) 1
(c) 4 (d) 2

41. The number of solutions of $\sin 3x=\cos 2x$, in the interval $\left(\frac{\pi}{2},\pi\right)$ is **[Online April 15, 2018]**

(a) 3 (b) 4
(c) 2 (d) 1

42. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)-1$ in $[0,\pi]$ is $k\pi$, then k is equal to : **[2018]**

(a) $\frac{13}{9}$ (b) $\frac{8}{9}$
(c) $\frac{20}{9}$ (d) $\frac{2}{3}$

43. If $0\le x<2\pi$, then the number of real values of x, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$ is: **[2016]**

(a) 7 (b) 9
(c) 3 (d) 5

44. The number of $x\in[0,2\pi]$ for which $\left|\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}\right|=1$ is **[Online April 9, 2016]**

(a) 2 (b) 6
(c) 4 (d) 8

45. The number of values of α in $[0,2\pi]$ for which $2\sin^3\alpha-7\sin^2\alpha+7\sin\alpha=2$, is: **[Online April 9, 2014]**

(a) 6 (b) 4
(c) 3 (d) 1

46. Let $A=\{\theta:\sin(\theta)=\tan(\theta)\}$ and $B=\{\theta:\cos(\theta)=1\}$ be two sets. Then : **[Online April 25, 2013]**

(a) $A=B$
(b) $A\not\subset B$
(c) $B\not\subset A$
(d) $A\subset B$ and $B-A\neq\phi$

47. The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is : **[Online April 23, 2013]**

(a) 3 (b) 5
(c) 4 (d) 6

48. **Statement-1:** The number of common solutions of the trigonometric equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is two.

Statement-2: The number of solutions of the equation, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, \pi]$ is two. **[Online April 22, 2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 is true; Statement-2 is false.

49. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[2012]**

(a) infinite number of real roots
(b) no real roots
(c) exactly one real root
(d) exactly four real roots

50. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[2011RS]**

(a) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(b) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

51. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[2006]**

(a) 4 (b) 6
(c) 1 (d) 2

52. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is **[2002]**

(a) 2 (b) 3
(c) 0 (d) 1

Hints & Solutions

1. **(b)** $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$

$=3(1-2\sin\theta\cos\theta)^2+6(1+2\sin\theta\cos\theta)+4\sin^6\theta$

$=3(1+4\sin^2\theta\cos^2\theta-4\sin\theta\cos\theta)+6-12\sin\theta\cos\theta+4\sin^6\theta$

$=9+12\sin^2\theta\cos^2\theta+4\sin^6\theta$

$=9+12\cos^2\theta(1-\cos^2\theta)+4(1-\cos^2\theta)^3$

$=9+12\cos^2\theta-12\cos^4\theta+4(1-\cos^6\theta-3\cos^2\theta+3\cos^4\theta)$

$=9+4-4\cos^6\theta$

$=13-4\cos^6\theta$

2. **(b)** Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$

Consider $f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$

$=\frac{1}{4}[1-2\sin^2 x\cos^2 x]-\frac{1}{6}[1-3\sin^2 x\cos^2 x]$

$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$

3. **(d)** Given $2\cos\theta+\sin\theta=1$

Squaring both sides, we get

$(2\cos\theta+\sin\theta)^2=1^2$

$\Rightarrow 4\cos^2\theta+\sin^2\theta+4\sin\theta\cos\theta=1$

$\Rightarrow 3\cos^2\theta+(\cos^2\theta+\sin^2\theta)+4\sin\theta\cos\theta=1$

$\Rightarrow 3\cos^2\theta+\cancel{1}+4\sin\theta\cos\theta=\cancel{1}$

$\Rightarrow 3\cos^2\theta+4\sin\theta\cos\theta=0$

$\Rightarrow \cos\theta(3\cos\theta+4\sin\theta)=0$

$\Rightarrow 3\cos\theta+4\sin\theta=0\Rightarrow 3\cos\theta=-4\sin\theta$

$\Rightarrow \frac{-3}{4}=\tan\theta=\sqrt{\sec^2\theta-1}=\frac{-3}{4}$

$\left(\because \tan\theta=\sqrt{\sec^2\theta-1}\right)$

$\Rightarrow \sec^2\theta-1=\left(\frac{-3}{4}\right)^2=\frac{9}{16}$

$\Rightarrow \sec^2\theta=\frac{9}{16}+1=\frac{25}{16}\Rightarrow \sec\theta=\frac{5}{4}$

or $\boxed{\cos\theta=\frac{4}{5}}$ ...(1)

Now, $\sin^2\theta+\cos^2\theta=1\Rightarrow \sin^2\theta+\left(\frac{4}{5}\right)^2=1$

$\sin^2\theta+\frac{4}{5}=1\Rightarrow \sin^2\theta=1-\frac{16}{25}=\frac{9}{25}$

$\sin\theta=\pm\frac{3}{5}$ ...(2)

Taking $\left(\sin\theta=+\frac{3}{5}\right)$ because $\left(\sin\theta=-\frac{3}{5}\right)$ cannot satisfy the given equation.

Therefore; $7\cos\theta+6\sin\theta$

$=7\times\frac{4}{5}+6\times\frac{3}{5}=\frac{28}{5}+\frac{18}{5}=\frac{46}{5}$

4. **(b)** Given expression can be written as

$\frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$

$\left(\because \tan A=\frac{\sin A}{\cos A} \text{ and } \cot A=\frac{\cos A}{\sin A}\right)$

$=\frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=\frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A}$

$=1+\sec A\operatorname{cosec} A$

5. **(c)** Consider $\cos 255°+\sin 195°$

$=\cos(270°-15°)+\sin(180°+15°)$

$=-\sin 15°-\sin 15°$

$=-2\sin 15°=-2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)=-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$

6. **(c)** Let $f(x)=\sin x$ and $g(x)=x$

Statement-1: $f(x)\le g(x)\ \forall x\in(0,\infty)$

i.e., $\sin x\le x\ \forall x\in(0,\infty)$

which is true

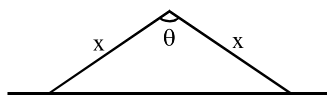
Statement-2: $f(x)\le 1\ \forall\ x\in(0,\infty)$

i.e., $\sin x\le 1\ \forall\ x\in(0,\infty)$

It is true and

$g(x)=x\to\infty$ as $x\to\infty$ also true.

7. **(c)** Area $= \frac{1}{2}x^2 \sin\theta$



Maximum value of $\sin\theta$ is 1 at $\theta = \frac{\pi}{2}$

$A_{\max} = \frac{1}{2}x^2$

8. **(b)** $\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right]$

$+\sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$

$= 4\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]$

$\left[\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]$

$-3\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)\right]$

$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right]$

$= \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}$

9. **(1)** $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos^2\beta}{2}} = \frac{1}{10}$

$\Rightarrow \frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\therefore \tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$

$\tan\beta = \frac{1}{3}$

$\therefore \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2.\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$

$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta}$

$= \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7}.\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$

10. **(d)** $L + M = 1 - 2\sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ...(i)

and $L - M = -\cos\frac{\pi}{8}$ ...(ii)

From equations (i) and (ii),

$L = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$ and

$M = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

11. **(a)** Let $f(x, y) = x + y - 1$

Given $(1, 2)$ and $(\sin\theta, \cos\theta)$ are lies on same side.

$\therefore f(1, 2)\cdot f(\sin\theta, \cos\theta) > 0$

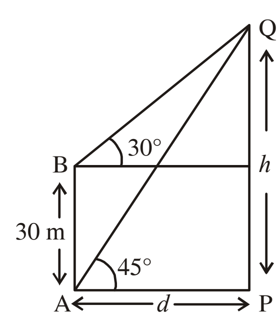
$\Rightarrow 2[\sin\theta + \cos\theta - 1] > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

$\Rightarrow \theta + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

12. **(a)** Let the height of the tower be h and distance of the foot of the tower from the point A is d.

By the diagram,



$\tan 45° = \frac{h}{d} = 1$

$h = d$ ...(i)

$\tan 30° = \frac{h-30}{d}$

$\sqrt{3}(h-30) = d$ ...(ii)

Put the value of h from (i) to (ii),

$\sqrt{3}d = d + 30\sqrt{3}$

$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}\left(\sqrt{3}+1\right) = 15\left(3+\sqrt{3}\right)$

13. **(b)** $\cos^2 10° - \cos10° \cos50° + \cos^2 50°$

$$= \left(\frac{1+\cos 20°}{2}\right) + \left(\frac{1+\cos 100°}{2}\right) - \frac{1}{2}(2\cos 10° \cos 50°)$$

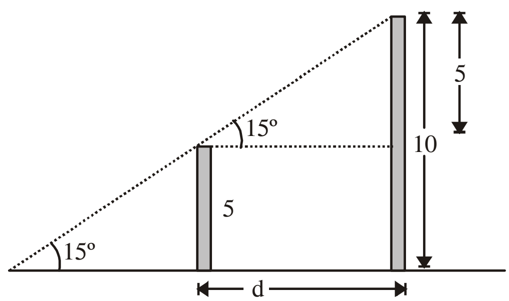
$$= 1 + \frac{1}{2}(\cos 20° + \cos 100°) - \frac{1}{2}[\cos 60° + \cos 40°]$$

$$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20° + \cos 100° - \cos 40°]$$

$$= \frac{3}{4} + \frac{1}{2}[2\cos 60° \times \cos 40° - \cos 40°]$$

$$= \frac{3}{4}$$

14. **(a)**



By the diagram,

$$\tan 15° = \frac{5}{d} \Rightarrow d = \frac{5}{\tan 15°} = \frac{5(\sqrt{3}+1)}{\sqrt{3}-1}$$

$$= \frac{5(4+2\sqrt{3})}{2} = 5(2+\sqrt{3})$$

15. **(a)** $\because \sin(60° + A).\sin(60° - A) \sin A = \frac{1}{4} \sin 3A$

$\therefore \sin 10° \sin 50° \sin 70° = \sin 10° \sin(60° - 10°)$

$\sin(60° + 10°) = \frac{1}{4} \sin 30°$

$\Rightarrow \sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{4} \sin^2 30 = \frac{1}{16}$

16. **(b)** $\because \alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$\cos(\alpha+\beta) = \frac{3}{5}$, then $\sin(\alpha+\beta) = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\tan(\alpha+\beta) = \frac{4}{3}$

And $\sin(\alpha - \beta) = \frac{5}{13}$, then

$$\cos(\alpha-\beta) = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$$

$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

Now, $\tan 2\alpha = \tan((\alpha+\beta) + (\alpha-\beta))$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)} = \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}} = \frac{63}{16}$$

17. **(d)** $\because$ The given equation is

$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha \cdot \cos\beta,\ \alpha, \beta \in [0, \pi]$

Then, by A.M., G.M. ineqality;

A.M. $\geq$ G.M.

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq \left(\sin^4\alpha \cdot 4\cos^4\beta \cdot 1 \cdot 1\right)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2}\sin\alpha \cdot |\cos\beta|$

Inequality still holds when $\cos\beta < 0$ but L.H.S. is positive than $\cos\beta > 0$, then

L.H.S. = R.H.S

$\therefore \sin^4\alpha = 1$ and $\cos^4\beta = \frac{1}{4}$

$\Rightarrow \alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

$\therefore \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$$= \cos\left(\frac{\pi}{2}+\beta\right) - \cos\left(\frac{\pi}{2}-\beta\right)$$

$$= -\sin\beta - \sin\beta = -2\sin\frac{\pi}{4} = -\sqrt{2}$$

18. **(a)** $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$$f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x]$$

$$= \frac{1}{4}\left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2}\right]$$

$$= \frac{1}{4}\left[1 - \frac{(\sin 2x)^2}{2}\right]$$

$$f_6(x) = \frac{1}{6}[\sin^6 x + \cos^6 x]$$

$$= \frac{1}{6}\left[(\sin^2 x + (\cos^2 x) - \frac{3}{4}(\sin^2 x)^2\right]$$

$=\frac{1}{6}\left[1-\frac{3}{4}(\sin 2x)^2\right]$

Now $f_4(x)-f_{(6)}(x)=\frac{1}{4}-\frac{1}{6}-\frac{(\sin 2x)^2}{8}+\frac{1}{8}(\sin 2x)^2$

$=\frac{1}{12}$

19. (a) $A=\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$

$=\frac{1}{2}\left(\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^9}\sin\frac{\pi}{2^9}\right)$

$=\frac{1}{2^8}\left(\cos\frac{\pi}{2^2}.\sin\frac{\pi}{2^2}\right)=\frac{1}{2^9}\sin\frac{\pi}{2}$

$=\frac{1}{512}$

20. (a) We have

$5\tan^2 x-5\cos^2 x=2(2\cos^2 x-1)+9$

$\Rightarrow 5\tan^2 x-5\cos^2 x=4\cos^2 x-2+9$

$\Rightarrow 5\tan^2 x=9\cos^2 x+7$

$\Rightarrow 5(\sec^2 x-1)=9\cos^2 x+7$

Let $\cos^2 x=t$

$\Rightarrow \frac{5}{t}-9t-12=0$

$\Rightarrow 9t^2+12t-5=0$

$\Rightarrow 9t^2+15t-3t-5=0$

$\Rightarrow (3t-1)(3t+5)=0$

$\Rightarrow t=\frac{1}{3}$ as $t\neq-\frac{5}{3}$.

$\cos 2x=2\cos^2 x-1=2\left(\frac{1}{3}\right)-1=-\frac{1}{3}$

$\cos 4x=2\cos^2 2x-1=2\left(-\frac{1}{3}\right)^2-1=-\frac{7}{9}$

21. (b) $4+\frac{1}{2}\sin^2 2x-2\cos^4 x$

$4+2(1-\cos^2 x)\cos^2 x-2\cos^4 x$

$-4\left\{\cos^4 x-\frac{\cos^2 x}{2}-1+\frac{1}{16}-\frac{1}{16}\right\}$

$-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}$

$0\le\cos^2 x\le 1$

$-\frac{1}{4}\le\cos^2 x-\frac{1}{4}\le\frac{3}{4}$

$0\le\left(\cos^2 x-\frac{1}{4}\right)^2\le\frac{9}{16}$

$-\frac{17}{16}\le\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\le\frac{9}{16}-\frac{17}{16}$

$\frac{17}{4}\ge-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}\ge\frac{1}{2}$

$M=\frac{17}{4}$

$m=\frac{1}{2}$

$M-m=\frac{17}{4}-\frac{2}{4}=\frac{15}{4}$

22. (b) Let $\cos\alpha+\cos\beta=\frac{3}{2}$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{3}{2}$ ...(i)

and $\sin\alpha+\sin\beta=\frac{1}{2}$

$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{1}{2}$ ...(ii)

On dividing (ii) by (i), we get

$\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

Given : $\theta=\frac{\alpha+\beta}{2}\Rightarrow 2\theta=\alpha+\beta$

Consider $\sin 2\theta+\cos 2\theta=\sin(\alpha+\beta)+\cos(\alpha+\beta)$

$=\frac{\frac{2}{3}}{1+\frac{1}{9}}+\frac{1-\frac{1}{9}}{1+\frac{1}{9}}=\frac{6}{10}+\frac{8}{10}=\frac{7}{5}$

23. (b) $\operatorname{cosec}\theta=\frac{p+q}{p-q}$, $\sin\theta=\frac{p-q}{p+q}$

$\cos\theta=\pm\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\frac{p-q}{p+q}\right)^2}=\frac{2\sqrt{pq}}{(p+q)}$

$\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|=\left|\frac{\cot\frac{\pi}{4}\cot\frac{\theta}{2}-1}{\cot\frac{\pi}{4}+\cot\frac{\theta}{2}}\right|=\left|\frac{\cot\frac{\theta}{2}-1}{\cot\frac{\theta}{2}+1}\right|$

$=\left|\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right|$

On rationalizing denominator, we get

$$\left|\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\right|$$

$$=\left|\frac{\cos\theta}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right|$$

$$=\left|\frac{\cos\theta}{1+\sin\theta}\right|=\left|\frac{2\sqrt{pq}/(p+q)}{1+\frac{(p-q)}{p+q}}\right|=\frac{\sqrt{pq}}{p}=\sqrt{\frac{q}{p}}$$

24. (d) $A=\sin^2 x+\cos^4 x$

$=\sin^2 x+\cos^2 x(1-\sin^2 x)$

$=\sin^2 x+\cos^2 x-\frac{1}{4}(2\sin x.\cos x)^2$

$=1-\frac{1}{4}\sin^2(2x)$

$\because -1\le \sin 2x\le 1$

$\Rightarrow\ 0\le \sin^2(2x)\le 1$

$\Rightarrow\ 0\ge -\frac{1}{4}\sin^2(2x)\ge -\frac{1}{4}$

$\Rightarrow 1\ge 1-\frac{1}{4}\sin^2(2x)\ge 1-\frac{1}{4}$

$\Rightarrow 1\ge A\ge \frac{3}{4}$

25. (a) $\cos(\alpha+\beta)=\frac{4}{5}\ \Rightarrow \tan(\alpha+\beta)=\frac{3}{4}$

$\sin(\alpha-\beta)=\frac{5}{13}\Rightarrow \tan(\alpha-\beta)=\frac{5}{12}$

$\tan 2\alpha=\tan\left[(\alpha+\beta)+(\alpha-\beta)\right]$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}}=\frac{56}{33}$$

26. (b) Given that

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$

$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3=0$

$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]$
$+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta$
$+\sin^2\gamma+\cos^2\alpha=0$

$\Rightarrow\ [\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\ \sin\beta$
$+2\sin\beta\sin\gamma+2\sin\gamma\sin\alpha]+[\cos^2\alpha+\cos^2\beta$
$+\cos^2\gamma+2\cos\alpha\cos\beta+2\cos\beta\cos\gamma$
$+2\cos\gamma\ \cos\alpha]=0$

$[\because \cos(A-B)=\cos A.\cos B+\sin A.\sin B]$

$\Rightarrow\ [\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$

$\Rightarrow\ \sin\alpha+\sin\beta+\sin\gamma=0$ and $\cos\alpha+\cos\beta+\cos\gamma=0$

$\therefore$ A and B both are true.

27. (c) Given that $p^2+q^2=1$

$\therefore\ p=\cos\theta$ and $q=\sin\theta$ satisfy the given equation

Then $p+q=\cos\theta+\sin\theta$

We know that

$-\sqrt{a^2+b^2}\le a\cos\theta+b\sin\theta\le\sqrt{a^2+b^2}$

$\therefore\ -\sqrt{2}\le\cos\theta+\sin\theta\le\sqrt{2}$

Hence max. value of $p+q$ is $\sqrt{2}$

28. (c) $\cos x+\sin x=\frac{1}{2}\ \Rightarrow\ 1+\sin 2x=\frac{1}{4}$

$\Rightarrow\ \sin 2x=-\frac{3}{4}$,

$\therefore \pi<2x<2\pi$

$\Rightarrow\ \frac{\pi}{2}<x\le\pi$ ...(i)

$\frac{2\tan x}{1+\tan^2 x}=-\frac{3}{4}$

$\Rightarrow\ 3\tan^2 x+8\tan x+3=0$

$\therefore\ \tan x=\frac{-8\pm\sqrt{64-36}}{6}=-\frac{-4\pm\sqrt{7}}{3}$

for $\frac{\pi}{2}<x<\pi$, $\tan x<0$

$\therefore\ \tan x=\frac{-4-\sqrt{7}}{3}$

29. (a) $u^2=a^2+b^2+2\sqrt{\begin{matrix}(a^4+b^4)\cos^2\theta\sin^2\theta\\ +a^2b^2(\cos^4\theta+\sin^4\theta)\end{matrix}}$ …(1)

Now, $(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(\cos^4\theta+\sin^4\theta)$

$=(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(1-2\cos^2\theta\sin^2\theta)$

$=(a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta+a^2b^2$

$=(a^2-b^2)^2.\frac{\sin^2 2\theta}{4}+a^2b^2$ …(2)

$\because 0\le\sin^2 2\theta\le 1$

$\Rightarrow 0\le(a^2-b^2)^2\frac{\sin^2 2\theta}{4}\le\frac{(a^2-b^2)^2}{4}$

$\Rightarrow a^2b^2\le(a^2-b^2)^2\frac{\sin^2 2\theta}{4}+a^2b^2$

$\le(a^2-b^2)^2.\frac{1}{4}+a^2b^2$ ....(3)

From (1)

$$a^2+b^2+2\sqrt{a^2b^2} \le u^2 \le a^2+b^2+\frac{2}{2}\sqrt{\left(a^2+b^2\right)^2}$$

$$(a+b)^2 \le u^2 \le 2\left(a^2+b^2\right)$$

$\therefore$ Max. value – Min. value

$= 2(a^2+b^2)-(a+b^2)=(a-b)^2$

30. **(d)** $\pi < \alpha - \beta < 3\pi$

$$\Rightarrow \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2} \Rightarrow \cos\frac{\alpha-\beta}{2} < 0 \qquad ....(1)$$

$$\sin\alpha + \sin\beta = -\frac{21}{65}$$

$$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{21}{65} \qquad ....(2)$$

$$\cos\alpha + \cos\beta = -\frac{27}{65}$$

$$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{27}{65} \qquad ....(3)$$

Squaring and adding (2) and (3), we get

$$4\cos^2\frac{\alpha-\beta}{2} = \frac{(21)^2+(27)^2}{(65)^2} = \frac{1170}{65\times 65}$$

$$\therefore \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}} \qquad \text{[from (1)]}$$

31. **(c)** Given $f(x)=\log(x+\sqrt{x^2+1})$

$$f(-x)=\log\left\{-x+\sqrt{x^2+1}\right\} = \log\left\{\frac{x^2-x^2+1}{x+\sqrt{x^2+1}}\right\}$$

$$= -\log(x+\sqrt{x^2+1}) = -f(x)$$

$\Rightarrow f(x)$ is an odd function.

32. **(b)** We know that $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$;

Since period of $\cos 2\theta = \dfrac{2\pi}{2} = \pi$

Hence period of $\sin^2\theta$ is also π.

33. **(b)** we know that $\cos\sqrt{x}$ is non periodic

$\therefore$ $\cos\sqrt{x}+\cos^2 x$ can not be periodic.

34. **(b)** $\sin^4\theta+\cos^4\theta = -\lambda$

$$\Rightarrow (\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cdot\cos^2\theta = -\lambda$$

$$\Rightarrow 1-2\sin^2\theta\cos^2\theta = -\lambda$$

$$\Rightarrow \lambda = \frac{(\sin 2\theta)^2}{2} - 1$$

$\Rightarrow$ as $\sin^2 2\theta \in [0, 1]$

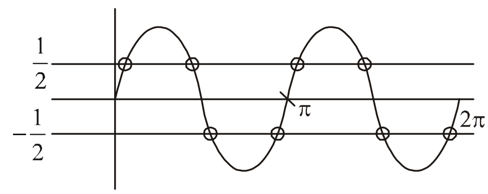
$$\Rightarrow \lambda \in \left[-1, \frac{-1}{2}\right]$$

35. **(8)** $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$

$\Rightarrow$ $\log_{1/2}|\sin x\cos x| = 2$

$\Rightarrow$ $|\sin x\cos x| = \dfrac{1}{4}$

$\Rightarrow$ $\sin 2x = \pm\dfrac{1}{2}$



Hence, total number of solutions = 8.

36. **(c)** Consider equation, $1+\sin^4 x = \cos^2 3x$

L.H.S. $= 1+\sin^4 x$ and R.H.S. $= \cos^2 3x$

$\because$ L.H.S. ≥ 1 and R.H.S. $\le 1 \Rightarrow$ L.H.S. = R.H.S. = 1

$\sin^4 x = 0$, and $\cos^2 3x = 1$

$\Rightarrow$ $\sin x = 0$ and $(4\cos^2 x - 3)^2\cos^2 x = 1$

$\Rightarrow$ $\sin x = 0$ and $\cos^2 x = 1 \Rightarrow x = 0, \pm\pi, \pm 2\pi$

Hence, total number of solutions is 5.

37. **(d)** Given equation is, $\cos 2x + \alpha\sin x = 2\alpha - 7$

$1-2\sin^2 x + \alpha\sin x = 2\alpha - 7$

$2\sin^2 x - \alpha\sin x + (2\alpha - 8) = 0$

$$\Rightarrow \quad \sin x = \frac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha+8)}}{4}$$

$$\Rightarrow \quad \sin x = \frac{\alpha\pm(\alpha-8)}{4} \Rightarrow \sin x = \frac{\alpha-4}{4}$$

[$\sin x = 2$ (rejected)]

$\because$ equation has solution, then $\dfrac{\alpha-4}{4} \in [-1, 1]$

$\Rightarrow \alpha \in [2, 6]$

38. **(a)** According to the question, there are two cases.

Case 1 : $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$

In this interval, $[\sin\theta] = 0$, $[-\cos\theta] = 0$ and $[\cot\theta] = -1$

Then the system of equations will be ;

$0\cdot x + 0\cdot y = 0$ and $-x + y = 0$

Which have infinitely many solutions.

Case 2 : $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$

In this interval, $[\sin\theta] = -1$ and $[-\cos\theta] = 0$,

Then the system of equations will be ;

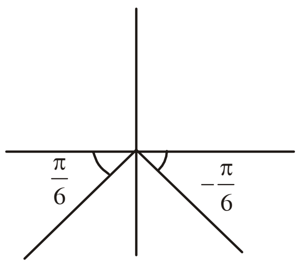
$-x+0\cdot y=0$ and $[\cot\theta]\,x+y=0$

Clearly, $x=0$ and $y=0$ which has unique solution.

39. **(c)** $2\cos^2\theta+3\sin\theta=0$

$(2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\dfrac{1}{2}$ or $\sin\theta=2\rightarrow$ Not possibe



The required sum of all solutions in $[-2\pi, 2\pi]$ is

$$=\left(\pi+\frac{\pi}{6}\right)+\left(2\pi-\frac{\pi}{6}\right)+\left(-\frac{\pi}{6}\right)+\left(-\pi+\frac{\pi}{6}\right)=2\pi$$

40. **(d)** $\sin x-\sin 2x+\sin 3x=0$

$\Rightarrow \sin x-2\sin x.\cos x+3\sin x-4\sin^3 x=0$

$\Rightarrow 4\sin x-4\sin^3 x-2\sin x.\cos x=0$

$\Rightarrow 2\sin x(1-\sin^2 x)-\sin x.\cos x=0$

$\Rightarrow 2\sin x.\cos^2 x-\sin x.\cos x=0$

$\Rightarrow \sin x.\cos x(2\cos x-1)=0$

$\therefore \sin x=0, \cos x=0, \cos x=\dfrac{1}{2}$

$\therefore x=0,\dfrac{\pi}{3} \quad \because x\in\left[0,\dfrac{\pi}{2}\right)$

41. **(d)** $\sin 3x=\cos 2x$

$\Rightarrow 3\sin x-4\sin^3 x=1-2\sin^2 x$

$\Rightarrow 4\sin^3 x-2\sin^2 x-3\sin x+1=0$

$\Rightarrow \sin x=1,\ \dfrac{-2\pm 2\sqrt{5}}{8}$

In the interval $\left(\dfrac{\pi}{2},\pi\right)$, $\sin x=\dfrac{-2+2\sqrt{5}}{8}$

So, there is only one solution.

42. **(a)** $\because 8\cos x\left(\cos^2\dfrac{\pi}{6}-\sin^2 x-\dfrac{1}{2}\right)=1$

$\Rightarrow 8\cos x\left(\dfrac{3}{4}-\dfrac{1}{2}-\sin^2 x\right)=1$

$\Rightarrow 8\cos x\left(\dfrac{1}{4}-(1-\cos^2 x)\right)=1$

$\Rightarrow 8\cos x\left(\dfrac{1}{4}-1+\cos^2 x\right)=1$

$\Rightarrow 8\cos x\left(\cos^2 x-\dfrac{3}{4}\right)=1$

$\Rightarrow 8\left(\dfrac{4\cos^3 x-3\cos x}{4}\right)=1$

$\Rightarrow 2(4\cos^3 x-3\cos x)=1$

$\Rightarrow 2\cos 3x=1\Rightarrow \cos 3x=\dfrac{1}{2}$

$\therefore 3x=2n\pi\pm\dfrac{\pi}{3}, n\in I$

$\Rightarrow x=\dfrac{2n\pi}{3}\pm\dfrac{\pi}{9}$

In $x\in[0,\pi]: x=\dfrac{\pi}{9},\dfrac{2\pi}{3}+\dfrac{\pi}{9},\dfrac{2\pi}{3}-\dfrac{\pi}{9}$, only

Sum of all the solutions of the equation

$$=\left(\frac{1}{9}+\frac{2}{3}+\frac{1}{9}+\frac{2}{3}-\frac{1}{9}\right)\pi=\frac{13}{9}\pi$$

43. **(a)** $\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$\Rightarrow 2\cos 2x\cos x+2\cos 3x\cos x=0$

$\Rightarrow 2\cos x\left(2\cos\dfrac{5x}{2}\cos\dfrac{x}{2}\right)=0$

$\cos x=0,\ \cos\dfrac{5x}{2}=0,\ \cos\dfrac{x}{2}=0$

$x=\pi,\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{\pi}{5},\dfrac{3\pi}{5},\dfrac{7\pi}{5},\dfrac{9\pi}{5}$

44. **(d)** $\left|\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}\right|=1$

$\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}=\pm 1$

$\sqrt{2\sin^4 x+18\cos^2 x}=\pm 1+\sqrt{2\cos^4 x+18\sin^2 x}$

by squaring both the sides we will get 8 solutions

45. **(c)** $2\sin^3\alpha-7\sin^2\alpha+7\sin\alpha-2=0$

$\Rightarrow 2\sin^2\alpha(\sin\alpha-1)-5\sin\alpha(\sin\alpha-1)$
$+2(\sin\alpha-1)=0$

$\Rightarrow (\sin\alpha-1)(2\sin^2\alpha-5\sin\alpha+2)=0$

$\Rightarrow \sin\alpha-1=0$ or $2\sin^2\alpha-5\sin\alpha+2=0$

$\sin\alpha=1$ or $\sin\alpha=\dfrac{5\pm\sqrt{25-16}}{4}=\dfrac{5\pm 3}{4}$

$\alpha=\dfrac{\pi}{2}$ or $\sin\alpha=\dfrac{1}{2},2$

Now, $\sin\alpha\neq 2$

for, $\sin\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$

There are three values of α between $[0, 2\pi]$

46. **(b)** Let $A = \{\theta : \sin\theta = \tan\theta\}$

and $B = \{\theta : \cos\theta = 1\}$

Now, $A = \left\{\theta : \sin\theta = \frac{\sin\theta}{\cos\theta}\right\}$

$= \{\theta : \sin\theta(\cos\theta - 1) = 0\}$

$= \{\theta = 0, \pi, 2\pi, 3\pi, ...\}$

For $B : \cos\theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, ...$

This shows that A is not contained in B. i.e. $A \not\subset B$. but $B \subset A$.

47. **(a)** $\sin 2x - 2\cos x + 4\sin x = 4$

$\Rightarrow 2\sin x \cdot \cos x - 2\cos x + 4\sin x - 4 = 0$

$\Rightarrow (\sin x - 1)(\cos x - 2) = 0$

$\because \cos x - 2 \neq 0, \therefore \sin x = 1$

$\therefore x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

48. **(b)** $2\sin^2\theta - \cos 2\theta = 0$

$\Rightarrow 2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$

$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\frac{1}{2}$

$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \theta \in [0, 2\pi]$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Now $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$

$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$

$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$

$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}, -2$

But $\sin\theta = -2$, is not possible

$\therefore \sin\theta = \frac{1}{2}, -2 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, there are two common solution, there each of the statement-1 and 2 are true but statement-2 is not a correct explanation for statement-1.

49. **(b)** Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$

Put $e^{\sin x} = t$ in the given equation, we get

$t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5} \quad (\because t = e^{\sin x})$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5}$ and $e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0$

and $\sin x = \ln(2 + \sqrt{5}) > 1$

So, rejected.

Hence, given equation has no solution.

$\therefore$ The equation has no real roots.

50. **(d)** $\sin 4\theta + 2\sin 4\theta\cos 3\theta = 0$

$\sin 4\theta(1 + 2\cos 3\theta) = 0$

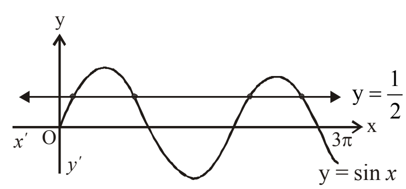
$\sin 4\theta = 0$ or $\cos 3\theta = -\frac{1}{2}$

$4\theta = n\pi; n \in I$

or $3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$ $[\because \theta, \in (0, \pi)]$

51. **(a)**



$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ and $\sin x \neq -3$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

52. **(b)** $\because \tan x + \sec x = 2\cos x;$

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x);$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0;$

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

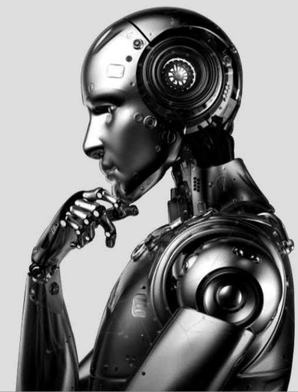
$\Rightarrow \sin x = \frac{1}{2}, -1.;$

$\Rightarrow x = 30°, 150°, 270°.$

Number of solution = 3

4

Principle of Mathematical Induction



TOPIC 1 Problems Based on Sum of Series, Problems Based on Inequality and Divisibility

1. Consider the statement: "P(n) : $n^2 - n + 41$ is prime." Then which one of the following is true? **[Jan. 10, 2019 (II)]**

(a) Both P(3) and P(5) are true.
(b) P(3) is false but P(5) is true.
(c) Both P(3) and P(5) are false.
(d) P(5) is false but P(3) is true.

2. Let $S(K) = 1+3+5...+(2K-1) = 3+K^2$. Then which of the following is true **[2004]**

(a) Principle of mathematical induction can be used to prove the formula
(b) $S(K) \Rightarrow S(K+1)$
(c) $S(K) \not\Rightarrow S(K+1)$
(d) $S(1)$ is correct

3. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+......}}}$ having n radical signs then by methods of mathematical induction which is true **[2002]**

(a) $a_n > 7 \ \forall \ n \ge 1$
(b) $a_n < 7 \ \forall \ n \ge 1$
(c) $a_n < 4 \ \forall \ n \ge 1$
(d) $a_n < 3 \ \forall \ n \ge 1$

Hints & Solutions

1. **(a)** $P(n) = n^2 - n + 41$

$\Rightarrow$ $P(3) = 9 - 3 + 41 = 47$ (prime)

& $P(5) = 25 - 5 + 41 = 61$ (prime)

$\therefore$ $P(3)$ and $P(5)$ are both prime i.e., true.

2. **(b)** $S(K) = 1+3+5+...+(2K-1) = 3 + K^2$

$S(1): 1 = 3 + 1,$ which is not true

$\because$ $S(1)$ is not true.

$\therefore$ P.M.I cannot be applied

Let $S(K)$ is true, i.e.

$S(K): 1 + 3 + 5 .... + (2K - 1) = 3 + K^2$

Adding $2K + 1$ on both sides

$\Rightarrow 1 + 3 + 5 .... + (2K - 1) + 2K + 1$

$= 3 + K^2 + 2K + 1 = 3 + (K + 1)^2 = S(K + 1)$

$\therefore S(K) \Rightarrow S(K + 1)$

3. **(b)** For $n = 1$, $a_1 = \sqrt{7} < 7$. Let $a_m < 7$.

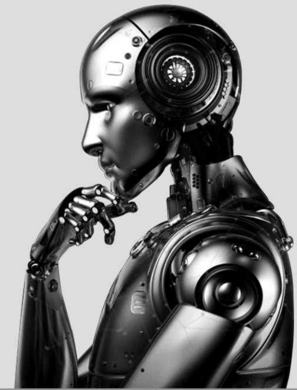
Then $a_{m+1} = \sqrt{7 + a_m}$

$\Rightarrow$ $a^2_{m+1} = 7 + a_m < 7 + 7 < 14.$

$\Rightarrow$ $a_{m+1} < \sqrt{14} < 7;$ So, by the principle of mathematical induction $a_n < 7$, $\forall\ n$.

5

Complex Numbers and Quadratic Equations



TOPIC 1 **Integral Powers of Iota, Algebraic Operations of Complex Numbers, Conjugate, Modulus and Argument or Amplitude of a Complex Number**

1. If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta \in [0, 2\pi]$, is *a* real number, then an argument of $\sin\theta + i\cos\theta$ is: **[Jan. 7, 2020 (II)]**

(a) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (b) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$

(c) $-\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

2. If the four complex numbers $z, \bar{z}, \bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to : **[Sep. 05, 2020 (I)]**

(a) $4\sqrt{2}$ (b) 4 (c) $2\sqrt{2}$ (d) 2

3. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is : **[Sep. 05, 2020 (II)]**

(a) -2^{15} (b) $2^{15} i$ (c) $-2^{15} i$ (d) 6^5

4. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1, (m, n \in \mathbf{N})$, then the greatest common divisor of the least values of m and n is __________. **[Sep. 03, 2020 (I)]**

5. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to : **[Sep. 03, 2020 (II)]**

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

6. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z + 3i|$ is : **[Jan. 9, 2020 (I)]**

(a) $\sqrt{10}$ (b) $\frac{7}{2}$ (c) $\frac{15}{4}$ (d) $2\sqrt{3}$

7. If z be a complex number satisfying $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be: **[Jan. 9, 2020 (II)]**

(a) $\sqrt{\frac{17}{2}}$ (b) $\sqrt{10}$ (c) $\sqrt{7}$ (d) $\sqrt{8}$

8. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then : **[April 12, 2019 (II)]**

(a) $n = 20$ and $\text{Re}(z) = -10$

(b) $n = 40$ and $\text{Re}(z) = 10$

(c) $n = 40$ and $\text{Re}(z) = -10$

(d) $n = 20$ and $\text{Re}(z) = 10$

9. The equation $|z - i| = |z - 1|, i = \sqrt{-1}$, represents: **[April 12, 2019 (I)]**

(a) a circle of radius $\frac{1}{2}$.

(b) the line through the origin with slope 1.

(c) a circle of radius 1.

(d) the line through the origin with slope – 1.

10. If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to : **[April 10, 2019 (I)]**

(a) $-\frac{1}{5} - \frac{3}{5}i$ (b) $-\frac{3}{5} - \frac{1}{5}i$

(c) $\frac{1}{5} - \frac{3}{5}i$ (d) $-\frac{1}{5} + \frac{3}{5}i$

11. If z and ω are two complex numbers such that $|z\omega|=1$ and $\arg(z)-\arg(\omega)=\frac{\pi}{2}$, then: **[April 10, 2019 (II)]**

(a) $\bar{z}\omega=i$ (b) $z\bar{\omega}=\frac{-1+i}{\sqrt{2}}$

(c) $\bar{z}\omega=-i$ (d) $z\bar{\omega}=\frac{1-i}{\sqrt{2}}$

12. Let $z \in C$ be such that $|z|<1$. If $\omega=\frac{5+3z}{5(1-z)}$, then : **[April 09, 2019 (II)]**

(a) $5\,\text{Re}(\omega)>4$ (b) $4\,\text{Im}(\omega)>5$

(c) $5\,\text{Re}(\omega)>1$ (d) $5\,\text{Im}(\omega)<1$

13. If $\frac{z-\alpha}{z+\alpha}(\alpha \in R)$ is a purely imaginary number and $|z|=2$, then a value of α is : **[Jan. 12, 2019 (I)]**

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

14. Let z_1 and z_2 be two complex numbers satisfying $|z_1|=9$ and $|z_2| - |3|-|4i||=|4$. Then the minimum value of $|z_1-z_2|$ is : **[Jan. 12, 2019 (II)]**

(a) 0 (b) $\sqrt{2}$ (c) 1 (d) 2

15. Let z be a complex number such that $|z|+z=3+i$ $\left(\text{where } i=\sqrt{-1}\right)$.

Then $|z|$ is equal to : **[Jan. 11, 2019 (II)]**

(a) $\frac{\sqrt{34}}{3}$ (b) $\frac{5}{3}$ (c) $\frac{\sqrt{41}}{4}$ (d) $\frac{5}{4}$

16. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1|=4|z_2|$. If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$ then: **[Jan. 10 2019 (II)]**

(a) $\text{Re}(z)=0$ (b) $|z|=\sqrt{\frac{5}{2}}$

(c) $|z|=\frac{1}{2}\sqrt{\frac{17}{2}}$ (d) $\text{Im}(z)=0$

17. Let $A=\left\{\theta \in \left(-\frac{\pi}{2},\pi\right): \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$.

Then the sum of the elements in A is: **[Jan. 9 2019 (I)]**

(a) $\frac{5\pi}{6}$ (b) π (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

18. The set of all $\alpha \in R$, for which $w=\frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z|=1$ and $Re\, z \neq 1$, is **[Online April 15, 2018]**

(a) $\{0\}$ (b) an empty set

(c) $\left\{0,\frac{1}{4},-\frac{1}{4}\right\}$ (d) equal to R

19. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: **[2016]**

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

20. If z is a non-real complex number, then the minimum value of $\frac{lmz^5}{(lmz)^5}$ is : **[Online April 11, 2015]**

(a) –1 (b) –4 (c) –2 (d) –5

21. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$: **[2014]**

(a) is strictly greater than $\frac{5}{2}$

(b) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(c) is equal to $\frac{5}{2}$

(d) lie in the interval $(1, 2)$

22. For all complex numbers z of the form $1+i\alpha$, $\alpha \in R$, if $z^2=x+iy$, then **[Online April 19, 2014]**

(a) $y^2-4x+2=0$ (b) $y^2+4x-4=0$

(c) $y^2-4x-4=0$ (d) $y^2+4x+2=0$

23. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number.

Then $z+\frac{1}{z}$ is: **[Online April 12, 2014]**

(a) zero

(b) any non-zero real number other than 1.

(c) any non-zero real number.

(d) a purely imaginary number.

24. If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then

$\arg\left(\frac{z_1}{z_4}\right)+\arg\left(\frac{z_2}{z_3}\right)$ equals: **[Online April 11, 2014]**

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

25. Let w (Im w $\neq 0$) be a complex number. Then the set of all complex number z satisfying the equation

$w-\overline{w}z=k(1-z)$, for some real number k, is **[Online April 9, 2014]**

(a) $\{z:|z|=1\}$ (b) $\{z:z=\overline{z}\}$

(c) $\{z:z\neq 1\}$ (d) $\{z:|z|=1, z\neq 1\}$

26. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\overline{z}}\right)$ equals: **[2013]**

(a) $-\theta$ (b) $\frac{\pi}{2}-\theta$ (c) θ (d) $\pi-\theta$

27. Let z satisfy $|z|=1$ and $z=1-\overline{z}$.
Statement 1 : z is a real number.

Statement 2 : Principal argument of z is $\frac{\pi}{3}$

[Online April 25, 2013]

(a) Statement 1 is true Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
(b) Statement 1 is false; Statement 2 is true
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

28. Let $a=\operatorname{Im}\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. **[Online April 23, 2013]**

The set $A=\{a:|z|=1 \text{ and } z\neq \pm 1\}$ is equal to:
(a) $(-1,1)$ (b) $[-1,1]$ (c) $[0,1)$ (d) $(-1,0]$

29. If $Z_1 \neq 0$ and Z_2 be two complex numbers such that $\frac{Z_2}{Z_1}$ is a purely imaginary number, then $\left|\frac{2Z_1+3Z_2}{2Z_1-3Z_2}\right|$ is equal to:

[Online April 9, 2013]

(a) 2 (b) 5 (c) 3 (d) 1

30. $|z_1+z_2|^2+|z_1-z_2|^2$ is equal to **[Online May 26, 2012]**

(a) $2(|z_1|+|z_2|)$ (b) $2(|z_1|^2+|z_2|^2)$

(c) $|z_1||z_2|$ (d) $|z_1|^2+|z_2|^2$

31. Let Z and W be complex numbers such that $|Z|=|W|$, and arg Z denotes the principal argument of Z.
[Online May 19, 2012]

Statement 1: If arg Z + arg $W = \pi$, then $Z=-\overline{W}$.

Statement 2: $|Z|=|W|$, implies arg $Z-$ arg $\overline{W}=\pi$.
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(d) Statement 1 is false, Statement 2 is true.

32. Let Z_1 and Z_2 be any two complex number.

Statement 1: $|Z_1-Z_2|\geq |Z_1|-|Z_2|$

Statement 2: $|Z_1+Z_2|\leq |Z_1|+|Z_2|$ **[Online May 7, 2012]**
(a) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is false, Statement 2 is true.

33. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ equals **[2010]**
(a) 1 (b) 2 (c) ∞ (d) 0

34. The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is **[2008]**

(a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

35. If $z=x-iy$ and $z^{\frac{1}{3}}=p+iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to **[2004]**
(a) –2 (b) –1 (c) 2 (d) 1

36. Let z and w be complex numbers such that $\overline{z}+i\overline{w}=0$ and arg $zw=\pi$. Then arg z equals **[2004]**

(a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

37. If $\left(\frac{1+i}{1-i}\right)^x=1$ then **[2003]**

(a) $x=2n+1$, where n is any positive integer
(b) $x=4n$, where n is any positive integer
(c) $x=2n$, where n is any positive integer
(d) $x=4n+1$, where n is any positive integer.

38. If z and ω are two non-zero complex numbers such that $|z\omega|=1$ and $Arg(z)-Arg(\omega)=\frac{\pi}{2}$, then $\bar{z}\omega$ is equal to **[2003]**

(a) -1 (b) 1 (c) $-i$ (d) i

39. If $|z-4|<|z-2|$, its solution is given by **[2002]**

(a) $\text{Re}(z)>0$ (b) $\text{Re}(z)<0$

(c) $\text{Re}(z)>3$ (d) $\text{Re}(z)>2$

40. z and w are two non zero complex numbers such that $|z|=|w|$ and Arg z + Arg w = π then z equals **[2002]**

(a) $\bar{\omega}$ (b) $-\bar{\omega}$ (c) ω (d) $-\omega$

TOPIC 2 Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moiver's Theorem, Powers of Complex Numbers

41. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i=\sqrt{-1}$, then z lies on the: **[Sep. 06, 2020 (II)]**

(a) line, $y=-x$ (b) imaginary axis

(c) line, $y=x$ (d) real axis

42. If a and b are real numbers such that $(2+\alpha)^4 = a+b\alpha$, where $\alpha=\frac{-1+i\sqrt{3}}{2}$, then $a+b$ is equal to : **[Sep. 04, 2020 (II)]**

(a) 9 (b) 24 (c) 33 (d) 57

43. The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is : **[Sep. 02, 2020 (I)]**

(a) $\frac{1}{2}(1-i\sqrt{3})$ (b) $\frac{1}{2}(\sqrt{3}-i)$

(c) $-\frac{1}{2}(\sqrt{3}-i)$ (d) $-\frac{1}{2}(1-i\sqrt{3})$

44. The imaginary part of $(3+2\sqrt{-54})^{1/2}-(3-2\sqrt{-54})^{1/2}$ can be : **[Sep. 02, 2020 (II)]**

(a) $-\sqrt{6}$ (b) $-2\sqrt{6}$ (c) 6 (d) $\sqrt{6}$

45. Let $\alpha=\frac{-1+i\sqrt{3}}{2}$. If $a=(1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b=\sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation: **[Jan. 8, 2020 (II)]**

(a) $x^2+101x+100=0$ (b) $x^2-102x+101=0$

(c) $x^2-101x+100=0$ (d) $x^2+102x+101=0$

46. If $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$, where $z=x+iy$, then the point (x,y) lies on a: **[Jan. 7, 2020 (I)]**

(a) circle whose centre is at $\left(-\frac{1}{2},-\frac{3}{2}\right)$.

(b) straight line whose slope is $-\frac{2}{3}$.

(c) straight line whose slope is $\frac{3}{2}$.

(d) circle whose diameter is $\frac{\sqrt{5}}{2}$.

47. If $z=\frac{\sqrt{3}}{2}+\frac{i}{2}\left(i=\sqrt{-1}\right)$, then $(1+iz+z^5+iz^8)^9$ is equal to: **[April 08, 2019 (II)]**

(a) 0 (b) 1

(c) $(-1+2i)^9$ (d) -1

48. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}\left(i=\sqrt{-1}\right)$, where x and y are real numbers then y – x equals : **[Jan. 11, 2019 (I)]**

(a) 91 (b) –85 (c) 85 (d) –91

49. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R($z$) and I($z$) respectively denote the real and imaginary parts of z, then: **[Jan. 10, 2019 (II)]**

(a) I(z) = 0 (b) R(z) > 0 and I(z) > 0

(c) R(z) < 0 and I(z) > 0 (d) R(z) = –(c)

50. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n=1$, is **[Online April 16, 2018]**

(a) 2 (b) 6 (c) 5 (d) 3

51. The point represented by 2 + i in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south–westwards direction. Then its new position in the Argand plane is at the point represented by : **[Online April 9, 2016]**

(a) 1 + i (b) 2 + 2i (c) –2 – 2i (d) –1 – i

52. A complex number z is said to be unimodular if |z| = 1. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: **[2015]**

(a) circle of radius 2.
(b) circle of radius $\sqrt{2}$.
(c) straight line parallel to x-axis
(d) straight line parallel to y-axis.

53. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[2012]**
(a) either on the real axis or on a circle passing through the origin.
(b) on a circle with centre at the origin
(c) either on the real axis or on a circle not passing through the origin.
(d) on the imaginary axis.

54. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals **[2011]**
(a) (1, 1) (b) (1, 0) (c) (–1, 1) (d) (0, 1)

55. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is **[2007]**
(a) 6 (b) 0 (c) 4 (d) 10

56. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on **[2005]**
(a) an ellipse (b) a circle
(c) a straight line (d) a parabola

57. If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to **[2005]**
(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

58. If the cube roots of unity are **1**, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are **[2005]**
(a) $-1, -1+2\omega, -1-2\omega^2$
(b) $-1, -1, -1$
(c) $-1, 1-2\omega, 1-2\omega^2$
(d) $-1, 1+2\omega, 1+2\omega^2$

59. If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[2004]**
(a) an ellipse (b) the imaginary axis
(c) a circle (d) the real axis

60. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be **[2002]**
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

TOPIC 3 Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots.

61. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is: **[Sep. 06, 2020 (I)]**
(a) 2 (b) 3 (c) 1 (d) 4

62. If α and β are the roots of the equation $2x(2x+1) = 1$, then β is equal to: **[Sep. 06, 2020 (II)]**
(a) $2\alpha(\alpha+1)$ (b) $-2\alpha(\alpha+1)$
(c) $2\alpha(\alpha-1)$ (d) $2\alpha^2$

63. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is : **[Sep. 05, 2020 (I)]**
(a) $\frac{5}{9}$ (b) $\frac{25}{81}$ (c) $\frac{5}{27}$ (d) $\frac{25}{9}$

64. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, the the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to : **[Sep. 05, 2020 (II)]**
(a) $\frac{27}{32}$ (b) $\frac{1}{24}$ (c) $\frac{3}{8}$ (d) $\frac{27}{16}$

65. Let $u = \frac{2z+i}{z-ki}$, $z = x+iy$ and $k > 0$. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where $PQ = 5$, then the value of k is : **[Sep. 04, 2020 (I)]**
(a) 3/2 (b) 1/2 (c) 4 (d) 2

66. Let $\lambda \neq 0$ be in **R**. If α and β are roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to : **[Sep. 04, 2020 (II)]**
(a) 27 (b) 18 (c) 9 (d) 36

67. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to : **[Sep. 03, 2020 (I)]**

(a) $\frac{9}{4}(9+q^2)$ (b) $\frac{9}{4}(9-q^2)$

(c) $\frac{9}{4}(9+p^2)$ (d) $\frac{9}{4}(9-p^2)$

68. The set of all real values of λ for which the quadratic equations, $(\lambda^2+1)x^2-4\lambda x+2=0$ always have exactly one root in the interval (0, 1) is : **[Sep. 03, 2020 (II)]**

(a) $(0,2)$ (b) $(2,4]$ (c) $(1,3]$ (d) $(-3,-1)$

69. Let α and β be the roots of the equation, $5x^2+6x-2=0$. If $S_n=\alpha^n+\beta^n$, $n=1,2,3,\ldots$, then : **[Sep. 02, 2020 (I)]**

(a) $6S_6+5S_5=2S_4$ (b) $6S_6+5S_5+2S_4=0$

(c) $5S_6+6S_5=2S_4$ (d) $5S_6+6S_5+2S_4=0$

70. The number of real roots of the equation, $e^{4x}+e^{3x}-4e^{2x}+e^x+1=0$ is: **[Jan. 9, 2020 (I)]**

(a) 1 (b) 3 (c) 2 (d) 4

71. The least positive value of 'a' for which the equation, $2x^2+(a-10)x+\frac{33}{2}=2a$ has real roots is _______. **[Jan. 8, 2020 (I)]**

72. If the equation, $x^2+bx+45=0$ $(b\in R)$ has conjugate complex roots and they satisfy $|z+1|=2\sqrt{10}$, then: **[Jan. 8, 2020 (I)]**

(a) $b^2-b=30$ (b) $b^2+b=72$

(c) $b^2-b=42$ (d) $b^2+b=12$

73. Let α and β be the roots of the equation $x^2-x-1=0$. If $p_k=(\alpha)^k+(\beta)^k$, $k\geq 1$, then which one of the following statements is not true ? **[Jan. 7, 2020 (II)]**

(a) $p_3=p_5-p_4$

(b) $P_5=11$

(c) $(p_1+p_2+p_3+p_4+p_5)=26$

(d) $p_5=p_2\cdot p_3$

74. Let α and β be two real roots of the equation $(k+1)\tan^2 x-\sqrt{2}\,.\,\lambda\tan x=(1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta)=50$, then a value of λ is: **[Jan. 7, 2020 (I)]**

(a) $10\sqrt{2}$ (b) 10 (c) 5 (d) $5\sqrt{2}$

75. If α and β are the roots of the quadratic equation, $x^2+x\sin\theta-2\sin\theta=0$ $0,\theta\in\left(0,\frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})(\alpha-\beta)^{24}}$ is equal to : **[April 10, 2019 (I)]**

(a) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (b) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

(c) $\frac{2^{12}}{(\sin\theta-8)^{6}}$ (d) $\frac{2^{6}}{(\sin\theta+8)^{12}}$

76. The number of real roots of the equation $5+|2^x-1|=2^x(2^x-2)$ is: **[April 10, 2019 (II)]**

(a) 3 (b) 2 (c) 4 (d) 1

77. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2+px+q=0$, then: **[April 9, 2019 (I)]**

(a) $p^2-4q+12=0$ (b) $q^2-4p-16=0$

(c) $q^2+4p+14=0$ (d) $p^2-4q-12=0$

78. If m is chosen in the quadratic equation $(m^2+1)x^2-3x+(m^2+1)^2=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is: **[April 09, 2019 (II)]**

(a) $10\sqrt{5}$ (b) $8\sqrt{3}$ (c) $8\sqrt{5}$ (d) $4\sqrt{3}$

79. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0,\ (x>0)$ is equal to: **[April 8, 2019 (I)]**

(a) 9 (b) 12 (c) 4 (d) 10

80. If α and β be the roots of the equation $x^2-2x+2=0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n=1$ is : **[April 8, 2019 (I)]**

(a) 2 (b) 5 (c) 4 (d) 3

81. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2=0$, then the least value of m for which $\lambda+\frac{1}{\lambda}=1$, is : **[Jan. 12, 2019 (I)]**

(a) $2-\sqrt{3}$ (b) $4-3\sqrt{2}$

(c) $-2+\sqrt{2}$ (d) $4-2\sqrt{3}$

82. If one real root of the quadratic equation $81x^2+kx+256=0$ is cube of the other root, then a value of k is : **[Jan. 11, 2019 (I)]**

(a) –81 (b) 100 (c) 144 (d) –300

83. Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0$, $c\neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is: **[Jan. 10, 2019 (I)]**

(a) 18 (b) 12 (c) 10 (d) 11

84. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2+(3-\lambda)x+2=\lambda$ has the least value is: **[Jan. 10, 2019 (II)]**

(a) $\frac{15}{8}$ (b) 1 (c) $\frac{4}{9}$ (d) 2

85. Let α and β be two roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to: **[Jan. 9, 2019 (I)]**

(a) –256 (b) 512 (c) –512 (d) 256

86. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha=0$ are rational numbers is: **[Jan. 09, 2019 (II)]**

(a) 3 (b) 2 (c) 4 (d) 5

87. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval: **[Jan. 09, 2019 (II)]**

(a) $(-5, -4)$ (b) $(4, 5)$
(c) $(5, 6)$ (d) $(3, 4)$

88. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6i\, z_0^{81} - 3i\, z_0^{93}$, then arg z is equal to: **[Jan. 09, 2019 (II)]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) 0

89. Let p, q and r be real numbers $(p \neq q, r \neq 0)$, such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to. **[Online April 16, 2018]**

(a) $p^2 + q^2 + r^2$ (b) $p^2 + q^2$
(c) $2(p^2 + q^2)$ (d) $\frac{p^2 + q^2}{2}$

90. If an angle A of a ΔABC satisfies $5 \cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are. **[Online April 16, 2018]**

(a) $\sin A, \sec A$ (b) $\sec A, \tan A$
(c) $\tan A, \cos A$ (d) $\sec A, \cot A$

91. If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3 \sin^2(A+B) - 10 \sin(A+B).\cos(A+B) - 25 \cos^2(A+B)$ is **[Online April 15, 2018]**

(a) 25 (b) -25 (c) -10 (d) 10

92. If $f(x)$ is a quadratic expression such that $f(a) + f(b) = 0$, and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is **[Online April 15, 2018]**

(a) $-\frac{5}{8}$ (b) $-\frac{8}{5}$ (c) $\frac{5}{8}$ (d) $\frac{8}{5}$

93. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to: **[2018]**

(a) 0 (b) 1 (c) 2 (d) -1

94. If, for a positive integer n, the quadratic equation,
$x(x+1) + (x+1)(x+2) + ... + (x+\overline{n-1})(x+n) = 10n$
has two consecutive integral solutions, then n is equal to: **[2017]**

(a) 11 (b) 12 (c) 9 (d) 10

95. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is: **[Online April 9, 2017]**

(a) 16 (b) 14 (c) -4 (d) -5

96. Let p(x) be a quadratic polynomial such that p(0)=1. If p(x) leaves remainder 4 when divided by x–1 and it leaves remainder 6 when divided by x+1; then : **[Online April 8, 2017]**

(a) p(b) = 11 (b) p(b) = 19
(c) p(–2) = 19 (d) p(–2) = 11

97. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is : **[2016]**

(a) 6 (b) 5 (c) 3 (d) -4

98. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to : **[Online April 10, 2016]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) 2

99. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to: **[2015]**

(a) 3 (b) -3 (c) 6 (d) -6

100. If the two roots of the equation, $(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is : **[Online April 11, 2015]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$
(c) $\left(-\frac{1}{2}, 0\right)$ (d) $(-\infty, -2) \cup (2, \infty)$

101. If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation : **[Online April 10, 2015]**

(a) exists and is equal to $-\frac{1}{2}$.
(b) exists and is equal to $\frac{1}{2}$.
(c) exists and is equal to 1.
(d) does not exist.

102. If $a \in R$ and the equation
$$-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$$
(where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: **[2014]**

(a) $(-2, -1)$ (b) $(-\infty, -2) \cup (2, \infty)$
(c) $(-1, 0) \cup (0, 1)$ (d) $(1, 2)$

103. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has; **[Online April 19, 2014]**

(a) no solution (b) exactly one solution
(c) exactly two solution (d) exactly four solution

104. The sum of the roots of the equation, $x^2+|2x-3|-4=0$, is: **[Online April 12, 2014]**

(a) 2 (b) -2 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

105. If α and β are roots of the equation, $x^2-4\sqrt{2}kx+2e^{4\ln k}-1=0$ for some k, and $\alpha^2+\beta^2=66$, then $\alpha^3+\beta^3$ is equal to: **[Online April 11, 2014]**

(a) $248\sqrt{2}$ (b) $280\sqrt{2}$ (c) $-32\sqrt{2}$ (d) $-280\sqrt{2}$

106. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2+bx+1=0$ ($a \neq 0$, a, b, $\in$ R), then the equation, $x\left(x+b^3\right)+\left(a^3-3abx\right)=0$ as roots : **[Online April 9, 2014]**

(a) $\alpha^{3/2}$ and $\beta^{3/2}$ (b) $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$

(c) $\sqrt{\alpha\beta}$ and $\alpha\beta$ (d) $\alpha^{-\frac{3}{2}}$ and $\beta^{-\frac{3}{2}}$

107. If p and q are non-zero real numbers and $\alpha^3+\beta^3=-p, \alpha\beta=q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is : **[Online April 25, 2013]**

(a) $px^2-qx+p^2=0$ (b) $qx^2+px+q^2=0$

(c) $px^2+qx+p^2=0$ (d) $qx^2-px+q^2=0$

108. If α and β are roots of the equation $x^2+px+\frac{3p}{4}=0$, such that $|\alpha-\beta|=\sqrt{10}$, then p belongs to the set : **[Online April 22, 2013]**

(a) $\{2,-5\}$ (b) $\{-3,2\}$ (c) $\{-2,5\}$ (d) $\{3,-5\}$

109. If a complex number z statisfies the equation $z+\sqrt{2}\,|z+1|+i=0$, then $|z|$ is equal to : **[Online April 22, 2013]**

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) 1

110. Let $p, q, r \in R$ and $r>p>0$. If the quadratic equation $px^2+qx+r=0$ has two complex roots α and β, then $|\alpha|+|\beta|$ is **[Online May 19, 2012]**

(a) equal to1

(b) less than 2 but not equal to 1

(c) greater than 2

(d) equal to 2

111. If the sum of the square of the roots of the equation $x^2-(\sin\alpha-2)x-(1+\sin\alpha)=0$ is least, then α is equal to **[Online May 12, 2012]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

112. The value of k for which the equation $(k-2)x^2+8x+k+4=0$ has both roots real, distinct and negative is **[Online May 7, 2012]**

(a) 6 (b) 3 (c) 4 (d) 1

113. Let for $a \neq a_1 \neq 0$,

$f(x)=ax^2+bx+c$, $g(x)=a_1x^2+b_1x+c_1$

and $p(x)=f(x)-g(x)$.

If $p(x)=0$ only for $x=-1$ and $p(-2)=2$, then the value of p (b) is : **[2011 RS]**

(a) 3 (b) 9 (c) 6 (d) 18

114. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3). Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are : **[2011 RS]**

(a) 6, 1 (b) 4, 3 (c) $-6,-1$ (d) $-4,-3$

115. Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line Re $z=1$, then it is necessary that : **[2011]**

(a) $\beta\in(-1,0)$ (b) $|\beta|=1$

(c) $\beta\in(1,\infty)$ (d) $\beta\in(0,1)$

116. If α and β are the roots of the equation $x^2-x+1=0$, then $\alpha^{2009}+\beta^{2009}=$ **[2010]**

(a) -1 (b) 1 (c) 2 (d) -2

117. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is : **[2009]**

(a) less than $4ab$ (b) greater than $-4ab$

(c) less than $-4ab$ (d) greater than $4ab$

118. If the difference between the roots of the equation $x^2+ax+1=0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**

(a) $(3,\infty)$ (b) $(-\infty,-3)$ (c) $(-3,3)$ (d) $(-3,\infty)$

119. All the values of m for which both roots of the equation $x^2-2mx+m^2-1=0$ are greater than -2 but less than 4, lie in the interval **[2006]**

(a) $-2<m<0$ (b) $m>3$

(c) $-1<m<3$ (d) $1<m<4$

120. If the roots of the quadratic equation $x^2+px+q=0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2+q-p$ is **[2006]**

(a) 2 (b) 3 (c) 0 (d) 1

121. If $z^2+z+1=0$, where z is complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots\ldots..+\left(z^6+\frac{1}{z^6}\right)^2$ is **[2006]**

(a) 18 (b) 54 (c) 6 (d) 12

122. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $-\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0, a \neq 0$ then **[2005]**

(a) $a = b + c$ (b) $c = a + b$
(c) $b = c$ (d) $b = a + c$

123. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals **[2005]**

(a) -2 (b) 3 (c) 2 (d) 1

124. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is **[2004]**

(a) 4 (b) 12 (c) 3 (d) $\frac{49}{4}$

125. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its root are **[2004]**

(a) $-1, 2$ (b) $-1, 1$ (c) $0, -1$ (d) $0, 1$

126. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is **[2003]**

(a) 3 (b) 2 (c) 4 (d) 1

127. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a-1)x + 2 = 0$ is twice as large as the other is **[2003]**

(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

128. Let Z_1 and Z_2 be two roots of the equation $Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then **[2003]**

(a) $a^2 = 4b$ (b) $a^2 = b$
(c) $a^2 = 2b$ (d) $a^2 = 3b$

129. If p and q are the roots of the equation $x^2 + px + q = 0$, then **[2002]**

(a) $p = 1, q = -2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

130. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ **[2002]**

(a) is always positive (b) is always negative
(c) does not exist (d) none of these

131. Difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then **[2002]**

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

132. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is **[2002]**

(a) $3x^2 - 19x + 3 = 0$ (b) $3x^2 + 19x - 3 = 0$
(c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 5x + 3 = 0$.

TOPIC 4 Condition for Common Roots, Maximum and Minimum value of Quadratic Equation, Quadratic Expression in two Variables, Solution of Quadratic Inequalities.

133. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to: **[Jan. 10, 2019 (I)]**

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{3}{2}$

134. Let $a, b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : **[Jan. 9, 2020 (II)]**

(a) 25 (b) 26 (c) 28 (d) 24

135. If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2-\lambda)x + (10-\lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is **[Online April 15, 2018]**

(a) 20 (b) $2\sqrt{5}$ (c) $2\sqrt{7}$ (d) $4\sqrt{2}$

136. If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is **[Online April 15, 2018]**

(a) $\sqrt{13}$ (b) $2\sqrt{13}$ (c) 8 (d) $4 + \sqrt{13}$

137. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to : **[Online April 9, 2016]**

(a) 2 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$

138. If non-zero real numbers b and c are such that min f(x) > max g(x), where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ $(x \in R)$;

then $\left|\frac{c}{b}\right|$ lies in the interval: **[Online April 19, 2014]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

(c) $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$ (d) $\left(\sqrt{2}, \infty\right)$

139. If equations $ax^2+bx+c=0\ (a,b,c \in R, a \neq 0)$ and $2x^2+3x+4=0$ have a common root, then a : b : c equals: **[Online April 9, 2014]**

(a) $1:2:3$ (b) $2:3:4$ (c) $4:3:2$ (d) $3:2:1$

140. If the equations $x^2+2x+3=0$ and $ax^2+bx+c=0$, $a,b,c \in R$, have a common root, then $a:b:c$ is **[2013]**

(a) $1:2:3$ (b) $3:2:1$ (c) $1:3:2$ (d) $3:1:2$

141. The least integral value α of x such that $\frac{x-5}{x^2+5x-14}>0$, satisfies : **[Online April 23, 2013]**

(a) $\alpha^2+3\alpha-4=0$ (b) $\alpha^2-5\alpha+4=0$
(c) $\alpha^2-7\alpha+6=0$ (d) $\alpha^2+5\alpha-6=0$

142. The values of 'a' for which one root of the equation $x^2-(a+1)x+a^2+a-8=0$ exceeds 2 and the other is lesser than 2, are given by : **[Online April 9, 2013]**

(a) $3<a<10$ (b) $a \geq 10$
(c) $-2<a<3$ (d) $a \leq -2$

143. If $\left|z-\frac{4}{z}\right|=2$, then the maximum value of $|z|$ is equal to : **[2009]**

(a) $\sqrt{5}+1$ (b) 2 (c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

144. The quadratic equations $x^2-6x+a=0$ and $x^2-cx+6=0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[2009]**

(a) 1 (b) 4 (c) 3 (d) 2

145. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is **[2006]**

(a) $\frac{1}{4}$ (b) 41 (c) 1 (d) $\frac{17}{7}$

146. If both the roots of the quadratic equation $x^2-2kx+k^2+k-5=0$ are less than 5, then k lies in the interval **[2005]**

(a) $(5,6]$ (b) $(6,\infty)$ (c) $(-\infty,4)$ (d) $[4,5]$

147. The value of a for which the sum of the squares of the roots of the equation $x^2-(a-2)x-a-1=0$ assume the least value is **[2005]**

(a) 1 (b) 0 (c) 3 (d) 2

Hints & Solutions

1. (b) Let $z=\dfrac{3+i\sin\theta}{4-i\cos\theta}$, after rationalising

$$z=\frac{(3+i\sin\theta)}{(4-i\cos\theta)}\times\frac{(4+i\cos\theta)}{(4+i\cos\theta)}$$

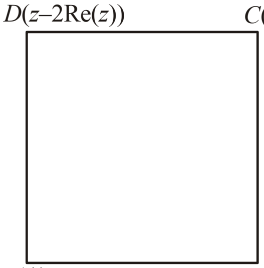
As z is purely real

$$\Rightarrow\quad 3\cos\theta+4\sin\theta=0\quad\Rightarrow\tan\theta=-\frac{3}{4}$$

$$arg(\sin\theta+i\cos\theta)=\pi+\tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$=\pi+\tan^{-1}\left(-\frac{4}{3}\right)=\pi-\tan^{-1}\left(\frac{4}{3}\right)$$

2. (c) $D(z-2\text{Re}(z))$ $\quad$ $C(\bar{z}-2\text{Re}(\bar{z}))$



$A(\bar{z})$ $\quad$ $B(\bar{z})$

Let $z=x+iy$

$\because$ Length of side of square = 4 units

Then, $|z-\bar{z}|=4\Rightarrow|2iy|=4\Rightarrow|y|=2$

Also, $|z-(z-2\text{Re}(z))|=4$

$\Rightarrow|2\text{Re}(z)|=4\Rightarrow|2x|=4\Rightarrow|x|=2$

$\therefore|z|=\sqrt{x^2+y^2}=\sqrt{4+4}=2\sqrt{2}$

3. (c) $\because\ -1+\sqrt{3}i=2\cdot e^{\frac{2\pi}{3}i}$ and $1-i=\sqrt{2}\cdot e^{-\frac{i\pi}{4}}$

$$\therefore\left(\frac{-1+\sqrt{3}i}{1-i}\right)^{30}=\left(\sqrt{2}e^{\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)i}\right)^{30}$$

$$=2^{15}\cdot e^{-\frac{\pi}{2}i}=-2^{15}\cdot i.$$

4. (4)

Given that $\left(\dfrac{1+i}{1-i}\right)^{m/2}=\left(\dfrac{1+i}{i-1}\right)^{n/3}=1$

$$\Rightarrow\left(\frac{(1+i)^2}{2}\right)^{m/2}=\left(\frac{(1+i)^2}{-2}\right)^{n/3}=1$$

$$\Rightarrow i^{m/2}=(-i)^{n/3}=1$$

m (least) = 8, n (least) = 12

GCD (8, 12) = 4.

5. (b) Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$

$\because|z_1-1|=\text{Re}(z_1)$

$\Rightarrow(x_1-1)^2+y_1^2=x_1^2$

$\Rightarrow y_1^2-2x_1+1=0$ $\quad$ ...(i)

$|z_2-1|=\text{Re}(z_2)\ \Rightarrow(x_2-1)^2+y_2^2=x_2^2$

$\Rightarrow y_2^2-2x_2+1=0$ $\quad$ ...(ii)

From eqn. (i) – (ii),

$y_1^2-y_2^2-2(x_1-x_2)=0$

$\Rightarrow y_1+y_2=2\left(\dfrac{x_1-x_2}{y_1-y_2}\right)$ $\quad$ ...(iii)

$\because\arg(z_1-z_2)=\dfrac{\pi}{6}$

$\Rightarrow\tan^{-1}\left(\dfrac{y_1-y_2}{x_1-x_2}\right)=\dfrac{\pi}{6}$

$\Rightarrow\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{1}{\sqrt{3}}$

$\Rightarrow\dfrac{2}{y_1+y_2}=\dfrac{1}{\sqrt{3}}$ $\quad\left[\text{From, }\dfrac{y_1-y_2}{x_1-x_2}=\dfrac{2}{y_1+y_2}\right]$

$\therefore y_1+y_2=2\sqrt{3}\Rightarrow\text{Im}(z_1+z_2)=2\sqrt{3}$

6. (b) Let $z=x+iy$

Then, $\left|\dfrac{z-i}{z+2i}\right|=1\ \Rightarrow x^2+(y-1)^2$

$=x^2+(y+2)^2\Rightarrow-2y+1=4y+4$

$\Rightarrow\quad 6y=-3\ \Rightarrow y=-\dfrac{1}{2}$

$\because\quad|z|=\dfrac{5}{2}\ \Rightarrow x^2+y^2=\dfrac{25}{4}$

$\Rightarrow \quad x^2 = \dfrac{24}{4} = 6$

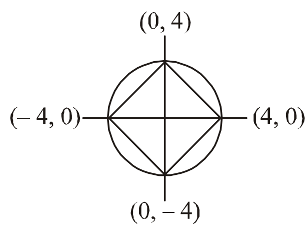
$\therefore \quad z = x + iy \qquad \Rightarrow z = \pm\sqrt{6} - \dfrac{i}{2}$

$|z + 3i| = \sqrt{6 + \dfrac{25}{4}} = \sqrt{\dfrac{49}{4}}$

$\Rightarrow \quad |z + 3i| = \dfrac{7}{2}$

7. **(c)** $z = x + iy$
$|x| + |y| = 4$

$|z| = \sqrt{x^2 + y^2}$
Minimum value of
$|z| = 2\sqrt{2}$
Maximum value of
$|z| = 4$



$|z| \in \left[\sqrt{8}, \sqrt{16}\right]$

So, $|z|$ can't be $\sqrt{7}$.

8. **(c)** Let Re $(z) = x$ i.e., $z = x + 10i$
$2z - n = (2i - 1)(2z + n)$
$(2x - n) + 20i = (2i - 1)((2x + n) + 20i)$
On comparing real and imaginary parts,
$-(2x + n) - 40 = 2x - n$ and $20 = 4x + 2n - 20$
$\Rightarrow 4x = -40$ and $40 = -40 + 2n$
$\Rightarrow x = -10$ and $n = 40$
Hence, $\text{Re}(z) = -10$

9. **(b)** Given equation is, $|z - 1| = |z - i|$
$\Rightarrow \quad (x - 1)^2 + y^2 = x^2 + (y - 1)^2 \qquad$ [Here, $z = x + iy$]
$\Rightarrow \quad 1 - 2x = 1 - 2y \Rightarrow x - y = 0$
Hence, locus is straight line with slope 1.

10. **(a)** $z = \dfrac{(1+i)^2}{a - i} \times \dfrac{a + i}{a + i}$

$z = \dfrac{(1 - 1 + 2i)(a + i)}{a^2 + 1} = \dfrac{2ai - 2}{a^2 + 1}$

$|z| = \sqrt{\left(\dfrac{-2}{a^2+1}\right)^2 + \left(\dfrac{2a}{a^2+1}\right)^2} = \sqrt{\dfrac{4 + 4a^2}{(a^2+1)^2}}$

$\Rightarrow |z| = \sqrt{\dfrac{4(1 + a^2)}{(1 + a^2)^2}} = \dfrac{2}{\sqrt{1 + a^2}} \qquad$ ...(i)

Since, it is given that $|z| = \sqrt{\dfrac{2}{5}}$

Then, from equation (i),

$\sqrt{\dfrac{2}{5}} = \dfrac{2}{\sqrt{1 + a^2}}$

Now, square on both side; we get
$\Rightarrow 1 + a^2 = 10 \Rightarrow a = \pm 3$
Since, it is given that $a > 0 \Rightarrow a = 3$

Then, $z = \dfrac{(1+i)^2}{a - i} = \dfrac{1 + i^2 + 2i}{3 - i} = \dfrac{2i}{3 - i}$

$= \dfrac{2i(3 + i)}{10} = \dfrac{-1 + 3i}{5}$

Hence, $\bar{z} = \dfrac{-1}{5} - \dfrac{3}{5}i$

11. **(c)** Given $|z\omega| = 1 \qquad$ ...(i)

and $\arg\left(\dfrac{z}{\omega}\right) = \dfrac{\pi}{2} \qquad$ ...(ii)

$\therefore \dfrac{z}{\omega} + \dfrac{\bar{z}}{\bar{\omega}} = 0 \qquad \left[\because \text{Re}\left(\dfrac{z}{\omega}\right) = 0\right]$

$\Rightarrow z\bar{\omega} = -\bar{z}\bar{\omega}$

from equation (i), $z\bar{z}\omega\bar{\omega} = 1 \quad$ [using $z\bar{z} = |z|^2$]

$(\bar{z}\omega)^2 = -1 \Rightarrow \bar{z}\omega = \pm i$

from equation (ii), $-\arg(\bar{z}) - \arg\omega = \dfrac{\pi}{2} \; -\arg(\bar{z}\omega) = \dfrac{-\pi}{2}$

Hence, $\bar{z}w = -i$

12. **(c)** $\omega = \dfrac{5 + 3z}{5 - 5z} \Rightarrow 5\omega - 5\omega z = 5 + 3z$

$\Rightarrow 5\omega - 5 = z(3 + 5\omega) \Rightarrow z = \dfrac{5(\omega - 1)}{3 + 5\omega}$

$\because |z| < 1, \therefore 5|\omega - 1| < |3 + 5\omega|$

$\Rightarrow 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) < 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega}$

$\left(\because |z|^2 = z\bar{z}\right)$

$\Rightarrow 16 < 40\omega + 40\bar{\omega} \Rightarrow \omega + \bar{\omega} > \dfrac{2}{5} \Rightarrow 2\text{Re}(\omega) > \dfrac{2}{5}$

$\Rightarrow \text{Re}(\omega) > \dfrac{1}{5}$

13. **(a)** Let $t = \dfrac{z-\alpha}{z+\alpha}$

$\because$ t is purely imaginary number.

$\therefore$ $t + \bar{t} = 0$

$\Rightarrow \dfrac{z-\alpha}{z+\alpha} + \dfrac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$

$\Rightarrow (z-\alpha)(\bar{z}+\alpha) + (\bar{z}-\alpha)(z+\alpha) = 0$

$\Rightarrow z\bar{z} - \alpha^2 + z\bar{z} - \alpha^2 = 0$

$\Rightarrow z\bar{z} - \alpha^2 = 0$

$\Rightarrow |z|^2 - \alpha^2 = 0$

$\Rightarrow \alpha^2 = 4$

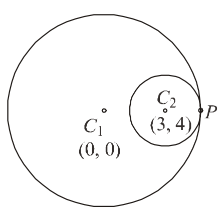
$\Rightarrow \alpha = \pm 2$

14. **(a)** $|z_1| = 9, |z_2 - 3 - 4i| = 4$

z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$

z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$

So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



15. **(b)** Since, $|z| + z = 3 + i$

Let $z = a + ib$, then

$|z| + z = 3 + i \Rightarrow \sqrt{a^2+b^2} + a + ib = 3 + i$

Compare real and imaginary coefficients on both sides

$b = 1,\ \sqrt{a^2+b^2} + a = 3$

$\sqrt{a^2+1} = 3 - a$

$a^2 + 1 = a^2 + 9 - 6a$

$6a = 8 \Rightarrow a = \dfrac{4}{3}$

Then,

$|z| = \sqrt{\left(\dfrac{4}{3}\right)^2 + 1} = \sqrt{\dfrac{16}{9} + 1} = \dfrac{5}{3}$

16. **(none)** Let $z_1 = r_1 e^{i\theta}$ and $z_2 = r_2 e^{i\phi}$

$3|z_1| = 4|z_2| \Rightarrow 3r_1 = 4r_2$

$z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1} = \dfrac{3r_1}{2r_2} e^{i(\theta-\phi)} + \dfrac{2}{3}\dfrac{r_2}{r_1} e^{i(\phi-\theta)}$

$= \dfrac{3}{2} \times \dfrac{4}{3} (\cos(\theta-\phi) + i\sin(\theta-\phi)) +$

$\dfrac{2}{3} \times \dfrac{3}{4} [\cos(\theta-\phi) - i\sin(\theta-\phi)]$

$z = \left(2 + \dfrac{1}{2}\right)\cos(\theta-\phi) + i\left(2 - \dfrac{1}{2}\right)\sin(\theta-\phi)$

$\therefore\ |z| = \sqrt{\dfrac{25}{4}\cos^2(\theta-\phi) + \dfrac{9}{4}\sin^2(\theta-\phi)}$

$= \sqrt{\dfrac{16}{4}\cos^2(\theta-\phi) + \dfrac{9}{4}} \Rightarrow \dfrac{3}{2} \le |z| \le \dfrac{5}{2}$

17. **(d)** Suppose $z = \dfrac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$

Since, z is purely imaginary, then $z + \bar{z} = 0$

$\Rightarrow \dfrac{3+2i\sin\theta}{1-2i\sin\theta} + \dfrac{3-2i\sin\theta}{1+2i\sin\theta} = 0$

$\Rightarrow \dfrac{(3+2i\sin\theta)(1+2i\sin\theta) + (3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta}$

$= 0$

$\Rightarrow \sin^2\theta = \dfrac{3}{4} \Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = -\dfrac{\pi}{3}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

Now, the sum of elements in $A = -\dfrac{\pi}{3} + \dfrac{\pi}{3} + \dfrac{2\pi}{3} = \dfrac{2\pi}{3}$

18. **(a)** $\because |z| = 1\ \&\ Re\, z \ne 1$

Suppose $z = x + iy \Rightarrow x^2 + y^2 = 1$ .....(i)

Now, $w = \dfrac{1 + (1-8\alpha)z}{1-z}$

$\Rightarrow w = \dfrac{1 + (1-8\alpha)(x+iy)}{1-(x+iy)}$

$\Rightarrow w = \dfrac{1 + (1-8\alpha)(x+iy))((1-x)+iy)}{1-(x+iy))((1-x)+iy)}$

$\Rightarrow w = \dfrac{[(1 + x(1-8\alpha)(1-x) - (1-8\alpha)y^2]}{(1-x)^2 + y^2}$

$+ i\dfrac{[(1 + x(1-8\alpha))y - (1-8\alpha)y(1-x)]}{(1-x)^2 + y^2}$

As, w is purely imaginary. So,

$$Re\,w = \frac{\left[(1+x(1-8\alpha))(1-x)-(1-8\alpha)y^2\right]}{(1-x)^2+y^2} = 0$$

$\Rightarrow (1-x)+x(1-8\alpha)(1-x) = (1-8\alpha)y^2$

$\Rightarrow (1-x)+x(1-8\alpha)-x^2(1-8\alpha) = (1-8x)y^2$

$\Rightarrow (1-x)+x(1-8\alpha) = 1-8\alpha$ [From (i), $x^2+y^2=1$]

$\Rightarrow 1-8\alpha = 1$

$\Rightarrow \alpha = 0$

$\therefore \alpha \in \{0\}$

19. **(b)** Rationalizing the given expression

$$\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$$\Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta} = 0 \quad \Rightarrow \sin^2\theta = \frac{1}{3}$$

$$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

20. **(b)** Let $z = re^{i\theta}$

Consider $\dfrac{\text{Im}z^5}{(\text{Im}z)^5} = \dfrac{r^5(\sin 5\theta)}{r^5(\sin\theta)^5}$

$(\because e^{i\theta} = \cos\theta + i\sin\theta)$

$$= \frac{\sin 5\theta}{\sin^5\theta} = \frac{16\sin^5\theta - 20\sin^3\theta + 5\sin\theta}{\sin^5\theta}$$

$$= \frac{16\sin^5\theta}{\sin^5\theta} - \frac{20\sin^3\theta}{\sin^5\theta} + \frac{5\sin\theta}{\sin^5\theta}$$

$= 5\,\text{cosec}^4\theta - 20\,\text{cosec}^2\theta + 16$

minimum value of $\dfrac{\text{Im}z^5}{(\text{Im}z)^5}$ is -4.

21. **(d)** We know minimum value of $|Z_1 + Z_2|$ is

$||Z_1| - |Z_2||$. Thus minimum value of $\left|Z+\frac{1}{2}\right|$ is $\left||Z|-\frac{1}{2}\right|$

$$\le \left|Z+\frac{1}{2}\right| \le |Z| + \frac{1}{2}$$

Since, $|Z| \ge 2$ therefore

$$2-\frac{1}{2} < \left|Z+\frac{1}{2}\right| < 2+\frac{1}{2}$$

$$\Rightarrow \quad \frac{3}{2} < \left|Z+\frac{1}{2}\right| < \frac{5}{2}$$

22. **(b)** Let $z = 1 + i\alpha$, $\alpha \in R$

$z^2 = (1+i\alpha)(1+i\alpha)$

$x + iy = (1 + 2i\alpha - \alpha^2)$

On comparing real and imaginary parts, we get

$x = 1 - \alpha^2$, $y = 2\alpha$

Now, consider option (b), which is

$y^2 + 4x - 4 = 0$

LHS : $y^2 + 4x - 4 = (2\alpha)^2 + 4(1-\alpha^2) - 4$

$= 4\alpha^2 + 4 - 4\alpha^2 - 4$

$= 0 =$ R.H.S.

Hence, $y^2 + 4x - 4 = 0$

23. **(c)** Let $z = x + iy$

$\dfrac{z-i}{z+i}$ is purely imaginary means its real part is zero.

$$\frac{x+iy-i}{x+iy+i} = \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$$

$$= \frac{x^2 - 2ix(y+1) + xi(y-1) + y^2 - 1}{x^2+(y+1)^2}$$

$$= \frac{x^2+y^2-1}{x^2+(y+1)^2} - \frac{2xi}{x^2+(y+1)^2}$$

for pure imaginary, we have

$$\frac{x^2+y^2-1}{x^2+(y+1)^2} = 0$$

$\Rightarrow \quad x^2 + y^2 = 1$

$\Rightarrow \quad (x+iy)(x-iy) = 1$

$$\Rightarrow \quad x + iy = \frac{1}{x-iy} = z$$

and $\dfrac{1}{z} = x - iy$

$$z + \frac{1}{z} = (x+iy)+(x-iy) = 2x$$

$\left(z+\dfrac{1}{z}\right)$ is any non-zero real number

24. **(a)** Consider $\arg\left(\dfrac{z_1}{z_4}\right) + \arg\left(\dfrac{z_2}{z_3}\right)$

$= \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$

$= (\arg(z_1) + \arg(z_2)) - (\arg(z_3) + \arg(z_4))$

given $\begin{pmatrix} z_2 = \bar{z}_1\ \& \\ z_4 = \bar{z}_3 \end{pmatrix}$

$= (\arg(z_1)+\arg(\bar{z}_1)) - (\arg(z_3)+\arg(\bar{z}_3))$

$$\left\{\begin{array}{l}\text{also}\,(\arg(\bar{z}_1) = -\arg(z_1)\\ \arg(\bar{z}_3) = -\arg(z_3)\end{array}\right\}$$

$= (\arg(z_1)-\arg(z_1)) - (\arg(z_3)-\arg(z_3))$

$= 0 - 0 = 0$

25. **(d)** Consider the equation

$w - \bar{w}z = k(1-z), k \in R$

Clearly $z \neq 1$ and $\dfrac{w-\bar{w}z}{1-z}$ is purely real

$\therefore \overline{\left(\dfrac{w-\bar{w}z}{1-z}\right)} = \dfrac{w-\bar{w}z}{1-z}$

$\Rightarrow \dfrac{\bar{w}-w\bar{z}}{1-\bar{z}} = \dfrac{w-\bar{w}z}{1-z}$

$\Rightarrow \bar{w} - \bar{w}z - w\bar{z} + wz\bar{z} = w - w\bar{z} - \bar{w}z + \bar{w}z\bar{z}$

$\Rightarrow \bar{w} + w|z|^2 = w + \bar{w}|z|^2$

$\Rightarrow (w-\bar{w})(|z|^2) = w - \bar{w}$

$\Rightarrow |z|^2 = 1 \quad (\because \text{Im}\, w \neq 0)$

$\Rightarrow |z| = 1$ and $z \neq 1$

$\therefore$ The required set is $\{z : |z| = 1, z \neq 1\}$

26. **(c)** Given $|z| = 1, \arg z = \theta$

$\Rightarrow \bar{z} = \dfrac{1}{z}$

$\therefore \arg\left(\dfrac{1+z}{1+\bar{z}}\right) = \arg\left(\dfrac{1+z}{1+\dfrac{1}{z}}\right) = \arg(z) = \theta.$

27. **(b)** Let $z = x + iy$, $\bar{z} = x - iy$

Now, $z = 1 - \bar{z}$

$\Rightarrow x + iy = 1 - (x - iy)$

$\Rightarrow 2x = 1 \Rightarrow x = \dfrac{1}{2}$

Now, $|z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$\Rightarrow y = \pm\dfrac{\sqrt{3}}{2}$

Now, $\tan\theta = \dfrac{y}{x}$ (θ is the argument)

$= \dfrac{\sqrt{3}}{2} \div \dfrac{1}{2}$ (+ve since only principal argument)

$= \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

Hence, z is not a real number

So, statement-1 is false and 2 is true.

28. **(a)** Let $z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$

Now, $\dfrac{1+z^2}{2iz} = \dfrac{1+x^2-y^2+2ixy}{2i(x+iy)} = \dfrac{(x^2-y^2+1)+2ixy}{2ix-2y}$

$= \dfrac{(x^2-y^2+1)+2ixy}{-2y+2ix} \times \dfrac{-2y-2ix}{-2y-2ix}$

$= \dfrac{y(x^2+y^2-1)+x(x^2+y^2+1)i}{2(x^2+y^2)}$

$a = \dfrac{x(x^2+y^2+1)}{2(x^2+y^2)}$

Since, $|z| = 1 \Rightarrow \sqrt{x^2+y^2} = 1$

$\Rightarrow x^2 + y^2 = 1$

$\therefore a = \dfrac{x(1+1)}{2\times 1} = x$

Also $z \neq 1 \Rightarrow x + iy \neq 1$

$\therefore A = (-1, 1)$

29. **(d)** Let $z_1 = 1 + i$ and $z_2 = 1 - i$

$\dfrac{z_2}{z_1} = \dfrac{1-i}{1+i} = \dfrac{(1-i)(1-i)}{(1+i)(1-i)} = -i$

$\dfrac{2z_1+3z_2}{2z_1-3z_2} = \dfrac{2+3\left(\dfrac{z_2}{z_1}\right)}{2-3\left(\dfrac{z_2}{z_1}\right)} = \dfrac{2-3i}{2+3i}$

$\left|\dfrac{2z_1+3z_2}{2z_1-3z_2}\right| = \left|\dfrac{2-3i}{2+3i}\right| = \left|\dfrac{2-3i}{2+3i}\right| \quad \left[\because \left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}\right]$

$= \dfrac{\sqrt{4+9}}{\sqrt{4+9}} = 1$

30. **(b)** $|z_1+z_2|^2 + |z_1-z_2|^2$

$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2| + |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$= 2|z_1|^2 + 2|z_2|^2 = 2\left[|z_1|^2 + |z_2|^2\right]$

31. **(a)** Let $|Z| = |W| = r$

$\Rightarrow Z = re^{i\theta}, W = re^{i\phi}$

where $\theta + \phi = \pi$

$\therefore \overline{W} = re^{-i\phi}$

Now, $Z = re^{i(\pi-\phi)} = re^{i\pi} \times e^{-i\phi} = -re^{-i\phi}$

$= -\overline{W}$

Thus, statement-1 is true but statement-2 is false.

32. **(b)** Statement - 1 and 2 both are true.

It is fundamental property.

But Statement - 2 is not correct explanation for Statement - 1.

33. **(a)** Let $z = x + iy$

$|z-1| = |z+1| \Rightarrow (x-1)^2 + y^2 = (x+1)^2 + y^2$

$\Rightarrow x = 0 \quad \Rightarrow \operatorname{Re} z = 0$

$|z-1| = |z-i| \Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow x = y$

$|z+1| = |z-i| \Rightarrow (x+1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow x = -y$

Only (0, 0) will satisfy all conditions.

$\Rightarrow$ Number of complex number $z = 1$

34. **(c)** $\overline{\left(\frac{1}{i-1}\right)} = \frac{1}{\overline{(i-1)}} = \frac{1}{-i-1} = \frac{-1}{i+1}$

35. **(a)** Given that $z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p+iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

Comparing both side, we get

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2 \quad$ ...(i)

and $y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2 \quad$ ...(ii)

Adding (i) and (ii), we get

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \quad \therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$

36. **(c)** Given that $\arg zw = \pi$

$\Rightarrow \arg z + \arg w = \pi \quad$ ...(i)

$\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$

Replace i by $-i$, we get

$\therefore \; z = iw \Rightarrow \arg z = \frac{\pi}{2} + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z \quad$ (from (i))

$\therefore \arg z = \frac{3\pi}{4}$

37. **(b)** Given that

$\left(\frac{1+i}{1-i}\right)^x = 1 \quad \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$

$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1; \; \therefore x = 4n; \; n \in I^+$

38. **(a)** $|\bar{z}\omega| = |\bar{z}||\omega| = |z||\omega| = |z\omega| = 1 \; [\because |\bar{z}| = |z|]$

$\operatorname{Arg}(\bar{z}\omega) = \arg(\bar{z}) + \arg(\omega)$

$= -\arg(z) + \arg\omega = -\frac{\pi}{2}$

$[\because \arg(\bar{z}) = -\arg(z)]$

$\therefore \bar{z}\omega = -1$

39. **(c)** Given that $|z-4| < |z-2|$

Let $z = x + iy$

$\Rightarrow |(x-4) + iy)| < |(x-2) + iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$

$\Rightarrow x > 3 \Rightarrow \operatorname{Re}(z) > 3$

40. **(b)** Let $|z| = |\omega| = r$

$\therefore z = re^{i\theta}, \; \omega = re^{i\phi}$ where $\theta + \phi = \pi$.

$\therefore z = re^{i(\pi-\phi)} = re^{i\pi} \cdot e^{-i\phi} = -re^{-i\phi} = -\overline{\omega}$.

$[\because e^{i\pi} = -1$ and $\overline{\omega} = re^{-i\phi}]$

41. **(c)** Let $z = x + iy$

$\because z^2 = i|z|^2$

$\therefore x^2 - y^2 + 2ixy = i(x^2 + y^2)$

$\Rightarrow x^2 - y^2 = 0$ and $2xy = x^2 + y^2$

$\Rightarrow (x-y)(x+y) = 0$ and $(x-y)^2 = 0$

$\Rightarrow x = y$

42. **(a)** Given that, $\alpha = \frac{-1+\sqrt{3}i}{2} = \omega$

$\therefore (2+\omega)^4 = a + b\omega \Rightarrow (4 + \omega^2 + 4\omega)^2 = a + b\omega$

$\Rightarrow (\omega^2 + 4(1+\omega))^2 = a + b\omega$

$\Rightarrow (\omega^2 - 4\omega^2)^2 = a + b\omega$

$[\because 1 + \omega = -\omega^2]$

$\Rightarrow (-3\omega^2)^2 = a + b\omega \Rightarrow 9\omega^4 = a + b\omega$

$\Rightarrow 9\omega = a + b\omega \quad (\because \omega^3 = 1)$

On comparing, $a = 0, b = 9$

$\Rightarrow a + b = 0 + 9 = 9$.

43. **(c)** $\left(\frac{1 + \cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}}{1 + \cos\frac{5\pi}{18} - i\sin\frac{5\pi}{18}}\right)^3$

$= \left(\frac{2\cos^2\frac{5\pi}{36} + i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36} - i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}\right)^3$

$$=\left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3=\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)^6$$

$$=\cos\left(6\times\frac{5\pi}{36}\right)+i\sin\left(6\times\frac{5\pi}{36}\right)=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$=-\frac{\sqrt{3}}{2}+i\frac{1}{2}=-\frac{1}{2}(\sqrt{3}-i)$$

44. **(b)** $3+2\sqrt{-54}=3+6\sqrt{6}i$

Let $\sqrt{3+6\sqrt{6}i}=a+ib$

$\Rightarrow a^2-b^2=3$ and $ab=3\sqrt{6}$

$\Rightarrow a^2+b^2=\sqrt{(a^2-b^2)^2+4a^2b^2}=15$

So, $a=\pm 3$ and $b=\pm\sqrt{6}$

$\sqrt{3+6\sqrt{6}\,i}=\pm(3+\sqrt{6}\,i)$

Similarly, $\sqrt{3-6\sqrt{6}\,i}=\pm(3-\sqrt{6}\,i)$

$\text{Im}\,(\sqrt{3+6\sqrt{6}i}-\sqrt{3-6\sqrt{6}i})=\pm 2\sqrt{6}$

45. **(b)** Let $\alpha=\omega$, $b=1+\omega^3+\omega^6+\ldots..=101$

$a=(1+\omega)(1+\omega^2+\omega^4+\ldots..\omega^{198}+\omega^{200})$

$$=(1+\omega)\frac{\left(1-(\omega^2)^{101}\right)}{1-\omega^2}=\frac{(\omega+1)(\omega^{202}-1)}{(\omega^2-1)}$$

$$\Rightarrow\quad a=\frac{(1+\omega)(1-\omega)}{1-\omega^2}=1$$

Required equation $=x^2-(101+1)x+(101)\times 1=0$

$\Rightarrow\quad x^2-102x+101=0$

46. **(d)** $\because\quad z=x+iy$

$$\left(\frac{z-1}{2z+i}\right)=\frac{(x-1)+iy}{2(x+iy)+i}$$

$$=\frac{(x-1)+iy}{2x+(2y+1)i}\times\frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re}\left(\frac{z+1}{2z+i}\right)=\frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2}=1$$

$$\Rightarrow\quad\left(x+\frac{1}{2}\right)^2+\left(y+\frac{3}{4}\right)^2=\left(\frac{\sqrt{5}}{4}\right)^2.$$

47. **(d)** $\frac{\sqrt{3}}{2}+\frac{i}{2}=-i\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=-i\omega$

where ω is imaginary cube root of unity.

Now, $(1+iz+z^5+iz^8)^9$

$=(1+\omega-i\omega^2+i\omega^2)^9=(1+\omega)^9$

$=(-\omega^2)^9=-\omega^{18}=-1 \qquad (\because 1+\omega+\omega^2=0)$

48. **(a)** $-(6+i)^3=x+iy$

$\Rightarrow\quad -[216+i^3+18i(6+i)]=x+iy$

$\Rightarrow\quad -[216-i+108i-18]=x+iy$

$\Rightarrow\quad -216+i-108i+18=x+iy$

$\Rightarrow\quad -198-107i=x+iy$

$\Rightarrow\quad x=-198, y=-107$

$\Rightarrow\quad y-x=-107+198=91$

49. **(a)** $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$

$$=\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^5+\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^5$$

$$=\left(e^{i\frac{\pi}{6}}\right)^5+\left(e^{-i\frac{\pi}{6}}\right)^5=2\cos\frac{\pi}{6}=\sqrt{3}$$

$\Rightarrow\quad I(z)=0$, $\text{Re}(z)=\sqrt{3}$

50. **(d)** Let $l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$.

$$\therefore\ l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1+i\sqrt{3}}\right)$$

$$=\left(\frac{-2+i2\sqrt{3}}{4}\right)=\left(\frac{1-i\sqrt{3}}{-2}\right)$$

Also, $l=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right)$

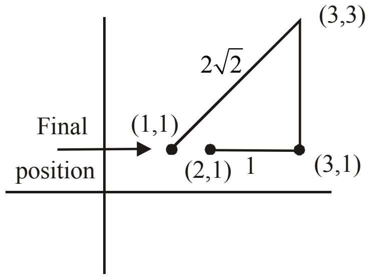
$$=\left(\frac{4}{-2-i2\sqrt{3}}\right)=\left(\frac{-2}{1+i\sqrt{3}}\right)$$

Now, $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^3=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$

$$=\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{-2}{1+i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{-2}\right)=1$$

$\therefore$ least positive integer n is 3.

51. (a)



So new position is at the point $1 + i$

52. (a) $\left|\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$

$\Rightarrow \quad |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

$\Rightarrow \quad (z_1 - 2z_2)\overline{(z_1 - 2z_2)} = (2 - z_1\bar{z}_2)\overline{(2 - z_1\bar{z}_2)}$

$\Rightarrow \quad (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$

$\Rightarrow \quad (z_1\bar{z}_1) - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4z_2\bar{z}_2$

$= 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 + z_1\bar{z}_1 z_2\bar{z}_2$

$\Rightarrow \quad |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$

$\Rightarrow \quad |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$

$$\left(|z_1|^2 - 4\right)\left(1 - |z_2|^2\right) = 0$$

$\because \quad |z_2| \neq 1$

$\therefore \quad |z_1|^2 = 4$

$\Rightarrow \quad |z_1| = 2$

$\Rightarrow$ Point z_1 lies on circle of radius 2.

53. (a) $\dfrac{z^2}{z-1} = \dfrac{\bar{z}^2}{\bar{z}-1} \quad \left[\because \overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}\right]$

$\Rightarrow z\bar{z}\bar{z} - z^2 = z.\bar{z}.\bar{z} - \bar{z}^2$

$\Rightarrow |z|^2.z - z^2 = |z|^2.\bar{z} - \bar{z}^2$

$\Rightarrow |z|^2(z - \bar{z}) - (z - \bar{z})(z + \bar{z}) = 0$

$\Rightarrow (z - \bar{z})\left(|z|^2 - (z + \bar{z})\right) = 0$

Either $z - \bar{z} = 0$ or $|z|^2 - (z + \bar{z}) = 0$

Either $z = \bar{z} \Rightarrow$ real axis

or $|z|^2 = z + \bar{z} \Rightarrow z\bar{z} - z - \bar{z} = 0$

represents a circle passing through origin.

54. (a) $(1 + \omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega \quad (\because \omega^{14} = \omega^{12}.\omega^2 = \omega^2)$

$-\omega^2 = A + B\omega$

$1 + \omega = A + B\omega$

$\Rightarrow A = 1, B = 1.$

55. (a) $|z+1| = |z+4-3| \leq |z+4| + |-3| \leq |3| + |-3|$

$\Rightarrow |z+1| \leq 6 \Rightarrow |z+1|_{max} = 6$

56. (c) Given that $w = \dfrac{z}{z - \frac{1}{3}i}$

$\Rightarrow |w| = \dfrac{|z|}{\left|z - \frac{1}{3}i\right|} = 1 \qquad \left[\because \left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}\right]$

$\Rightarrow |z| = \left|z - \dfrac{1}{3}i\right|$

$\Rightarrow$ distance of z from origin and point $\left(0, \dfrac{1}{3}\right)$ is same

hence z lies on bisector of the line joining points (0, 0) and (0, 1/3).

Hence z lies on a straight line.

57. (c) $|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

58. (c) $\because (x-1)^3 + 8 = 0 \Rightarrow (x-1) = (-2)\,(1)^{1/3}$

$\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$

or $x = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$.

59. (b) Given that $|z^2 - 1| = |z|^2 + 1 \Rightarrow |z^2 - 1|^2 = (z\bar{z} + 1)^2$

$$[\because |z|^2 = z\bar{z}]$$

$\Rightarrow (z^2 - 1)(\bar{z}^2 - 1) = (z\bar{z} + 1)^2 \left(\because \overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2\right)$

$\Rightarrow z^2\bar{z}^2 - z^2 - \bar{z}^2 + 1 = z^2\bar{z}^2 + 2z\bar{z} + 1$

$\Rightarrow z^2 + 2z\bar{z} + \bar{z}^2 = 0$

$\Rightarrow (z + \bar{z})^2 = 0 \Rightarrow z = -\bar{z}$

$\Rightarrow z$ is purely imaginary

60. (b) Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a$ ...(i)

$|z_0 - z_2| = r + b$ ...(ii)

Subtract (ii) from (i)

we get $|z_0 - z_1| - |z_0 - z_2| = a - b$.

$\therefore$ Locus of centre z_0 is $|z - z_1| - |z - z_2|$ $= a - b$, which represents a hyperbola.

61. **(a)** $\because \alpha+\beta=64, \alpha\beta=256$

$$\frac{\alpha^{3/8}}{\beta^{5/8}}+\frac{\beta^{3/8}}{\alpha^{5/8}}=\frac{\alpha+\beta}{(\alpha\beta)^{5/8}}=\frac{64}{(2^8)^{5/8}}=\frac{64}{32}=2$$

62. **(b)** Let α and β be the roots of the given quadratic equation,

$2x^2+2x-1=0$ ...(i)

Then, $\alpha+\beta=-\frac{1}{2} \Rightarrow -1=2\alpha+2\beta$

and $4\alpha^2+2\alpha-1=0$ [$\because \alpha$ is root of eq. (i)]

$\Rightarrow 4\alpha^2+2\alpha+2\alpha+2\beta=0 \Rightarrow \beta=-2\alpha(\alpha+1)$

63. **(b)** Let $|x|=y$ then

$9y^2-18y+5=0$

$\Rightarrow 9y^2-15y-3y+5=0$

$\Rightarrow (3y-1)(3y-5)=0$

$\Rightarrow y=\frac{1}{3}$ or $\frac{5}{3} \Rightarrow |x|=\frac{1}{3}$ or $\frac{5}{3}$

Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$

$\therefore$ Product $=\frac{25}{81}$

64. **(d)** Let α and β be the roots of the quadratic equation

$7x^2-3x-2=0$

$\therefore \alpha+\beta=\frac{3}{7}, \alpha\beta=\frac{-2}{7}$

Now, $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$

$$=\frac{\alpha-\alpha\beta(\alpha+\beta)+\beta}{1-(\alpha^2+\beta^2)+(\alpha\beta)^2}$$

$$=\frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{1-(\alpha+\beta)^2+2\alpha\beta+(\alpha\beta)^2}$$

$$=\frac{\frac{3}{7}+\frac{2}{7}\times\frac{3}{7}}{1-\frac{9}{49}+2\times\frac{-2}{7}+\frac{4}{49}}=\frac{27}{16}$$

65. **(d)** $u=\frac{2(x+iy)+i}{(x+iy)-ki}=\frac{2x+i(2y+1)}{x+i(y-k)}$

Real part of $u=\text{Re}(u)=\frac{2x^2+(y-K)(2y+1)}{x^2+(y-K)^2}$

Imaginary part of u

$$=\text{Im}(u)=\frac{-2x(y-K)+x(2y+1)}{x^2+(y-K)^2}$$

$\because \text{Re}(u)+\text{Im}(u)=1$

$\Rightarrow 2x^2+2y^2-2Ky+y-K-2xy+2Kx+2xy+x$

$=x^2+y^2+K^2-2Ky$

Since, the curve intersect at y-axis

$\therefore x=0$

$\Rightarrow y^2+y-K(K+1)=0$

Let y_1 and y_2 are roots of equations if $x=0$

$\because y_1+y_2=-1$

$y_1y_2=-(K^2+K)$

$\therefore (y_1-y_2)^2=(1+4K^2+4K)$

Given $PQ=5 \Rightarrow |y_1-y_2|=5$

$\Rightarrow 4K^2+4K-24=0 \Rightarrow K=2$ or -3

as $K>0, \therefore K=2$

66. **(b)** Since α is common root of $x^2-x+2\lambda=0$ and $3x^2-10x+27\lambda=0$

$\therefore 3\alpha^2-10\alpha+27\lambda=0$ ...(i)

$3\alpha^2-3\alpha+6\lambda=0$ ...(ii)

$\therefore$ On subtract, we get $\alpha=3\lambda$

Now, $\alpha\beta=2\lambda \Rightarrow 3\lambda\cdot\beta=2\lambda \Rightarrow \beta=\frac{2}{3}$

$\Rightarrow \alpha+\beta=1 \Rightarrow 3\lambda+\frac{2}{3}=1 \Rightarrow \lambda=\frac{1}{9}$ and

$\alpha\gamma=9\lambda \Rightarrow 3\lambda\cdot\gamma=9\lambda \Rightarrow \gamma=3$

$\therefore \frac{\beta\gamma}{\lambda}=18$

67. **(d)** $\alpha\cdot\beta=2$ and $\alpha+\beta=-p$ also $\frac{1}{\alpha}+\frac{1}{\beta}=-q$

$\Rightarrow p=2q$

Now $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$$=\left[\alpha\beta+\frac{1}{\alpha\beta}-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right]\left[\alpha\beta+\frac{1}{\alpha\beta}+2\right]$$

$= \frac{9}{2}\left[\frac{5}{2} - \frac{\alpha^2+\beta^2}{2}\right] = \frac{9}{4}[5-(p^2-4)]$

$= \frac{9}{4}(9-p^2)$ $\quad [\because \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta]$

68. **(c)** The given quadratic equation is

$(\lambda^2+1)x^2 - 4\lambda x + 2 = 0$

$\because$ One root is in the interval (0, 1)

$\therefore f(0)f(1) \le 0$

$\Rightarrow 2(\lambda^2+1-4\lambda+2) \le 0$

$\Rightarrow 2(\lambda^2-4\lambda+3) \le 0$

$(\lambda-1)(\lambda-3) \le 0 \Rightarrow \lambda \in [1, 3]$

But at $\lambda = 1$, both roots are 1 so $\lambda \ne 1$

$\therefore \lambda \in (1, 3]$

69. **(c)** Since, α and β are the roots of the equaton $5x^2+6x-2=0$

Then, $5\alpha^2+6\alpha-2=0$, $5\beta^2+6\beta-2=0$

$5\alpha^2+6\alpha = 2$

$5S_6+6S_5 = 5(\alpha^6+\beta^6)+6(\alpha^5+\beta^5)$

$= (5\alpha^4+6\alpha^5)+(5\beta^6+6\beta^5)$

$= \alpha^4(5\alpha^2+6\alpha)+\beta^4(5\beta^2+6\beta)$

$= 2(\alpha^4+\beta^4) = 2S_4$

70. **(a)** Let $e^x = t \in (0, \infty)$

Given equation

$t^4+t^3-4t^2+t+1=0$

$\Rightarrow \quad t^2+t-4+\frac{1}{t}+\frac{1}{t^2}=0$

$\Rightarrow \quad \left(t^2+\frac{1}{t^2}\right)+\left(t+\frac{1}{t}\right)-4=0$

Let $t+\frac{1}{t}=y$

$(y^2-2)+y-4=0 \quad \Rightarrow \quad y^2+y-6=0$

$y^2+y-6=0 \quad \Rightarrow \quad y=-3, 2$

$\Rightarrow \quad y=2 \quad \Rightarrow \quad t+\frac{1}{t}=2$

$\Rightarrow \quad e^x+e^{-x}=2$

$x = 0$, is the only solution of the equation

Hence, there only one solution of the given equation.

71. **(8)** Since, $2x^2+(a-10)x+\frac{33}{2}=2a$ has real roots,

$\therefore \quad D \ge 0$

$\Rightarrow \quad (a-10)^2-4(2)\left(\frac{33}{2}-2a\right) \ge 0$

$\Rightarrow \quad (a-10)^2-4(33-4a) \ge 0$

$\Rightarrow \quad a^2-4a-32 \ge 0$

$\Rightarrow \quad (a-8)(a+4) \ge 0$

$\Rightarrow \quad a \le -4 \cup a \ge 8$

$\Rightarrow \quad a \in (-\infty, -4] \cup [8, \infty)$

72. **(a)** Let $z = \alpha \pm i\beta$ be the complex roots of the equation

So, sum of roots $= 2\alpha = -b$ and

Product of roots $= \alpha^2+\beta^2 = 45$

$(\alpha+1)^2+\beta^2 = 40$

Given, $|z+1| = 2\sqrt{10}$

$\Rightarrow \quad (\alpha+1)^2-\alpha^2 = -5 \quad [\because \beta^2 = 45-\alpha^2]$

$\Rightarrow \quad 2\alpha+1 = -5 \quad \Rightarrow \quad 2\alpha = -6$

Hence, $b = 6$ and $b^2 - b = 30$

73. **(d)** $\alpha^5 = 5\alpha+3$

$\beta^5 = 5\beta+3$

$p_5 = 5(\alpha+\beta)+6 = 5(1)+6$

$[\because \text{ from } x^2-x-1=0, \alpha+\beta = \frac{-b}{a} = 1]$

$p_5 = 11$ and $p_5 = \alpha^2+\beta^2 = \alpha+1+\beta+1$

$p_2 = 3$ and $p_3 = \alpha^3+\beta^3 = 2\alpha+1+2\beta+1$

$= 2(1)+2 = 4$

$p_2 \times p_3 = 12$ and $p_5 = 11 \quad \Rightarrow \quad p_5 \ne p_2 \times p_3$

74. **(b)** $(k+1)\tan^2 x - \sqrt{2}\lambda \tan x + (k-1) = 0$

$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$ [Sum of roots]

$\tan\alpha \cdot \tan\beta = \frac{k-1}{k+1}$ [Product of roots]

$\therefore \quad \tan(\alpha+\beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$

$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$

$\lambda = 10.$

75. **(b)** Given equation is, $x^2+x\sin\theta-2\sin\theta=0$

$\alpha+\beta = -\sin\theta$ and $\alpha\beta = -2\sin\theta$

$$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$\therefore\ |\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\sin^2\theta+8\sin\theta}$

$\therefore\ \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$

76. **(d)** Let $2^x-1=t$

$5+|t|=(t+1)(t-1)\Rightarrow|t|=t^2-6$

When $t>0$, $t^2-t-6=0\Rightarrow t=3$ or -2

$t=-2$ (rejected)

When $t<0$, $t^2+t-6=0\Rightarrow t=-3$ or 2 (both rejected)

$\therefore\ 2^x-1=3\Rightarrow 2^x=4\Rightarrow x=2$

77. **(d)** Since $2-\sqrt{3}$ is a root of the quadratic equation $x^2+px+q=0$

$\therefore 2+\sqrt{3}$ is the other root

$\Rightarrow x^2+px+q=[x-(2-\sqrt{3})][x-(2+\sqrt{3})]$

$=x^2-(2+\sqrt{3})x-(2-\sqrt{3})x+(2^2-(\sqrt{3})^2)$

$=x^2-4x+1$

Now, by comparing $p=-4$, $q=1$

$\Rightarrow p^2-4q-12=16-4-12=0$

78. **(c)** Sum of roots $=\frac{3}{m^2+1}$

$\because$ sum of roots is greatest. $\therefore\ m=0$

Hence equation becomes $x^2-3x+1=0$

Now, $\alpha+\beta=3,\ \alpha\beta=1\Rightarrow|-\alpha-\beta|=\sqrt{5}$

$\left|\alpha^3-\beta^3\right|=\left|(\alpha-\beta)\left(\alpha^2+\beta^2+\alpha\beta\right)\right|=\sqrt{5}(9-1)=8\sqrt{5}$

79. **(d)** Let $\sqrt{x}=a$

$\therefore$ given equation will become:

$|a-2|+a(a-4)+2=0$

$\Rightarrow\ |a-2|+a^2-4a+4-2=0$

$\Rightarrow\ |a-2|+(a-2)^2-2=0$

Let $|a-2|=y$ (Clearly $y\geq 0$)

$\Rightarrow\ y+y^2-2=0$

$\Rightarrow\ y=1$ or -2 (rejected)

$\Rightarrow\ |a-2|=1\Rightarrow a=1,3$

When $\sqrt{x}=1\Rightarrow x=1$

When $\sqrt{x}=3\Rightarrow x=9$

Hence, the required sum of solutions of the equation $=10$

80. **(c)** The given quadratic equation is $x^2-2x+2=0$

Then, the roots of the this equation are $\frac{2\pm\sqrt{-4}}{2}=1\pm i$

Now, $\frac{\alpha}{\beta}=\frac{1-i}{1+i}=\frac{(1-i)^2}{1-i^2}=i$

or $\frac{\alpha}{\beta}=\frac{1-i}{1+i}=\frac{(1-i)^2}{1-i^2}=-i$ So, $\frac{\alpha}{\beta}=\pm i$

Now, $\left(\frac{\alpha}{\beta}\right)^n=1\Rightarrow(\pm i)^n=1$

$\Rightarrow$ n must be a multiple of 4.

Hence, the required least value of $n=4$.

81. **(b)** Let roots of the quadratic equation are α,β.

Given, $\lambda=\frac{\alpha}{\beta}$ and $\lambda+\frac{1}{\lambda}=1\Rightarrow\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=1$

$\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=1$...(i)

The quadratic equation is, $3m^2x^2+m(m-4)x+2=0$

$\therefore\ \alpha+\beta=\frac{m(4-m)}{3m^2}=\frac{4-m}{3m}$ and $\alpha\beta=\frac{2}{3m^2}$

Put these values in eq (1),

$$\frac{\left(\frac{4-m}{3m}\right)^2}{\frac{2}{3m^2}}=3$$

$\Rightarrow\ (m-4)^2=18\Rightarrow m=4\pm\sqrt{18}$

Therefore, least value is $4-\sqrt{18}=4-3\sqrt{2}$

82. **(d)** Let α and β be the roots of the equation, $81x^2+kx+256=0$

Given $(\alpha)^{\frac{1}{3}}=\beta\Rightarrow\alpha=\beta^3$

$\because$ Product of the roots $=\frac{256}{81}$

$\therefore\ (\alpha)(\beta)=\frac{256}{81}$

$\Rightarrow\ \beta^4=\left(\frac{4}{3}\right)^4\Rightarrow\beta=\frac{4}{3}\Rightarrow\alpha=\frac{64}{27}$

$\because$ Sum of the roots $=-\frac{k}{81}$

$\therefore \quad \alpha+\beta=-\frac{k}{81} \Rightarrow \frac{4}{3}+\frac{64}{27}=-\frac{k}{81}$

$\Rightarrow \quad k=-300$

83. **(d)** Consider the quadratic equation

$(c-5)x^2-2cx+(c-4)=0$

Now, $f(0).f(3)>0$ and $f(0).f(2)<0$

$\Rightarrow \quad (c-4)(4c-49)>0$ and $(c-4)(c-24)<0$

$\Rightarrow \quad c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty\right)$ and $c \in (4, 24)$

$\Rightarrow \quad c \in \left(\frac{49}{4}, 24\right)$

Integral values in the interval $\left(\frac{49}{4}, 24\right)$ are 13, 14, ..., 23.

$\therefore \quad S=\{13, 14, ..., 23\}$

84. **(d)** The given quadratic equation is

$x^2+(3-\lambda)x+2=\lambda$

Sum of roots $=\alpha+\beta=\lambda-3$

Product of roots $=\alpha\beta=2-\lambda$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(\lambda-3)^2-2(2-\lambda)$

$=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

For least $(\alpha^2+\beta^2)$ $\lambda=2$.

85. **(a)** Consider the equation

$x^2+2x+2=0$

$$x=\frac{-2\pm\sqrt{4-8}}{2}=-1\pm i$$

Let $\alpha=-1+i, \beta=-1-i$

$\alpha^{15}+\beta^{15}=(-1+i)^{15}+(-1-i)^{15}$

$$=\left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15}+\left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$$

$$=\left(\sqrt{2}\right)^{15}\left[e^{\frac{i45\pi}{4}}+e^{\frac{-i45\pi}{4}}\right]$$

$$=\left(\sqrt{2}\right)^{15}.2\cos\frac{45\pi}{4}=\left(\sqrt{2}\right)^{15}.2\cos\frac{3\pi}{4}$$

$$=\frac{-2}{\sqrt{2}}\left(\sqrt{2}\right)^{15}$$

$$=-2\left(\sqrt{2}\right)^{14}=-256$$

86. **(a)** The roots of $6x^2-11x+\alpha=0$ are rational numbers.

$\therefore$ Discriminant D must be perfect square number.

$D=(-11)^2-4\cdot 6\cdot\alpha$

$=121-24\alpha$ must be a perfect square

Hence, possible values for α are

$\alpha=3, 4, 5.$

$\therefore$ 3 positive integral values are possible.

87. **(b)** Given quadratic equation is: $x^2-mx+4=0$

Both the roots are real and distinct.

So, discriminant $B^2-4AC>0$.

$\therefore \quad m^2-4\cdot 1\cdot 4>0$

$\therefore \quad (m-4)(m+4)>0$

$\therefore \quad m\in(-\infty,-4)\cup(4,\infty)$ ...(i)

Since, both roots lies in [1, 5]

$\therefore \quad -\frac{-m}{2}\in(1,5)$

$\Rightarrow \quad m\in(2,10)$ ...(ii)

And $1\cdot(1-m+4)>0 \Rightarrow m<5$

$\therefore \quad m\in(-\infty,5)$ ...(iii)

And $1\cdot(25-5m+4)>0 \Rightarrow m<\frac{29}{5}$

$\therefore \quad m\in\left(-\infty\frac{29}{5}\right)$ ...(iv)

From (i), (ii), (iii) and (iv), $m\in(4,5)$

88. **(a)** $\because z_0$ is a root of quadratic equation

$x^2+x+1=0$

$\therefore \quad z_0=\omega$ or $\omega^2 \Rightarrow z_0{}^3=1$

$\therefore \quad z=3+6i\,z_0{}^{81}-3i\,z_0{}^{93}$

$=3+6i((z_0)^3)^{27}-3i((z_0)^3)^{31}$

$=3+6i-3i$

$=3+3i$

$\therefore \quad \arg(z)\tan^{-1}\left(\frac{3}{3}\right)=\frac{\pi}{4}$

89. **(b)** $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$

$$\frac{x+p+x+q}{(x+p)(x+q)}=\frac{1}{r}$$

$(2x+p+q)r=x^2+px+qx+pq$

$x^2+(p+q-2r)x+pq-pr-qr=0$

Let α and β be the roots.

$\therefore \alpha+\beta=-(p+q-2r)$ .... (i)

$\& \alpha\beta=pq-pr-qr$ .... (ii)

$\because \alpha=-\beta$ (given)

$\therefore$ in eq. (1), we get

$\Rightarrow \quad -(p+q-2r)=0$ ... (iii)

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-(p+q-2r))^2-2(pq-pr-qr)$ .... (from (i) and (ii))

$=p^2+q^2+4r^2+2pq-4pr-4qr-2pq+2pr+2qr$

$=p^2+q^2+4r^2-2pr-2qr$

$=p^2+q^2+2r(2r-p-q)$ ... (from (iii))

$=p^2+q^2+0$

$=p^2+q^2$

90. **(b)** Here, $9x^2+27x+20=0$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-27\pm\sqrt{27^2-4\times9\times20}}{2\times9}$$

$$\Rightarrow x=-\frac{4}{3},-\frac{5}{3}$$

Given, $\cos A=-\frac{3}{5}$

$$\therefore \sec A=\frac{1}{\cos A}=-\frac{5}{3}$$

Here, A is an obtuse angle.

$$\therefore \tan A=-\sqrt{\sec^2 A-1}=-\frac{4}{3}.$$

Hence, roots of the equation are $\sec A$ and $\tan A$.

91. **(b)** As $\tan A$ and $\tan B$ are the roots of $3x^2-10x-25=0$,

$$\text{So, } \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{10}{3}}{1+\frac{25}{3}}=\frac{10/3}{28/3}=\frac{5}{14}$$

Now, $\cos 2(A+B)=-1+2\cos^2(A+B)$

$$=\frac{1-\tan^2(A+B)}{1+\tan^2(A+B)}\Rightarrow \cos^2(A+B)=\frac{196}{221}$$

$\therefore 3\sin^2(A+B)-10\sin(A+B)\cos(A+B)-25\cos^2(A+B)$

$=\cos^2(A+B)[3\tan^2(A+B)-10\tan(A+B)-25]$

$$=\frac{75-700-4900}{196}\times\frac{196}{221}=-\frac{5525}{196}\times\frac{196}{221}=-25$$

92. **(d)** If a and -1 are the roots of the polynomial, then we get

$f(x)=x^2+(1-a)x-a.$

$\therefore \quad f(1)=2-2a$

and $f(2)=6-3a$

As, $f(1)+f(2)=0 \Rightarrow 2-2a+6-3a=0 \Rightarrow a=\frac{8}{5}$

Therefore, the other root is $\frac{8}{5}$

93. **(b)** α,β are roots of $x^2-x+1=0$

$\therefore \quad \alpha=-\omega$ and $\beta=-\omega^2$

where ω is cube root of unity

$\therefore \quad \alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega)^{107}$

$=-[\omega^2+\omega]=-[-1]=1$

94. **(a)** We have, $\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$$\sum_{r=1}^{n}(x^2+xr+(r-1)x+r^2-r)=10n$$

$$\Rightarrow \sum_{r=1}^{n}(x^2+(2r-1)x+r(r-1))=10n$$

$\Rightarrow \quad nx^2+\{1+3+5+....+(2n-1)\}x$
$+\{1.2+2.3+....+(n-1)n\}=10n$

$$\Rightarrow nx^2+n^2x+\frac{(n-1)n(n+1)}{3}=10n$$

$$\Rightarrow x^2+nx+\frac{n^2-31}{3}=0$$

Let α and $\alpha+1$ be its two solutions

($\because$ it has two consequtive integral solutions)

$\Rightarrow \quad \alpha+(\alpha+1)=-n$

$$\Rightarrow \alpha=\frac{-n-1}{2} \quad ...(i)$$

$$\text{Also } \alpha(\alpha+1)=\frac{n^2-31}{3} \quad ...(ii)$$

Putting value of (i) in (ii), we get

$$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right)=\frac{n^2-31}{3}$$

$\Rightarrow \quad n^2=121 \Rightarrow n=11$

95. **(c)** $(x-1)(x^2+5x-50)=0$

$\Rightarrow \quad (x-1)(x+10)(x-5)=0$

$\Rightarrow \quad x=1,5,-10$

Sum $=-4$

96. **(c)** Let $p(x)=ax^2+bx+c$

$\because \quad p(0)=1\Rightarrow c=1$

Also, $p(1)=4$ & $p(-1)=6$

$\Rightarrow \quad a+b+1=4$ & $a-b+1=6$

$\Rightarrow \quad a+b=3$ & $a-b=5$

$\Rightarrow \quad a=4$ & $b=-1$

$p(x)=4x^2-x+1$

$p(b)=16-2+1=15$

$p(-2)=16+2+1=19$

97. **(c)** $(x^2-5x+5)^{x^2+4x-60}=1$

Case I

$x^2-5x+5=1$ and $x^2+4x-60$ can be any real number

$\Rightarrow x=1,4$

Case II

$x^2-5x+5=-1$ and $x^2+4x-60$ has to be an even number

$\Rightarrow x=2,3$

where 3 is rejected because for $x=3$,

$x^2+4x-60$ is odd.

Case III

x^2-5x+5 can be any real number and

$x^2+4x-60=0$

$\Rightarrow x=-10,6$

$\Rightarrow$ Sum of all values of x

$=-10+6+2+1+4=3$

98. **(a)** $\sqrt{2x+1}-\sqrt{2x-1}=1$ .....(i)

$\Rightarrow\ 2x+1+2x-1-2\sqrt{4x^2-1}=1$

$\Rightarrow\ 4x-1=2\sqrt{4x^2-1}$

$\Rightarrow\ 16x^2-8x+1=16x^2-4$

$\Rightarrow\ 8x=5$

$\Rightarrow\ x=\dfrac{5}{8}$ which satisfies equation (i)

So, $\sqrt{4x^2-1}=\dfrac{3}{4}$

99. **(a)** $\alpha,\beta=\dfrac{6\pm\sqrt{36+8}}{2}=3\pm\sqrt{11}$

$\alpha=3+\sqrt{11}$, $\beta=3-\sqrt{11}$

$\therefore a_n=\left(3+\sqrt{11}\right)^n-\left(3-\sqrt{11}\right)^n \quad \dfrac{a_{10}-2a_8}{2a_9}$

$$=\frac{\left(3+\sqrt{11}\right)^{10}-\left(3-\sqrt{11}\right)^{10}-2\left(3+\sqrt{11}\right)^{8}+2\left(3-\sqrt{11}\right)^{8}}{2\left[\left(3+\sqrt{11}\right)^{9}-\left(3-\sqrt{11}\right)^{9}\right]}$$

$$=\frac{\left(3+\sqrt{11}\right)^{8}\left[\left(3+\sqrt{11}\right)^{2}-2\right]+\left(3-\sqrt{11}\right)^{8}\left[2-\left(3-\sqrt{11}\right)^{2}\right]}{2\left[\left(3+\sqrt{11}\right)^{9}-\left(3-\sqrt{11}\right)^{9}\right]}$$

$$=\frac{\left(3+\sqrt{11}\right)^{8}\left(9+11+6\sqrt{11}-2\right)+\left(3-\sqrt{11}\right)^{8}\left(2-9-11+6\sqrt{11}\right)}{2\left[\left(3+\sqrt{11}\right)^{9}-\left(3-\sqrt{11}\right)^{9}\right]}$$

$$=\frac{6\left(3+\sqrt{11}\right)^{9}-6\left(3-\sqrt{11}\right)^{9}}{2\left[\left(3+\sqrt{11}\right)^{9}-\left(3-\sqrt{11}\right)^{9}\right]}=\frac{6}{2}=3$$

100. **(b)** $(a-1)(x^4+x^2+1)+(a+1)(x^2+x+1)^2=0$

$\Rightarrow (a-1)(x^2+x+1)(x^2-x+1)+(a+1)(x^2+x+1)^2=0$

$\Rightarrow (x^2+x+1)[(a-1)(x^2-x+1)+(a+1)(x^2+x+1)]=0$

$\Rightarrow (x^2+x+1)(ax^2+x+a)=0$

For roots to be distinct and real, $a\neq 0$ and $1-4a^2>0$

$\Rightarrow a\neq 0$ and $a^2<\dfrac{1}{4}$

$\Rightarrow a\in\left(-\dfrac{1}{2},0\right)\cup\left(0,\dfrac{1}{2}\right)$

101. **(b)** $\alpha=2+3i;\ \beta=2-3i,\ \gamma=?$

$\alpha\beta\gamma=\dfrac{13}{2}\quad\left[\text{since product of roots}=\dfrac{d}{a}\right]$

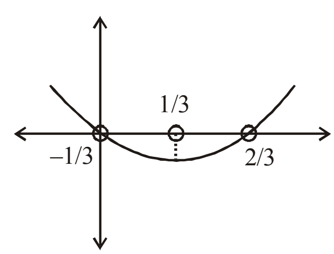
$\Rightarrow (4+9)\gamma=\dfrac{13}{2}\Rightarrow\gamma=\dfrac{1}{2}$

102. **(c)** Consider $-3(x-[x])^2+2[x-[x]]+a^2=0$

$\Rightarrow\ 3\{x\}^2-2\{x\}-a^2=0 \qquad (\because x-[x]=\{x\})$

$\Rightarrow\ 3\left(\{x\}^2-\dfrac{2}{3}\{x\}\right)=a^2, a\neq 0$

$\Rightarrow\ a^2=3\{x\}\left(\{x\}-\dfrac{2}{3}\right)$



Now, $\{x\}\in(0,1)$ and $\dfrac{-2}{3}\leq a^2<1$ (by graph)

Since , x is not an integer

$\therefore\ a\in(-1,1)-\{0\}$

$\Rightarrow a\in(-1,0)\cup(0,1)$

103. **(a)** Consider $\sqrt{3x^2+x+5}=x-3$

Squaring both the sides, we get

$3x^2+x+5=(x-3)^2$

$\Rightarrow 3x^2+x+5=x^2+9-6x$

$\Rightarrow 2x^2+7x-4=0$

$\Rightarrow 2x^2+8x-x-4=0$

$\Rightarrow 2x(x+4)-1(x+4)=0$

$\Rightarrow\ x=\dfrac{1}{2}$ or $x=-4$

For x $=\dfrac{1}{2}$ and $x=-4$

L.H.S. $\neq$ R.H.S. of equation, $\sqrt{3x^2+x+5}=x-3$

Also, for every $x\in R$, LHS $\neq$ RHS of the given equation.

$\therefore$ Given equation has no solution.

104. (c) $x^2+|2x-3|-4=0$

$$|2x-3|=\begin{cases}(2x-3) & \text{if}\quad x>\dfrac{3}{2}\\ -(2x-3) & \text{if}\quad x<\dfrac{3}{2}\end{cases}$$

for $x>\dfrac{3}{2}$, $x^2+2x-3-4=0$

$x^2+2x-7=0$

$$x=\frac{-2\pm\sqrt{4+28}}{2}=\frac{-2\pm4\sqrt{2}}{2}=-1\pm2\sqrt{2}$$

Here $x=2\sqrt{2}-1\quad\left\{2\sqrt{2}-1<\dfrac{3}{2}\right\}$

for $x<\dfrac{3}{2}$

$x^2-2x+3-4=0$

$\Rightarrow\ x^2-2x-1=0$

$$\Rightarrow\ x=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$$

Here $x=1-\sqrt{2}\quad\left\{(1-\sqrt{2})<\dfrac{3}{2}\right\}$

Sum of roots : $(2\sqrt{2}-1)+(1-\sqrt{2})=\sqrt{2}$

105. (d) $x^2-4\sqrt{2}kx+2e^{4\ln k}-1=0$

or, $x^2-4\sqrt{2}kx+2k^4-1=0$

$\alpha+\beta=4\sqrt{2}k$ and $\alpha.\beta=2k^4-1$

Squaring both sides, we get

$(\alpha+\beta)^2=(4\sqrt{2}k)^2\Rightarrow\alpha^2+\beta^2+2\alpha\beta=32k^2$

$66+2\alpha\beta=32k^2$

$66+2(2k^4-1)=32k^2$

$66+4k^4-2=32k^2\Rightarrow4k^4-32k^2+64=0$

or, $k^4-8k^2+16=0\Rightarrow(k^2)^2-8k^2+16=0$

$\Rightarrow(k^2-4)(k^2-4)=0\ \Rightarrow k^2=4,\ k^2=4$

$\Rightarrow k=\pm2$

Now, $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

$\therefore\ \alpha^3+\beta^3=(4\sqrt{2}k)\ [66-(2k^4-1)]$

Putting $k=-2$, ($k=+2$ cannot be taken because it does not satisfy the above equation)

$\therefore\ \alpha^3+\beta^3=(4\sqrt{2}(-2))[66-2(-2)^4-1]$

$\alpha^3+\beta^3=(-8\sqrt{2})\ (66-32+1)=(-8\sqrt{2})\ (35)$

$\therefore\ \alpha^3+\beta^3=-280\sqrt{2}$

106. (a) Let $\dfrac{1}{\sqrt{\alpha}}$ and $\dfrac{1}{\sqrt{\beta}}$ be the roots of $ax^2+bx+1=0$

$$\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=\left(\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}\right)=-\frac{b}{a}$$

$$\frac{1}{\sqrt{\alpha}\sqrt{\beta}}=\frac{1}{a}\Rightarrow a=\sqrt{\alpha\beta}$$

$b=-\left(\sqrt{\alpha}+\sqrt{\beta}\right)$

$x(x+b^3)+(a^3-3abx)=0$

$\Rightarrow\ x^2+(b^3-3ab)x+a^3=0$

Putting values of a and b, we get

$$x^2+\left[\left(-\sqrt{\alpha}+\sqrt{\beta}\right)^3+3\left(\sqrt{\alpha\beta}\right)\left(\sqrt{\alpha}+\sqrt{\beta}\right)\right]+(\alpha\beta)^{3/2}=0$$

$$\Rightarrow x^2-\left[\alpha^{3/2}+\beta^{3/2}+3\sqrt{\alpha\beta}\left(\sqrt{\alpha}+\sqrt{\beta}\right)-3\sqrt{\alpha\beta}(\sqrt{\alpha}+\sqrt{\beta})\right]x+(\alpha\beta)^{3/2}=0$$

$\Rightarrow\ x^2-(\alpha^{3/2}+\beta^{3/2})x+\alpha^{3/2}\beta^{3/2}=0$

Roots of this equation are $\alpha^{3/2},\beta^{3/2}$

107. (b) Given $\alpha^3+\beta^3=-p$ and $\alpha\beta=q$

Let $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$ be the root of required quadratic equation.

So, $\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}=\dfrac{\alpha^3+\beta^3}{\alpha\beta}=\dfrac{-p}{q}$

and $\dfrac{\alpha^2}{\beta}\times\dfrac{\beta^2}{\alpha}=\alpha\beta=q$

Hence, required quadratic equation is

$$x^2-\left(\frac{-p}{q}\right)x+q=0$$

$\Rightarrow x^2+\dfrac{p}{q}x+q=0\ \Rightarrow qx^2+px+q^2=0$

108. (c) Given quadratic eqn. is

$$x^2+px+\frac{3p}{4}=0$$

So, $\alpha+\beta=-p, \alpha\beta=\dfrac{3p}{4}$

Now, given $|\alpha-\beta|=\sqrt{10}$

$\Rightarrow \alpha-\beta=\pm\sqrt{10}$

$\Rightarrow (\alpha-\beta)^2=10 \Rightarrow \alpha^2+\beta^2-2\alpha\beta=10$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta=10$

$\Rightarrow p^2-4\times\dfrac{3p}{4}=10 \Rightarrow p^2-3p-10=0$

$\Rightarrow p=-2, 5 \Rightarrow p\in\{-2, 5\}$

109. **(c)** Given equation is

$z+\sqrt{2}\,|z+1|+i=0$

put $z=x+iy$ in the given equation.

$(x+iy)+\sqrt{2}\,|x+iy+1|+i=0$

$\Rightarrow x+iy+\sqrt{2}\left[\sqrt{(x+1)^2+y^2}\right]+i=0$

Now, equating real and imaginary part, we get

$x+\sqrt{2}\sqrt{(x+1)^2+y^2}=0$ and

$y+1=0 \Rightarrow y=-1$

$\Rightarrow x+\sqrt{2}\sqrt{(x+1)^2+(-1)^2}=0 \qquad (\because y=-1)$

$\Rightarrow \sqrt{2}\sqrt{(x+1)^2+1}=-x$

$\Rightarrow 2[(x+1)^2+1]=x^2$

$\Rightarrow x^2+4x+4=0$

$\Rightarrow x=-2$

Thus, $z=-2+i(-1) \Rightarrow |z|=\sqrt{5}$

110. **(c)** Given quadratic equation is

$px^2+qx+r=0 \qquad$ ...(i)

$D=q^2-4pr$

Since α and β are two complex root

$\therefore \beta=\bar{\alpha} \Rightarrow |\beta|=|\bar{\alpha}| \Rightarrow |\beta|=|\alpha| \qquad (\because|\bar{\alpha}|=|\alpha|)$

Consider

$|\alpha|+|\beta|=|\alpha|+|\alpha| \qquad (\because |\beta|=|\alpha|)$

$=2|\alpha|>2.1=2 \qquad (\because |\alpha|>1)$

Hence, $|\alpha|+|\beta|$ is greater than 2.

111. **(d)** Given equation is

$x^2-(\sin\alpha-2)x-(1+\sin\alpha)=0$

Let x_1 and x_2 be two roots of quadratic equation.

$\therefore x_1+x_2=\sin\alpha-2$ and $x_1x_2=-(1+\sin\alpha)$

$(x_1+x_2)^2=(\sin\alpha-2)^2=\sin^2\alpha+4-4\sin\alpha$

$\Rightarrow \quad x_1^2+x_2^2=\sin^2\alpha+4-4\sin\alpha-2x_1x_2$

$=\sin^2\alpha+4-4\sin\alpha+2(1+\sin\alpha)$

$=\sin^2\alpha-2\sin\alpha+6 \qquad$ ...(i)

Now, By putting

$\alpha=\dfrac{\pi}{6},\ \alpha=\dfrac{\pi}{4},\ \alpha=\dfrac{\pi}{3}$ and $\alpha=\dfrac{\pi}{2}$ in (i) one by one

We get least value of $x_1^2+x_2^2$ at $\dfrac{\pi}{2}$

Hence, $\alpha=\dfrac{\pi}{2}$

112. **(b)** $(k-2)\,x^2+8x+k+4=0$

If real roots then,

$8^2-4(k-2)(k+4)>0$

$\Rightarrow \quad k^2+2k-8<16$

$\Rightarrow k^2+6k-4k-24<0$

$\Rightarrow \quad (k+6)(k-4)<0$

$\Rightarrow \quad -6<k<4$

If both roots are negative

then $\alpha\beta$ is +ve

$\Rightarrow \quad \dfrac{k+4}{k-2}>0 \Rightarrow k>-4$

Also, $\dfrac{k-2}{k+4}>0 \Rightarrow k>2$

Roots are real so, $-6<k<4$

So, 6 and 4 are not correct.

Since, $k>2$, so 1 is also not correct value of k.

$\therefore k=3$

113. **(d)** $p(x)=0$

$\Rightarrow \quad f(x)=g(x)$

$\Rightarrow \quad ax^2+bx+c=a_1x^2+b_1x+c_1$

$\Rightarrow \quad (a-a_1)x^2+(b-b_1)x+(c-c_1)=0.$

It has only one solution, $x=-1$

$\Rightarrow \quad b-b_1=a-a_1+c-c_1 \qquad$ ...(i)

Sum of roots $\dfrac{-(b-b_1)}{(a-a_1)}=-1-1$

$\Rightarrow \quad \dfrac{b-b_1}{2(a-a_1)}=1$

$\Rightarrow \quad b-b_1=2(a-a_1) \qquad$ ...(ii)

Now $p(-2)=2$

$\Rightarrow \quad f(-2)-g(-2)=2$

$\Rightarrow \quad 4a-2b+c-4a_1+2b_1-c_1=2$

$\Rightarrow \quad 4(a-a_1)-2(b-b_1)+(c-c_1)=2$ ...(iii)

From equations, (i), (ii) and (iii)

$a-a_1=c-c_1=\dfrac{1}{2}(b-b_1)=2$

Now, $p(2) = f(2) - g(2)$

$= 4(a - a_1) + 2(b - b_1) + (c - c_1)$

$= 8 + 8 + 2 = 18$

114. **(a)** Let the correct equation be

$ax^2 + bx + c = 0$

Now, Sachin's equation

$ax^2 + bx + c' = 0$

Given that, roots found by Sachin's are 4 and 3

$\Rightarrow -\dfrac{b}{a} = 7$ ....(i)

Rahul's equation, $ax^2 + b'x + c = 0$

Given that roots found by Rahul's are 3 and 2

$\Rightarrow \dfrac{c}{a} = 6$ ...(ii)

From (i) and (ii), roots of the correct equation $x^2 - 7x + 6 = 0$ are 6 and 1.

115. **(c)** Since both the roots of given quadratic equation lie in the line $Re\, z = 1$ i.e., $x = 1$, hence real part of both the roots are 1.

Let both roots be $1 + i\alpha$ and $1 - i\alpha$

Product of the roots, $1 + \alpha^2 = \beta$

$\because\ \alpha^2 + 1 \geq 1$

$\therefore \beta \geq 1 \Rightarrow \because \beta \in (1, \infty)$

116. **(b)** $x^2 - x + 1 = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1-4}}{2}$

$x = \dfrac{1 \pm \sqrt{3}\, i}{2}$

$\alpha = \dfrac{1}{2} + i\dfrac{\sqrt{3}}{2} = -\omega^2$

$\beta = \dfrac{1}{2} - \dfrac{i\sqrt{3}}{2} = -\omega$

$\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$

$= -\omega^2 - \omega = 1$ $[\because \omega^3 = 1]$

117. **(b)** Given that roots of the equation $bx^2 + cx + a = 0$ are imaginary

$\therefore\ c^2 - 4ab < 0$ ....(i)

Let $y = 3b^2x^2 + 6\, bc\, x + 2c^2$

$\Rightarrow\ 3b^2x^2 + 6\, bc\, x + 2c^2 - y = 0$

As x is real, $D \geq 0$

$\Rightarrow\ 36\, b^2c^2 - 12\, b^2\, (2c^2 - y) \geq 0$

$\Rightarrow\ 12\, b^2\ (3\, c^2 - 2\, c^2 + y) \geq 0$ $[\because b^2 \geq 0]$

$\Rightarrow\ c^2 + y \geq 0 \quad \Rightarrow y \geq -c^2$

But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$

$\therefore$ we get $y \geq -c^2 > -4ab$

$\Rightarrow\ y > -4\, ab$

118. **(c)** Let α and β are roots of the equation $x^2 + ax + 1 = 0$

$\alpha + \beta = -a$ and $\alpha\beta = 1$

Given that $|\alpha - \beta| < \sqrt{5}$

$\Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} < \sqrt{5}$

$\left(\because\ (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta\right)$

$\Rightarrow \sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 - 4 < 5$

$\Rightarrow a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$

$\Rightarrow a \in (-3, 3)$

119. **(c)** Given equation is $x^2 - 2mx + m^2 - 1 = 0$

$\Rightarrow (x - m)^2 - 1 = 0$

$\Rightarrow (x - m + 1)(x - m - 1) = 0$

$\Rightarrow x = m - 1, m + 1$

$m - 1 > -2$ and $m + 1 < 4$

$\Rightarrow m > -1$ and $m < 3 \Rightarrow -1 < m < 3$

120. **(b)** Given that $x^2 + px + q = 0$

Sum of roots $= \tan 30° + \tan 15° = -p$

Product of roots $= \tan 30° \,.\, \tan 15° = q$

$\tan 45° = \dfrac{\tan 30° + \tan 15°}{1 - \tan 30°.\tan 15°} \Rightarrow \dfrac{-p}{1-q} = 1$

$\Rightarrow -p = 1 - q \Rightarrow q - p = 1$

$\therefore\ 2 + q - p = 3$

121. **(d)** $z^2 + z + 1 = 0 \Rightarrow z = \omega$ or ω^2

So, $z + \dfrac{1}{z} = \omega + \omega^2 = -1$

$\left[\because \dfrac{1}{z} = \omega^2 \text{ and } 1 + \omega + \omega^2 = 0\right]$

$z^2 + \dfrac{1}{z^2} = \omega^2 + \omega = -1,$ $[\because \omega^3 = 1]$

$z^3 + \dfrac{1}{z^3} = \omega^3 + \omega^3 = 2$

$z^4+\frac{1}{z^4}=-1,\ z^5+\frac{1}{z^5}=-1$

and $z^6+\frac{1}{z^6}=2$

$\therefore$ The given sum $=1+1+4+1+1+4=12$

122. **(b)** $\tan\left(\frac{P}{2}\right),\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2+bx+c=0$

$$\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)=-\frac{b}{a}$$

$$\tan\left(\frac{P}{2}\right)\cdot\tan\left(\frac{Q}{2}\right)=\frac{c}{a}$$

$$\frac{\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)}{1-\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)}=\tan\left(\frac{P}{2}+\frac{Q}{2}\right)=1$$

$$\left[\because P+Q=\frac{\pi}{2}\right]$$

$$\Rightarrow \frac{-\frac{b}{a}}{1-\frac{c}{a}}=1 \Rightarrow -\frac{b}{a}=\frac{a-c}{a}$$

$\Rightarrow -b=a-c \Rightarrow c=a+b$

123. **(d)** Let α, $\alpha+1$ be roots

Then $\alpha+\alpha+1=b=$ sum of roots

$\alpha(\alpha+1)=c=$ product of roots

$\therefore \quad b^2-4c=(2\alpha+1)^2-4\alpha(\alpha+1)=1$.

124. **(d)** Given that 4 is a root of $x^2+px+12=0$

$\Rightarrow 16+4p+12=0 \Rightarrow p=-7$

Now, the equation $x^2+px+q=0$

has equal roots.

$\therefore D=0$

$$\Rightarrow p^2-4q=0 \Rightarrow q=\frac{p^2}{4}=\frac{49}{4}$$

125. **(c)** Let the second root be α.

Then $\alpha+(1-p)=-p \Rightarrow \alpha=-1$

Also $\alpha.(1-p)=1-p$

$\Rightarrow (\alpha-1)(1-p)=0 \Rightarrow p=1\ [\because \alpha=-1]$

$\therefore$ Roots are $\alpha=-1$ and $1-p=0$

126. **(c)** Given that

$x^2-3|x|+2=0 \Rightarrow |x|^2-3|x|+2=0$

$\Rightarrow (|x|-2)(|x|-1)=0$

$\Rightarrow |x|=1,2 \Rightarrow x=\pm1,\pm2$

$\therefore$ No. of solution $=4$

127. **(b)** Let one roots of given equation be α

$\therefore$ Second roots be 2α then

$$\alpha+2\alpha=3\alpha=\frac{1-3a}{a^2-5a+3}$$

$$\Rightarrow \alpha=\frac{1-3a}{3\left(a^2-5a+3\right)} \quad \text{....(i)}$$

and $\alpha.2\alpha=2\alpha^2=\frac{2}{a^2-5a+3}$

$$\therefore\ 2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right]=\frac{2}{a^2-5a+3}$$

[from (i)]

$$\frac{(1-3a)^2}{(a^2-5a+3)}=9$$

$\Rightarrow 9a^2-6a+1=9a^2-45a+27$

$\Rightarrow 39a=26 \Rightarrow a=\frac{2}{3}$

128. **(d)** Given that $Z^2+aZ+b=0$;

$Z_1+Z_2=-a\ \&\ Z_1Z_2=b$

$0, Z_1, Z_2$ form an equilateral triangle

$\therefore\ 0^2+Z_1{}^2+Z_2{}^2=0.Z_1+Z_1.Z_2+Z_2.0$

(for an equilateral triangle,

$Z_1{}^2+Z_2{}^2+Z_3{}^2=Z_1Z_2+Z_2Z_3+Z_3Z_1$)

$\Rightarrow Z_1{}^2+Z_2{}^2=Z_1Z_2$

$\Rightarrow (Z_1+Z_2)^2=3Z_1Z_2$

$\therefore\ a^2=3b$

129. **(a)** $p+q=-p \Rightarrow q=2p$

and $pq=q \Rightarrow q(p-1)=0$

$\Rightarrow q=0$ or $p=1$.

If $q=0$, then $p=0$.

or $p=1$, then $q=-2$.

130. **(a)** Product of real roots $=\frac{c}{a}=\frac{9}{t^2}>0,\ \forall\ t\in R$

$\therefore$ Product of real roots is always positive.

131. (a) Let α and β are roots of the equation $x^2 + ax + b = 0$ and γ and δ be the roots of the equation $x^2 + bx + a = 0$ respectively.

$\therefore\ \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a$.

Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0 \qquad (\because a \neq b)$

132. (a) Given that $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;

$\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$

or $x^2 - 5x + 3 = 0$

$\therefore\ \alpha + \beta = 5$ and $\alpha\beta = 3$

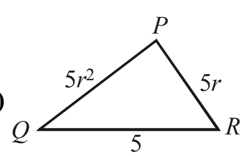
Thus, the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

or $3x^2 - 19x + 3 = 0$

133. (c)



ΔPQR is possible if

$5 + 5r > 5r^2$

$\Rightarrow\ 1 + r > r^2$

$\Rightarrow\ r^2 - r - 1 < 0$

$$\Rightarrow\ \left(r - \frac{1}{2} + \frac{\sqrt{5}}{2}\right)\left(r - \frac{1}{2} - \frac{\sqrt{5}}{2}\right) < 0$$

$$\Rightarrow\ r \in \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

$$\because\ \frac{7}{4} \notin \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2}\right) \quad \therefore\ r \neq \frac{7}{4}$$

134. (a) $ax^2 - 2bx + 5 = 0$,

If α and α are roots of equations, then sum of roots

$$2\alpha = \frac{2b}{a} \quad \Rightarrow \quad \alpha = \frac{b}{a}$$

and product of roots $= \alpha^2 = \frac{5}{a} \Rightarrow \frac{b^2}{a^2} = \frac{5}{a}$

$\Rightarrow\ b^2 = 5a \qquad (a \neq 0) \qquad$ ...(i)

For $x^2 - 2bx - 10 = 0$

$\alpha + \beta = 2b \qquad$ ...(ii)

and $\alpha\beta = -10 \qquad$ ...(iii)

$\alpha = \frac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\Rightarrow\ b^2 - 2ab^2 - 10a^2 = 0$

By eqn. (i) $\Rightarrow\ 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow\ 20a^2 = 5a$

$\Rightarrow\ a = \frac{1}{4}$ and $b^2 = \frac{5}{4}$

$\alpha^2 = 20$ and $\beta^2 = 5$

Now, $\alpha^2 + \beta^2 = 5 + 20 = 25$

135. (b) Let, the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ are α and β.

Also roots of the given equation are

$$\frac{\lambda - 2 \pm \sqrt{4 - 4\lambda + \lambda^2 - 40 + 4\lambda}}{2} = \frac{\lambda - 2 \pm \sqrt{\lambda^2 - 36}}{2}$$

The magnitude of the difference of the roots is $\left|\sqrt{\lambda^2 - 36}\right|$

$$\text{So, } \alpha^3 + \beta^3 = \frac{(\lambda - 2)^3}{4} + \frac{3(\lambda - 2)(\lambda^2 - 36)}{4}$$

$$= \frac{(\lambda - 2)(4\lambda^2 - 4\lambda - 104)}{4} = (\lambda - 2)(\lambda^2 - \lambda - 26) = f(\lambda)$$

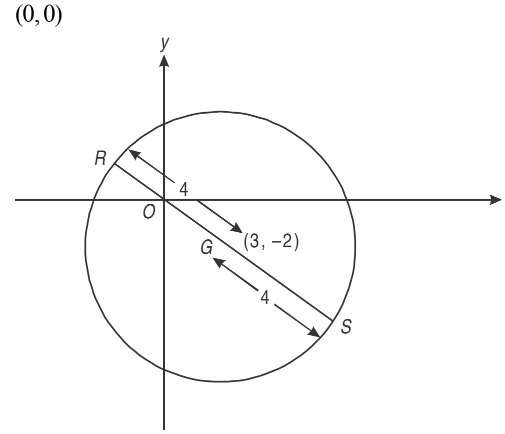
As $f(\lambda)$ attains its minimum value at $\lambda = 4$.

Therefore, the magintude of the difference of the roots is

$|i\ \sqrt{20}| = 2\sqrt{5}$

136. (b) $|z - (3 - 2i)| \leq 4$ represents a circle whose centre is $(3, -2)$ and radius $= 4$.

$|z| = |z - 0|$ represents the distance of point 'z' from origin $(0, 0)$



Suppose RS is the normal of the circle passing through origin 'O' and G is its center $(3, -2)$.

Here, OR is the least distance

and OS is the greatest distance

$OR = RG - OG$ and $OS = OG + GS \qquad$ ...(i)

As, $RG = GS = 4$

$OG = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

From (i), $OR = 4 - \sqrt{13}$ and $OS = 4 + \sqrt{13}$

So, required difference $= \left(4+\sqrt{13}\right) - \left(4-\sqrt{13}\right)$

$= \sqrt{13} + \sqrt{13} = 2\sqrt{13}$

137. (c) $x^2 + bx - 1 = 0$ common root

$$\begin{array}{r} x^2 + x + b = 0 \\ - \quad - \quad - \\ \hline x = \dfrac{b+1}{b-1} \end{array}$$

Put $x = \dfrac{b+1}{b-1}$ in equation

$\left(\dfrac{b+1}{b-1}\right)^2 + \left(\dfrac{b+1}{b-1}\right) + b = 0$

$(b+1)^2 + (b+1)(b-1) + b(b-1)^2 = 0$

$b^2 + 1 + 2b + b^2 - 1 + b(b^2 - 2b + 1) = 0$

$2b^2 + 2b + b^3 - 2b^2 + b = 0$

$b^3 + 3b = 0$

$b(b^2 + 3) = 0$

$b^2 = -3$

$b = \pm\sqrt{3}i$

$|b| = \sqrt{3}$

138. (d) We have

$f(x) = x^2 + 2bx + 2c^2$

and $g(x) = -x^2 - 2cx + b^2, (x \in R)$

$\Rightarrow f(x) = (x+b)^2 + 2c^2 - b^2$

and $g(x) = -(x+c)^2 + b^2 + c^2$

Now, $f_{min} = 2c^2 - b^2$ and $g_{max} = b^2 + c^2$

Given : $\min f(x) > \max g(x)$

$\Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow c^2 > 2b^2$

$\Rightarrow |c| > |b|\sqrt{2}$

$\Rightarrow \dfrac{|c|}{|b|} > \sqrt{2} \Rightarrow \left|\dfrac{c}{b}\right| > \sqrt{2}$

$\Rightarrow \left|\dfrac{c}{b}\right| \in (\sqrt{2}, \infty)$.

139. (b) Let α, β be the common roots of both the equations.

For first equation $ax^2 + bx + c = 0$,
we have

$\alpha + \beta = \dfrac{-b}{a}$ ...(i)

$\alpha.\beta = \dfrac{c}{a}$ ...(ii)

For second equation $2x^2 + 3x + 4 = 0$,
we have

$\alpha + \beta = \dfrac{-3}{2}$ ...(iii)

$\alpha\,.\,\beta = \dfrac{2}{1}$ ...(iv)

Now, from (i) & (iii) & from (ii) & (iv)

$\dfrac{-b}{a} = \dfrac{-3}{2}$ $\qquad \dfrac{c}{a} = \dfrac{2}{1}$

$\dfrac{b}{a} = \dfrac{3/2}{1}$

Therefore on comparing we get $a = 1$, $b = \dfrac{3}{2}$ & $c = 2$

putting these values in first equation, we get

$x^2 + \dfrac{3}{2}x + 2 = 0$ or $2x^2 + 3x + 4 = 0$

from this, we get $a = 2$, $b = 3$; $c = 4$

or $a : b : c = 2 : 3 : 4$

140. (a) Given equations are

$x^2 + 2x + 3 = 0$ …(i)

$ax^2 + bx + c = 0$ …(ii)

Roots of equation (i) are imaginary roots in order pair.
According to the question (ii) will also have both roots same as (i). Thus

$\dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3} = \lambda$ (say)

$\Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$

Hence, required ratio is 1 : 2 : 3

141. (a) $\dfrac{x-5}{x^2+5x-14} > 0 \Rightarrow x^2 + 5x - 14 < x - 5$

$\Rightarrow x^2 + 4x - 9 < 0$

$\Rightarrow \alpha = -5, -4, -3, -2, -1, 0, 1$

$\alpha = -5$ does not satisfy any of the options

$\alpha = -4$ satisfy the option (a) $\alpha^2 + 3\alpha - 4 = 0$

142. (c) $x^2-(a+1)x+a^2+a-8=0$

Since roots are different, therefore $D>0$

$\Rightarrow (a+1)^2-4(a^2+a-8)>0$

$\Rightarrow (a-3)(3a+1)<0$

There are two cases arises.

Case I. $a-3>0$ and $3a+1<0$

$\Rightarrow\ a>3$ and $a<-\dfrac{11}{3}$

Hence, no solution in this case

Case II : $a-3<0$ and $3a+11>0$

$\Rightarrow a<3$ and $a>-\dfrac{11}{3}$

$\therefore\ -\dfrac{11}{3}<a<3 \Rightarrow -2<a<3$

143. (a) Given that $\left|z-\dfrac{4}{z}\right|=2$

$$|z| = \left|z-\frac{4}{z}+\frac{4}{z}\right| \le \left|z-\frac{4}{z}\right|+\frac{4}{|z|}$$

$$\Rightarrow |z| \le 2+\frac{4}{|z|}$$

$$\Rightarrow |z|^2-2|z|-4\le 0$$

$$\Rightarrow \left(|z|-\frac{2+\sqrt{20}}{2}\right)\left(|z|-\frac{2-\sqrt{20}}{2}\right)\le 0$$

$$\Rightarrow \left(|z|-(1+\sqrt{5})\right)\left(|z|-(1-\sqrt{5})\right)\le 0$$

$+\quad -\quad +$

$-\infty \quad (1-\sqrt{5}) \quad (1+\sqrt{5}) \quad \infty$

$$\Rightarrow \left(-\sqrt{5}+1\right)\le |z| \le \left(\sqrt{5}+1\right)$$

$$\Rightarrow |z|_{\max}=\sqrt{5}+1$$

144. (d) Let the roots of equation $x^2-6x+a=0$ be α and 4 β and that of the equation $x^2-cx+6=0$ be α and 3 β. Then

$\alpha+4\beta=6$ ...(i) $4\alpha\beta=a$ ...(ii)

and $\alpha+3\beta=c$ ...(iii) $3\alpha\beta=6$ ...(iv)

$\Rightarrow\ a=8$ (from (ii) and (iv))

$\therefore$ The equation becomes $x^2-6x+8=0$

$\Rightarrow\ (x-2)(x-4)=0$

$\Rightarrow$ roots are 2 and 4

$\Rightarrow\ \alpha=2, \beta=1\ \therefore$ Common root is 2.

145. (b) $y=\dfrac{3x^2+9x+17}{3x^2+9x+7}$

$3x^2(y-1)+9x(y-1)+7y-17=0$

$D\ge 0 \quad \because$ x is real

$81(y-1)^2-4\times 3(y-1)(7y-17)\ge 0$

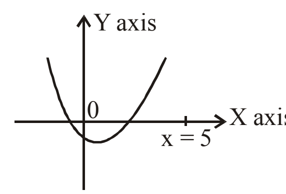
$\Rightarrow\ (y-1)(y-41)\le 0 \Rightarrow\ 1\le y\le 41$

$\therefore$ Max value of y is 41

$+\quad -\quad +$

$-8 \quad 1 \quad 41 \quad 8$

146. (c) Given that both roots of quadratic equation are less than 5 then (i)



Discriminant ≥ 0

$4k^2-4(k^2+k-5)\ge 0$

$4k^2-4k^2-4k+20\ge 0$

$4k\le 20 \Rightarrow k\le 5$

(ii) $p(5)>0$

$\Rightarrow f(5)>0\,;\ 25-10k+k^2+k-5>0$

$\Rightarrow k^2-9k+20>0$

$\Rightarrow k(k-4)-5(k-4)>0$

$\Rightarrow (k-5)(k-4)>0$

$+\quad -\quad +$

$-\infty \quad 4 \quad 5 \quad \infty$

$\Rightarrow k\in(-\infty, 4)\cup(5,\infty)$

(iii) $\dfrac{\text{Sum of roots}}{2}<5$

$\Rightarrow -\dfrac{b}{2a}=\dfrac{2k}{2}<5$

$\Rightarrow k<5$

The intersection of (i), (ii) & (iii) gives $k\in(-\infty, 4)$.

147. (a) Given equation is $x^2-(a-2)x-a-1=0$

$\Rightarrow\ \alpha+\beta=a-2\,;\ \alpha\beta=-(a+1)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

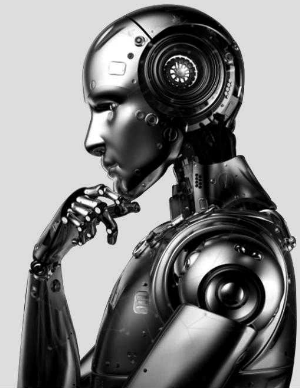
$=a^2-2a+6=(a-1)^2+5$

For min. value of $\alpha^2+\beta^2$, $a-1=0$

$\Rightarrow\ a=1.$

6

Linear Inequalities



TOPIC 1 **Solution of Linear Inequality and System of Linear Inequalities, Representation of Solution of Linear Inequality in One Variable on a Number Line, Representation of Solution of a Linear Inequality and System of Linear Inequalities in a Cartesian Plane, Equations and Inequations Involving Absolute Value Functions, Greatest Integer Functions, Logarithmic Functions**

1. The region represented by $\{z = x + iy \in C : |z| - \text{Re}(z) \le 1\}$ is also given by the inequality: **[Sep. 06, 2020 (I)]**

   (a) $y^2 \ge 2(x+1)$ (b) $y^2 \le 2\left(x+\frac{1}{2}\right)$

   (c) $y^2 \le x+\frac{1}{2}$ (d) $y^2 \ge x+1$

2. Consider the two sets :

   $A = \{m \in \mathbf{R}$: both the roots of $x^2 - (m+1)x + m + 4 = 0$ are real$\}$ and $B = [-3, 5)$.

   Which of the following is **not** true? **[Sep. 03, 2020 (I)]**

   (a) $A - B = (-\infty, -3) \cup (5, \infty)$

   (b) $A \cap B = \{-3\}$

   (c) $B - A = (-3, 5)$

   (d) $A \cup B = \mathbf{R}$

3. If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x-2| \ge 3\}$; then : **[Jan. 9, 2020 (II)]**

   (a) $A \cap B = (-2, -1)$ (b) $B - A = R - (-2, 5)$

   (c) $A \cup B = R - (2, 5)$ (d) $A - B = [-1, 2)$

4. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S: **[Jan. 8, 2020 (II)]**

   (a) contains exactly two elements.
   (b) is a singleton.
   (c) is an empty set.
   (d) contains at least four elements.

5. All the pairs (x,y) that satisfy the inequality $2\sqrt{\sin^2 x - 2\sin x + 5}.\frac{1}{4\sin^2 y} \le 1$ also satisfy the equation: **[April 10, 2019 (I)]**

   (a) $2|\sin x| = 3\sin y$ (b) $2\sin x = \sin y$

   (c) $\sin x = 2\sin y$ (d) $\sin x = |\sin y|$

6. The number of integral values of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is : **[April 08, 2019 (II)]**

   (a) 1 (b) 2

   (c) infinitely many (d) 3

7. The number of integral values of m for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is : **[Jan. 12, 2019 (II)]**

   (a) 3 (b) 8

   (c) 7 (d) 6

8. If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$, $x \in R$, then the equation $f(x) = 0$ has : **[Online April 9, 2014]**

   (a) no solution
   (b) one solution
   (c) two solutions
   (d) more than two solutions

9. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is **[2002]**

   (a) less than 1 (b) equal to 1

   (c) greater than 1 (d) any real no.

Hints & Solutions

1. **(b)** $\because |z| - \text{Re}(z) \leq 1 \qquad (\because z = x + iy)$

$\Rightarrow \sqrt{x^2+y^2} - x \leq 1 \Rightarrow \sqrt{x^2+y^2} \leq 1 + x$

$\Rightarrow x^2 + y^2 \leq 1 + x^2 + 2x$

$\Rightarrow y^2 \leq 1 + 2x \Rightarrow y^2 \leq 2\left(x + \frac{1}{2}\right)$

2. **(a)** $A = \{m \in \mathbf{R} : x^2 - (m+1)x + m + 4 = 0$ has real roots$\}$

$D \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$

$$\begin{array}{ccccccc} & + & & - & & + & \\ \infty & & -3 & & 5 & & -\infty \end{array}$$

$A = \{(-\infty, -3] \cup [5, \infty)\}$

$B = [-3, 5) \Rightarrow A - B = (-\infty, -3) \cup [5, \infty)$

3. **(b)** $A = \{x : x \in (-2, 2)\}$

$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$

$A \cap B = \{x : x \in (-2, -1]\}$

$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$

$A - B = \{x : x \in (-1, 2)\}$

$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

4. **(b)** Let $3^x = y$

$\therefore \quad y(y-1) + 2 = |y-1| + |y-2|$

Case 1: when $y > 2$

$y^2 - y + 2 = y - 1 + y - 2$

$y^2 - 3y + 5 = 0$

$\because \quad D < 0$ [$\therefore$ Equation not satisfy.]

Case 2: when $1 \leq y \leq 2$

$y^2 - y^2 + 2 = y - 1 - y + 2$

$y^2 - y + 1 = 0$

$\because \quad D < 0$ [$\therefore$ Equation not satisfy.]

Case 3: when $y \leq 1$

$y^2 - y + 2 = -y + 1 - y + 2$

$y^2 + y - 1 = 0$

$\therefore y = \frac{-1+\sqrt{5}}{2}$

$= \frac{-1-\sqrt{5}}{2} \qquad [\therefore$ Equation not Satisfy]

$\therefore$ Only one $-1 + \frac{\sqrt{5}}{2}$ satisfy equation

5. **(d)** Given inequality is,

$2\sqrt{\sin^2 x - 2\sin x + 5} \leq 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x - 2\sin x + 5} \leq 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \leq 2\sin^2 y$

It is true if $\sin x = 1$ and $|\sin y| = 1$

Therefore, $\sin x = |\sin y|$

6. **(c)** Given equation is

$(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$

$\because$ equation has no real solution

$\therefore \; D < 0$

$\Rightarrow 4(1+3m)^2 < 4(1+m^2)(1+8m)$

$\Rightarrow 1 + 9m^2 + 6m < 1 + 8m + m^2 + 8m^3$

$\Rightarrow 8m^3 - 8m^2 + 2m > 0$

$\Rightarrow 2m(4m^2 - 4m + 1) > 0 \Rightarrow 2m(2m-1)^2 > 0$

$\Rightarrow m > 0$ and $m \neq \frac{1}{2} \qquad \left[\because \frac{1}{2} \text{ is not an integer}\right]$

$\Rightarrow$ number of integral values of m are infinitely many.

7. **(3)** Let the given quadratic expression $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, is positive for all $x \in R$, then

$1 + 2m > 0 \qquad$ ...(i)

$D < 0$

$\Rightarrow \quad 4(1+3m)^2 - 4(1+2m)4(1+m) < 0$

$\Rightarrow \quad 1 + 9m^2 + 6m - 4[1 + 2m^2 + 3m] < 0$

$\Rightarrow \quad m^2 - 6m - 3 < 0$

$\Rightarrow \quad m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$

From (i)

$\therefore \quad m > -\frac{1}{2}$

$m \in \left(3 - 2\sqrt{3}, 3 + 2\sqrt{3}\right)$

Then, integral values of $m = \{0, 1, 2, 3, 4, 5, 6\}$

Hence, number of integral values of $m = 7$

8. **(b)** $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

Put $f(x) = 0$

$\Rightarrow \quad 0 = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

$\Rightarrow \quad \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x = 1$

$\Rightarrow \quad 3^x + 4^x = 5^x$ ...(i)

For $x = 1$

$3^1 + 4^1 > 5^1$

For $x = 3$

$3^3 + 4^3 = 91 < 5^3$

Only for $x = 2$, equation (i) Satisfy

So, only one solution $(x = 2)$

9. **(a)** $\because (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$

$\Rightarrow 2(a^2 + b^2 + c^2 - ab - bc - ca) > 0$

$[\because a^2 + b^2 + c^2 = 1]$

$\Rightarrow 2 > 2(ab + bc + ca) \Rightarrow ab + bc + ca < 1$

7

Permutations and Combinations

TOPIC 1 Fundamental Principle of Counting, Factorials, Permutations, Counting Formula for Permutations, Permutations in Which Things may be Repeated, Permutations in Which all Things are Different, Number of Permutations Under Certain Restricted Conditions, Circular Permutations

1. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated? **[Sep. 06, 2020 (I)]**

(a) $2!\,3!\,4!$ (b) $(3!)^3 \cdot (4!)$
(c) $(3!)^2 \cdot (4!)$ (d) $3!(4!)^3$

2. The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \ldots$ up to 51^{th} term$)$ $+ (1! - 2! + 3! - \ldots$ up to 51^{th} term$)$ is equal to : **[Sep. 03, 2020 (I)]**

(a) $1 - 51(51)!$ (b) $1 + (51)!$
(c) $1 + (52)!$ (d) 1

3. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ________. **[NA Sep. 02, 2020 (I)]**

4. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to: **[Jan. 9, 2020 (I)]**

(a) 4 (b) 6
(c) 7 (d) 8

5. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is: **[Jan. 7, 2020 (I)]**

(a) $\frac{1}{2}(6!)$ (b) $6!$
(c) 5^6 (d) $\frac{5}{2}(6!)$

6. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is: **[April 10, 2019 (I)]**

(a) 72 (b) 60
(c) 48 (d) 36

7. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is: **[April 08, 2019 (II)]**

(a) 288 (b) 360
(c) 306 (d) 310

8. Consider three boxes, each containing 10 balls labelled 1, 2,..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is : **[Jan. 12, 2019 (I)]**

(a) 120 (b) 82
(c) 240 (d) 164

9. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to: **[Jan. 09, 2019 (II)]**

(a) 374 (b) 372
(c) 375 (d) 250

10. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is: **[Jan. 09, 2019 (II)]**

(a) 9 (b) 18
(c) 36 (d) 32

11. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4, (repetition of digits is not allowed) and are multiple of 3 is? **[Online April 16, 2018]**

(a) 30 (b) 48
(c) 24 (d) 36

12. n – digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is **[Online April 15, 2018]**

(a) 6 (b) 8

(c) 9 (d) 7

13. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is : **[Online April 9, 2017]**

(a) $5 \times 6!$ (b) $6 \times 6!$

(c) $7!$ (d) $5 \times 7!$

14. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is : **[Online April 8, 2017]**

(a) 44^{th} (b) 45^{th}

(c) 46^{th} (d) 47^{th}

15. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[2016]**

(a) 52^{nd} (b) 58^{th}

(c) 46^{th} (d) 59^{th}

16. The sum $\sum_{r=1}^{10}(r^2+1)\times(r!)$ is equal to : **[Online April 10, 2016]**

(a) $11 \times (11!)$ (b) $10 \times (11!)$

(c) $(11!)$ (d) $101 \times (10!)$

17. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is : **[Online April 9, 2016]**

(a) 110 (b) 59

(c) $\dfrac{11!}{(2!)^3}$ (d) 56

18. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is : **[2015]**

(a) 820 (b) 780

(c) 901 (d) 861

19. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[2015]**

(a) 120 (b) 72

(c) 216 (d) 192

20. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is: **[Online April 10, 2015]**

(a) 1120 (b) 1880

(c) 1960 (d) 1240

21. Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval: **[Online April 19, 2014]**

(a) [8, 9] (b) [10, 12)

(c) (11, 13] (d) (14, 17)

22. 8-digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such numbers in which the odd digits do no occupy odd places, is: **[Online April 12, 2014]**

(a) 160 (b) 120

(c) 60 (d) 48

23. An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is: **[Online April 11, 2014]**

(a) 72 (7!) (b) 18 (7!)

(c) 40 (7!) (d) 36 (7!)

24. The sum of the digits in the unit's place of all the 4-digit numbers formed by using the numbers 3, 4, 5 and 6, without repetition, is: **[Online April 9, 2014]**

(a) 432 (b) 108

(c) 36 (d) 18

25. 5 - digit numbers are to be formed using 2, 3, 5, 7, 9 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, then $p : q$ is : **[Online April 25, 2013]**

(a) 6 : 5 (b) 3 : 2

(c) 4 : 3 (d) 5 : 3

26. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is: **[2012]**

(a) 880 (b) 629

(c) 630 (d) 879

27. If seven women and seven men are to be seated around a circular table such that there is a man on either side of every woman, then the number of seating arrangements is **[Online May 26, 2012]**

(a) 6! 7! (b) $(6!)^2$

(c) $(7!)^2$ (d) 7!

28. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number **[2005]**

(a) 601 (b) 600

(c) 603 (d) 602

29. How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order **[2004]**

(a) 480 (b) 240
(c) 360 (d) 120

30. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is **[2004]**

(a) $\{1, 2, 3, 4, 5\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3, 4,\}$ (d) $\{1, 2, 3,\}$

31. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[2003]**

(a) $6! \times 5!$ (b) 6×5
(c) 30 (d) 5×4

32. The sum of integers from 1 to 100 that are divisible by 2 or 5 is **[2002]**

(a) 3000 (b) 3050
(c) 3600 (d) 3250

33. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4 (repetition allowed). Their number is **[2002]**

(a) 125 (b) 105
(c) 374 (d) 625

34. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are **[2002]**

(a) 216 (b) 375
(c) 400 (d) 720

TOPIC 2 Combinations, Counting Formula for Combinations, Division and Distribution of Objects, Dearrangement Theorem, Sum of Numbers, Important Result About Point

35. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ______. **[NA Sep. 06, 2020 (II)]**

36. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ______. **[NA Sep. 05, 2020 (I)]**

37. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is : **[Sep. 05, 2020 (II)]**

(a) 3000 (b) 1500
(c) 2255 (d) 2250

38. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is __________. **[NA Sep. 04, 2020 (II)]**

39. The total number of 3-digit numbers, whose sum of digits is 10, is _________. **[NA Sep. 03, 2020 (II)]**

40. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is : **[Sep. 02, 2020 (II)]**

(a) 201 (b) 200
(c) 101 (d) 199

41. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5\cdot C_1 + 9\cdot C_2 + ... + (101)\cdot C_{25} = 2^{25}\cdot k$, then k is equal to ________. **[NA Jan. 9, 2020 (II)]**

42. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is _________. **[NA Jan. 8, 2020 (I)]**

43. If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then: **[Jan. 8, 2020 (I)]**

(a) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$ (b) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

(c) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$ (d) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$

44. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _________. **[NA Jan. 8, 2020 (II)]**

45. The number of ordered pairs (r, k) for which $6.{}^{35}C_r = (k^2 - 3).{}^{36}C_{r+1}$, where k is an integer, is: **[Jan. 7, 2020 (II)]**

(a) 3 (b) 2
(c) 6 (d) 4

46. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct is: **[April 12, 2019 (I)]**

(a) $2^{20} - 1$ (b) 2^{21}
(c) 2^{20} (d) $2^{20}+1$

47. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to : **[April 12, 2019 (II)]**

(a) 28 (b) 27
(c) 25 (d) 24

48. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is : **[April 10, 2019 (II)]**

(a) 170 (b) 180
(c) 210 (d) 190

49. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then: **[April 9, 2019 (I)]**
(a) $m+n=68$ (b) $m=n=78$
(c) $n=m-8$ (d) $m=n=68$

50. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is : **[April 8, 2019 (I)]**
(a) 180 (b) 175
(c) 160 (d) 162

51. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is **[Jan. 12, 2019 (II)]**
(a) 12 (b) 11
(c) 9 (d) 7

52. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$, then k equals: **[Jan. 10, 2019 (I)]**
(a) 400 (b) 50
(c) 200 (d) 100

53. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is: **[Jan. 9, 2019 (I)]**
(a) 500 (b) 200
(c) 300 (d) 350

54. The number of four letter words that can be formed using the letters of the word BARRACK is **[Online April 15, 2018]**
(a) 144 (b) 120
(c) 264 (d) 270

55. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is : **[2018]**
(a) less than 500
(b) at least 500 but less than 750
(c) at least 750 but less than 1000
(d) at least 1000

56. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[2017]**
(a) 484 (b) 485
(c) 468 (d) 469

57. If $\frac{{}^{n+2}C_6}{{}^{n-2}P_2}=11$, then n satisfies the equation : **[Online April 10, 2016]**
(a) $n^2+n-110=0$ (b) $n^2+2n-80=0$
(c) $n^2+3n-108=0$ (d) $n^2+5n-84=0$

58. The value of $\sum_{r=1}^{15} r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$ is equal to : **[Online April 9, 2016]**
(a) 1240 (b) 560
(c) 1085 (d) 680

59. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is : **[2015]**
(a) 275 (b) 510
(c) 219 (d) 256

60. If in a regular polygon the number of diagonals is 54, then the number of sides of this polygon is **[Online April 11, 2015]**
(a) 12 (b) 6
(c) 10 (d) 9

61. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is **[2013]**
(a) 256 (b) 220
(c) 219 (d) 211

62. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1}-T_n=10$, then the value of n is : **[2013]**
(a) 7 (b) 5
(c) 10 (d) 8

63. On the sides AB, BC, CA of a ΔABC, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are : **[Online April 23, 2013]**
(a) 210 (b) 205
(c) 215 (d) 220

64. The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is : **[Online April 22, 2013]**
(a) ${}^{30}C_7$ (b) ${}^{21}C_8$
(c) ${}^{21}C_7$ (d) ${}^{30}C_8$

65. A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is : **[Online April 9, 2013]**
(a) 40 (b) 41
(c) 16 (d) 32

66. The number of arrangements that can be formed from the letters a, b, c, d, e, f taken 3 at a time without repetition and each arrangement containing at least one vowel, is **[Online May 19, 2012]**

(a) 96 (b) 128
(c) 24 (d) 72

67. If $n = {}^mC_2$, then the value of ${}^nC_2$ is given by **[Online May 19, 2012]**

(a) $3({}^{m+1}C_4)$ (b) ${}^{m-1}C_4$
(c) ${}^{m+1}C_4$ (d) $2({}^{m+2}C_4)$

68. **Statement 1:** If A and B be two sets having p and q elements respectively, where $q > p$. Then the total number of functions from set A to set B is q^p. **[Online May 12, 2012]**
Statement 2: The total number of selections of p different objects out of q objects is ${}^qC_p$.
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
(c) Statement 1 is false, Statement 2 is true
(d) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.

69. If the number of 5-element subsets of the set $A = \{a_1, a_2, ..., a_{20}\}$ of 20 distinct elements is k times the number of 5-element subsets containing a_4, then k is **[Online May 7, 2012]**

(a) 5 (b) $\frac{20}{7}$
(c) 4 (d) $\frac{10}{3}$

70. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. Then : **[2011RS]**

(a) $N \le 100$ (b) $100 < N \le 140$
(c) $140 < N \le 190$ (d) $N > 190$

71. **Statement-1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.
Statement-2: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$. **[2011]**
(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

72. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[2010]**

(a) 36 (b) 66
(c) 108 (d) 3

73. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is: **[2009]**

(a) at least 500 but less than 750
(b) at least 750 but less than 1000
(c) at least 1000
(d) less than 500

74. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent? **[2008]**

(a) $8.{}^6C_4.{}^7C_4$ (b) $6.7.{}^8C_4$
(c) $6.8.{}^7C_4$. (d) $7.{}^6C_4.{}^8C_4$

75. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size.
Thus $A \cup B \cup C = S,\ A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is **[2007]**

(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$
(c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$

76. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is **[2006]**

(a) 5040 (b) 6210
(c) 385 (d) 1110

77. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is **[2005]**

(a) ${}^{55}C_4$ (b) ${}^{55}C_3$
(c) ${}^{56}C_3$ (d) ${}^{56}C_4$

78. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[2004]**

(a) ${}^8C_3$ (b) 21
(c) 3^8 (d) 5

79. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is **[2003]**

(a) 346 (b) 140
(c) 196 (d) 280

80. If ${}^nC_r$ denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals **[2003]**

(a) ${}^{n+1}C_{r+1}$ (b) ${}^{n+2}C_r$
(c) ${}^{n+2}C_{r+1}$ (d) ${}^{n+1}C_r$

81. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are **[2002]**

(a) 312 (b) 3125
(c) 120 (d) 216

Hints & Solutions

1. **(b)** Number of arrangement

$= (3! \times 3! \times 4!) \times 3! = (3!)^3 4!$

2. **(c)** We know, $(r+1)\cdot {}^{r}P_{r-1} = (r+1)\cdot \dfrac{r!}{1!} = (r+1)!$

So, $(2\cdot {}^{1}P_0 - 3\cdot {}^{2}P_1 + ...51 \text{ terms}) +$
$(1! - 2! + 3! - ... \text{ upto 51 terms})$
$= [2! - 3! + 4! - ... + 52!] + [1! - 2! + 3! - ... + 51!]$
$= 52! + 1! = 52! + 1$

3. **(309)**

M	O	T	H	E	R
3	4	6	2	1	5

$\Rightarrow 2\times 5! + 2\times 4! + 3\times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1 = 309$

4. **(d)** Number of five digit numbers with 2 at 10[th] place
$= 8\times 8\times 7\times 6 = 2688$
$\because$ It is given that, number of five digit number with 2 at 10[th] place $= 336k$
$\therefore$ $336k = 2688 \Rightarrow k = 8$

5. **(d)** Five digits numbers be 1, 3, 5, 7, 9
For selection of one digit, we have ${}^5C_1$ choice.

And six digits can be arrange in $\dfrac{6!}{2!}$ ways.

Hence, total such numbers $= \dfrac{5.6!}{2!} = \dfrac{5}{2}.6!$

6. **(b)** Given digit 0, 1, 2, 5, 7, 9

a_1	a_2	a_3	a_4	a_5	a_6

$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11$ K
Therefore, (1, 2, 9) (0, 5, 7)
Number of ways to arranging them
$= 3! \times 3! + 3! \times 2 \times 2 = 6\times 6 + 6\times 4 = 6\times 10 = 60$

7. **(d)** 0, 1, 2, 3, 4, 5
Number of four-digit number starting with 5 is,

5			
	6	6	6

$= 6\times 6\times 6 = 216$

Number of four-digit numbers starting with 45 is,

4	5		
		6	6

$= 6\times 6 = 36$

Number of four-digit numbers starting with 44 is,

4	4		
		6	6

$= 6\times 6 = 36$

Number of four-digit numbers starting with 43 and greater than 4321 is,

4	3		
		3 (5/4/3)	6

$= 3\times 6 = 18$

Number of four-digit numbers starting with 432 and greater than 4321 is,

4	3	2	
			4

$= 4$

Hence, required numbers $= 216 + 36 + 36 + 18 + 4 = 310$.

8. **(a)** Collecting different labels of balls drawn $= 10\times 9\times 8$
$\because$ arrangement is not required.
$\therefore$ the number of ways in which the balls can be chosen is,

$\dfrac{10\times 9\times 8}{3!} = 120$

9. **(a)** Number of numbers with one digit $= 4 = 4$
Number of numbers with two digits $= 4\times 5 = 20$
Number of numbers with three digits $= 4\times 5\times 5$
$= 100$
Number of numbers with four digits $= 2\times 5\times 5\times 5$
$= 250$
$\therefore$ Total number of numbers $= 4 + 20 + 100 + 250$
$= 374$

10. **(c)** One of the possible ΔOAB is $A(a, 0)$ and $B(0, b)$.

Area of $\Delta OAB = \dfrac{1}{2}|ab|$.

$\therefore$ $|ab| = 100$
$|a|\,|b| = 100$
But $100 = 1\times 100, 2\times 50, 4\times 25, 5\times 20$ or 10×10
$\therefore$ For 1×100, $a = 1$ or -1 and $b = 100$ or -100
$\therefore$ Total possible pairs are 8.
Total possible pairs for $1\times 100, 2\times 50, 4\times 25$ or 5×20 are 4×8.
And for 10×10 total possible pairs are 4.
$\therefore$ Total number of possible triangles with integral coordinates are $4\times 8 + 4 = 36$.

11. **(a)** The thousands place can only be filled with 2, 3 or 4, since the number is greater than 2000.
For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3.
It the sum of digits of the number is divisible by 3, then the number itself is divisible by 3.
Case 1: If we take 2 at thousands place.

The remaining digits can be filled as:

0, 1 and 3 as $2 + 1 + 0 + 3 = 6$ is divisible by 3.

0, 3 and 4 as $2+ 3 + 0 + 4 = 9$ is divisible by 3.

In both the above combinations the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case $= 2 \times 3! = 12$.

Case 2: If we take 3 at thousands place. The remaining digits can be filled as:

0, 1 and 2 as $3 + 1 + 0 + 2 = 6$ is divisible by 3.

0, 2 and 4 as $3 + 2 + 0 + 4 = 9$ is divisible by 3.

In both the above combinations, the remaining three digits can be arranged in 3! ways. Total number of numbers in this case $= 2 \times 3! = 12$.

Case 3: If we take 4 at thousands place.

The remaining digits can be filled as:

0, 2 and 3 as $4 + 2 + 0 + 3 = 9$ is divisible by 3.

In the above combination, the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case $= 3! = 6$.

$\therefore$ Total number of numbers between 2000 and 5000 divisible by 3 are $12 + 12 + 6 = 30$.

12. **(d)** Required n digit numbers is 3^n as each place can be filled by 2, 5, 7.

So smallest value of n such that $3^n > 900$. Therefore $n = 7$.

13. **(a)** 4 boys and 2 girls in circle

$\Rightarrow 5! \times \dfrac{6!}{4!\,2!} \times 2!$

$\Rightarrow 5 \times 6!$

14. **(c)** E, E, N, Q, U
(i) E .... $= 4! = 24$

(ii) N .... $= \dfrac{4!}{2} = 12$

(iii) QE .... $= 3! = 6$

(iv) QN .... $= \dfrac{3!}{2!} = 3$

(v) QUEEN $= 1$

$\therefore$ Required rank
$= (24) + (12) + (6) + (c) + (a) = 46$th

15. **(b)** ALLMS
No. of words starting with

A : $\underline{A}\_\_\_\_\ \dfrac{4!}{2!} = 12$

L : $\underline{L}\_\_\_\_\ 4! = 24$

M : $\underline{M}\_\_\_\_\ \dfrac{4!}{2!} = 12$

S : $\underline{S}\,\underline{A}\_\_\_\_\ \dfrac{3!}{2!} = 3$

: $\underline{S}\,\underline{L}\_\_\_\ 3! = 6$

SMALL $\rightarrow 58^{\text{th}}$ word

16. **(b)** $\sum_{R-1}^{10} (r^2 + 1)\lfloor r$

$T_1 = (r^2 + 1 + r - r)\lfloor r = (r^2 + r)\lfloor r - (r - r)\lfloor r$

$T_1 = r\lfloor r + r - (r - 1)\lfloor r$

$T_1 = 1\lfloor 2 - 0$

$T_2 = 2\lfloor 3 - 1\lfloor 2$

$T_3 = 3\lfloor 4 - 2\lfloor 3$

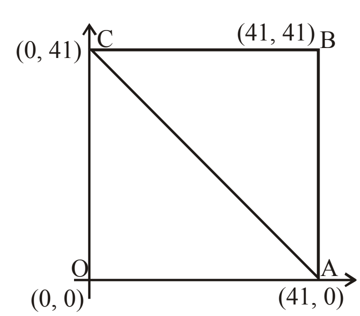
$T_{10} = 10\lfloor 11 - 9\lfloor 10$

$\sum_{R-1}^{10} (r^2 + 1)\lfloor r = 10\lfloor 11$

17. **(b)** M, EEE, D. I, T, RR, AA, NN
R – – E
Two empty places can be filled with identical letters [EE, AA, NN] $\Rightarrow$ 3 ways
Two empty places, can be filled with distinct letters [M, E, D, I, T, R, A, N] $\Rightarrow {}^8P_2$
$\therefore$ Number of words $3 + {}^8P_2 = 59$

18. **(b)** Total number of integral points inside the square OABC $= 40 \times 40 = 1600$

No. of integral points on AC



= No. of integral points on OB

$= 40$ [namely $(1, 1), (2, 2) \ldots (40, 40)$]

$\therefore$ No. of integral points inside the ΔOAC

$= \dfrac{1600 - 40}{2} = 780$

19. **(d)** Four digits number can be arranged in $3 \times 4!$ ways.
Five digits number can be arranged in 5! ways.
Number of integers $= 3 \times 4! + 5! = 192$.

20. **(d)** Number of ways of selecting a man and a woman for a team from 15 men and 15 women
$= 15 \times 15 = (15)^2$
Number of ways of selecting a man and a woman for next team out of the remaining 14 men and 14 women.
$= 14 \times 14 = (14)^2$
Similarly for other teams
Hence required number of ways

$= (15)^2 + (14)^2 + \ldots + (1)^2 = \dfrac{15 \times 16 \times 31}{6} = 1240$

21. **(b)** Let no. of men $= n$
No. of women = 2
Total participants $= n + 2$
No. of games that M_1 plays with all other men
$= 2(n - 1)$
These games are played by all men
$M_2, M_3, \ldots\ldots, M_n$.
So, total no. of games among men $= n.2(n - 1)$.
However, we must divide it by '2', since each game is counted twice (for both players).
So, total no. of games among all men
$= n(n - 1)$ .... (i)
Now, no. of games M_1 plays with W_1 and $W_2 = 4$
(2 games with each)
Total no. of games that $M_1, M_2, \ldots\ldots, M_n$ play with W_1 and $W_2 = 4n$ .... (ii)
Given : $n(n - 1) - 4n = 66$
$\Rightarrow$ $n = 11, -6$
As the number of men can't be negative.
So, $n = 11$

22. **(b)** In 8 digits numbers, 4 places are odd places.
Also, in the given 8 digits, there are three odd digits 1, 1 and 3.
No. of ways three odd digits arranged at four even places $= \dfrac{{}^4P_3}{2!} = \dfrac{4!}{2!}$
No. of ways the remaining five digits 2, 2, 2, 4 and 4 arranged at remaining five places $= \dfrac{5!}{3!2!}$
Hence, required number of 8 digits number

$= \dfrac{4!}{2!} \times \dfrac{5!}{3!2!} = 120$

23. **(d)** We know that any number is divisible by 9 if sum of the digits of the number is divisible by 9.
Now sum of the digits from 0 to 9
$= 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$
Hence to form 8 digits numbers which are divisible by 9, a pair of digits either 0 and 9, 1 and 8, 2 and 7, 3 and 6 or 4 and 5 are not used.

Digits which are not used to form 8 digits number divisible by 9	Number of 8 digits numbers which are divisible by 9
0 and 9	$8 \times 7!$
1 and 8	$7 \times 7!$
2 and 7	$7 \times 7!$
3 and 6	$7 \times 7!$
4 and 5	$7 \times 7!$

Hence total number of 8 digits numbers which are divisible by 9
$= 8 \times (7!) + 7 \times (7!) + 7 \times (7!) + 7 \times (7!) + 7 \times (7!)$
$= 36 \times (7!)$

24. **(b)** With 3 at unit place,
total possible four digit number (without repetition) will be $3! = 6$
With 4 at unit place,
total possible four digit numbers will be $3! = 6$
With 5 at unit place,
total possible four digit numbers will be $3! = 6$
With 6 at unit place,
total possible four digit numbers will be $3! = 6$
Sum of unit digits of all possible numbers
$= 6 \times 3 + 6 \times 4 + 6 \times 5 + 6 \times 6$
$= 6 [3 + 4 + 5 + 6]$
$= 6 [18] = 108$

25. **(d)** p :

0	0	0	0	0	place
5	4	3	2	1	ways

Total no. of ways $= 5! = 120$
Since all numbers are $> 20{,}000$
$\therefore$ all numbers 2, 3, 5, 7, 9 can come at first place.

q :

0	0	0	0	0	place
3	4	3	2	1	ways

Total no. of ways $= 3 \times 4! = 72$
($\because$ 2 and 9 can not be put at first place)
So, $p : q = 120 : 72 = 5 : 3$

26. **(d)** Given that number of white balls = 10
Number of green balls = 9
and Number of black balls = 7
$\therefore$ Required probability

$= (10+1)(9+1)(7+1) - 1$

$= 11.10.8 - 1 = 879$

[$\because$ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind is

$(p+1)(q+1)(r+1) - 1$]

27. **(a)** 7 women can be arranged around a circular table in 6! ways.
Among these 7 men can sit in 7! ways.
Hence, number of seating arrangement
$= 7! \times 6!$

28. **(a)** Alphabetical order is
A, C, H, I, N, S
No. of words starting with A = 5! = 120
No. of words starting with C = 5! = 120
No. of words starting with H = 5! = 120
No. of words starting with I = 5! = 120
No. of words starting with N = 5! = 120
SACHIN – 1
$\therefore$ Sachin appears at serial no. 601

29. **(c)** Total number of arrangements of letters in the word GARDEN = 6 ! = 720 there are two vowels A and E, in half of the arrangements A preceeds E and other half A follows E. So, numbers of word with vowels in alphabetical order in

$\frac{1}{2} \times 720 = 360$

30. **(d)** ${}^{7-x}P_{x-3}$ is defined if

$7 - x \geq 0, x - 3 \geq 0$ and $7 - x \geq x - 3$

$\Rightarrow 3 \leq x \leq 5$ and $x \in \mathbf{I}$

$\therefore x = 3, 4, 5$

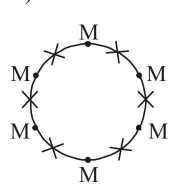
$\therefore f(3) = {}^{7-3}P_{3-3} = {}^{4}P_{0} = 1$

$\therefore f(4) = {}^{7-4}P_{4-3} = {}^{3}P_{1} = 3$

$\therefore f(5) = {}^{7-5}P_{5-3} = {}^{2}P_{2} = 2$

Hence range = {1, 2, 3}

31. **(a)** No. of ways in which 6 men can be arranged at a round table = (6 – 1)! = 5!



Now women can be arranged in ${}^{6}P_{5}$
= 6! ways.
Total Number of ways = 6! × 5!

32. **(b)** Required sum

$= (2+4+6+...+100) + (5+10+15+...+100)$
$\quad - (10+20+...+100)$
$= 2(1+2+3...+50) + 5(1+2+3+...+50)$
$\quad -10(1+2+3+....+10)$
$= 2550 + 1050 - 530 = 3050.$

33. **(c)** Total number of numbers
$= 3 \times 5 \times 5 \times 5 - 1 = 374$

34. **(d)** Total number of numbers formed using 0, 1, 2, 3, 5, 7
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

35. **(120.00)**
For vowels not together

Number of ways to arrange L, T, T, R $= \frac{4!}{2!}$

Then put both E in 5 gaps formed in ${}^{5}C_{2}$ ways.

$\therefore$ No. of ways $= \frac{4!}{2!} \cdot {}^{5}C_{2} = 120$

36. **(240)**
$S \to 2,\ L \to 2,\ A, B, Y, U.$

$\therefore$ Required number of ways $= {}^{2}C_{1} \times {}^{5}C_{2} \times \frac{4!}{2!} = 240.$

37. **(d)** Since, each section has 5 questions.
$\therefore$ Total number of selection of 5 questions

$= 3 \times {}^{5}C_{1} \times {}^{5}C_{1} \times {}^{5}C_{3} + 3 \times {}^{5}C_{1} \times {}^{5}C_{2} \times {}^{5}C_{2}$
$= 3 \times 5 \times 5 \times 10 + 3 \times 5 \times 10 \times 10$
$= 750 + 1500 = 2250.$

38. **(135)**
Select any 4 correct questions in ${}^{6}C_{4}$ ways.
Number of ways of answering wrong question = 3

$\therefore$ Required number of ways $= {}^{6}C_{4}(1)^{4} \times 3^{2} = 135.$

39. **(54)**
Let xyz be the three digit number

$x + y + z = 10,\ x \leq 1,\ y \geq 0,\ z \geq 0$

$x - 1 = t \Rightarrow x = 1 + t \qquad x - 1 \geq 0,\ t \geq 0$

$t + y + z = 10 - 1 = 9 \qquad 0 \leq t,\ z,\ z \leq 9$

$\therefore$ Total number of non-negative integral solution

$= {}^{9+3-1}C_{3-1} = {}^{11}C_{2} = \frac{11 \cdot 10}{2} = 55$

But for $t = 9$, $x = 10$, so required number of integers
$= 55 - 1 = 54.$

40. **(a)** Number of two consecutive stations (Blue lines) = n
Number of two non-consecutive stations (Red lines)

$= {}^{n}C_{2} - n$

Now, according to the question, ${}^{n}C_{2} - n = 99n$

$\Rightarrow \dfrac{n(n-1)}{2} - 100n = 0 \Rightarrow n(n-1-200) = 0$

$\Rightarrow n-1-200 = 0 \Rightarrow n = 201$

41. **(51)** $\sum_{r=0}^{25}(4r+1)\,{}^{25}C_r = 4\sum_{r=0}^{25} r.\,{}^{25}C_r + \sum_{r=0}^{25}{}^{25}C_r$

$= 4\sum_{r=1}^{25} r \times \dfrac{25}{r}\,{}^{24}C_{r-1} + 2^{25} = 100\sum_{r=1}^{25}{}^{24}C_{r-1} + 2^{25}$

$= 100.2^{24} + 2^{25} = 2^{25}(50+1) = 51.2^{25}$

Hence, by comparison $k = 51$

42. **(490)**

0 Red, 1 Red, 2 Red, 3 Red

Number of ways of selecting atmost three red balls

$= {}^7C_4 + {}^5C_1 \cdot {}^7C_3 + {}^5C_2 \cdot {}^7C_2 + {}^5C_3 \cdot {}^7C_1$

$= 35 + 175 + 210 + 70 = 490$

43. **(c)** We know ${}^nC_r$ is greatest at middle term.

So, $a = ({}^{19}C_p)_{\max} = {}^{19}C_{10} = {}^{19}C_9$

$b = ({}^{20}C_q)_{\max} = {}^{20}C_{10}$

$c = ({}^{21}C_6)_{\max} = {}^{21}C_{10} = {}^{21}C_{11}$

Now, $\dfrac{a}{{}^{19}C_9} = \dfrac{b}{\frac{20}{10}.\,{}^{19}C_9} = \dfrac{c}{\frac{21}{11}.\frac{20}{10}\,{}^{19}C_9}$

$\Rightarrow \dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{42/11}$ $\therefore \dfrac{a}{11} = \dfrac{b}{22} = \dfrac{c}{42}$

44. **(2454)** EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I : If all are different, then

${}^8p_4 = \dfrac{8!}{4!} = 8.7.6.5 = 1680$

Case II : If two are same and two are different, then

${}^3C_1 . {}^7C_2 . \dfrac{4!}{2!} = 3.21.12 = 756$

Case III : If two are same and other two are same, then

${}^3C_2 . \dfrac{4!}{2!2!} = 3.6 = 18$

$\therefore$ Total cases $= 1680 + 756 + 18 = 2454$

45. **(1)** $\dfrac{36}{r+1} \times {}^{35}C_r(k^2-3) = {}^{35}C_r.6$

$\Rightarrow k^2 - 3 = \dfrac{r+1}{6}$

$\Rightarrow k^2 = 3 + \dfrac{r+1}{6}$

r can be 5, 35 for $k \in I$

$r = 5, k = \pm 2$

$r = 35, k = \pm 3$

Hence, number of ordered pairs = 4.

46. **(c)** Number of ways of selecting 10 objects

$= (10I, 0D)$ or $(9I, 1D)$ or $(8I, 1D)$ or ... $(0I, 10D)$

Here, D signifies distinct object and I indicates identical object

$= 1 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10} = \dfrac{2^{21}}{2} = 2^{20}$

47. **(c)** Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons

$= {}^{n+5}C_3 - {}^nC_3 - {}^5C_3 = 1750$

$\Rightarrow \dfrac{(n+5)!}{3!(n+2)!} - \dfrac{n!}{3!(n-3)!} - \dfrac{5!}{3!\,2!} = 1750$

$\Rightarrow \dfrac{(n+5)(n+4)(n+3)}{6} - \dfrac{n(n-1)(n-2)}{6} = 1760$

$\Rightarrow n^2 + 3n - 700 = 0 \Rightarrow n = 25$ $[n = -28$ rejected$]$

48. **(a)** Total number of beams $= {}^{20}C_2 - 20 = 190 - 20 = 170$

49. **(b)** Since, m = number of ways the committee is formed with at least 6 males

$= {}^8C_6 . {}^5C_5 + {}^8C_7 . {}^5C_4 + {}^8C_8 . {}^5C_3 = 78$

and n = number of ways the committee is formed with at least 3 females

$= {}^5C_3 . {}^8C_8 + {}^5C_4 . {}^8C_7 + {}^5C_5 . {}^8C_6 = 78$

Hence, m = n = 78

50. **(a)** $\because$ There are total 9 digits and out of which only 3 digits are odd.

$\therefore$ Number of ways to arrange odd digits first $= {}^4C_3 . \dfrac{3!}{2!}$

Hence, total number of 9 digit numbers

$= \left({}^4C_3 . \dfrac{3!}{2!}\right) . \dfrac{6!}{2!\,4!} = 180$

51. **(a)** ${}^mC_2 \times 2 = {}^mC_1 \cdot {}^2C_1 \times 2 + 84$

$m(m-1) = 4m + 84$

$m^2 - 5m - 84 = 0$

$m^2 - 12m - 7m - 84 = 0$

$m(m-12) + 7(m-12) = 0$

$m = 12, m = -7$

$\because m > 0$

$m = 12$

52. **(d)** Consider the expression,

$\dfrac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} = \dfrac{{}^{20}C_{i-1}}{{}^{21}C_1}$

$$=\frac{20!}{(i-1)!(21-i)!}\times\frac{i!(21-i)!}{21!}=\frac{i}{21}$$

$$\therefore \quad \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\sum_{i=1}^{20}\left(\frac{i}{21}\right)^3=\frac{(1)}{(21)^3}\sum_{i=1}^{20}i^3$$

$$=\frac{1}{(21)^3}\times\left(\frac{20\times 21}{2}\right)^2=\frac{100}{21}$$

$$\because \quad \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$$

$\therefore \quad k=100$

53. **(c)** Since, the number of ways to select 2 girls is ${}^5C_2$.

Now, 3 boys can be selected in 3 ways.

(a) Selection of A and selection of any 2 other boys (except B) in ${}^5C_2$ ways

(b) Selection of B and selection of any 2 two other boys (except A) in ${}^5C_2$ ways

(c) Selection of 3 boys (except A and B) in ${}^5C_3$ ways

Hence, required number of different teams

$={}^5C_2({}^5C_2+{}^5C_2+{}^5C_3)=300$

54. **(d)** If all four letters are different then the number of words ${}^5C_4\times 4!=120$

If two letters are R and other two different letters are chosen from B, A, C, K then the number of words

$$={}^4C_2\times\frac{4!}{2!}=72$$

If two letters are A and other two different letters are chosen from B, R, C, K then the number of words

$$={}^4C_2\times\frac{4!}{2!}=72$$

If word is formed using two R's and two A's then the number of words $=\dfrac{4!}{2!\,2!}=6$

Therefore, the number of four-letter words that can be formed $=120+72+72+6=270$

55. **(d)** $\therefore$ Required number of ways $={}^6C_4\times{}^3C_1\times 4!$

$=15\times 3\times 24=1080$

56. **(b)**

X → 4 ladies, 3 men

Y → 3 ladies, 4 men

Possible cases for X are

(1) 3 ladies, 0 man

(2) 2 ladies, 1 man

(3) 1 lady, 2 men

(4) 0 ladies, 3 men

Possible cases for Y are

(1) 0 ladies, 3 men

(2) 1 lady, 2 men

(3) 2 ladies, 1 man

(4) 3 ladies, 0 man

No. of ways $={}^4C_3\,.\,{}^4C_3+({}^4C_2\,.\,{}^3C_1)^2+({}^4C_1\,.\,{}^3C_2)^2+({}^3C_3)^2$

$=16+324+144+1=485$

57. **(c)** $\dfrac{{}^{n+2}C_6}{{}^{n-2}P_2}=11$

$$\Rightarrow \quad \frac{\dfrac{(n+2)(n+1)n(n-1)(n-2)(n-3)}{6.5.4.3.2.1}}{\dfrac{(n-2)(n-3)}{2.1}}=11$$

$\Rightarrow \quad (n+2)(n+1)n(n-1)=11\,.\,10.\,9.\,4$

$\Rightarrow \quad n=9$

$n^2+3n-108=(9)^2+3(9)-108$

$=81+27-108$

$=108-108=0$

58. **(d)** $\displaystyle\sum_{r=1}^{15}r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$

$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}}=\frac{\dfrac{\lfloor 15}{\lfloor 15-r\,\lfloor r}}{\dfrac{\lfloor 15}{\lfloor r-1\,\lfloor 16-r}}=\frac{\dfrac{1}{\lfloor 15-r\,\lfloor r-1}}{\dfrac{1}{\lfloor r-1\,\lfloor 15-r\,.(16-r)}}$$

$$=\frac{16-r}{r}$$

$$=\sum_{r=1}^{15}r^2\left(\frac{16-r}{r}\right)=\sum_{r=1}^{15}r(16-r)$$

$$=16\sum_{r=1}^{15}r-\sum_{r=1}^{15}r^2$$

$$=\frac{16\times 15\times 16}{2}-\frac{15\times 31\times 16}{6}$$

$=8\times 15\times 16-5\times 8\times 31=1920-1240=680$

59. **(c)** Given

$n(A)=4, n(B)=2, n(A\times B)=8$

Required number of subsets

$={}^8C_3+{}^8C_4+....+{}^8C_8=2^8-{}^8C_0-{}^8C_1-{}^8C_2$

$=256-1-8-28=219$

60. **(a)** Number of diagonal = 54

$\Rightarrow \frac{n(n-3)}{2} = 54$

$\Rightarrow n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$

$\Rightarrow n(n-12) + 9(n-12) = 0$

$\Rightarrow n = 12, -9 \Rightarrow n = 12 \ (\because n \neq -9)$

61. **(c)** Given

$n(A) = 2, n(B) = 4, n(A \times B) = 8$

Required number of subsets =

${}^8C_3 + {}^8C_4 + ... + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$

$= 256 - 1 - 8 - 28 = 219$

62. **(b)** We know,

$T_n = {}^nC_3, T_{n+1} = {}^{n+1}C_3$

ATQ, $T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$

$\Rightarrow {}^nC_2 = 10$

$\Rightarrow n = 5.$

63. **(b)** Required number of triangles

$= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 205$

64. **(c)** 30 marks to be alloted to 8 questions. Each question has to be given ≥ 2 marks

Let questions be a, b, c, d, e, f, g, h

and $a + b + c + d + e + f + g + h = 30$

Let $a = a_1 + 2$ so, $a_1 \geq 0$

$b = a_2 + 2$ so, $a_2 \geq 0, ..., a_8 \geq 0$

So, $\left.\begin{matrix} a_1 + a_2 + ... + a_8 \\ +2 + 2 + ... + 2 \end{matrix}\right\} = 30$

$\Rightarrow a_1 + a_2 + ... + a_8 = 30 - 16 = 14$

So, this is a problem of distributing 14 articles in 8 groups.

Number of ways = ${}^{14+8-1}C_{8-1} = {}^{21}C_7$

65. **(b)**

L	O	Y
2	2	4
≥ 1	≥ 1	2 ≤

$\Rightarrow$

L	O	Y
1	1	2
1	2	1
2	1	1
2	2	0

Required number of ways

$= {}^2C_1 \times {}^2C_1 \times {}^2C_2 + {}^2C_1 \times {}^2C_2 \times {}^4C_1 + {}^2C_2 \times {}^2C_1 \times {}^4C_1 + {}^2C_2 \times {}^2C_2 \times {}^4C_0$

$= 2 \times 2 \times \frac{4 \times 3}{2} + 2 \times 1 \times 4 + 1 \times 2 \times 4 + 1 \times 1 \times 1$

$= 24 + 8 + 8 + 1 = 41$

66. **(a)** There are 2 vowels and 4 consonants in the letters a, b, c, d, e, f.

If we select one vowel, then number of arrangements

$= {}^2C_1 \times {}^4C_2 \times 3! = 2 \times \frac{4 \times 3}{2} \times 3 \times 2 = 72$

If we select two vowels, then number of arrangements

$= {}^2C_2 \times {}^4C_1 \times 3! = 1 \times 4 \times 6 = 24$

Hence, total number of arrangements

$= 72 + 24 = 96$

67. **(a)** $n = {}^mC_2 = \frac{m(m-1)}{2}$

Since m and $(m-1)$ are two consecutive natural numbers, therefore their product is an even natural number. So $\frac{m(m-1)}{2}$ is also a natural number.

Now $\frac{m(m-1)}{2} = \frac{m^2 - m}{2}$

$\therefore {}^{\frac{m(m-1)}{2}}C_2 = \frac{\left(\frac{m^2-m}{2}\right)\left(\frac{m^2-m}{2} - 1\right)}{2}$

$= \frac{m(m-1)(m^2 - m - 2)}{8}$

$= \frac{m(m-1)[m^2 - 2m + m - 2]}{8}$

$= \frac{m(m-1)[m(m-2) + 1(m-2)]}{8}$

$= \frac{m(m-1)(m-2)(m+1)}{8}$

$= \frac{3 \times (m+1)m(m-1)(m-2)}{4 \times 3 \times 2 \times 1} = 3\left({}^{m+1}C_4\right)$

68. **(d)** **Statement - 1 :** $n(A) = p, n(B) = q, q > p$

Total number of functions from $A \to B = q^p$

It is a true statement.

Statement - 2 : The total number of selections of p different objects out of q objects is ${}^qC_p$.

It is also a true statement and it is a correct explanation for statement - 1 also.

69. **(c)** Set $A = \{a_1, a_2, ..., a_{20}\}$ has 20 distinct elements.

We have to select 5-element subset.

$\therefore$ Number of 5-element subsets = ${}^{20}C_5$

According to question

${}^{20}C_5 = \left({}^{19}C_4\right).k$

$\Rightarrow \frac{20!}{5!\ 15!} = k.\left(\frac{19!}{4!\ 15!}\right)$

$\Rightarrow \frac{20}{5} = k \Rightarrow k = 4$

70. **(a)** Number of required triangles = ${}^{10}C_3 - {}^{6}C_3$

$= \dfrac{10\times9\times8}{6} - \dfrac{6\times5\times4}{6} = 120 - 20 = 100$

71. **(a)** The number of ways of distributing 10 identical balls in 4 distinct boxes

$= {}^{10-1}C_{4-1} = {}^{9}C_3$

72. **(c)** Two balls are taken from each urn

Total number of ways = ${}^{3}C_2 \times {}^{9}C_2$

$= 3 \times \dfrac{9\times8}{2} = 3\times36 = 108$

73. **(c)** 4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in ${}^{6}C_4 \times {}^{3}C_1$ ways

Then 4 novels with one dictionary in the middle can be arranged in 4! ways.

$\therefore$ Total ways of arrangement

$= {}^{6}C_4 \times {}^{3}C_1 \times 4! = 1080$

74. **(d)** First let us arrange M, I, I, I, P, P

Which can be done in $\dfrac{7!}{4!2!}$ ways

$* M * I * I * I * P * P *$

Now 4 S can be kept at any of the * places in ${}^{8}C_4$ ways so that no two S are adjacent.

Total required ways

$= \dfrac{7!}{4!2!}\,{}^{8}C_4 = \dfrac{7!}{4!2!}\,{}^{8}C_4 = 7 \times {}^{6}C_4 \times {}^{8}C_4$

75. **(a)** Set S = {1, 2, 3, ... 12}

$A \cup B \cup C = S,\ A \cap B = B \cap C = A \cap C = \phi$

$\therefore$ Each sets contain 4 elements.

$\therefore$ The number of ways to partition

$= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$

$= \dfrac{12!}{4!8!} \times \dfrac{8!}{4!4!} \times \dfrac{4!}{4!0!} = \dfrac{12!}{(4!)^3}$

76. **(c)** The number of ways can vote

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210 = 385$

77. **(d)** ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

$= {}^{50}C_4 + \left[\begin{array}{r} {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 \\ + {}^{51}C_3 + {}^{50}C_3 \end{array}\right]$

We know $\left[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

Proceeding in the same way, we get

$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$.

78. **(b)** We know that the number of ways of distributing n identical items among r persons, when each one of them receives at least one item is ${}^{n-1}C_{r-1}$

$\therefore$ The required number of ways

$= {}^{8-1}C_{3-1} = {}^{7}C_2 = \dfrac{7!}{2!5!} \qquad = \dfrac{7\times6}{2\times1} = 21$

79. **(c)** According to given question two cases are possible.

(i) Selecting 4 out of first five question and 6 out of remaining question

$= {}^{5}C_4 \times {}^{8}C_6 = 140$ ways

(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions $= {}^{5}C_5 \times {}^{8}C_5 = 56$ ways

Therefore, total number of choices

$= 140 + 56 = 196$.

80. **(c)** ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2\,{}^{n}C_r$

$= {}^{n}C_{r-1} + {}^{n}C_r + {}^{n}C_r + {}^{n}C_{r+1}$

$\left[\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$

$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

81. **(d)** We know that a number is divisible by 3 only when the sum of the digits is divisible by 3. The given digits are 0, 1, 2, 3, 4, 5.

Here the possible number of combinations of 5 digits out of 6 are ${}^{5}C_4 = 5$, which are as follows–

$1 + 2 + 3 + 4 + 5 = 15 = 3 \times 5$ (divisible by 3)

$0 + 2 + 3 + 4 + 5 = 14$ (not divisible by 3)

$0 + 1 + 3 + 4 + 5 = 13$ (not divisible by 3)

$0 + 1 + 2 + 4 + 5 = 12 = 3 \times 4$ (divisible by 3)

$0 + 1 + 2 + 3 + 5 = 11$ (not divisible by 3)

$0 + 1 + 2 + 3 + 4 = 10$ (not divisible by 3)

Thus the number should contain the digits 1, 2, 3, 4, 5 or the digits 0, 1, 2, 4, 5.

Taking 1, 2, 3, 4, 5, the 5 digit numbers are

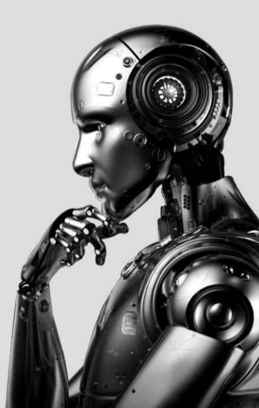
$= 5! = 120$

Taking 0, 1, 2, 4, 5, the 5 digit numbers are

$= 5! - 4! = 96$

$\therefore$ Total number of numbers $= 120 + 96 = 216$

8



Binomial Theorem

TOPIC 1 Binomial Theorem for a Positive Integral Index 'x', Expansion of Binomial, General Term, Coefficient of any Power of 'x'

1. If $\{p\}$ denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to : **[Sep. 06, 2020 (I)]**

(a) $\frac{5}{8}$ (b) $\frac{7}{8}$

(c) $\frac{3}{8}$ (d) $\frac{1}{8}$

2. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ______.

[NA Sep. 05, 2020 (I)]

3. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ____________. **[NA Sep. 05, 2020 (II)]**

4. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ___________. **[NA Sep. 04, 2020 (I)]**

5. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of

$\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$, then: **[Jan. 8, 2020 (II)]**

(a) $\alpha + \beta = 60$ (b) $\alpha + \beta = -30$

(c) $\alpha - \beta = 60$ (d) $\alpha - \beta = -132$

6. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

[April 10, 2019 (II)]

(a) 38 (b) 58

(c) 23 (d) 35

7. If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is:

[April 9, 2019 (I)]

(a) 8^3 (b) 8^2

(c) 8 (d) 8^{-2}

8. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2:15:70, then the average of these three coefficients is:

[April 09, 2019 (II)]

(a) 964 (b) 232

(c) 227 (d) 625

9. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$, $(x > 1)$ is equal to : **[April 8, 2019 (I)]**

(a) 29 (b) 32

(c) 26 (d) 24

10. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is: **[April 08, 2019 (II)]**

(a) 100 (b) 10

(c) 10^3 (d) 10^4

11. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1 x + a_2 x^2 + ... + a_{50} x^{50}$, for all $x \in \mathbf{R}$; then $\frac{a_2}{a_0}$ is equal to : **[Jan. 11, 2019 (II)]**

(a) 12.50 (b) 12.00

(c) 12.25 (d) 12.75

12. If the third term in the binomial expansion of $\left(1 + x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is:

[Jan. 10, 2019 (I)]

(a) $\frac{1}{4}$ (b) $4\sqrt{2}$

(c) $\frac{1}{8}$ (d) $2\sqrt{2}$

13. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is: **[Jan. 10, 2019 (II)]**

(a) 4 (b) $2\sqrt{2}$

(c) $\sqrt{5}$ (d) 3

14. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to: **[Jan. 9, 2019 (I)]**

(a) 6 (b) 8

(c) 4 (d) 14

15. The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to **[Online April 15, 2018]**

(a) 52 (b) 44

(c) 50 (d) 56

16. If n is the degree of the polynomial,

$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8 \text{ and}$$

m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to **[Online April 15, 2018]**

(a) $(12,(20)^4)$ (b) $(8,5(10)^4)$

(c) $(24,(10)^8)$ (d) $(12,8(10)^4)$

17. The coefficient of x^2 in the expansion of the product $(2-x^2).((1+2x+3x^2)^6+(1-4x^2)^6)$ is **[Online April 16, 2018]**

(a) 106 (b) 107

(c) 155 (d) 108

18. The sum of the co-efficients of all odd degree terms in the expansion of **[2018]**

$(x+\sqrt{x^3-1})^5+(x-\sqrt{x^3-1})^5, (x>1)$ is :

(a) 0 (b) 1

(c) 2 (d) -1

19. The coefficient of x^{-5} in the binomial expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10} \text{ where } x \neq 0, 1, \text{ is :}$$

[Online April 9, 2017]

(a) 1 (b) 4

(c) -4 (d) -1

20. If $(27)^{999}$ is divided by 7, then the remainder is : **[Online April 8, 2017]**

(a) 1 (b) 2

(c) 3 (d) 6

21. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, $(x>0)$, are m and n respectively, then $\frac{m}{n}$ is equal to : **[Online April 10, 2016]**

(a) 27 (b) 182

(c) $\frac{5}{4}$ (d) $\frac{4}{5}$

22. If the coefficients of the three successive terms in the binomial expansion of $(1+x)^n$ are in the ratio 1 : 7 : 42, then the first of these terms in the expansion is: **[Online April 10, 2015]**

(a) 8^{th} (b) 6^{th}

(c) 7^{th} (d) 9^{th}

23. If the coefficents of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to: **[2014]**

(a) $\left(14,\frac{272}{3}\right)$ (b) $\left(16,\frac{272}{3}\right)$

(c) $\left(16,\frac{251}{3}\right)$ (d) $\left(14,\frac{251}{3}\right)$

24. If $X=\{4^n-3n-1 : n\in N\}$ and $Y=\{9(n-1): n\in N\}$, where N is the set of natural numbers, then $X\cup Y$ is equal to: **[2014]**

(a) X (b) Y

(c) N (d) $Y-X$

25. If $1+x^4+x^5=\sum_{i=0}^{5}a_i(1+x)^i$, for all x in R, then a_2 is: **[Online April 12, 2014]**

(a) -4 (b) 6

(c) -8 (d) 10

26. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are: **[Online April 12, 2014]**

(a) 7^{th} and 8^{th} (b) 8^{th} and 9^{th}

(c) 28^{th} and 29^{th} (d) 27^{th} and 28^{th}

27. The number of terms in the expansion of $(1+x)^{101}(1+x^2-x)^{100}$ in powers of x is: **[Online April 9, 2014]**

(a) 302 (b) 301

(c) 202 (d) 101

28. If for positive integers $r > 1$, $n > 2$, the coefficients of the $(3r)^{th}$ and $(r+2)^{th}$ powers of x in the expansion of $(1+x)^{2n}$ are equal, then n is equal to : **[Online April 25, 2013]**

(a) $2r+1$ (b) $2r-1$

(c) $3r$ (d) $r+1$

29. The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}}+3^{\frac{1}{5}}\right)^{10}$ is : **[Online April 23, 2013]**

(a) 25 (b) 32

(c) 9 (d) 41

30. If the 7th term in the binomial expansion of $\left(\frac{3}{\sqrt[3]{84}}+\sqrt{3}\ln x\right)^9$, $x > 0$, is equal to 729, then x can be : **[Online April 22, 2013]**

(a) e^2 (b) e

(c) $\frac{e}{2}$ (d) $2e$

31. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is : **[2012]**

(a) an irrational number

(b) an odd positive integer

(c) an even positive integer

(d) a rational number other than positive integers

32. The number of terms in the expansion of $\left(y^{1/5}+x^{1/10}\right)^{55}$, in which powers of x and y are free from radical signs are **[Online May 12, 2012]**

(a) six (b) twelve

(c) seven (d) five

33. If $f(y) = 1-(y-1)+(y-1)^2-(y-1)^3 + ... -(y-1)^{17}$, then the coefficient of y^2 in it is **[Online May 7, 2012]**

(a) ${}^{17}C_2$ (b) ${}^{17}C_3$

(c) ${}^{18}C_2$ (d) ${}^{18}C_3$

34. **Statement - 1 :** For each natural number n, $(n+1)^7-1$ is divisible by 7.

Statement - 2 : For each natural number n, n^7-n is divisible by 7. **[2011 RS]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

35. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is **[2011]**

(a) –132 (b) –144

(c) 132 (d) 144

36. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is: **[2009]**

(a) 2 (b) 7

(c) 8 (d) 0

37. **Statement -1 :** $\sum_{r=0}^{n}(r+1)\ {}^nC_r = (n+2)2^{n-1}$.

Statement-2: $\sum_{r=0}^{n}(r+1)\ {}^nC_r x^r = (1+x)^n + nx(1+x)^{n-1}$. **[2008]**

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

38. In the binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then a/b equals **[2007]**

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$.

39. For natural numbers m, n if $(1-y)^m(1+y)^n = 1+a_1y+a_2y^2+.......$ and $a_1 = a_2 = 10$, then (m,n) is **[2006]**

(a) $(20,45)$ (b) $(35,20)$

(c) $(45,35)$ (d) $(35,45)$

40. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation **[2005]**

(a) $a-b=1$ (b) $a+b=1$

(c) $\frac{a}{b}=1$ (d) $ab=1$

41. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is **[2004]**

(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$

(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

42. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is **[2003]**

(a) 35 (b) 32
(c) 33 (d) 34

43. r and n are positive integers $r>1$, $n>2$ and coefficient of $(r+2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals **[2002]**

(a) $3r$ (b) $3r+1$
(c) $2r$ (d) $2r+1$

44. The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are **[2002]**

(a) equal
(b) equal with opposite signs
(c) reciprocals of each other
(d) none of these

TOPIC 2 Middle Term, Greatest Term, Independent Term, Particular Term from end in Binomial Expansion, Greatest Binomial Coefficients

45. If the constant term in the binomial expansion of $\left(\sqrt{x}\frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals: **[Sep. 06, 2020 (II)]**

(a) 9 (b) 1
(c) 3 (d) 2

46. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is : **[Sep. 04, 2020 (II)]**

(a) 462 (b) 330
(c) 792 (d) 252

47. If the number of integral terms in the expansion of $(3^{1/2}+5^{1/8})^n$ is exactly 33, then the least value of n is : **[Sep. 03, 2020 (I)]**

(a) 264 (b) 128
(c) 256 (d) 248

48. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then $18k$ is equal to : **[Sep. 03, 2020 (II)]**

(a) 5 (b) 9
(c) 7 (d) 11

49. Let $\alpha>0, \beta>0$ be such that $\alpha^3+\beta^2=4$. If the maximum value of the term independent of x in the binomial expansion of $(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}})^{10}$ is $10k$, then k is equal to : **[Sep. 02, 2020 (I)]**

(a) 336 (b) 352
(c) 84 (d) 176

50. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to __________. **[NA Sep. 02, 2020 (II)]**

51. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to : **[Jan. 9, 2020 (II)]**

(a) $1:8$ (b) $16:1$
(c) $8:1$ (d) $1:16$

52. The total number is irrational terms in the binomial expansion of $\left(7^{\frac{1}{5}}-3^{\frac{1}{10}}\right)^{60}$ is : **[Jan. 12, 2019 (II)]**

(a) 55 (b) 49
(c) 48 (d) 54

53. A ratio of the 5^{th} term from the begining to the 5th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is: **[Jan. 12, 2019 (I)]**

(a) $1:2(6)^{\frac{1}{3}}$ (b) $1:4(16)^{\frac{1}{3}}$
(c) $4(36)^{\frac{1}{3}}:1$ (d) $2(36)^{\frac{1}{3}}:1$

54. The term independent of x in the binomial expansion of $\left(1-\frac{1}{x}+3x^5\right)\left(2x^2-\frac{1}{x}\right)^8$ is : **[Online April 11, 2015]**

(a) 496 (b) –496
(c) 400 (d) –400

55. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is **[2013]**

(a) 4 (b) 120
(c) 210 (d) 310

56. The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2+\frac{2}{x}\right)^{15}$ is : **[Online April 9, 2013]**

(a) 7 : 16 (b) 7 : 64
(c) 1 : 4 (d) 1 : 32

57. The middle term in the expansion of $\left(1-\frac{1}{x}\right)^n(1-x)^n$ in powers of x is **[Online May 26, 2012]**

(a) $-{}^{2n}C_{n-1}$ (b) $-{}^{2n}C_n$
(c) ${}^{2n}C_{n-1}$ (d) ${}^{2n}C_n$

58. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals **[2004]**

(a) $\frac{3}{5}$ (b) $\frac{10}{3}$
(c) $\frac{-3}{10}$ (d) $\frac{-5}{3}$

TOPIC 3 Properties of Binomial Coefficients, Number of Terms in the Expansion of $(x+y+z)^n$, Binomial theorem for any Index, Multinomial theorem, Infinite Series

59. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to : **[Sep. 04, 2020 (I)]**

(a) ${}^{51}C_7 - {}^{30}C_7$ (b) ${}^{50}C_7 - {}^{30}C_7$
(c) ${}^{50}C_6 - {}^{30}C_6$ (d) ${}^{51}C_7 + {}^{30}C_7$

60. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is ________. **[NA Jan. 9, 2020 (I)]**

61. If the sum of the coefficients of all even powers of x in the product

$(1+x+x^2+...+x^{2n})(1-x+x^2-x^3+...+x^{2n})$ is 61, then n is equal to ______. **[NA Jan. 7, 2020 (I)]**

62. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to : **[NA April 12, 2019 (II)]**

(a) –72 (b) 36
(c) –36 (d) –108

63. If ${}^{20}C_1+(2^2)\,{}^{20}C_2+(3^2)\,{}^{20}C_3+\ldots\ldots+(20^2)\,{}^{20}C_{20}=A(2^\beta)$, then the ordered pair (A, β) is equal to : **[April 12, 2019 (II)]**

(a) (420, 19) (b) (420, 18)
(c) (380, 18) (d) (380, 19)

64. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is : **[April 12, 2019 (I)]**

(a) 84 (b) –126
(c) –84 (d) 126

65. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1–3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to: **[April 10, 2019 (I)]**

(a) (28, 861) (b) (–54, 315)
(c) (28, 315) (d) (–21, 714)

66. The sum of the series

$2\cdot{}^{20}C_0+5\cdot{}^{20}C_1+8\cdot{}^{20}C_2+11\cdot{}^{20}C_3+\ldots+62\cdot{}^{20}C_{20}$

is equal to : **[April 8, 2019 (I)]**

(a) 2^{26} (b) 2^{25}
(c) 2^{23} (d) 2^{24}

67. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is : **[Jan. 11, 2019 (I)]**

(a) 0 (b) 6
(c) 4 (d) 8

68. The value of r for which

${}^{20}C_r\,{}^{20}C_0+{}^{20}C_{r-1}\,{}^{20}C_1+{}^{20}C_{r-2}\,{}^{20}C_2+\ldots+{}^{20}C_0\,{}^{20}C_r$

is maximum, is : **[Jan. 11, 2019 (I)]**

(a) 15 (b) 20
(c) 11 (d) 10

69. If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to: **[Jan. 10, 2019 (II)]**

(a) $(25)^2$ (b) $2^{25}-1$
(c) 2^{24} (d) 2^{25}

70. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ **[Jan. 09, 2019 (II)]**

(a) 14 (b) 15
(c) 10 (d) 12

71. The value of

$({}^{21}C_1-{}^{10}C_1)+({}^{21}C_2-{}^{10}C_2)+({}^{21}C_3-{}^{10}C_3)+({}^{21}C_4-{}^{10}C_4)+\ldots+({}^{21}C_{10}-{}^{10}C_{10})$ is : **[2017]**

(a) $2^{20}-2^{10}$ (b) $2^{21}-2^{11}$
(c) $2^{21}-2^{10}$ (d) $2^{20}-2^{9}$

72. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **[2016]**

(a) 243 (b) 729
(c) 64 (d) 2187

73. The sum of coefficients of integral power of x in the binomial expansion $\left(1-2\sqrt{x}\right)^{50}$ is : **[2015]**

(a) $\frac{1}{2}\left(3^{50}-1\right)$ (b) $\frac{1}{2}\left(2^{50}+1\right)$
(c) $\frac{1}{2}\left(3^{50}+1\right)$ (d) $\frac{1}{2}\left(3^{50}\right)$

74. The coefficient of x^{1012} in the expansion of $(1+x^n+x^{253})^{10}$, (where $n \leq 22$ is any positive integer), is **[Online April 19, 2014]**

(a) 1 (b) $^{10}C_4$
(c) 4n (d) $^{253}C_4$

75. Let $S_1=\sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2=\sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3=\sum_{j=1}^{10} j^2\,{}^{10}C_j$. **[2010]**

Statement -1 : $S_3=55\times 2^9$.
Statement - 2: $S_1=90\times 2^8$ and $S_2=10\times 2^8$.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

76. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1 : The number of different ways the child can buy the six ice-creams is $^{10}C_5$.

Statement -2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. **[2008]**

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement - 2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

77. The sum of the series **[2007]**

$^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-.....+{}^{20}C_{10}$ is

(a) 0 (b) $^{20}C_{10}$
(c) $-^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$

78. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as **[2005]**

(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$
(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

79. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is **[2003]**

(a) 6th term (b) 7th term
(c) 5th term (d) 8th term

80. The positive integer just greater than $(1+0.0001)^{10000}$ is **[2002]**

(a) 4 (b) 5
(c) 2 (d) 3

81. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is **[2002]**

(a) 1594 (b) 792
(c) 924 (d) 2924

Hints & Solutions

1. **(d)** $\dfrac{3^{200}}{8}=\dfrac{1}{8}(9^{100})$

$$=\frac{1}{8}(1+8)^{100}=\frac{1}{8}\left[1+n\cdot 8+\frac{n(n+1)}{2}\cdot 8^2+....\right]$$

$$=\frac{1}{8}+\text{Integer}$$

$$\therefore \left\{\frac{3^{200}}{8}\right\}=\left\{\frac{1}{8}+\text{integer}\right\}=\frac{1}{8}$$

2. **(13)**

$$T_{r+1}={}^{22}C_r\cdot(x^m)^{22-r}\cdot\left(\frac{1}{x^2}\right)^r$$

$$T_{r+1}={}^{22}C_r\cdot x^{22m-mr-2r}$$

$\because 22m-mr-2r=1$

$$\Rightarrow r=\frac{22m-1}{m+2}\Rightarrow r=22-\frac{3\cdot3\cdot5}{m+2}$$

So, possible value of $m=1, 3, 7, 13, 43$

But ${}^{22}C_r=1540$

$\therefore$ Only possible value of $m=13$.

3. **(120.00)**

Coefficient of x^4 in $\left(\dfrac{1-x^4}{1-x}\right)^6$ = coefficient of x^4 in $(1-6x^4)(1-x)^{-6}$

= coefficient of x^4 in $(1-6x^4)\left[1+{}^6C_1x+{}^7C_2x^2+....\right]$

$$={}^9C_4-6\cdot1=126-6=120.$$

4. **(8.00)**

The given expression is $(2x^2+3x+4)^{10}=\displaystyle\sum_{r=0}^{20}a_rx^r$

General term $=\dfrac{10!}{r_1!r_2!r_3!}(2x^2)^{r_1}(3x)^{r_2}(4)^{r_3}$

Since, a_7 = Coeff. of x^7

$2r_1+r_2=7$ and $r_1+r_2+r_3=10$

Possibilities are

r_1	r_2	r_3
0	7	3
1	5	4
2	3	5
3	1	6

$$a_7=\frac{10!3^74^3}{7!3!}+\frac{10!(2)(3)^5(4)^4}{5!4!}+\frac{10!(2)^2(3)^3(4)^5}{2!3!5!}+\frac{10!(2)^3(3)(4)^6}{3!6!}$$

a_{13} = Coeff. of x^{13}

$2r_1+r_2=13$ and $r_1+r_2+r_3=10$

Possibilities are

r_1	r_2	r_3
3	7	0
4	5	1
5	3	2
6	1	3

$$a_{13}=\frac{10!(2^3)(3^7)}{3!7!}+\frac{10!(2^4)(3^5)(4)}{4!5!}+\frac{10!(2^5)(3^3)(4^2)}{5!\,3!\,2!}+\frac{10!(2^6)(3)(4^3)}{6!\,1!\,3!}$$

$$\therefore \frac{a_7}{a_{13}}=2^3=8$$

5. **(d)** Using Binomial expansion

$(x+a)^n+(x-a)^n=2(T_1+T_3+T_5+T_7...)$

$$\therefore\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6=2(T_1+T_3+T_5+T_7)$$

$2[{}^6C_0x^5+{}^6C_2x^4(x^2-1)+{}^6C_4x^2(x^2-1)^2+{}^6C_6(x^2-1)^3]$

$=2[x^6+15(x^6-x^4)+15x^2(x^4-2x^2+1)+(-1+3x^2-3x^4+x^6)]$

$=2(32x^6-48x^4+18x^2-1)$

$\alpha=-96$ and $\beta=36$

$\therefore\quad \alpha-\beta=-132$

6. **(a)** $\left(x^2+\frac{1}{x^2}\right)^n$

General term $T_{r+1}={}^nC_r\left(x^2\right)^{n-r}\left(\frac{1}{x^3}\right)^r={}^nC_r\cdot x^{2n-5r}$

To find coefficient of x, $2n-5r=1$

Given ${}^nC_r={}^nC_{23}\Rightarrow r=23$ or $n-r=23$

$\therefore$ $n=58$ or $n=38$

Minimum value is $n=38$

7. **(b)** $\because T_4=20\times 8^7$

$$\Rightarrow {}^6C_3\left(\frac{2}{x}\right)^3\times(x^{\log_8 x})^3=20\times 8^7$$

$$\Rightarrow 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3=20\times 8^7\Rightarrow\frac{x^{\log_8 x}}{x}=64$$

Now, take $\log_8$ on both sides, then

$(\log_8 x)^2-(\log_8 x)=2$

$\Rightarrow \log_8 x=-1$ or $\log_8 x=2$

$\Rightarrow x=\frac{1}{8}$ or $x=8^2$

8. **(b)** Given ${}^nC_{r-1}:{}^nC_r:{}^nC_{r+1}=2:15:70$

$$\Rightarrow\frac{{}^nC_{r-1}}{{}^nC_r}=\frac{2}{15}\text{ and }\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{15}{70}$$

$$\Rightarrow\frac{r}{n-r+1}=\frac{2}{15}\text{ and }\frac{r+1}{n-r}=\frac{3}{14}$$

$\Rightarrow 17r=2n+2$ and $17r=3n-14$

i.e., $2n+2=3n-14\Rightarrow n=16$ & $r=2$

$$\therefore\text{ Average }=\frac{{}^{16}C_1+{}^{16}C_2+{}^{16}C_3}{3}=\frac{16+120+560}{3}$$

$$=\frac{696}{3}=232$$

9. **(d)** $(x+\sqrt{x^3-1})^6+(x-\sqrt{x^3-1})^6$

$=2[{}^6C_0x^6+{}^6C_2x^4(x^3-1)+{}^6C_4x^2(x^3-1)^2+{}^6C_6(x^3-1)^3]$

$=2[x^6+15x^7-15x^4+15x^8-30x^5+15x^2+x^9-3x^6+3x^3-1]$

Hence, the sum of coefficients of even powers of $x=2[1-15+15+15-3-1]=24$

10. **(b)** $\therefore$ fourth term is equal to 200.

$$T_4={}^6C_3\left(\sqrt{x^{\left(\frac{1}{1+\log_{10}x}\right)}}\right)^3\left(x^{\frac{1}{12}}\right)^3=200$$

$$\Rightarrow 20x^{\frac{3}{2(1+\log_{10}x)}}.x^{\frac{1}{4}}=200$$

$$x^{\frac{1}{4}+\frac{3}{2(1+\log_{10}x)}}=10$$

Taking $\log_{10}$ on both sides and putting $\log_{10}x=t$

$$\left(\frac{1}{4}+\frac{3}{2(1+t)}\right)t=1\Rightarrow t^2+3t-4=0$$

$\Rightarrow t^2+4t-t-4=0\Rightarrow t(t+4)-1(t+4)=0$

$\Rightarrow t=1$ or $t=-4$

$\log_{10}x=1\Rightarrow x=10$

or $\log_{10}x=-4\Rightarrow x=10^{-4}$

According to the question $x>1$, $\therefore$ $x=10$.

11. **(c)** $(x+10)^{50}+(x-10)^{50}$

$=a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$

$\therefore$ $a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$

$=2({}^{50}C_0x^{50}+{}^{50}C_2x^{48}.10^2+{}^{50}C_4x^{46}.10^4+\ldots)$

$\therefore$ $a_0=2.{}^{50}C_{50}10^{50}$

$a_2=2.{}^{50}C_2.10^{48}$

$$\therefore\ \frac{a_2}{a_0}=\frac{{}^{50}C_2\times 10^{48}}{{}^{50}C_{50}10^{50}}$$

$$=\frac{50\times 49}{2\times 100}=\frac{49}{4}=12.25$$

12. **(a)** Third term of $(1+x^{\log_2 x})^5={}^5C_2(x^{\log_2 x})^{5-3}$

$={}^5C_2(x^{\log_2 x})^2$

Given, ${}^5C_2(x^{\log_2 x})^2=2560$

$\Rightarrow$ $(x^{\log_2 x})^2=256=(\pm 16)^2$

$\Rightarrow$ $x^{\log_2 x}=16$ or $x^{\log_2 x}=-16$ (rejected)

$\Rightarrow$ $x^{\log_2 x}=16\Rightarrow\log_2 x\log_2 x=\log_2 16=4$

$\Rightarrow$ $\log_2 x=\pm 2\Rightarrow x=2^2$ or 2^{-2}

$\Rightarrow$ $x=4$ or $\frac{1}{4}$

13. **(a)** Since, coefficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)$ is a constant term, then

Coefficient of x^2 in $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

$=$ co-efficient of constant term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

General term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}={}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r$

$= {}^{10}C_r (x)^{\frac{10-r}{2}-2r} \cdot \lambda^2$

Then, for constant term,

$\frac{10-r}{2} - 2r = 0 \Rightarrow r = 2$

Co-efficient is x^2 in expression = ${}^{10}C_2 \lambda^2 = 720$

$\Rightarrow \quad \lambda^2 = \frac{720}{5 \times 9} = 16 \Rightarrow \lambda = 4d$

Hence, required value of λ is 4.

14. **(b)** $2^{403} = 2^{400} \cdot 2^3$

$= 2^{4 \times 100} \cdot 2^3$

$= (2^4)^{100} \cdot 8$

$= 8(2^4)^{100} = 8(16)^{100}$

$= 8\,(1 + 15)^{100}$

$= 8 + 15\,\mu$

When 2^{403} is divided by 15, then remainder is 8.

Hence, fractional part of the number is $\frac{8}{15}$

Therefore value of k is 8

15. **(a)** $\because (1+x)^2 = 1 + 2x + x^2$,

$(1+x^2)^3 = 1 + 3x^2 + 3x^4 + x^6$,

and $(1+x^3)^4 = 1 + 4x^3 + 6x^6 + 4x^9 + x^{12}$

So, the possible combinations for x^{10} are:

$x \,.\, x^9, x \,.\, x^6 \,.\, x^3, x^2 \,.\, x^2 \,.\, x^6, x^4 \,.\, x^6$

Corresponding coefficients are $2 \times 4, 2 \times 1 \times 4, 1 \times 3 \times 6, 3 \times 6$ or 8, 8, 18, 18.

$\therefore$ Sum of the coefficient is $8 + 8 + 18 + 18 = 52$

Therefore, the coefficient of x^{10} in the expansion of $(1+x)^2 (1+x^2)^3 (1+x^3)^4$ is 52.

16. **(d)** $\left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1} + \sqrt{5x^3-1}}\right]^8$

After rationalise the polynomial we get

$$\left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1} + \sqrt{5x^3-1}}{\sqrt{5x^3+1} + \sqrt{5x^3-1}}\right]^8$$

$$+\left[\frac{1}{\sqrt{5x^3+1} + \sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1} - \sqrt{5x^3-1}}{\sqrt{5x^3+1} - \sqrt{5x^3-1}}\right]^8$$

$$= \left[\frac{\sqrt{5x^3+1} + \sqrt{5x^3-1}}{(5x^3+1) - (5x^3-1)}\right]^8 + \left[\frac{\sqrt{5x^3+1} - \sqrt{5x^3-1}}{(5x^3+1) - (5x^3-1)}\right]^8$$

$$= \frac{1}{2^8}\left[\left(\sqrt{5x^3+1} + \sqrt{5x^3-1}\right)^8 + \left(\sqrt{5x^3+1} - \sqrt{5x^3-1}\right)^8\right]$$

$$= \frac{1}{2^8}\left[\begin{array}{l} {}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(\sqrt{5x^3-1}\right)^2 \\ + {}^8C_4\left(\sqrt{5x^3+1}\right)^4\left(\sqrt{5x^3-1}\right)^4 \\ + {}^8C_6\left(\sqrt{5x^3+1}\right)^2\left(\sqrt{5x^3-1}\right)^6 + {}^8C_8\left(\sqrt{5x^3-1}\right)^8 \end{array}\right]$$

$$= \frac{1}{2^8}\left[\begin{array}{l} {}^8C_0(5x^3+1)^4 + {}^8C_2\,(5x^3+1)^3\,(5x^3-1) + 8_{C_4} \\ (5x^3+1)^2\,(5x^3-1)^2 \\ + {}^8C_6(5x^3+1)\,(5x^3-1)^3 + 8_{C_8}(5x^3-1)^4 \end{array}\right]$$

So, the degree of polynomial is 12,

Now, coefficient of $x^{12} = [{}^8C_0 5^4 + {}^8C_2 5^4 + {}^8C_4 5^4 + {}^8C_6 5^4$

$+ {}^8C_8 5^4]$

$= 5^4 \times \frac{2^8}{2} = 5^4 \times 2^4 \times \frac{2^4}{2}$

$= 10^4 \times 2^3 = 8\,(10)^4$

17. **(a)** Let $a = ((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$\therefore$ Coefficient of x^2 in the expansion of the product

$(2 - x^2)\,((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$= 2$ (Coefficient of x^2 in a) $- 1$ (Constant of expansion)

In the expansion of $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$.

Constant $= 1 + 1 = 2$

Coefficient of x^2 = [Coefficient of x^2 in $({}^6C_0\,(1+2x)^6\,(3x^2)^0)$] + [Cofficient of x^2 in $({}^6C_1\,(1+2x)^5\,(3x^2)^1)$]

$- [{}^6C_1\,(4x^2)]$

$= 60 + 6 \times 3 - 24 = 54$

$\therefore$ The coefficient of x^2 in $(2 - x^2)\,((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$= 2 \times 54 - 1\,(2) = 108 - 2 = 106$

18. **(c)** Since we know that,

$(x + a)^5 + (x - a)^5$

$= 2[{}^5C_0\,x^5 + {}^5C_2\,x^3 \cdot a^2 + {}^5C_4 x \cdot a^4]$

$\therefore \quad \left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$

$= 2[{}^5C_0\,x^5 + {}^5C_2\,x^3(x^3 - 1) + {}^5C_4 x(x^3 - 1)^2]$

$\Rightarrow \quad 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

$\therefore$ Sum of coefficients of odd degree terms = 2.

19. **(a)** $\left[\dfrac{\left(x^{1/3}+1\right)\left(x^{2/3}-x^{1/3}+1\right)}{\left(x^{2/3}-x^{1/3}+1\right)}-\dfrac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{\sqrt{x}\left(\sqrt{x}-1\right)}\right]^{10}$

$=\left(x^{1/3}+1-1-1/x^{1/2}\right)^{10}=\left(x^{1/3}-1/x^{1/2}\right)^{10}$

$T_{r+1}={}^{10}C_r\ x^{\frac{20-5r}{6}}$

for r = 10

$T_{11}={}^{10}C_{10}\ x^{-5}$

Coefficient of $x^{-5}={}^{10}C_{10}(1)(-1)^{10}=1$

20. **(d)** $\dfrac{(28-1)^{999}}{7}=\dfrac{28\lambda-1}{7}$

$\Rightarrow\ \dfrac{28\lambda-7+7-1}{7}=\dfrac{7(4\lambda-1)+6}{7}$

$\therefore$ Remainder = 6

21. **(b)** $T_{r+1}={}^{18}C_r\left(x^{\frac{1}{3}}\right)^{18-r}\left(\dfrac{1}{2x^{\frac{1}{3}}}\right)^r={}^{18}C_r x^{6-\frac{2r}{3}}\dfrac{1}{2^r}$

$\left\{\begin{array}{l}6-\dfrac{2r}{3}=-2\ \Rightarrow r=12\\ \&\ 6-\dfrac{2r}{3}=-4\ \Rightarrow\ r=15\end{array}\right\}$

$\Rightarrow\ \dfrac{\text{coefficient of } x^{-2}}{\text{coefficient of } x^{-4}}=\dfrac{{}^{18}C_{12}\dfrac{1}{2^{12}}}{{}^{18}C_{15}\dfrac{1}{2^{15}}}=182$

22. **(c)** $\dfrac{{}^nC_r}{1}=\dfrac{{}^nC_{r+1}}{7}=\dfrac{{}^nC_{r+2}}{42}$

By solving we get r = 6

so, it is 7th term.

23. **(b)** Consider $(1+ax+bx^2)(1-2x)^{18}$

$=(1+ax+bx^2)[{}^{18}C_0-{}^{18}C_1(2x)+{}^{18}C_2(2x)^2-{}^{18}C_3(2x)^3+{}^{18}C_4(2x)^4-\ldots\ldots.]$

Coeff. of $x^3={}^{18}C_3(-2)^3+a.(-2)^2.{}^{18}C_2+b(-2).{}^{18}C_1=0$

Coeff. of $x^3=-{}^{18}C_3.8+a\times 4.\ {}^{18}C_2-2b\times 18=0$

$=-\dfrac{18\times 17\times 16}{6}.8+\dfrac{4a+18\times 17}{2}-36b=0$

$=-51\times 16\times 8+a\times 36\times 17-36b=0$

$=-34\times 16+51a-3b=0$

$=51a-3b=34\times 16=544$

$=51a-3b=544$ ....(i)

Only option number (b) satisfies the equation number (i)

24. **(b)** $4^n-3n-1=(1+3)^n-3n-1$

$=[{}^nC_0+{}^nC_1.3+{}^nC_2.3^2+\ldots\ldots+{}^nC_n3^n]-3n-1$

$=9\,[{}^nC_2+{}^nC_3.3+\ldots.+{}^nC_n.3^{n-2}]$

$\therefore\ 4^n-3n-1$ is a multiple of 9 for all n.

$\therefore\ X=\{x : x \text{ is a multiple of } 9\}$

Also, $Y=\{9(n-1) : n\in\mathbf{N}\}$

$=\{\text{All multiples of } 9\}$

Clearly $X\subset Y$. $\therefore\ X\cup Y=Y$

25. **(a)** $1+x^4+x^5=\sum_{i=0}^{5}a_i(1+x)^i$

$=a_0+a_1(1+x)^1+a_2(1+x)^2+a_3(1+x)^3+a_4(1+x)^4+a_5(1+x)^5$

$\Rightarrow 1+x^4+x^5$

$=a_0+a_1(1+x)+a_2(1+2x+x^2)+a_3(1+3x+3x^2+x^3)+a_4(1+4x+6x^2+4x^3+x^4)+a_5(1+5x+10x^2+10x^3+5x^4+x^5)$

$\Rightarrow 1+x^4+x^5$

$=a_0+a_1+a_1x+a_2+2a_2x+a_2x^2+a_3+3a_3x+3a_3x^2+a_3x^3+a_4+4a_4x+6a_4x^2+4a_4x^3+a_4x^4+a_5+5a_5x+10a_5x^2+10a_5x^3+5a_5x^4+a_5x^5$

$\Rightarrow 1+x^4+x^5$

$=(a_0+a_1+a_2+a_3+a_4+a_5)+x(a_1+2a_2+3a_3+4a_4+5a_5)+x^2(a_2+3a_3+6a_4+10a_5)+x^3(a_3+4a_4+10a_5)+x^4(a_4+5a_5)+x^5(a_5)$

On comparing the like coefficients, we get

$\boxed{a_5=1}$ ...(i) ; $\boxed{a_4+5a_5=1}$ ...(ii);

$\boxed{a_3+4a_4+10a_5=0}$ ...(iii)

and $\boxed{a_2+3a_3+6a_4+10a_5=0}$ ...(iv)

from (i) & (ii), we get

$\boxed{a_4=-4}$ ...(v) from (i), (iii) & (v), we get

$\boxed{a_3=+6}$ ...(vi)

Now, from (i), (v) and (vi), we get

$a_2=-4$

26. **(a)** Let r^{th} and $(r+1)^{\text{th}}$ term has equal coefficient

$\left(2+\dfrac{x}{3}\right)^{55}=2^{55}\left(1+\dfrac{x}{6}\right)^{55}$

r^{th} term $=2^{55}\ {}^{55}C_r\left(\dfrac{x}{6}\right)^r$

Coefficient of x^r is $2^{55}\,{}^{55}C_r \frac{1}{6^r}$

$(r+1)^{\text{th}}$ term $= 2^{55}\,{}^{55}C_{r+1}\left(\frac{x}{6}\right)^{r+1}$

Coefficient of x^{r+1} is $2^{55}\,{}^{55}C_{r+1}\cdot\frac{1}{6^{r+1}}$

Both coefficients are equal

$$2^{55}\,{}^{55}C_r \frac{1}{6^r} = 2^{55}\,{}^{55}C_{r+1}\frac{1}{6^{r+1}}$$

$$\frac{1}{\underline{|r}\,\underline{|55-r}} = \frac{1}{\underline{|r+1}\,\underline{|54-r}}\cdot\frac{1}{6}$$

$6(r+1) = 55 - r$
$6r + 6 = 55 - r$
$7r = 49$
$r = 7$
$(r+1) = 8$

Coefficient of 7^{th} and 8^{th} terms are equal.

27. **(c)** Given expansion is
$(1+x)^{101}(1-x+x^2)^{100}$
$= (1+x)(1+x)^{100}(1-x+x^2)^{100}$
$= (1+x)[(1+x)(1-x+x^2)]^{100}$
$= (1+x)[(1-x^3)^{100}]$

Expansion $(1-x^3)^{100}$ will have $100 + 1 = 101$ terms

So, $(1+x)(1-x^3)^{100}$ will have $2 \times 101 = 202$ terms

28. **(a)** Expansion of $(1+x)^{2n}$ is $1 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots\ldots + {}^{2n}C_r x^r + {}^{2n}C_{r+1} x^{r+1} + \ldots\ldots + {}^{2n}C_{2n} x^{2n}$

As given ${}^{2n}C_{r+2} = {}^{2n}C_{3r}$

$$\Rightarrow \frac{(2n)!}{(r+2)!\,(2n-r-2)!} = \frac{(2n)!}{(3r)!\,(2n-3r)!}$$

$\Rightarrow (3r)!\,(2n-3r)! = (r+2)!\,(2n-r-2)!$ ...(1)

Now, put value of n from the given choices.

Choice (a) put $n = 2r+1$ in (1)

LHS : $(3r)!\,(4r+2-3r)! = (3r)!\,(r+2)!$

RHS : $(r+2)!\,(3r)!$

$\Rightarrow$ LHS = RHS

29. **(d)** $(2^{1/2}+3^{1/5})^{10} = {}^{10}C_0(2^{1/2})^{10} + {}^{10}C_1(2^{1/2})^9(3^{1/5}) + \ldots\ldots + {}^{10}C_{10}(3^{1/5})^{10}$

There are only two rational terms – first term and last term.

Now sum of two rational terms

$= (2)^5 + (3)^2 = 32 + 9 = 41$

30. **(b)** Let $r+1 = 7 \Rightarrow r = 6$

Given expansion is

$$\left(\frac{3}{\sqrt[3]{84}} + \sqrt{3}\ln x\right)^9, x > 0$$

We have

$T_{r+1} = {}^nC_r (x)^{n-r} a^r$ for $(x+a)^n$.

$\therefore$ According to the question

$$729 = {}^9C_6\left(\frac{3}{\sqrt[3]{84}}\right)^3.(\sqrt{3}\ln x)^6$$

$$\Rightarrow 3^6 = 84 \times \frac{3^3}{84} \times 3^3 \times (6\ln x)$$

$\Rightarrow (\ln x)^6 = 1 \Rightarrow (\ln x)^6 = (\ln e)^6$
$\Rightarrow x = e$

31. **(a)** Consider $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$

$$= 2\left[{}^{2n}C_1\left(\sqrt{3}\right)^{2n-1} + {}^{2n}C_3\left(\sqrt{3}\right)^{2n-3} + {}^{2n}C_5\left(\sqrt{3}\right)^{2n-5} + \ldots\right]$$

$\because (a+b)^n - (a-b)^n$

$= 2[{}^nC_1 a^{n-1} b + {}^nC_3 a^{n-3} b^3 \ldots]$

= which is an irrational number.

32. **(a)** Given expansion is $\left(y^{\frac{1}{5}} + x^{\frac{1}{10}}\right)^{55}$

The general term is

$$T_{r+1} = {}^{55}C_r\left(y^{\frac{1}{5}}\right)^{55-r}.\left(x^{\frac{1}{10}}\right)^r$$

T_{r+1} would free from radical sign if powers of y and x are integers.

i.e. $\frac{55-r}{5}$ and $\frac{r}{10}$ are integer.

$\Rightarrow r$ is multiple of 10.

Hence, $r = 0, 10, 20, 30, 40, 50$

It is an A.P.

Thus, $50 = 0 + (k-1)\,10$

$50 = 10k - 10 \Rightarrow k = 6$

Thus, the six terms of the given expansion in which x and y are free from radical signs.

33. **(d)** Given function is

$f(y) = 1-(y-1)+(y-1)^2-(y-1)^3 + ... -(y-1)^{17}$

In the expansion of $(y-1)^n$

$T_{r+1} = {}^nC_r\, y^{n-r}(-1)^r$

coeff of y^2 in $(y-1)^2 = {}^2C_0$

coeff of y^2 in $(y-1)^3 = -{}^3C_1$

coeff of y^2 in $(y-1)^4 = {}^4C_2$

So, coeff of termwise is

${}^2C_0 + {}^3C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= 1 + {}^3C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= ({}^3C_0 + {}^3C_1) + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= {}^4C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= {}^5C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= {}^{18}C_{15} = {}^{18}C_3$

34. **(a)** **Statement 2 :**

$P(n): n^7 - n$ is divisible by 7

Put $n = 1$, $1 - 1 = 0$ is divisible by 7, which is true

Let $n = k$, $P(k): k^7 - k$ is divisible by 7, true

Put $n = k + 1$

$\therefore \quad P(k+1):(k+1)^7-(k+1)$ is div. by 7

$P(k+1): k^7 + {}^7C_1k^6 + {}^7C_2k^2 + ...+ {}^7C_6k + 1 - k - 1$, is div. by 7.

$P(k+1): (k^7 - k) + ({}^7C_1k^6 + {}^7C_2k^5 + ...+ {}^7C_6k)$ is div. by 7.

Since 7 is coprime with 1, 2, 3, 4, 5, 6.

So ${}^7C_1, {}^7C_2, ...{}^7C_6$ are all divisible by 7

$\therefore \quad P(k+1)$ is divisible by 7

Hence $P(n): n^7 - n$ is divisible by 7

Statement 1 : $n^7 - n$ is divisible by 7

$\Rightarrow \quad (n+1)^7-(n+1)$ is divisible by 7

$\Rightarrow \quad (n+1)^7 - n^7 - 1 + (n^7 - n)$ is divisible by 7

$\Rightarrow \quad (n+1)^7 - n^7 - 1$ is divisible by 7

Hence both Statements 1 and 2 are correct and Statement 2 is the correct explanation of Statement -1.

35. **(b)** $(1-x-x^2+x^3)^6 = [(1-x)-x^2(1-x)]^6$

$= (1-x)^6(1-x^2)^6$

$= (1-6x+15x^2-20x^3+15x^4-6x^5+x^6) \times (1-6x^2+15x^4 - 20x^6+15x^8-6x^{10}+x^{12})$

Coefficient of $x^7 = (-6)(-20) + (-20)(15) + (-6)(-6)$

$= -144$

36. **(a)** $(8)^{2n} - (62)^{2n+1}$

$= (64)^n - (62)^{2n+1}$

$= (63+1)^n - (63-1)^{2n+1}$

$$= \left[{}^nC_0(63)^n + {}^nC_1(63)^{n-1} + {}^nC_2(63)^{n-2} + ...+ {}^nC_{n-1}(63) + {}^nC_n\right]$$

$$- \left[{}^{2n+1}C_0(63)^{2n+1} - {}^{2n+1}C_1(63)^{2n} + {}^{2n+1}C_2(63)^{2n-1} - ....+ (-1)^{2n+1}\,{}^{2n+1}C_{2n+1}\right]$$

$$= 63 \times \left[{}^nC_0(63)^{n-1} + {}^nC_1(63)^{n-2} + {}^nC_2(63)^{n-3} + ...+ {}^nC_{n-1}\right] + 1 - 63 \times \left[{}^{2n+1}C_0(63)^{2n} - {}^{2n+1}C_1(63)^{2n-1} + ...+ {}^{2n+1}C_{2n}\right] + 1$$

$= 63 \times$ some integral value $+ 2$

Hence, when divided by 9 leaves 2 as the remainder.

37. **(b)** **From statement 2:**

$$\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = \sum_{r=0}^{n} r.{}^nC_r x^r + \sum_{r=0}^{n}{}^nC_r x^r$$

$$= \sum_{r=1}^{n} r \cdot \frac{n}{r}\,{}^{n-1}C_{r-1}x^r + (1+x)^n$$

$$= nx\sum_{r=1}^{n}{}^{n-1}C_{r-1}x^{r-1} + (1+x)^n$$

$= nx(1+x)^{n-1} + (1+x)^n =$ RHS

$\therefore$ Statement 2 is correct.

Putting $x = 1$, we get

$$\sum_{r=0}^{n}(r+1)\,{}^nC_r = n \cdot 2^{n-1} + 2^n = (n+2)\cdot 2^{n-1}.$$

$\therefore$ Statement 1 is also true and statement 2 is a correct explanation for statement 1.

38. **(b)** $T_{r+1} = (-1)^r.\ {}^nC_r\,(a)^{n-r}.\,(b)^r$ is an expansion of $(a-b)^n$

$\therefore \quad$ 5th term $= t_5 = t_{4+1}$

$= (-1)^4.\,{}^nC_4\,(a)^{n-4}.(b)^4 = {}^nC_4 \,.\, a^{n-4}\,.\,b^4$

6th term $= t_6 = t_{5+1} = (-1)^5\,{}^nC_5\,(a)^{n-5}(b)^5$

Given $t_5 + t_6 = 0$

$\therefore\ {}^nC_4 \,.\, a^{n-4}\,.\,b^4 + (-{}^nC_5\,.\,a^{n-5}\,.\,b^5) = 0$

$$\Rightarrow \frac{n!}{4!(n-4)!}.\frac{a^n}{a^4}.b^4 - \frac{n!}{5!(n-5)!}.\frac{a^nb^5}{a^5} = 0$$

$$\Rightarrow \frac{n!.a^nb^4}{4!(n-5)!.a^4}\left[\frac{1}{(n-4)} - \frac{b}{5.a}\right] = 0 \quad [\because a \neq 0, b \neq 0]$$

$$\Rightarrow \frac{1}{n-4} - \frac{b}{5a} = 0 \Rightarrow \frac{a}{b} = \frac{n-4}{5}$$

39. **(d)** $(1-y)^m(1+y)^n$

$=[1-{}^mC_1y+{}^mC_2y^2-......]\ [1+{}^nC_1y+{}^nC_2y^2+.....]$

$=1+(n-m)y+\left\{\frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn\right\}y^2+.....$

$\therefore\ a_1=n-m=10$ ...(i)

and $a_2=\frac{m^2+n^2-m-n-2mn}{2}=10$

$(m-n)^2-(m+n)=20$

$\Rightarrow\ m+n=80$ ...(ii) [from (i)]

Solving (i) and (ii), we get

$\therefore\ m=35, n=45$

40. **(d)** T_{r+1} in the expansion

$\left[ax^2+\frac{1}{bx}\right]^{11}={}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$={}^{11}C_r\,(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$

For the Coefficient of x^7, we have

$22-3r=7\Rightarrow r=5$

$\therefore$ Coefficient of x^7

$={}^{11}C_5(a)^6\ (b)^{-5}$ ...(i)

Again T_{r+1} in the expansion

$\left[ax-\frac{1}{bx^2}\right]^{11}={}^{11}C_r(ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$

$={}^{11}C_r(a)^{11-r}(-1)^r\times(b)^{-r}(x)^{-2r+11-r}$

For the Coefficient of x^{-7}, we have

Now $11-3r=-7\Rightarrow 3r=18\ \Rightarrow\ r=6$

$\therefore$ Coefficient of x^{-7}

$={}^{11}C_6\,a^5\times1\times(b)^{-6}$ ...(ii)

$\because$ Coefficient of x^7 = Coefficient of x^{-7}

From (i) and (ii),

$\therefore\ {}^{11}C_5\,(a)^6\,(b)^{-5}={}^{11}C_6\,a^5\times(b)^{-6}$

$\Rightarrow ab=1.$

41. **(b)** Coeff. of x^n in $(1+x)(1-x)^n$

= coeff of x^n in

$(1+x)(1-{}^nC_1x+{}^nC_2x^2-....+(-1)^{n\,n}C_nx^n)$

$=(-1)^{n\,n}C_n+(-1)^{n-1\,n}C_{n-1}$

$=(-1)^n+(-1)^{n-1}.n$

$=(-1)^n(1-n)$

42. **(c)** $T_{r+1}={}^{256}C_r(\sqrt{3})^{256-r}(\sqrt[8]{5})^r$

$={}^{256}C_r(3)^{\frac{256-r}{2}}(5)^{r/8}$

Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer.

It is possible if r is an integral multiple of 8 and $0\le r\le256$

$\therefore\ r=33$

43. **(c)** $t_{r+2}={}^{2n}C_{r+1}\ x^{r+1}; t_{3r}={}^{2n}C_{3r-1}\ x^{3r-1}$

Given that, ${}^{2n}C_{r+1}={}^{2n}C_{3r-1}$;

$\Rightarrow\ r+1+3r-1=2n$

$\Rightarrow\ 2n=4r\Rightarrow\ n=2r$

44. **(a)** We know that $t_{p+1}={}^{p+q}C_p\,x^p$ and

$t_{q+1}={}^{p+q}C_q\,x^q$

$\because\ {}^{p+q}C_p={}^{p+q}C_q$.[Remember ${}^nC_r={}^nC_{n-r}$]

45. **(c)** General term $=T_{r+1}={}^{10}C_r(\sqrt{x})^{10-r}\cdot\left(-\frac{k}{x^2}\right)^r$

$={}^{10}C_r(-k)^r\cdot x^{\frac{10-r}{2}-2r}$

$={}^{10}C_r(-k)^r\cdot x^{\frac{10-5r}{2}}$

Since, it is constant term, then

$\frac{10-5r}{2}=0\Rightarrow r=2$

$\therefore {}^{10}C_2(-k)^2=405$

$\Rightarrow k^2=\frac{405\times2}{10\times9}=\frac{81}{9}=9$

$\therefore |k|=3$

46. **(a)** Consider the three consecutive coefficients of $(1+x)^{n+5}$ be ${}^{n+5}C_r,\ {}^{n+5}C_{r+1},\ {}^{n+5}C_{r+2}$

$\because\ \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}}=\frac{1}{2}$ (Given)

$\Rightarrow\ \frac{r+1}{n+5-r}=\frac{1}{2}\Rightarrow 3r=n+3$ ...(i)

and $\frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_{r+2}}=\frac{5}{7}$

$\Rightarrow\ \frac{r+2}{n+4-r}=\frac{5}{7}\Rightarrow 12r=5n+6$ ...(ii)

Solving (i) and (ii) we get $r=4$ and $n=6$

$\therefore$ Largest coefficient in the expansion is ${}^{11}C_6=462.$

47. **(c)** Here, $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^n$

$$T_{r+1}={}^nC_r(3)^{\frac{n-r}{2}}(5)^{\frac{r}{8}}$$

$\because \dfrac{n-r}{2}$ and $\dfrac{r}{8}$ are integer

So, r must be 0, 8, 16, 24 ......

Now $n=t_{33}=a+(n-1)d=0+32\times 8=256$

$\Rightarrow n=256$

48. **(c)** General term $=T_{r+1}={}^9C_r\left(\dfrac{3x^2}{2}\right)^{9-r}\left(-\dfrac{1}{3x}\right)^r$

$$={}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

The term is independent of x, then

$18-3r=0\Rightarrow r=6$

$$\therefore T_7={}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6={}^9C_3\left(\frac{1}{6}\right)^3$$

$$=\frac{9\times 8\times 7}{3\times 2\times 1}\left(\frac{1}{6}\right)^3=\left(\frac{7}{18}\right).$$

$$\therefore 18k=18\times\frac{7}{18}=7.$$

49. **(a)** General term of

$$(\alpha x^{\frac{1}{9}}+\beta x^{\frac{-1}{6}})^{10}={}^{10}C_r(\alpha x^{\frac{1}{9}})^{10-r}(\beta x^{\frac{-1}{6}})^r$$

$$={}^{10}C_r\alpha^{10-r}\beta^r(x)^{\frac{10-r}{9}-\frac{r}{6}}$$

Term independent of x if $\dfrac{10-r}{9}-\dfrac{r}{6}=0\Rightarrow r=4.$

$\therefore$ Term independent of $x={}^{10}C_4\alpha^6\beta^4$

Since $\alpha^3+\beta^2=4$

Then, by AM-GM inequality

$$\frac{\alpha^3+\beta^2}{2}\ge(\alpha^3b^2)^{\frac{1}{2}}$$

$$\Rightarrow (2)^2\ge\alpha^3\beta^2\Rightarrow\alpha^6\beta^4\le 16$$

$\because$ The maximum value of the term independent of $x=10k$

$$\therefore 10k={}^{10}C_4\cdot 16\Rightarrow k=336.$$

50. **(118)**

According to the question,

$${}^nC_{r-1}:{}^nC_r:{}^nC_{r+1}=2:5:12$$

$$\Rightarrow\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{5}{2}\Rightarrow\frac{n-r+1}{r}=\frac{5}{2}$$

$$\Rightarrow 2n-7r+2=0 \quad ...(i)$$

$$\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{12}{5}\Rightarrow\frac{n-r}{r+1}=\frac{12}{5}$$

$$\Rightarrow 5n-17r-12=0 \quad ...(ii)$$

Solving eqns. (i) and (ii),

$n=118, r=34$

51. **(b)** General term of the given expansion

$$T_{r+1}={}^{16}C_r\left(\frac{x}{\sin\theta}\right)^{16-r}\left(\frac{1}{x\cos\theta}\right)^r$$

For $r=8$ term is free from 'x'

$$T_9={}^{16}C_8\frac{1}{\sin^8\theta\cos^8\theta}$$

$$T_9={}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$$

When $\theta\in\left[\dfrac{\pi}{8},\dfrac{\pi}{4}\right]$, then least value of the term independent of x,

$l_1={}^{16}C_8 2^8$ [$\because$ min. value of l_1 at $\theta=\pi/4$]

When $\theta\in\left[\dfrac{\pi}{16},\dfrac{\pi}{8}\right]$, then least value of the term independent of x,

$$l_2={}^{16}C_8=\frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8}={}^{16}C_8.2^8.2^4$$

[$\because$ min. value of l_2 at $\theta=\pi/8$]

Now, $\dfrac{l_2}{l_1}=\dfrac{{}^{16}C_8.2^8.2^4}{{}^{16}C_8.2^8}=16$

52. **(d)** Let the general term of the expansion

$$T_{r+1}={}^{60}C_r\left(7^{\frac{1}{5}}\right)^{60-r}\left(-3^{\frac{1}{10}}\right)^r$$

$$={}^{60}C_r\cdot(7)^{12-\frac{r}{5}}(-1)^r\cdot(3)^{\frac{r}{10}}$$

Then, for getting rational terms, r should be multiple of L.C.M. of (5, 10)

Then, r can be 0, 10, 20, 30, 40, 50, 60.

Since, total number of terms = 61

Hence, total irrational terms = 61 – 7 = 54

53. (c) $\left(2^{\frac{1}{3}}+\dfrac{1}{2(3)^{\frac{1}{3}}}\right)^{10} = {}^{10}C_0\left(2^{\frac{1}{3}}\right)^0\left(\dfrac{1}{2(3)^{1/3}}\right)^{10}+$

$\cdots + {}^{10}C_{10}\left(2^{\frac{1}{3}}\right)^{10}\left(\dfrac{1}{2(3)^{1/3}}\right)^0$

5th term from beginning $T_5 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^6 \dfrac{1}{\left(2.3^{\frac{1}{3}}\right)^4}$

and 5th term from end $T_{11-5+1} = {}^{10}C_6\left(2^{\frac{1}{3}}\right)^4\left(\dfrac{1}{2.3^{\frac{1}{3}}}\right)^6$

$\therefore \quad T_5 : T_7 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^6\left(\dfrac{1}{2.3^{\frac{1}{3}}}\right)^4 : {}^{10}C_6\left(2^{\frac{1}{3}}\right)^4\left(\dfrac{1}{2.3^{\frac{1}{3}}}\right)^6$

$=\left(2^{\frac{1}{3}}\right)^2 : \left(\dfrac{1}{2.3^{\frac{1}{3}}}\right)^2$

$=\dfrac{2^{\frac{2}{3}}\cdot 2^2\cdot 3^{\frac{2}{3}}}{1} = 4(6)^{\frac{2}{3}} : 1 = 4.(36)^{\frac{1}{3}} : 1$

54. (c) General term of $\left(2x^2-\dfrac{1}{x}\right)^8$ is

${}^8C_r(2x^2)^{8-r}\left(\dfrac{-1}{x}\right)^r$

$\therefore$ Given expression is equal to

$\left(1-\dfrac{1}{x}+3x^5\right){}^8C_r(2x^2)^{8-r}\left(-\dfrac{1}{x}\right)^r$

$= {}^8C_r(2x^2)^{8-r}\left(-\dfrac{1}{x}\right)^r - \dfrac{1}{x}{}^8C_r(2x^2)^{8-r}\left(-\dfrac{1}{x}\right)^r$

$+3x^5.\,{}^8C_r(2x^2)^{8-r}\left(-\dfrac{1}{x}\right)^r$

$= {}^8C_r 2^{8-r}(-1)^r x^{16-3r} - {}^8C_r 2^{8-r}(-1)^r x^{15-3r}$

$+3.\,{}^8C_r 2^{(8-r)}\left(-\dfrac{1}{x}\right)^r(-1)^r x^{21-3r}$

For the term independent of x, we should have

$16-3r=0,\ 15-3r=0,\ 21-3r=0$

From the simplification we get r = 5 and r = 7

$\therefore -{}^8C_5(2^3)(-1)^5 - 3.\,{}^8C_7.2$

$+\left[\dfrac{8!}{5!3!}\times 8\right]-3\times\left[\dfrac{8!}{7!1!}\times 2\right]$

$=(56\times 8)-48$

$=448-6\times 8=448-48=400$

55. (c) Given expression can be written as

$\left[\dfrac{\left(x^{1/3}\right)^3+1^3}{x^{2/3}-x^{1/3}+1}-\dfrac{\left(\sqrt{x}\right)^2-1^2}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$

$=\left((x^{1/3}+1)-\left(\dfrac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10} = \left(x^{1/3}+1-1-\dfrac{1}{\sqrt{x}}\right)^{10}$

$=(x^{1/3}-x^{-1/2})^{10}$

General term $= T_{r+1}$

$= {}^{10}C_r\,(x^{1/3})^{10-r}(-x^{-1/2})^r = {}^{10}C_r\,x^{\frac{10-r}{3}}\cdot(-1)^r\cdot x^{-\frac{r}{2}}$

$= {}^{10}C_r(-1)^r\cdot x^{\frac{10-r}{3}-\frac{r}{2}}$

Term will be independent of x when $\dfrac{10-r}{3}-\dfrac{r}{2}=0$

$\Rightarrow \quad r=4$

So, required term $= T_5 = {}^{10}C_4 = 210$

56. (d) $T_{r+1} = {}^{15}C_r(x^2)^{15-r}.\,(2x^{-1})^r = {}^{15}C_r\times(2)^r\times x^{30-3r}$

For independent term, $30-3r=0 \Rightarrow r=10$

Hence the term independent of x,

$T_{11} = {}^{15}C_{10}\times(2)^{10}$

For term involve x^{15}, $30-3r=15 \Rightarrow r=5$

Hence coefficient of $x^{15} = {}^{15}C_5\times(2)^5$

Required ratio $=\dfrac{{}^{15}C_5\times(2)^5}{{}^{15}C_{10}\times(2)^{10}} = \dfrac{\dfrac{15!}{10!5!}}{\dfrac{15!}{5!10!}\times(2)^5}$

$=1:32$

57. (d) Given expansion can be written as

$\left(\dfrac{x-1}{x}\right)^n.(1-x)^n = (-1)^n x^{-n}(1-x)^{2n}$

Total number of terms will be $2n+1$ which is odd ($\because$ $2n$ is always even)

$\therefore$ Middle term $=\dfrac{2n+1+1}{2}=(n+1)$ th

Now, $T_{r+1} = {}^nC_r(1)^r x^{n-r}$

So, $\dfrac{{}^{2n}C_n\cdot x^{2n-n}}{x^n.(-1)^n} = {}^{2n}C_n\,.(-1)^n$

Middle term is an odd term. So, $n+1$ will be odd.

So, n will be even.

$\therefore$ Required answer is ${}^{2n}C_n$.

58. **(c)** The middle term in the expansion of

$(1+\alpha x)^4 = T_3 = {}^4C_2(\alpha x)^2 = 6\alpha^2 x^2$

The middle term in the expansion of

$(1-\alpha x)^6 = T_4 = {}^6C_3(-\alpha x)^3 = -20\alpha^3 x^3$

According to the question

$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\frac{3}{10}$

59. **(a)** The given series, $\sum_{r=0}^{20} {}^{50-r}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + {}^{47}C_6 + ... + {}^{32}C_6 + {}^{31}C_6 + {}^{30}C_6$

$= ({}^{30}C_7 + {}^{30}C_6) + {}^{31}C_6 + {}^{32}C_6 + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$= ({}^{31}C_7 + {}^{31}C_6) + {}^{32}C_6 + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$= ({}^{32}C_7 + {}^{32}C_6) + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

....

....

....

$= {}^{51}C_7 - {}^{30}C_7$

60. **(615)** General term of the expansion $= \frac{10!}{\alpha!\beta!\gamma!} x^{\beta+2\gamma}$

For coefficient of x^4; $\beta + 2\gamma = 4$

Here, three cases arise

Case-1 : When $\gamma = 0, \beta = 4, \alpha = 6$

$\Rightarrow \frac{10!}{\alpha!\beta!\gamma!} x^{\beta+2\gamma}$

Case-2 : When $\gamma = 1, \beta = 2, \alpha = 7$

$\Rightarrow \frac{10!}{7!2!1!} = 360$

Case-3 : When $\gamma = 2, \beta = 0, \alpha = 8$

$\Rightarrow \frac{10!}{8!0!2!} = 45$

Hence, total = 615

61. **(30)** Let $(1 - x + x^2 ..... x^{2n})(1 + x + x^2 ..... x^{2n})$

$= a_0 + a_1x + a_2x^2 + ...$

put $x = 1$

$1(2n+1) = a_0 + a_1 + a_2 + ... a_{2n}$ ...(i)

put $x = -1$

$(2n+1) \times 1 = a_0 - a_1 + a_2 + ... a_{2n}$ ...(ii)

Adding (i) and (ii), we get,

$4n + 2 = 2(a_0 + a_2 + ...) = 2 \times 61$

$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30.$

62. **(d)** Given expression is,

$\left(\frac{1}{60} - \frac{x^8}{81}\right)\left(2x^2 - \frac{3}{x^2}\right)^6$

$= \frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{x^8}{81}\left(2x^2 - \frac{3}{x^2}\right)^6$

Term independent of x,

$=$ Coefficient of x° in $\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81}$.

coefficient of x^{-8} in $\left(2x^2 - \frac{3}{x^2}\right)^6$

$= \frac{-1}{60}\ {}^6C_3(2)^3(3)^3 + \frac{1}{81}\ {}^6C_5(2)(3)^5$

$= -72 + 36 = -36$

63. **(b)** Given, ${}^{20}C_1 + 2^2.{}^{20}C_2 + 3^2.{}^{20}C_3 + ... + 20^2.{}^{20}C_{20}$

$= A(2^\beta)$

Taking L.H.S.,

$= \sum_{r=1}^{20} r^2.\ {}^{20}C_r = 20\sum_{r=1}^{20} r.\ {}^{19}C_{r-1}$

$= 20\left[\sum_{r=1}^{20}(r-1)\ {}^{19}C_{r-1} + \sum_{r=1}^{20}{}^{19}C_{r-1}\right]$

$= 20\left[19\sum_{r=2}^{20}{}^{18}C_{r-2} + 2^{19}\right] = 20[19 \,.\, 2^{18} + 2^{19}]$

$= 420 \times 2^{18}$

Now, compare it with R.H.S., A = 420 and $\beta = 18$

64. **(a)** Given expression,

$(1-x)^{10}(1+x+x^2)^9(1+x) = (1-x^3)^9(1-x^2)$

$= (1-x^3)^9 - x^2(1-x^3)^9$

$\Rightarrow$ Coefficient of x^{18} in $(1-x^3)^9$ – coeff. of x^{16} in $(1-x^3)^9$

$= {}^9C_6 - 0 = \frac{9!}{6!3!} = \frac{7\times8\times9}{6} = 84$

65. **(c)** Given expression is $(1 + ax + bx^2)(1 - 3x)^{15}$

Co-efficient of $x^2 = 0$

$\Rightarrow {}^{15}C_2(-3)^2 + a\,.\,{}^{15}C_1(-3) + b\,.\,{}^{15}C_0 = 0$

$\Rightarrow \frac{15\times14}{2}\times9 - 15\times3a + b = 0$

$\Rightarrow 945 - 45a + b = 0$ ...(i)

Now, co-efficient of $x^3 = 0$

$\Rightarrow {}^{15}C_3(-3)^3 + a\,.\,{}^{15}C_2(-3)^2 + b\,.{}^{15}C_1(-3) = 0$

$\Rightarrow \frac{15\times14\times13}{3\times2}\times(-3\times3\times3) + a\times\frac{15\times14\times9}{2} - b\times3\times15 = 0$

$\Rightarrow 15 \times 3[-3 \times 7 \times 13 + a \times 7 \times 3 - b] = 0$

$\Rightarrow 21a - b = 273$ ...(ii)

From (i) and (ii), we get,

$a = 28, b = 315 \Rightarrow (a, b) \equiv (28, 31{,}5)$

66. **(b)** $2.{}^{20}C_0+5.{}^{20}C_1+8.{}^{20}C_2+......+62.{}^{20}C_{20}$

$$=\sum_{r=0}^{20}(3r+2)\ {}^{20}C_r=3\sum_{r=0}^{20}r.\ {}^{20}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=60\sum_{r=1}^{20}{}^{19}C_{n-1}+2\sum_{r=0}^{20}{}^{20}C_r$$

$=60\times 2^{19}+2\times 2^{20}=2^{21}[15+1]=2^{25}$

67. **(a)** Middle Term, $\left(\frac{n}{2}+1\right)^{\text{th}}$ term in the binomial

expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ is,

$$T_{4+1}={}^8C_4\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4=5670$$

$$\Rightarrow\ \frac{8\times7\times6\times5}{4\times3\times2}\times x^{12-4}=5670$$

$\Rightarrow\ x^8=81$

$\Rightarrow\ x^8-81=0$

$\therefore$ sum of all values of x = sum of roots of equation ($x^8-81=0$).

68. **(b)** Consider the expression

${}^{20}C_r\,{}^{20}C_0+{}^{20}C_{r-1}\,{}^{20}C_1+{}^{20}C_{r-2}\,{}^{20}C_2+...+{}^{20}C_0\cdot{}^{20}C_r$

For maximum value of above expression r should be equal to 20.

as ${}^{20}C_{20}\cdot{}^{20}C_0+{}^{20}C_{19}\cdot{}^{20}C_1+...+{}^{20}C_{20}\cdot{}^{20}C_0$

$=({}^{20}C_0)^2+({}^{20}C_1)^2+...+({}^{20}C_{20})^2={}^{40}C_{20}$.

Which is the maximum value of the expression,

So, $r=20$.

69. **(d)** $$\sum_{r=0}^{25}\left({}^{50}C_r\,.{}^{50-r}C_{25-r}\right)=\sum_{r=0}^{25}\left(\frac{\lfloor 50}{\lfloor 50-r\,\lfloor r}\,\frac{\lfloor 50-r}{\lfloor 25\,\lfloor 25-r}\right)$$

$$=\sum_{r=0}^{25}\left(\frac{\lfloor 50}{\lfloor 25}\times\frac{1}{\lfloor 25}\times\left(\frac{\lfloor 25}{\lfloor 25-r\,\lfloor r}\right)\right)$$

$$={}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r={}^{50}C_{25}(2^{25})$$

Then, by comparison, $K=2^{25}$

70. **(b)** Consider the expression

$$\left(\frac{1-t^6}{1-t}\right)^3=(1-t^6)^3(1-t)^{-3}$$

$$=(1-3t^6+3t^{12}-t^{18})\left(1+3t+\frac{3\cdot4}{2!}t^2+\frac{3\cdot4\cdot5}{3!}t^3+\frac{3\cdot4\cdot5\cdot6}{4!}t^4+...\infty\right)$$

Hence, the coefficient of $t^4=1\cdot\frac{3\cdot4\cdot5\cdot6}{4!}$

$$=\frac{3\times4\times5\times6}{4\times3\times2\times1}=15$$

71. **(a)** We have $({}^{21}C_1+{}^{21}C_2\ ......+{}^{21}C_{10})$

$-({}^{10}C_1+{}^{10}C_2\ .....\ {}^{10}C_{10})$

$$=\frac{1}{2}[({}^{21}C_1+....+{}^{21}C_{10})+({}^{21}C_{11}+....{}^{21}C_{20})]-(2^{10}-1)$$

$(\because\ {}^{10}C_1+{}^{10}C_2+....+{}^{10}C_{10}=2^{10}-1)$

$$=\frac{1}{2}[2^{21}-2]-(2^{10}-1)$$

$=(2^{20}-1)-(2^{10}-1)=2^{20}-2^{10}$

72. **(b)** Total number of terms $={}^{n+2}C_2=28$

$(n+2)(n+1)=56;\ n=6$

$\therefore$ Put $x=1$ in expansion $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^6$,

we get sum of coefficient $=(1-2+4)^6$

$=3^6=729$.

73. **(c)** We know that $(a+b)^n+(a-b)^n$

$=2[{}^nC_0a^nb^0+{}^nC_2a^{n-2}b^2+{}^nC_4a^{n-4}b^4...]$

$(1-2\sqrt{x})^{50}+(1+2\sqrt{x})^{50}$

$2\left[{}^{50}C_0+{}^{50}C_2(2\sqrt{x})^2+{}^{50}C_4(2\sqrt{x})^4...\right]$

$=2\left[{}^{50}C_0+{}^{50}C_2 2^2x+{}^{50}C_4 2^4x^2+...\right]$

Putting $x=1$, we get,

$${}^{50}C_0+{}^{50}C_2 2^2+{}^{50}C_4 2^4...=\frac{3^{50}+1}{2}$$

74. **(b)** Given expansion $(1+x^n+x^{253})^{10}$

Let $x^{1012}=(1)^a\ (x^n)^b.\ (x^{253})^c$

Here $a,\ b,\ c,\ n$ are all +ve integers and $a\le 10$, $b\le 10$, $c\le 4$, $n\le 22$, $a+b+c=10$

Now $bn+253c=1012$

$\Rightarrow bn=253\ (4-c)$

For $c<4$ and $n\le 22$; $b>10$, which is not possible.

$\therefore\ c=4,\ b=0,\ a=6$

$\therefore\ x^{1012}=(1)^6.\ (x^n)^0\ .\ (x^{253})^4$

Hence the coefficient of $x^{1012}=\frac{10!}{6!0!4!}={}^{10}C_4$

75. **(b)** $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j = \sum_{j=1}^{10} 10\,{}^{9}C_{j-1}$

$\left[\because {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}\right]$

$= 10\left[{}^9C_0 + {}^9C_1 + {}^9C_2 + ... + {}^9C_9\right] = 10.2^9$

76. **(a)** The given situation in statement 1 is equivalent to find the non negative integral solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 6$
which is coeff. of x^6 in the expansion of
$(1 + x + x^2 + x^3 + ...\infty)^5$= coeff. of x^6 in $(1-x)^{-5}$

= coeff. of x^6 in $1 + 5x + \frac{5.6}{2!}x^2$..........

$= \frac{5\cdot6\cdot7\cdot8\cdot9\cdot10}{6!} = \frac{10!}{6!4!} = {}^{10}C_6$

$\therefore$ Statement 1 is wrong.
Number of ways of arranging 6A's and 4B's in a row

$= \frac{10!}{6!4!} = {}^{10}C_6$ which is same as the number of ways the child can buy six icecreams.

$\therefore$ Statement 2 is true.

77. **(d)** We know that, $(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + ...\ {}^{20}C_{10}x^{10} + ...\ {}^{20}C_{20}x^{20}$

Put $x = -1$, $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} - {}^{20}C_{11} .... + {}^{20}C_{20}$

$\Rightarrow\ 0 = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9] + {}^{20}C_{10}$

$\Rightarrow\ {}^{20}C_{10} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10}]$

$\Rightarrow\ {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10}$

$= \frac{1}{2}\ {}^{20}C_{10}$

78. **(c)** $\because$ x^3 and higher powers of x may be neglected

$$\therefore \frac{(1+x)^{\frac{3}{2}} - \left(1+\frac{x}{2}\right)^3}{\left(1-x^{\frac{1}{2}}\right)}$$

$$= (1-x)^{\frac{-1}{2}}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}\cdot\frac{1}{2}}{2!}x^2\right) - \left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$= \left[1+\frac{x}{2}+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right] = \frac{-3}{8}x^2$$

79. **(d)** $T_{r+1} = \frac{n(n-1)(n-2).........(n-r+1)}{r!}(x)^r$

For first negative term,
$n - r + 1 < 0 \Rightarrow r > n + 1$

$\Rightarrow r > \frac{32}{5}\ \therefore r = 7.\ \left(\because n = \frac{27}{5}\right)$

Therefore, first negative term is T_8.

80. **(d)** $(1 + 0.0001)^{10000} = \left(1+\frac{1}{n}\right)^n,\ n = 10000$

$= 1 + n.\frac{1}{n} + \frac{n(n-1)}{2!}\frac{1}{n^2} + \frac{n(n-1)(n-2)}{3!}\frac{1}{n^3} + ... + \frac{1}{n^n}$

$= 1 + 1 + \frac{1}{2!}\left(1-\frac{1}{n}\right) + \frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) + ... + \frac{1}{n^n}$

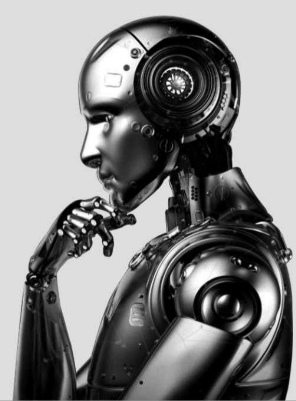
$< 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{(9999)!}$

$= 1 + \frac{1}{1!} + \frac{1}{2!} + ...\infty = e < 3$

81. **(c)** Take $a = 1$ and $b = 1$ in $(a + b)^n$.
$2^n = 4096 = 2^{12} \Rightarrow n = 12$;
The greatest coeff = coeff of middle term.
So middle term = t_7.
$\Rightarrow t_7 = t_{6+1} = {}^{12}C_6a^6b^6$

$\Rightarrow$ Coeff of $t_7 = {}^{12}C_6 = \frac{12!}{6!6!} = 924.$

9



Sequences and Series

TOPIC 1 Arithmetic Progression

1. The common difference of the A.P. $b_1, b_2, ..., b_m$ is 2 more than the common difference of A.P. $a_1, a_2, ..., a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to: **[Sep. 06, 2020 (II)]**
(a) 81 (b) -127 (c) -81 (d) 127

2. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P is: **[Sep. 05, 2020 (I)]**
(a) 66 (b) 81 (c) 65 (d) 78

3. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + ...$ is 460, then x is equal to : **[Sep. 05, 2020 (II)]**
(a) 7^2 (b) $7^{1/2}$ (c) e^2 (d) $7^{46/21}$

4. Let $a_1, a_2, ..., a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ... + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to : **[Sep. 04, 2020 (II)]**
(a) (2490, 249) (b) (2480, 249)
(c) (2480, 248) (d) (2490, 248)

5. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is : **[Sep. 03, 2020 (I)]**
(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{7}$

6. In the sum of the series
$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$ upto n^{th} term is 488 and then n^{th} term is negative, then : **[Sep. 03, 2020 (II)]**
(a) $n = 60$ (b) n^{th} term is -4
(c) $n = 41$ (d) n^{th} term is $-4\frac{2}{5}$

7. If the sum of first 11 terms of an A.P., $a_1, a_2, a_3, ...$ is 0 $(a_1 \ne 0)$, then the sum of the A.P., $a_1, a_3, a_5, ..., a_{23}$ is ka_1, where k is equal to : **[Sep. 02, 2020 (II)]**
(a) $-\frac{121}{10}$ (b) $\frac{121}{10}$ (c) $\frac{72}{5}$ (d) $-\frac{72}{5}$

8. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is_____. **[NA Jan. 9, 2020 (II)]**

9. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is: **[Jan. 8, 2020 (II)]**
(a) 50 (b) $50\frac{1}{4}$ (c) 100 (d) $100\frac{1}{2}$

10. Let $f: R \to R$ be such that for all $x \in R$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is: **[Jan. 8, 2020 (I)]**
(a) 2 (b) 3 (c) 0 (d) 4

11. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is: **[Jan. 7, 2020 (I)]**
(a) 27 (b) 7 (c) $\frac{21}{2}$ (d) 16

12. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to : **[April 12, 2019 (I)]**
(a) –260 (b) –410 (c) –320 (d) –380

13. If $a_1, a_2, a_3, ...$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is : **[April 12, 2019 (II)]**
(a) 200 (b) 280 (c) 120 (d) 150

14. If $a_1, a_2, a_3, ... a_n$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to : **[April 10, 2019 (I)]**
(a) 98 (b) 76 (c) 38 (d) 64

15. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots\ldots\ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to: **[April 09, 2019 (I)]**

(a) $(50, 50+46A)$ (b) $(50, 50+45A)$
(c) $(A, 50+45A)$ (d) $(A, 50+46A)$

16. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function f satisfies $f(x+y) = f(x)\,f(y)$ for all natural numbers x, y and $f(a) = 2$. Then the natural number 'a' is: **[April 09, 2019 (I)]**

(a) 2 (b) 16 (c) 4 (d) 3

17. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} term is: **[April 09, 2019 (II)]**

(a) –35 (b) 25 (c) –36 (d) –25

18. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is : **[April 08, 2019 (I)]**

(a) 3203 (b) 3303 (c) 3221 (d) 3121

19. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be : **[Jan. 12, 2019 (II)]**

(a) 9 (b) 14 (c) 11 (d) 12

20. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is : **[Jan. 11, 2019 (II)]**

(a) 4 : 1 (b) 1 : 3 (c) 3 : 1 (d) 2 : 1

21. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is: **[Jan. 10, 2019 (I)]**

(a) 1256 (b) 1465 (c) 1365 (d) 1356

22. Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$.

If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to: **[Jan. 09, 2019 (I)]**

(a) 52 (b) 57 (c) 47 (d) 42

23. Let $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots\ldots$, ($x_i \neq 0$ for $i = 1, 2, \ldots., n$) be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to. **[Online April 16, 2018]**

(a) 3 (b) $\frac{13}{8}$ (c) $\frac{13}{4}$ (d) $\frac{1}{8}$

24. If $x_1, x_2, \ldots.., x_n$ and $\frac{1}{h_1}, \frac{1}{h^2}, \ldots.. \frac{1}{h_n}$ are two A.P's such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 . h_{10}$ equals. **[Online April 15, 2018]**

(a) 2560 (b) 2650 (c) 3200 (d) 1600

25. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to : **[2018]**

(a) 68 (b) 34 (c) 33 (d) 66

26. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then : **[2017]**

(a) a, b and c are in G.P.
(b) b, c and a are in G.P.
(c) b, c and a are in A.P.
(d) a, b and c are in A.P.

27. If three positive numbers a, b and c are in A.P. such that $abc = 8$, then the minimum possible value of b is : **[Online April 9, 2017]**

(a) 2 (b) $4^{\frac{1}{3}}$ (c) $4^{\frac{2}{3}}$ (d) 4

28. Let $a_1, a_2, a_3, \ldots., a_n$, be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to : **[Online April 10, 2016]**

(a) 306 (b) 204 (c) 153 (d) 612

29. Let α and β be the roots of equation $px^2 + qx + r = 0, p \neq 0$. If p, q, r are in A.P and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is: **[2014]**

(a) $\frac{\sqrt{34}}{9}$ (b) $\frac{2\sqrt{13}}{9}$ (c) $\frac{\sqrt{61}}{9}$ (d) $\frac{2\sqrt{17}}{9}$

30. The sum of the first 20 terms common between the series $3 + 7 + 11 + 15 + \ldots\ldots$ and $1 + 6 + 11 + 16 + \ldots\ldots$, is **[Online April 11, 2014]**

(a) 4000 (b) 4020 (c) 4200 (d) 4220

31. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4^{th} term is: **[Online April 9, 2014]**

(a) 8 (b) 16 (c) 20 (d) 24

32. If $a_1, a_2, a_3, \ldots., a_n, \ldots.$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P., is : **[Online April 23, 2013]**

(a) 10m (b) 12m (c) 13m (d) 15m

33. Given sum of the first n terms of an A.P. is $2n+3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is : **[Online April 22, 2013]**

(a) $n+4n^2$ (b) $6n^2-n$ (c) n^2+4n (d) $3n+2n^2$

34. Let $a_1, a_2, a_3,...$ be an A.P, such that

$$\frac{a_1+a_2+...+a_p}{a_1+a_2+a_3+...+a_q}=\frac{p^3}{q^3}; p \neq q. \text{ Then } \frac{a_6}{a_{21}} \text{ is equal to:}$$

[Online April 9, 2013]

(a) $\frac{41}{11}$ (b) $\frac{31}{121}$ (c) $\frac{11}{41}$ (d) $\frac{121}{1861}$

35. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is : **[2012]**

(a) -150 (b) 150 times its 50^{th} term

(c) 150 (d) Zero

36. If the A.M. between p^{th} and q^{th} terms of an A.P. is equal to the A.M. between r^{th} and s^{th} terms of the same A.P., then $p+q$ is equal to **[Online May 26, 2012]**

(a) $r+s-1$ (b) $r+s-2$ (c) $r+s+1$ (d) $r+s$

37. Suppose θ and $\phi\,(\neq 0)$ are such that $\sec(\theta+\phi)$, $\sec\theta$ and $\sec(\theta-\phi)$ are in A.P. If $\cos\theta = k\cos\left(\frac{\phi}{2}\right)$ for some k, then k is equal to **[Online May 19, 2012]**

(a) $\pm\sqrt{2}$ (b) ± 1 (c) $\pm\frac{1}{\sqrt{2}}$ (d) ± 2

38. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r}=\alpha$ and $\sum_{r=1}^{100} a_{2r-1}=\beta$, then the common difference of the A.P. is **[2011]**

(a) $\alpha-\beta$ (b) $\frac{\alpha-\beta}{100}$ (c) $\beta-\alpha$ (d) $\frac{\alpha-\beta}{200}$

39. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[2011]**

(a) 19 months (b) 20 months

(c) 21 months (d) 18 months

40. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference -2, then the time taken by him to count all notes is **[2010]**

(a) 34 minutes (b) 125 minutes

(c) 135 minutes (d) 24 minutes

41. Let a_1, a_2, a_3........... be terms on A.P. If

$$\frac{a_1+a_2+..........a_p}{a_1+a_2+..........+a_q}=\frac{p^2}{q^2},\ p\neq q, \text{ then } \frac{a_6}{a_{21}} \text{ equals}$$

[2006]

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

42. If the coefficients of rth, $(r+1)$th, and $(r+2)$th terms in the the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation **[2005]**

(a) $m^2-m(4r-1)+4r^2-2=0$

(b) $m^2-m(4r+1)+4r^2+2=0$

(c) $m^2-m(4r+1)+4r^2-2=0$

(d) $m^2-m(4r-1)+4r^2+2=0$

43. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n,\ m\neq n,\ T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals **[2004]**

(a) $\frac{1}{m}+\frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

44. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P. then x equals **[2002]**

(a) $\log_3 4$ (b) $1-\log_3 4$

(c) $1-\log_4 3$ (d) $\log_4 3$

TOPIC 2 Geometric Progression

45. If $f(x+y)=f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x)=2$, $x, y \in$ N, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is : **[Sep. 06, 2020 (I)]**

(a) $\frac{2}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$

46. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)=0$. Then : **[Sep. 06, 2020 (I)]**

(a) a, c, p are in A.P. (b) a, c, p are in G.P.

(c) a, b, c, d are in G.P. (d) a, b, c, d are in A.P.

47. Suppose that a function $f: R \to R$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(a) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to ______. **[NA Sep. 06, 2020 (II)]**

48. If $2^{10} + 2^9.3^1 + 2^8.3^2 + ... + 2 \times 3^9 + 3^{10} = S - 2^{11}$ then S is equal to: **[Sep. 05, 2020 (I)]**

(a) $3^{11} - 2^{12}$ (b) 3^{11}

(c) $\frac{3^{11}}{2} + 2^{10}$ (d) $2 \cdot 3^{11}$

49. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is: **[Sep. 05, 2020 (II)]**

(a) $\frac{1}{26}(3^{49} - 1)$ (b) $\frac{1}{26}(3^{50} - 1)$

(c) $\frac{2}{13}(3^{50} - 1)$ (d) $\frac{1}{13}(3^{50} - 1)$

50. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If α, β, γ, δ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is : **[Sep. 04, 2020 (I)]**

(a) 3 : 1 (b) 9 : 7 (c) 5 : 3 (d) 33 : 31

51. The value of $(0.16)\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ... \text{ to } \infty\right)$ is equal to ________ . **[NA Sep. 03, 2020 (I)]**

52. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in : **[Sep. 02, 2020 (I)]**

(a) $(-\infty, -9] \cup [3, \infty)$ (b) $[-3, \infty)$

(c) $(-\infty, -3] \cup [9, \infty)$ (d) $(-\infty, 9]$

53. If $|x| < 1, |y| < 1$ and $x \neq y$, then the sum to infinity of the following series **[Sep. 02, 2020 (I)]**

$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+....$ is :

(a) $\frac{x+y-xy}{(1+x)(1+y)}$ (b) $\frac{x+y+xy}{(1+x)(1+y)}$

(c) $\frac{x+y-xy}{(1-x)(1-y)}$ (d) $\frac{x+y+xy}{(1-x)(1-y)}$

54. Let S be the sum of the first 9 terms of the series :

$\{x+ka\}+\{x^2+(k+2)a\}+\{x^3+(k+4)a\}$ $+\{x^4+(k+6)a\}+...$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to : **[Sep. 02, 2020 (II)]**

(a) – 5 (b) 1 (c) – 3 (d) 3

55. The product $2^{\frac{1}{4}} . 4^{\frac{1}{16}} . 8^{\frac{1}{48}} . 16^{\frac{1}{128}} ...$ to ∞ is equal to: **[Jan. 9, 2020 (I)]**

(a) $2^{\frac{1}{2}}$ (b) $2^{\frac{1}{4}}$ (c) 1 (d) 2

56. Let a_n be the n^{th} term of a G.P. of positive terms.

If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to : **[Jan. 9, 2020 (II)]**

(a) 300 (b) 225 (c) 175 (d) 150

57. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then : **[Jan. 9, 2020 (II)]**

(a) $x(1+y) = 1$ (b) $y(1-x) = 1$

(c) $y(1+x) = 1$ (d) $x(1-y) = 1$

58. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + ... + 49^2 + 49 + 1$, is: **[Jan. 7, 2020 (I)]**

(a) 32 (b) 63 (c) 60 (d) 65

59. Let $a_1, a_2, a_3, ...$ be a G. P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to: **[Jan. 7, 2020 (II)]**

(a) –513 (b) –171 (c) 171 (d) $\frac{511}{3}$

60. The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + ... + x^{10}$ is: **[Jan. 7, 2020 (II)]**

(a) 210 (b) 330 (c) 120 (d) 420

61. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then α (β+ γ) is equal to : **[April 12, 2019 (II)]**

(a) 0 (b) αβ (c) αγ (d) βγ

62. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a$, $7b$ and $15c$ are the first three terms of an A.P., then the 4th term of this A.P. is : **[April 10, 2019 (II)]**

(a) $\frac{2}{3}a$ (b) $5a$ (c) $\frac{7}{3}a$ (d) a

63. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? **[April 08, 2019 (II)]**

(a) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (b) d, e, f are in A.P.

(c) d, e, f are in G.P. (d) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

64. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is : **[Jan. 12, 2019 (I)]**

(a) 36 (b) 32 (c) 24 (d) 28

65. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45°)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to : **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (b) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(c) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$ (d) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

66. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals : **[Jan. 11, 2019 (I)]**

(a) 5^4 (b) $4(5^2)$ (c) 5^3 (d) $2(5^2)$

67. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is : **[Jan. 11, 2019 (I)]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$

68. Let $S_n = 1 + q + q^2 + ... + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + ... + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$, then α is equal to : **[Jan. 11, 2019 (II)]**

(a) 2^{99} (b) 202 (c) 200 (d) 2^{100}

69. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to: **[Jan. 09, 2019 (II)]**

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{7}{13}$ (d) 4

70. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be: **[Jan. 09, 2019 (I)]**

(a) -2 (b) -3 (c) 4 (d) 2

71. If b is the first term of an infinite G. P whose sum is five, then b lies in the interval. **[Online April 15, 2018]**

(a) $(-\infty, -10)$ (b) $(10, \infty)$

(c) $(0, 10)$ (d) $(-10, 0)$

72. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - ... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \geq p$ is **[Online April 15, 2018]**

(a) 5 (b) 7 (c) 11 (d) 9

73. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is **[Online April 15, 2018]**

(a) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$ (b) $\frac{1}{4} - \frac{1}{4\sqrt{2}}$

(c) $\frac{1}{4} - \frac{1}{\sqrt{2}}$ (d) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

74. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : **[2016]**

(a) 1 (b) $\frac{7}{4}$ (c) $\frac{8}{5}$ (d) $\frac{4}{3}$

75. Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is areal number. Then the sum $1 + z + z^2 + ... + z^{11}$ is equal to: **(Online April 10, 2016)**

(a) $1365\sqrt{3}i$ (b) $-1365\sqrt{3}i$

(c) $-1250\sqrt{3}i$ (d) $1250\sqrt{3}i$

76. If m is the A.M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. **[2015]**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$ (c) $4\,l^2mn$ (d) $4\,lm^2n$

77. The sum of the 3rd and the 4th terms of a G.P. is 60 and the product of its first three terms is 1000. If the first term of this G.P. is positive, then its 7th term is : **[Online April 11, 2015]**

(a) 7290 (b) 640 (c) 2430 (d) 320

78. Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is: **[2014]**

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$

(c) $\sqrt{2}+\sqrt{3}$ (d) $3+\sqrt{2}$

79. The least positive integer n such that

$1-\frac{2}{3}-\frac{2}{3^2}-....-\frac{2}{3^{n-1}}<\frac{1}{100}$, is: **[Online April 12, 2014]**

(a) 4 (b) 5 (c) 6 (d) 7

80. In a geometric progression, if the ratio of the sum of first 5 terms to the sum of their reciprocals is 49, and the sum of the first and the third term is 35. Then the first term of this geometric progression is: **[Online April 11, 2014]**

(a) 7 (b) 21 (c) 28 (d) 42

81. The coefficient of x^{50} in the binomial expansion of $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+....+x^{1000}$ is: **[Online April 11, 2014]**

(a) $\frac{(1000)!}{(50)!(950)!}$ (b) $\frac{(1000)!}{(49)!(951)!}$

(c) $\frac{(1001)!}{(51)!(950)!}$ (d) $\frac{(1001)!}{(50)!(951)!}$

82. Given a sequence of 4 numbers, first three of which are in G.P. and the last three are in A.P. with common difference six. If first and last terms of this sequence are equal, then the last term is : **[Online April 25, 2013]**

(a) 16 (b) 8 (c) 4 (d) 2

83. If a, b, c, d and p are distinct real numbers such that $(a^2+b^2+c^2)\,p^2-2p\,(ab+bc+cd)+(b^2+c^2+d^2)\le 0$, then **[Online May 12, 2012]**

(a) a, b, c, d are in A.P. (b) $ab = cd$

(c) $ac = bd$ (d) a, b, c, d are in G.P.

84. The difference between the fourth term and the first term of a Geometrical Progresssion is 52. If the sum of its first three terms is 26, then the sum of the first six terms of the progression is **[Online May 7, 2012]**

(a) 63 (b) 189 (c) 728 (d) 364

85. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[2008]**

(a) –4 (b) –12 (c) 12 (d) 4

86. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals **[2007]**

(a) $\sqrt{5}$ (b) $\frac{1}{2}\left(\sqrt{5}-1\right)$

(c) $\frac{1}{2}\left(1-\sqrt{5}\right)$ (d) $\frac{1}{2}\sqrt{5}$

87. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)$ is **[2006]**

(a) i (b) 1 (c) -1 (d) $-i$

88. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3......$ then a_n is **[2006]**

(a) $\frac{b^n-a^n}{b-a}$ (b) $\frac{a^n-b^n}{b-a}$

(c) $\frac{a^{n+1}-b^{n+1}}{b-a}$ (d) $\frac{b^{n+1}-a^{n+1}}{b-a}$

89. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation **[2004]**

(a) $x^2-18x-16=0$ (b) $x^2-18x+16=0$

(c) $x^2+18x-16=0$ (d) $x^2+18x+16=0$

90. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is **[2002]**

(a) 5 (b) 3/5 (c) 8/5 (d) 1/5

91. Fifth term of a GP is 2, then the product of its 9 terms is **[2002]**

(a) 256 (b) 512

(c) 1024 (d) none of these

92. The sum of all values of $\theta\in\left(0,\frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$ is: **[Jan. 10, 2019 (I)]**

(a) π (b) $\frac{5\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{8}$

TOPIC 3 Harmonic Progression, Relation Between A. M., G. M. and H.M. of two Positive Numbers

93. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to __________. **[Sep. 03, 2020 (II)]**

94. If the arithmetic mean of two numbers a and b, a > b > 0, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to : **[Online April 8, 2017]**

(a) $\frac{\sqrt{6}}{2}$ (b) $\frac{3\sqrt{2}}{4}$ (c) $\frac{7\sqrt{3}}{12}$ (d) $\frac{5\sqrt{6}}{12}$

95. If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{6}$, then the minimum value of $\tan A + \tan B$ is : **[Online April 10, 2016]**

(a) $\sqrt{3}-\sqrt{2}$ (b) $4-2\sqrt{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $2-\sqrt{3}$

96. Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to : **[Online April 9, 2016]**

(a) 342 (b) 216 (c) 258 (d) 270

97. Let G be the geometric mean of two positive numbers a and b, and M be the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M}$: G is 4 : 5, then a : b can be: **[Online April 12, 2014]**

(a) 1 : 4 (b) 1 : 2 (c) 2 : 3 (d) 3 : 4

98. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to **[2006]**

(a) $n(a_1 - a_n)$ (b) $(n-1)(a_1 - a_n)$

(c) na_1a_n (d) $(n-1)a_1a_n$

99. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P and $|a| < 1, |b| < 1, |c| < 1$ then x, y, z are in **[2005]**

(a) G.P.
(b) A.P.
(c) Arithmetic - Geometric Progression
(d) H.P.

100. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in **[2003]**

(a) Arithmetic - Geometric Progression
(b) Arithmetic Progression
(c) Geometric Progression
(d) Harmonic Progression.

TOPIC 4 Arithmetic-Geometric Sequence (A.G.S.), Some Special Sequences

101. If $1+(1-2^2\cdot 1)+(1-4^2\cdot 3)+(1-6^2\cdot 5)+.......+(1-20^2\cdot 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to : **[Sep. 04, 2020 (I)]**

(a) (10,97) (b) (11,103)
(c) (10,103) (d) (11,97)

102. Let $f : \mathbf{R} \to \mathbf{R}$ be a function which satisfies $f(x+y) = f(x)+f(y)$, $\forall\ x, y \in \mathbf{R}$. If f (a) = 2 and $g(n) = \sum_{k=1}^{(n-1)} f(k)$, $n \in \mathbf{N}$, then the value of n, for which $g(n) = 20$, is : **[Sep. 02, 2020 (II)]**

(a) 5 (b) 20 (c) 4 (d) 9

103. The sum, $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to __________. **[Jan. 8, 2020 (II)]**

104. The sum $\sum_{k=1}^{20} (1+2+3+...+k)$ is _______. **[Jan. 8, 2020 (I)]**

105. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... is (102)m, then m is equal to: **[Jan. 7, 2020 (II)]**

(a) 20 (b) 25 (c) 5 (d) 10

106. For $x \in R$, let $[x]$ denote the greatest integer $\le x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\cdots+\left[-\frac{1}{3}-\frac{99}{100}\right]$$ is

[April 12, 2019 (I)]

(a) –153 (b) –133 (c) –131 (d) –135

107. The sum $\frac{3\times 1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2})+......$ upto 10^{th} term, is : **[April 10, 2019 (I)]**

(a) 680 (b) 600 (c) 660 (d) 620

108. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+......+$

$$\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}(1+2+3+...+15$$ is equal to :

[April 10, 2019 (II)]

(a) 620 (b) 1240 (c) 1860 (d) 660

109. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$ upto 11^{th} term is: **[April 09, 2019 (II)]**

(a) 915 (b) 946 (c) 945 (d) 916

110. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is: **[April 09, 2019 (II)]**

(a) 157 2) 262 (c) 225 (d) 190

111. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to : **[April 08, 2019 (II)]**

(a) $2-\frac{3}{2^{17}}$ (b) $1-\frac{11}{2^{20}}$ (c) $2-\frac{11}{2^{19}}$ (d) $2-\frac{21}{2^{20}}$

112. Let $S_k = \frac{1+2+3+\ldots+k}{2}$.

If $S_1^2 + S_2^2 + \ldots + S_{10}^2 = \frac{5}{12}A$. Then A is equal to **[Jan. 12, 2019 (I)]**

(a) 283 (b) 301 (c) 303 (d) 156

113. If the sum of the first 15 terms of the series

$\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots$ is equal to 225 k then k is equal to : **[Jan. 12, 2019 (II)]**

(a) 108 (b) 27 (c) 54 (d) 9

114. The sum of the following series **[Jan. 09, 2019 (II)]**

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+\ldots+5^2)}{11}+\ldots$$ up to 15 terms, is:

(a) 7520 (b) 7510 (c) 7830 (d) 7820

115. The sum of the first 20 terms of the series

$1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots$ is? **[Online April 16, 2018]**

(a) $38+\frac{1}{2^{20}}$ (b) $39+\frac{1}{2^{19}}$

(c) $39+\frac{1}{2^{20}}$ (d) $38+\frac{1}{2^{19}}$

116. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2\cdot 2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$

If $B - 2A = 100\lambda$, then λ is equal to : **[2018]**

(a) 248 (b) 464 (c) 496 (d) 232

117. Let a, b, c $\in$ R. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x+y) = f(x) + f(y) + xy, \forall\, x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to : **[2017]**

(a) 255 (b) 330 (c) 165 (d) 190

118. Let $S_n = \frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+\ldots+\frac{1+2+\ldots+n}{1^3+2^3+\ldots+n^3}$, If $100\, S_n = n$, then n is equal to : **[Online April 9, 2017]**

(a) 199 (b) 99 (c) 200 (d) 19

119. If the sum of the first n terms of the series

$\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+\ldots$ is $435\sqrt{3}$, then n equals: **[Online April 8, 2017]**

(a) 18 (b) 15 (c) 13 (d) 29

120. If the sum of the first ten terms of the series

$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots$, is $\frac{16}{5}$m, then m is equal to : **[2016]**

(a) 100 (b) 99 (c) 102 (d) 101

121. For $x \in R, x \neq -1$, if $(1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots + x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to : **[Online April 9, 2016]**

(a) $\frac{2017!}{17!2000!}$ (b) $\frac{2016!}{17!\,1999!}$

(c) $\frac{2016!}{16!}$ (d) $\frac{2017!}{2000!}$

122. The sum of first 9 terms of the series. **[2015]**

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$

(a) 142 (b) 192 (c) 71 (d) 96

123. If $\sum_{n=1}^{5}\frac{1}{n(n+1)(n+2)(n+3)}=\frac{k}{3}$, then k is equal to **[Online April 11, 2015]**

(a) $\frac{1}{6}$ (b) $\frac{17}{105}$ (c) $\frac{55}{336}$ (d) $\frac{19}{112}$

124. The value of $\sum_{r=16}^{30}(r+2)(r-3)$ is equal to : **[Online April 10, 2015]**

(a) 7770 (b) 7785 (c) 7775 (d) 7780

125. If $(10)^9+2(11)^1(10^8)+3(11)^2(10)^7+.....$ $+10(11)^9=k(10)^9$, then k is equal to: **[2014]**

(a) 100 (b) 110 (c) $\frac{121}{10}$ (d) $\frac{441}{100}$

126. The number of terms in an A.P. is even; the sum of the odd terms in it is 24 and that the even terms is 30. If the last term exceeds the first term by $10\frac{1}{2}$, then the number of terms in the A.P. is: **[Online April 19, 2014]**

(a) 4 (b) 8 (c) 12 (d) 16

127. If the sum

$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+......+$ up to 20 terms is equal to $\frac{k}{21}$, then k is equal to: **[Online April 9, 2014]**

(a) 120 (b) 180 (c) 240 (d) 60

128. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is **[2013]**

(a) $\frac{7}{81}(179-10^{-20})$ (b) $\frac{7}{9}(99-10^{-20})$

(c) $\frac{7}{81}(179+10^{-20})$ (d) $\frac{7}{9}(99+10^{-20})$

129. The value of $1^2+3^2+5^2+......................+25^2$ is : **[Online April 25, 2013]**

(a) 2925 (b) 1469 (c) 1728 (d) 1456

130. The sum of the series :
$(b)^2+2(d)^2+3(6)^2+...$ upto 10 terms is : **[Online April 23, 2013]**

(a) 11300 (b) 11200 (c) 12100 (d) 12300

131. The sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+....$ upto 11-terms is: **[Online April 22, 2013]**

(a) $\frac{7}{2}$ (b) $\frac{11}{4}$ (c) $\frac{11}{2}$ (d) $\frac{60}{11}$

132. The sum of the series : **[Online April 9, 2013]**

$1+\frac{1}{1+2}+\frac{1}{1+2+3}+.......$ upto 10 terms, is :

(a) $\frac{18}{11}$ (b) $\frac{22}{13}$ (c) $\frac{20}{11}$ (d) $\frac{16}{9}$

133. **Statement-1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-2: $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$, for any natural number n. **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

134. If the sum of the series $1^2+2.2^2+3^2+2.4^2+5^2+...2.6^2+...$ upto n terms, when n is even, is $\frac{n(n+1)^2}{2}$, then the sum of the series, when n is odd, is **[Online May 26, 2012]**

(a) $n^2(n+1)$ (b) $\frac{n^2(n-1)}{2}$

(c) $\frac{n^2(n+1)}{2}$ (d) $n^2(n-1)$

135. The sum of the series $1+\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+...$ upto n terms is **[Online May 19, 2012]**

(a) $\frac{7}{6}n+\frac{1}{6}-\frac{2}{3.2^{n-1}}$ (b) $\frac{5}{3}n-\frac{7}{6}+\frac{1}{2.3^{n-1}}$

(c) $n+\frac{1}{2}-\frac{1}{2.3^n}$ (d) $n-\frac{1}{3}-\frac{1}{3.2^{n-1}}$

136. The sum of the series

$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...$

upto 15 terms is **[Online May 12, 2012]**

(a) 1 (b) 2 (c) 3 (d) 4

137. The sum of the series
$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+....+2(2m)^2$ is **[Online May 7, 2012]**

(a) $m(2m+1)^2$ (b) $m^2(m+2)$
(c) $m^2(2m+1)$ (d) $m(m+2)^2$

138. The sum to infinite term of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+...$ is **[2009]**

(a) 3 (b) 4 (c) 6 (d) 2

139. The sum of series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-$ .... upto infinity is **[2007]**

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$ (c) e^{-2} (d) e^{-1}

140. The sum of the series

$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+$ ....ad inf. is **[2005]**

(a) $\frac{e-1}{\sqrt{e}}$ (b) $\frac{e+1}{\sqrt{e}}$ (c) $\frac{e-1}{2\sqrt{e}}$ (d) $\frac{e+1}{2\sqrt{e}}$

141. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+$..... is **[2004]**

(a) $\frac{(e^2-2)}{e}$ (b) $\frac{(e-1)^2}{2e}$

(c) $\frac{(e^2-1)}{2e}$ (d) $\frac{(e^2-1)}{2}$

142. The sum of the first n terms of the series

$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+...$

is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is **[2004]**

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

143. If $S_n=\sum_{r=0}^{n}\frac{1}{^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{^nC_r}$, then $\frac{t_n}{S_n}$ is equal to **[2004]**

(a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n-1$ (c) $n-1$ (d) $\frac{1}{2}n$

144. The sum of the series **[2003]**

$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$........... up to ∞ is equal to

(a) $\log_e\left(\frac{4}{e}\right)$ (b) $2\log_e 2$

(c) $\log_e 2-1$ (d) $\log_e 2$

145. $1^3-2^3+3^3-4^3+...+9^3=$ **[2002]**

(a) 425 (b) –425 (c) 475 (d) –475

146. The value of $2^{1/4}.\,4^{1/8}.\,8^{1/16}$ ... ∞ is **[2002]**

(a) 1 (b) 2 (c) 3/2 (d) 4

Hints & Solutions

1. (c) Let common difference of series

$a_1, a_2, a_3, \ldots\ldots, a_n$ be d.

$\because a_{40} = a_1 + 39d = -159$ ...(i)

and $a_{100} = a_1 + 99d = -399$ ...(ii)

From equations (i) and (ii),
$d = -4$ and $a_1 = -3$
Since, the common difference of $b_1, b_2, \ldots\ldots, b_n$ is 2 more than common difference of $a_1, a_2, \ldots\ldots, a_n$.
$\therefore$ Common difference of $b_1, b_2, b_3, \ldots..$ is (-2).

$\because b_{100} = a_{70}$

$\Rightarrow b_1 + 99(-2) = (-3) + 69(-4)$

$\Rightarrow b_1 = 198 - 279 \Rightarrow b_1 = -81$

2. (a) Given that $3^{2\sin 2\alpha - 1}, 14, 3^{4-2\sin 2\alpha}$ are in A.P.

So, $3^{2\sin 2\alpha - 1} + 3^{4-2\sin 2\alpha} = 28$

$$\Rightarrow \frac{3^{2\sin 2\alpha}}{3} + \frac{81}{3^{2\sin 2\alpha}} = 28$$

Let $3^{2\sin 2\alpha} = x$

$$\Rightarrow \frac{x}{3} + \frac{81}{x} = 28$$

$\Rightarrow x^2 - 84x + 243 = 0 \Rightarrow x = 81, x = 3$

When $x = 81 \Rightarrow \sin 2\alpha = 2$ (Not possible)

When $x = 3 \Rightarrow \alpha = \dfrac{\pi}{12}$

$\therefore a = 3^0 = 1, d = 14 - 1 = 13$

$a_6 = a + 5d = 1 + 65 = 66.$

3. (a) $S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + \ldots 20$ terms

$\because S = 460$

$\Rightarrow \log_7 (x^2 \cdot x^3 \cdot x^4 \cdot \ldots\ldots x^{21}) = 460$

$\Rightarrow \log_7 x^{(2+3+4\ldots\ldots 21)} = 460$

$\Rightarrow (2 + 3 + 4 + \ldots.. + 21)\log_7 x = 460$

$$\Rightarrow \frac{20}{2}(2 + 21)\log_7 x = 460$$

$$\Rightarrow \log_7 x = \frac{460}{230} = 2 \Rightarrow x = 7^2 = 49$$

4. (d) Given that $a_1 = 1$ and $a_n = 300$ and $d \in \mathbf{Z}$

$\therefore 300 = 1 + (n-1)d$

$$\Rightarrow d = \frac{299}{(n-1)} = \frac{23 \times 13}{(n-1)},$$

$\because$ d is an integer

$\therefore n - 1 = 13$ or 23

$\Rightarrow n = 14$ or 24 $(\because 15 \le n \le 50)$

$\Rightarrow n = 24$ and $d = 13$

$a_{20} = 1 + 19 \times 13 = 248$

$$s_{20} = \frac{20}{2}(2 + 19 \times 13) = 2490.$$

5. (a) Given $a = 3$ and $S_{25} = S_{40} - S_{25}$

$\Rightarrow 2S_{25} = S_{40}$

$$\Rightarrow 2 \times \frac{25}{2}[6 + 24d] = \frac{40}{2}[6 + 39d]$$

$\Rightarrow 25[6 + 24d] = 20[6 + 39d]$

$\Rightarrow 5(2 + 8d) = 4(2 + 13d)$

$\Rightarrow 10 + 40d = 8 + 52d$

$$\Rightarrow d = \frac{1}{6}$$

6. (b) $S_n = 20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots.$

$\because S_n = 488$

$$488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(-\frac{2}{5}\right)\right]$$

$$488 = \frac{n}{2}(101 - n) \Rightarrow n^2 - 101n + 2440 = 0$$

$\Rightarrow n = 61$ or 40

For $n = 40 \Rightarrow T_n > 0$

For $n = 61 \Rightarrow T_n < 0$

$$n^{\text{th}} \text{ term} = T_{61} = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$$

7. **(d)** Let common difference be d.

$\because S_{11}=0 \quad \therefore \frac{11}{2}\{2a_1+10\cdot d\}=0$

$\Rightarrow a_1+5d=0 \Rightarrow d=-\frac{a_1}{5}$ ...(i)

Now, $S=a_1+a_3+a_5+...+a_{23}$

$=a_1+(a_1+2d)+(a_1+4d)+....+(a_1+22d)$

$=12a_1+2d\frac{11\times 12}{2}$

$=12\left[a_1+11\cdot\left(-\frac{a_1}{5}\right)\right]$ (From (i))

$=12\times\left(-\frac{6}{5}\right)a_1=-\frac{72}{5}a_1$

8. **(14)** First common term of both the series is 23 and common difference is $7\times 4=28$

$\because$ Last term ≤ 407

$\Rightarrow 23+(n-1)\times 28\le 407$

$\Rightarrow (n-1)\times 28\le 384$

$\Rightarrow n\le\frac{384}{28}+1$

$\Rightarrow n\le 14.71$

Hence, $n=14$

9. **(d)** $T_{10}=\frac{1}{20}=a+9d$ ...(i)

$T_{20}=\frac{1}{10}=a+19d$ ...(ii)

Solving equations (i) and (ii), we get

$a=\frac{1}{200}, d=\frac{1}{200}$

$\Rightarrow S_{200}=\frac{200}{2}\left[\frac{2}{200}+\frac{199}{200}\right]=\frac{201}{2}=100\frac{1}{2}$

10. **(b)** If $2^{1-x}+2^{1+x}, f(x), 3^x+3^{-x}$ are in A.P., then

$f(x)=\left(\frac{2^{1+x}+2^{1-x}+3^x+3^{-x}}{2}\right)$

$2f(x)=2\left(2^x+\frac{1}{2^x}\right)+\left(3^x+\frac{1}{3^x}\right)$

Using AM $\ge$ GM

$f(x)\ge 3$

11. **(d)** Let 5 terms of A.P. be

$a-2d, a-d, a, a+d, a+2d.$

Sum $=25 \Rightarrow 5a=25 \Rightarrow a=5$

Product $=2520$

$(5-2d)(5-d)5(5+d)(5+2d)=2520$

$\Rightarrow (25-4d^2)(25-d^2)=504$

$\Rightarrow 625-100d^2-25d^2+4d^4=504$

$\Rightarrow 4d^4-125d^2+625-504=0$

$\Rightarrow 4d^4-125d^2+121=0$

$\Rightarrow 4d^4-121d^2-4d^2+121=0$

$\Rightarrow (d^2-1)(4d^2-121)=0$

$\Rightarrow d=\pm 1, d=\pm\frac{11}{2}$

$d=\pm 1$ and $d=-\frac{11}{2}$, does not give $\frac{-1}{2}$ as a term

$\therefore d=\frac{11}{2}$

$\therefore$ Largest term $=5+2d=5+11=16$

12. **(c)** Given, $S_4=16$ and $S_6=-48$

$\Rightarrow 2(2a+3d)=16 \Rightarrow 2a+3d=8$ ...(i)

And $3[2a+5d]=-48 \Rightarrow 2a+5d=-16$

$\Rightarrow 2d=-24$ [using equation (i)]

$\Rightarrow d=-12$ and $a=22$

$\therefore S_{10}\frac{10}{2}=(44+9(-12))=-320$

13. **(a)** Let the common difference of the A.P. is 'd'.

Given, $a_1+a_7+a_{16}=40$

$\Rightarrow a_1+a_1+6d+a_1+15d=40$

$\Rightarrow 3a_1+21d=40$

$\Rightarrow a_1+7d=\frac{40}{3}$ ...(i)

Now, sum of first 15 terms of this A.P. is,

$S_{15}=\frac{15}{2}[2a_1+14d]=15(a_1+7d)$

$=15\left(\frac{40}{3}\right)=200$ [Using (i)]

14. **(b)** $a_1+a_4+a_7+....+a_{16}=114$

$\Rightarrow 3(a_1+a_{16})=114$

$\Rightarrow a_1+a_{16}=38$

Now, $a_1+a_6+a_{11}+a_{16}=2(a_1+a_{16})=2\times 38=76$

15. **(d)** $\because S_n=\left(50-\frac{7A}{2}\right)n+n^2\times\frac{A}{2} \Rightarrow a_1=50-3S$

$\therefore d=a_2-a_1=S_{n_2}-S_{n_1}-S_{n_1} \Rightarrow d=\frac{A}{2}\times 2=A$

Now, $a_{50}=a_1+49\times d$

$=(50-3A)+49A=50+46A$

So, $(d, a_{50})=(A, 50+46A)$

16. **(d)** $\because f(x+y)=f(x).f(y)$

$\Rightarrow$ Let $f(x)=t^x$

$\because f(1)=2$

$\therefore t^{'}=2$

$\Rightarrow f(x)=2^x$

Since, $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$

Then, $\sum_{k=1}^{10} 2^{a+k}=16(2^{10}-1)$

$\Rightarrow 2^a\sum_{k=1}^{10} 2^k=16(2^{10}-1)$

$\Rightarrow 2^a\times\frac{((2^{10})-1)\times 2}{(2-1)}=16\times(2^{10}-1)2.2^a=16$

$\Rightarrow a=3$

17. **(d)** Let three terms of A.P. are $a-d, a, a+d$

Sum of terms is, $a-d+a+a+d=33\Rightarrow a=11$

Product of terms is, $(a-d)\,a\,(a+d)=11(121-d^2)=1155$

$\Rightarrow 121-d^2=105\Rightarrow d=\pm 4$

if $d=4$

$T_{11}=T_1+10d=7+10(4)=47$

if $d=-4$

$T_{11}=T_1+10d=15+10(-4)=-25$

18. **(d)** $\because 91=13\times 7$

Then, the required numbers are either divisible by 7 or 13.

$\therefore$ Sum of such numbers = Sum of no. divisible by 7 + sum of the no. divisible by 13 – Sum of the numbers divisible by 91

$=(105+112+...+196)+(104+117+...+195)-182$

$=2107+1196-182=3121$

19. **(b)** Since ${}^nC_4, {}^nC_5$ and ${}^nC_6$ are in A.P.

$2\,{}^nC_5={}^nC_4+{}^nC_6$

$2=\frac{{}^nC_4}{{}^nC_5}+\frac{{}^nC_6}{{}^nC_5}$

$2=\frac{5}{n-4}+\frac{n-5}{5}$

$\Rightarrow\ 12(n-4)=30+n^2-9n+20$

$\Rightarrow\ n^2-21n+98=0$

$(n-7)\,(n-14)=0$

$(n-7)\,(n-14)=0$

$n=7, n=14$

20. **(c)** Let first term and common difference of AP be a and d respectively, then

$t_n=a+(n-1)d$

$\therefore\ t_{19}=a+18d=0$

$\therefore\ a=-18d$ ...(i)

$\therefore\ \frac{t_{49}}{t_{29}}=\frac{a+48d}{a+28d}$

$=\frac{-18d+48d}{-18d+28d}=\frac{30d}{10d}=3$

$t_{49}:t_{29}=3:1$

21. **(d)** Two digit positive numbers which when divided by 7 yield 2 as remainder are 12 terms i.e,16, 23, 30, ..., 93

Two digit positive numbers which when divided by 7 yield 5 as remainder are 13 terms i.e,12, 19, 26, ..., 96

By using AP sum of 16, 23, ..., 93, we get

$S_1=16+23+30+...+93=654$

By using AP sum of 12, 19, 26, ..., 96, we get

$S_1=12+19+26+...+96=702$

$\therefore$ required Sum $=S_1+S_2=654+702=1356$

22. **(a)** $S=\sum_{i=1}^{30} a_i=\frac{30}{2}[2a_1+29d]$

$T=\sum_{i=1}^{15} a_{(2i-1)}=\frac{15}{2}[2a_1+28d]$

Since, $S-2T=75$

$\Rightarrow 30\,a_1+435d-30a_1-420d=75$

$\Rightarrow d=5$

Also, $a_5=27\Rightarrow a_1+4d=27\Rightarrow a_1=7$,

Hence, $a_{10}=a_1+9d=7+9\times 5=52$

23. **(c)** $\because\ \frac{1}{x_1},\frac{1}{x_2},\frac{1}{x_3},.....,\frac{1}{x_n}$ are in A.P

$x_1=4$ and $x_{21}=20$

Let 'd' be the common difference of this A.P

$\therefore$ its 21[st] term $=\frac{1}{x_{21}}=\frac{1}{x_1}+\left[(21-1)\times d\right]$

$\Rightarrow d=\frac{1}{20}\times\left(\frac{1}{20}-\frac{1}{4}\right)\ \Rightarrow d=-\frac{1}{100}$

Also $x_n>50$ (given).

$\therefore\ \frac{1}{x_n}=\frac{1}{x_1}+\left[(n-1)\times d\right]$

$\Rightarrow x_n=\frac{x_1}{1+(n-1)\times d\times x_1}$

$\therefore\ \frac{x_1}{1+(n-1)\times d\times x_1}>50$

$$\Rightarrow \frac{4}{1+(n-1)\times\left(-\frac{1}{100}\right)\times 4} > 50$$

$$\Rightarrow 1+(n-1)\times\left(-\frac{1}{100}\right)\times 4 < \frac{4}{50}$$

$$\Rightarrow -\frac{1}{100}(n-1) < -\frac{23}{100}$$

$\Rightarrow n-1 > 23 \quad \Rightarrow n > 24$

Therefore, $n = 25$.

$$\Rightarrow \sum_{i=1}^{25}\frac{1}{x_i} = \frac{25}{2}\left[\left(2\times\frac{1}{4}\right)+(25-1)\times\left(-\frac{1}{100}\right)\right] = \frac{13}{4}$$

24. **(a)** Suppose d_1 is the common difference of the A.P. $x_1, x_2, ... x_n$ then

$$\because x_8 - x_3 = 5d_1 = 12 \Rightarrow d_1 = \frac{12}{5} = 2.4$$

$$\Rightarrow x_5 = x_3 + 2d_1 = 8 + 2\times\frac{12}{5} = 12.8$$

Suppose d_2 is the common difference of the A.P $\frac{1}{h_1}, \frac{1}{h_2}, ... \frac{1}{h_n}$ then

$$5d_2 = \frac{1}{20} - \frac{1}{8} = \frac{-3}{40} \Rightarrow d_2 = \frac{-3}{200}$$

$$\because \frac{1}{h_{10}} = \frac{1}{h_7} + 3d_2 = \frac{1}{200} \Rightarrow h_{10} = 200$$

$\Rightarrow x_5 . h_{10} = 12.8 \times 200 = 2560$

25. **(b)** $\because \sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$

$\Rightarrow a_1 + 24d = 32$ ...(i)

Now, $a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ ...(ii)

From eq. (i) & (ii) we get; $d = 1$ and $a_1 = 8$

Also, $\sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17}[8+(r-1)1]^2 = 140\text{ m}$

$$\Rightarrow \sum_{r=1}^{17}(r+7)^2 = 140\text{ m}$$

$$\Rightarrow \sum_{r=1}^{17}(r^2+14r+49) = 140\text{ m}$$

$$\Rightarrow \left(\frac{17\times18\times35}{6}\right)+14\left(\frac{17\times18}{2}\right)+(49\times17) = 140$$

$\Rightarrow m = 34$

26. **(c)** We have

$9(25a^2+b^2)+25(c^2-3ac) = 15b(3a+c)$

$\Rightarrow 225a^2+9b^2+25c^2-75ac = 45ab+15bc$

$\Rightarrow (15a)^2+(3b)^2+(5c)^2-75ac-45ab-15bc = 0$

$$\frac{1}{2}[(15a-3b)^2+(3b-5c)^2+(5c-15a)^2] = 0$$

It is possible when $15a - 3b = 0$, $3b - 5c = 0$ and $5c - 15a = 0$

$\Rightarrow 15a = 3b \Rightarrow b = 5a$

$$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$$

$$\Rightarrow a+b = \frac{c}{3}+\frac{5c}{3} = \frac{6c}{3}$$

$\Rightarrow a + b = 2c$

$\Rightarrow$ b, c, a are in A.P.

27. **(a)** **By Arithmetic Mean**:

$a + c = 2b$

Consider $a = b = c = 2$

$\Rightarrow abc = 8$

$\Rightarrow a + b = 2b$

$\therefore$ minimum possible value of $b = 2$

28. **(a)** $a_3 + a_7 + a_{11} + a_{15} = 72$

$(a_3 + a_{15}) + (a_7 + a_{11}) = 72$

$a_3 + a_{15} + a_7 + a_{11} = 2(a_1 + a_{17})$

$a_1 + a_{17} = 36$

$$S_{17} = \frac{17}{2}[a_1 + a_{17}] = 17 \times 18 = 306$$

29. **(b)** Let p, q, r are in AP

$\Rightarrow 2q = p + r$ ...(i)

Given $\frac{1}{\alpha}+\frac{1}{\beta} = 4$

$$\Rightarrow \frac{\alpha+\beta}{\alpha\beta} = 4$$

We have $\alpha+\beta = -q/p$ and $\alpha\beta = \frac{r}{p}$

$$\Rightarrow \frac{-\frac{q}{p}}{\frac{r}{p}} = 4 \Rightarrow q = -4r \quad ....(ii)$$

From (i), we have

$2(-4r) = p + r$

$p = -9r$ ....(iii)

Now, $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}} = \frac{\sqrt{q^2-4pr}}{|p|}$$

From (ii) and (iii)

$$=\frac{\sqrt{16r^2+36r^2}}{|-9r|}=\frac{2\sqrt{13}}{9}$$

30. **(b)** Given n = 20; S_{20} = ?
Series (1) → 3, 7, <u>11</u>, 15, 19, 23, 27, <u>31</u>, 35, 39, 43, 47, <u>51</u>, 55, 59...
Series (2) → 1, 6, <u>11</u>, 16, 21, 26, <u>31</u>, 36, 41, 46, <u>51</u>, 56, 61, 66, 71.
The common terms between both the series are 11, 31, 51, 71...
Above series forms an Arithmetic progression (A.P).
Therefore, first term (a) = 11 and
common difference (d) = 20

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{20} = \frac{20}{2}$ [2 × 11 + (20 – 1) 20]

S_{20} = 10 [22 + 19 × 20]
S_{20} = 10 × 402 = 4020
∴ S_{20} = 4020

31. **(c)** Let a be the first term and d be the common difference of given A.P.
Second term, $a + d = 12$ ...(i)
Sum of first nine terms,

$S_9 = \frac{9}{2}(2a+8d) = 9(a+4d)$

Given that S_9 is more than 200 and less than 220
⇒ $200 < S_9 < 220$
⇒ $200 < 9(a + 4d) < 220$
⇒ $200 < 9(a + d + 3d) < 220$
Putting value of $(a + d)$ from equation (i)
$200 < 9(12 + 3d) < 220$
⇒ $200 < 108 + 27d < 220$
⇒ $200 - 108 < 108 + 27d - 108 < 220 - 108$
⇒ $92 < 27d < 112$
Possible value of d is 4
27 × 4 = 108
Thus, 92 < 108 < 112
Putting value of d in equation (i)
$a + d = 12$
$a = 12 - 4 = 8$
4^{th} term $= a + 3d = 8 + 3 \times 4 = 20$

32. **(c)** If d be the common difference, then
$m = a_4 - a_7 + a_{10} = a_4 - a_7 + a_7 + 3d = a_7$

$$S_{13}=\frac{13}{2}[a_1+a_{13}]=\frac{13}{2}[a_1+a_7+6d]$$

$$=\frac{13}{2}[2a_7]=13a_7=13\text{ m}$$

33. **(b)** Given $S_n = 2n + 3n^2$
Now, first term = 2 + 3 = 5
second term = 2(2) + 3(4) = 16
third term = 2(3) + 3 (9) = 33
Now, sum given in option (b) only has the same first term and difference between 2nd and 1st term is double also.

34. **(b)** $\frac{a_1+a_2+a_3+......+a_p}{a_1+a_2+a_3+......+a_q}=\frac{p^3}{q^3}$

$\Rightarrow \frac{a_1+a_2}{a_1}=\frac{8}{1} \Rightarrow a_1+(a_1+d)=8a_1$

$\Rightarrow d = 6a_1$

Now $\frac{a_6}{a_{21}}=\frac{a_1+5d}{a_1+20d}$

$=\frac{a_1+5\times 6a_1}{a_1+20\times 6a_1}=\frac{1+30}{1+120}=\frac{31}{121}$

35. **(d)** Let 'a' is the first term and 'd' is the common difference of an $A.P.$
Now, According to the question
$100a_{100} = 50a_{50}$
$100(a + 99d) = 50(a + 49d)$
$\Rightarrow 2a + 198d = a + 49d \Rightarrow a + 149d = 0$
Hence, $T_{150} = a + 149d = 0$

36. **(d)** Given : $\frac{a_p+a_q}{2}=\frac{a_r+a_s}{2}$

$\Rightarrow a+(p-1)d+a+(q-1)d$
$= a+(r-1)d + a+(s-1)d$
$\Rightarrow 2a+(p+q)d-2d = 2a+(r+s)d-2d$
$\Rightarrow (p+q)d=(r+s)d \Rightarrow p+q=r+s.$

37. **(a)** Since, sec (θ – φ), secθ and sec (θ + φ) are in A.P.,

$\therefore 2\sec\theta = \sec(\theta-\phi)+\sec(\theta+\phi)$

$\Rightarrow \frac{2}{\cos\theta}=\frac{\cos(\theta+\phi)+\cos(\theta-\phi)}{\cos(\theta-\phi)\cos(\theta+\phi)}$

$\Rightarrow 2\left(\cos^2\theta-\sin^2\phi\right)=\cos\theta[2\cos\theta\cos\phi]$

$\Rightarrow \cos^2\theta(1-\cos\phi)=\sin^2\phi=1-\cos^2\phi$

$\Rightarrow \cos^2\theta=1+\cos\phi=2\cos^2\frac{\phi}{2}$

$\therefore \cos\theta=\sqrt{2}\cos\frac{\phi}{2}$

But given $\cos\theta = k\cos\frac{\phi}{2}$

$\therefore k=\sqrt{2}$

38. **(b)** Let A.P. be $a, a+d, a+2d, ...$

$a_2 + a_4 + ... + a_{200} = \alpha$

$\Rightarrow \frac{100}{2}\left[2(a+d)+(100-1)2d\right] = \alpha$ ...(i)

and $a_1 + a_3 + a_5 + ... + a_{199} = \beta$

$\Rightarrow \frac{100}{2}[2a + (100 - 1)\,2d] = \beta$ ...(ii)

Subtracting (ii) from (i), we get

$d = \frac{\alpha - \beta}{100}$

39. **(c)** Let number of months $= n$

$\therefore 200 \times 3 + (240 + 280 + 320 + ... + (n-3)^{th}\text{ term}) = 11040$

$\Rightarrow \frac{n-3}{2}[2 \times 240 + (n-4) \times 40] = 11040 - 600$

$\Rightarrow (n-3)[240 + 20n - 80] = 10440$

$\Rightarrow (n-3)(20n + 160) = 10440$

$\Rightarrow (n-3)(n+8) = 522$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n+26)(n-21) = 0$

$\therefore n = 21$

40. **(a)** Till 10^{th} minute number of counted notes $= 1500$

Remaining notes $= 4500 - 1500 = 3000$

$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$

$n^2 - 149n + 3000 = 0$

$\Rightarrow n = 125, 24$

But $n = 125$ is not possible

$\therefore$ Total time $= 24 + 10 = 34$ minutes.

41. **(d)** Given that

$$\frac{S_p}{S_q} = \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$$

$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$

Put $p = 11$ and $q = 41$

$\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \quad \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

42. **(c)** Coeficient of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms is ${}^mC_{r-1}$, ${}^mC_r$ and ${}^mC_{r+1}$ resp.

Given that ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ are in A.P.

$2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = \frac{r}{m-r+1} + \frac{m-r}{r+1}$

$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$.

43. **(d)** $T_m = a + (m-1)\,d = \frac{1}{n}$ .....(i)

$T_n = a + (n-1)d = \frac{1}{m}$ .....(ii)

Subtracting (ii) from (i), we get

$(m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (i) $a = \frac{1}{mn} \Rightarrow a - d = 0$

44. **(b)** $1, \log_9(3^{1-x} + 2), \log_3(4.3^x - 1)$ are in A.P.

$\because$ a, b, c are in A.P then $b = a + c$

$\Rightarrow 2\log_9(3^{1-x} + 2) = 1 + \log_3(4.3^x - 1)$

$\because \log_{b^q} a^p = \frac{p}{q}\log_b a$

$\Rightarrow \log_3(3^{1-x} + 2) = \log_3 3 + \log_3(4.3^x - 1)$

$\Rightarrow \log_3(3^{1-x} + 2) = \log_3[3(4\,.\,3^x - 1)]$

$\Rightarrow 3^{1-x} + 2 = 3\,(4.3^x - 1)$

$\Rightarrow 3.3^{-x} + 2 = 12.3^x - 3.$

Put $3^x = t$

$\Rightarrow \frac{3}{t} + 2 = 12t - 3 \Rightarrow 12t^2 - 5t - 3 = 0;$

Hence $t = -\frac{1}{3}, \frac{3}{4}$

$\Rightarrow 3^x = \frac{3}{4}$ (as $3^x \neq -ve$)

$\Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x = \log_3 3 - \log_3 4$

$\Rightarrow x = 1 - \log_3 4$

45. **(d)** Let $f(1) = k$, then $f(2) = f(1+1) = k^2$

$f(3) = f(2+1) = k^3$

$\sum_{x=1}^{\infty} f(x) = 2 \Rightarrow k + k^2 + k^3 + ...\infty = 2$

$\Rightarrow \frac{k}{1-k} = 2 \Rightarrow k = \frac{2}{3}$

Now, $\frac{f(4)}{f(2)} = \frac{k^4}{k^2} = k^2 = \frac{4}{9}.$

46. **(c)** Rearrange given equation, we get

$$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) = 0$$

$\Rightarrow (ap-b)^2 + (bp-c)^2 + (cp-d)^2 = 0$

$\therefore ap - b = bp - c = cp - d = 0$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ $\quad\therefore a, b, c, d$ are in G.P.

47. (5.00)

$\because f(x+y)=f(x)\cdot f(y) \quad \forall x\in R$ and $f(1)=3$

$\Rightarrow f(x)=3^x \Rightarrow f(i)=3^i$

$\Rightarrow \sum_{i=1}^{n} f(i)=363$

$\Rightarrow 3+3^2+3^3+....+3^n=363$

$\Rightarrow \frac{3(3^n-1)}{3-1}=363 \qquad \left[\because S_n=\frac{a(r^n-1)}{(r-1)}\right]$

$\Rightarrow 3^n-1=\frac{363\times 2}{3}=242$

$\Rightarrow 3^n=243=3^5 \Rightarrow n=5$

48. (b) Given sequence are in G.P. and common ratio $\frac{3}{2}$

$\therefore \frac{2^{10}\left(\left(\frac{3}{2}\right)^{11}-1\right)}{\left(\frac{3}{2}-1\right)}=S-2^{11}$

$\Rightarrow 2^{10}\frac{\left(\frac{3^{11}-2^{11}}{2^{11}}\right)}{\frac{1}{2}}=S-2^{11}$

$\Rightarrow 3^{11}-2^{11}=S-2^{11} \Rightarrow S=3^{11}$

49. (b) Let the first term be 'a' and common ratio be 'r'.

$\because ar(1+r+r^2)=3$ ...(i)

and $ar^5(1+r+r^2)=243$ ...(ii)

From (i) and (ii),

$r^4=81 \Rightarrow r=3$ and $a=\frac{1}{13}$

$\therefore S_{50}=\frac{a(r^{50}-1)}{r-1}=\frac{3^{50}-1}{26} \qquad \left[\because S_n=\frac{a(r^n-1)}{(r-1)}\right]$

50. (b) Let $\alpha, \beta, \gamma, \delta$ be in G.P., then $\alpha\delta=\beta\gamma$

$\Rightarrow \frac{\alpha}{\beta}=\frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha-\beta}{\alpha+\beta}\right|=\left|\frac{\gamma-\delta}{\gamma+\delta}\right|$

$\Rightarrow \frac{\sqrt{9-4p}}{3}=\frac{\sqrt{36-4q}}{6}$

$\Rightarrow 36-16p=36-4q \Rightarrow q=4p$

$\therefore \frac{2q+p}{2q-p}=\frac{8p+p}{8p-p}=\frac{9p}{7p}=\frac{9}{7}$

51. (4)

$(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....\infty\right)}$

$=0.16^{\log_{2.5}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)} \qquad \left[\because S_\infty=\frac{a}{1-r}\right]$

$=0.16^{\log_{2.5}\left(\frac{1}{2}\right)}$

$=(2.5)^{-2\log_{2.5}\left(\frac{1}{2}\right)}=\left(\frac{1}{2}\right)^{-2}=4.$

52. (c) Let terms of G.P. be $\frac{a}{r}, a, ar$

$\therefore a\left(\frac{1}{r}+1+r\right)=S$ ... (i)

and $a^3=27$

$\Rightarrow a=3$ ... (ii)

Put $a=3$ in eqn. (1), we get

$S=3+3\left(r+\frac{1}{r}\right)$

If $f(x)=x+\frac{1}{x}$, then $f(x)\in(-\infty,-2]\cup[2,\infty)$

$\Rightarrow 3f(x)\in(-\infty,-6]\cup[6,\infty)$

$\Rightarrow 3+3f(x)\in(-\infty,-3]\cup[9,\infty)$

Then, it concludes that

$S\in(-\infty,-3]\cup[9,\infty)$

53. (c) $S=(x+y)+(x^2+y^2+xy)+(x^3+x^2y+xy^2+y^3)+....\infty$

$=\frac{1}{x-y}\left[(x^2-y^2)+(x^3-y^3)+(x^4-y^4+....\infty\right]$

$=\frac{1}{x-y}\left[\frac{x^2}{1-x}-\frac{y^2}{1-y}\right]=\frac{(x-y)(x+y-xy)}{(x-y)(1-x)(1-y)}$

$\left[\because S_\infty=\frac{a}{1-r}\right]$

$=\frac{x+y-xy}{(1-x)(1-y)}$

54. (c) $S=(x+x^2+x^3+....9\text{ terms})$

$+a[k+(k+2)++(k+4)+...9\text{ terms}]$

$\Rightarrow S=\frac{x(x^9-1)}{x-1}+\frac{9}{2}[2ak+8\times(2a)]$

$\Rightarrow S=\dfrac{x^{10}-x}{x-1}+\dfrac{9a(k+8)}{1}=\dfrac{x^{10}-x+45a(x-1)}{x-1}$ (Given)

$\Rightarrow \dfrac{x^{10}-x+9a(k+8)(x-1)}{x-1}=\dfrac{x^{10}-x+45a(x-1)}{x-1}$

$\Rightarrow 9a(k+8)=45a \Rightarrow k+8=5 \Rightarrow k=-3.$

55. **(a)** $2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+.....\infty}$

$=2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.....\infty}=\sqrt{2}$

56. **(d)** Let G.P. be a, ar, ar^2 .....

$\sum_{n=1}^{100} a_{2n+1}=a_3+a_5+.....+a_{201}=200$

$\Rightarrow \dfrac{ar^2(r^{200}-1)}{r^2-1}=200$ ...(i)

$\sum_{n=1}^{100} a_{2n}=a_2+a_4+.....+a_{200}=100$

$\Rightarrow \dfrac{ar(r^{200}-1)}{r^2-1}=100$ ...(ii)

From equations (i) and (ii), $r=2$ and

$a_2+a_3+.....+a_{200}+a_{201}=300$

$\Rightarrow r(a_1+.....+a_{200})=300$

$\Rightarrow \sum_{n=1}^{200} a_n=\dfrac{300}{r}=150$

57. **(b)** $y=1+\cos^2\theta+\cos^4\theta+.....$

$\Rightarrow y=\dfrac{1}{1-\cos^2\theta} \Rightarrow \dfrac{1}{y}=\sin^2\theta$

$x=1-\tan^2\theta+\tan^4\theta+.....$

$x=\dfrac{1}{1-(-\tan^2\theta)}=\dfrac{1}{\sec^2\theta} \Rightarrow x=\cos^2\theta$

$y=\dfrac{1}{\sin^2\theta} \Rightarrow y=\dfrac{1}{1-x}$

$\therefore y(1-x)=1$

58. **(b)** $\dfrac{(49)^{126}-1}{48}=\dfrac{((49)^{63}+1)(49^{63}-1)}{48}\left[\because S_n=\dfrac{a(r^n-1)}{r-1}\right]$

$\therefore$ K $=63$

59. **(b)** Since, $a_1+a_2=4 \Rightarrow a_1+a_1r=4$ ...(i)

$a_3+a_4=16 \Rightarrow a_1r^2+a_1r^3=16$ ...(ii)

From eqn. (i), $a_1=\dfrac{4}{1+r}$ and substituting the value of a_1, in eqn (ii),

$\left(\dfrac{4}{1+r}\right)^{r^2}+\left(\dfrac{4}{1+r}\right)^{r^3}=16$

$\Rightarrow 4r^2(1+r)=16(1+r)$

$\Rightarrow r^2=4 \qquad \therefore r=\pm 2$

$r=2, a_1(1+2)=4 \Rightarrow a_1=\dfrac{4}{3}$

$r=-2, a_1(1-2)=4 \Rightarrow a_1=-4$

$\sum_{i=1}^{a} a_i=\dfrac{a_1(r^q-1)}{r-1}=\dfrac{(-4)((-2)^9-1)}{-2-1}$

$=\dfrac{4}{3}(-513)=4\lambda \Rightarrow \lambda=-171$

60. **(b)** The given series is in G.P. then

$S_n=\dfrac{a(1-r^n)}{1-r}$

$\dfrac{(1+x)^{10}\left[1-\left(\dfrac{x}{1+x}\right)^{11}\right]}{\left(1-\dfrac{x}{1+x}\right)}$

$\Rightarrow \dfrac{(1+x)^{10}\left[(1+x)^{11}-x^{11}\right]}{(1+x)^{11}\times\dfrac{1}{(1+x)}}=(1+x)^{11}-x^{11}$

$\therefore$ Coefficient of x^7 is ${}^{11}C_7={}^{11}C_{11-7}={}^{11}C_4=330$

61. **(d)** $\because \alpha, \beta, \gamma$ are three consecutive terms of a non-constant G.P.

$\therefore \beta^2=\alpha\gamma$

So roots of the equation $\alpha x^2+2\beta x+\gamma=0$ are

$\dfrac{-2\beta\pm 2\sqrt{\beta^2-\alpha\gamma}}{2\alpha}=\dfrac{\beta}{\alpha}$

$\because \alpha x^2+2\beta x+\gamma=0$ and $x^2+x-1=0$ have a common root.

$\therefore$ this root satisfy the equation $x^2+x-1=0$

$\beta^2-\alpha\beta-\alpha^2=0$

$\Rightarrow \alpha\gamma-\alpha\beta-\alpha^2=0 \Rightarrow \alpha+\beta=\gamma$

Now, $\alpha(\beta+\gamma)=\alpha\beta+\alpha\gamma$

$=\alpha\beta+\beta^2=(\alpha+\beta)\beta=\beta\gamma$

62. **(d)** $\because a, b, c$ are in G.P. $\Rightarrow b=ar, c=ar^2$

$\because 3a, 7b, 15c$ are in A.P. $\Rightarrow 3a, 7ar, 15ar^2$ are in A.P.

$\therefore 14ar=3a+15ar^2$

$\Rightarrow 15r^2-14r+3=0 \Rightarrow r=\dfrac{1}{3}$ or $\dfrac{3}{5}$

$\because r < \frac{1}{2}$ $\therefore r = \frac{3}{5}$ rejected

Fourth term $= 15ar^2 + 7ar - 3a$

$= a\,(15r^2 + 7r - 3) = a\left(\frac{15}{9} + \frac{7}{3} - 3\right) = a$

63. **(a)** Since a, b, c are in G.P.

$\therefore$ $b^2 = ac$

Given equation is, $ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 \Rightarrow (\sqrt{a}x + \sqrt{c})^2 = 0$

$\Rightarrow x = -\sqrt{\frac{c}{a}}$

Also, given that $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root.

$\Rightarrow x = -\sqrt{\frac{c}{a}}$ must satisfy $dx^2 + 2ex + f = 0$

$\Rightarrow d.\frac{c}{a} + 2e\left(-\sqrt{\frac{c}{a}}\right) + f = 0$

$\frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0 \Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c} \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

64. **(d)** Let three terms of a G.P. be $\frac{a}{r}, a, ar$

$\frac{a}{r} \cdot a \cdot ar = 512$

$a^3 = 512 \Rightarrow a = 8$

4 is added to each of the first and the second of three terms then three terms are, $\frac{8}{r} + 4, 8 + 4, 8r.$

$\because \frac{8}{r} + 4, 12, 8r$ form an A.P.

$\therefore$ $2 \times 12 = \frac{8}{r} + 8r + 4$

$\Rightarrow$ $2r^2 - 5r + 2 = 0$

$\Rightarrow$ $(2r - 1)(r - 2) = 0$

$\Rightarrow$ $r = \frac{1}{2}$ or 2

Therefore, sum of three terms $= \frac{8}{2} + 8 + 16 = 28$

65. **(c)** $x^2 \sin\theta - x(\sin\theta . \cos\theta + 1) + \cos\theta = 0.$

$x^2 \sin\theta - x\sin\theta . \cos\theta - x + \cos\theta = 0.$

$x\sin\theta\,(x - \cos\theta) - 1\,(x - \cos\theta) = 0.$

$(x - \cos\theta)\,(x\sin\theta - 1) = 0.$

$\therefore$ $x = \cos\theta, \operatorname{cosec}\theta, \theta \in (0, 45°)$

$\therefore$ $\alpha = \cos\theta, \beta = \operatorname{cosec}\theta$

$$\sum_{n=0}^{\infty} \alpha^n = 1 + \cos\theta + \cos^2\theta + \ldots \infty = \frac{1}{1 - \cos\theta}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\beta^n} = 1 - \frac{1}{\operatorname{cosec}\theta} + \frac{1}{\operatorname{cosec}^2\theta} - \frac{1}{\operatorname{cosec}^3\theta} + \ldots \infty$$

$= 1 - \sin\theta + \sin^2\theta - \sin^3\theta + \ldots \infty.$

$= \frac{1}{1 + \sin\theta}$

$$\therefore \sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right) = \sum_{n=0}^{\infty}\alpha^n + \sum_{n=0}^{\infty}\frac{(-1)^n}{\beta^n}$$

$= \frac{1}{1 - \cos\theta} + \frac{1}{1 + \sin\theta}.$

66. **(a)** Let $a_1 = a, a_2 = ar, a_3 = ar^2 \ldots a_{10} = ar^9$

where r = common ratio of given G.P.

Given, $\frac{a_3}{a_1} = 25$

$\Rightarrow$ $\frac{ar^2}{a} = 25$

$\Rightarrow$ $r = \pm 5$

Now, $\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = (\pm 5)^4 = 5^4$

67. **(b)** Let the terms of infinite series are $a, ar, ar^2, ar^3, \ldots$

So, $\frac{a}{1 - r} = 3$

Since, sum of cubes of its terms is $\frac{27}{19}$ that is sum of a^3, $a^3r^3, \ldots \infty$ is $\frac{27}{19}$

So, $\frac{a^3}{1 - r^3} = \frac{27}{19}$

$\Rightarrow$ $\frac{a}{1 - r} \times \frac{a^2}{(1 + r^2 + r)} = \frac{27}{19}$

$\Rightarrow$ $\frac{9(1 + r^2 - 2r) \times 3}{1 + r^2 + r} = \frac{27}{19}$

$\Rightarrow$ $6r^2 - 13r + 6 = 0$

$\Rightarrow$ $(3r - 2)(2r - 3) = 0$

$\Rightarrow$ $r = \frac{2}{3}$, or $\frac{3}{2}$

As $|r| < 1$

So, $r = \frac{2}{3}$

68. **(d)** $S_n = \left(\dfrac{1-q^{n+1}}{1-q}\right), T_n = \dfrac{1-\left(\dfrac{q+1}{2}\right)^{n+1}}{1-\left(\dfrac{q+1}{2}\right)}$

$$\Rightarrow T_{100} = \frac{1-\left(\dfrac{q+1}{2}\right)^{101}}{1-\left(\dfrac{q+1}{2}\right)}$$

$$Sn = \frac{1}{1-q} - \frac{q^{n+1}}{1-q}, T_{100} = \frac{2^{101}-(q+1)^{101}}{2^{100}(1-q)}$$

Now, ${}^{101}C_1 + {}^{101}C_2 S_1 + {}^{101}C_3 S_2 + \ldots + {}^{101}C_{101} S_{100}$

$$= \left(\frac{1}{1-q}\right)({}^{101}C_2 + \ldots + {}^{101}C_{101})$$

$$-\frac{1}{1-q}({}^{101}C_2 q^2 + {}^{101}C_3 q^3 + \ldots + {}^{101}C_{101} q^{101}) + 101$$

$$= \frac{1}{1-q}(2^{101} - 1 - 101) - \left(\frac{1}{1-q}\right)((1+q)^{101} - 1 - {}^{101}C_1 q) + 101$$

$$= \frac{1}{1-q}[2^{101} - 102 - (1+q)^{101} + 1 + 101q] + 101$$

$$= \frac{1}{1-q}[2^{101} - 101 + 101q - (1+q)^{101}] + 101$$

$$= \left(\frac{1}{1-q}\right)[2^{101} - (1+q)^{101}] = 2^{100}\, T_{100}$$

Hence, by comparison $\alpha = 2^{100}$

69. **(d)** Let first term and common difference be A and D respectively.

$\therefore a = A + 6D, b = A + 10D$

and $c = A + 12D$

Since, a, b, c are in G.P.

Hence, relation between a, b and c is,

$\therefore b^2 = a.c.$

$\therefore (A + 10D)^2 = (A + 6D)(A + 12D)$

$\therefore 14D + A = 0$

$\therefore A = -14D$

$\therefore a = -8D, b = -4D$ and $c = -2D$

$$\therefore \frac{a}{c} = \frac{-8D}{-2D} = 4$$

70. **(d)** $\because a, b, c,$ are in G.P.

$\Rightarrow b^2 = ac$

Since, $a + b + c = xb$

$\Rightarrow a + c = (x-1)b$

Take square on both sides, we get

$a^2 + c^2 + 2ac = (x-1)^2 b^2$

$\Rightarrow a^2 + c^2 = (x-1)^2 ac - 2ac$ $[\because b^2 = ac]$

$\Rightarrow a^2 + c^2 = ac[(x-1)^2 - 2]$

$\Rightarrow a^2 + c^2 = ac[x^2 - 2x - 1]$

$\because a^2 + c^2$ are positive and $b^2 = ac$ which is also positive. Then, $x^2 - 2x - 1$ would be positive but for $x = 2$, $x^2 - 2x - 1$ is negative.

Hence, x cannot be taken as 2.

71. **(c)** First term $= b$ and common ratio $=$ r

For infinite series, Sum $= \dfrac{b}{1-r} = 5$

$\Rightarrow b = 5(1 - \text{r})$

So, interval of $b = (0, 10)$ as, $-1 < \text{r} < 1$ for infinite G.P.

72. **(b)** $A_n = \left(\dfrac{3}{4}\right) - \left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\dfrac{3}{4}\right)^n$

Which is a G.P. with $a = \dfrac{3}{4}, r = \dfrac{-3}{4}$ and number of terms $= n$

$$\therefore A_n = \frac{\frac{3}{4}\times\left(1-\left(\frac{-3}{4}\right)^n\right)}{1-\left(\frac{-3}{4}\right)} = \frac{\frac{3}{4}\times\left(1-\left(\frac{-3}{4}\right)^n\right)}{\frac{7}{4}}$$

$$\Rightarrow A_n = \frac{3}{7}\left[1-\left(\frac{-3}{4}\right)^n\right] \quad \ldots(i)$$

As, $B_n = 1 - A_n$

For least odd natural number p, such that $B_n > A_n$

$\Rightarrow 1 - A_n > A_n \quad \Rightarrow 1 > 2 \times A_n \Rightarrow A_n < \dfrac{1}{2}$

From eqn. (i), we get

$$\frac{3}{7}\times\left[1-\left(\frac{-3}{4}\right)^n\right] < \frac{1}{2} \Rightarrow 1 - \left(\frac{-3}{4}\right)^n < \frac{7}{6}$$

$$\Rightarrow 1 - \frac{7}{6} < \left(\frac{-3}{4}\right)^n \Rightarrow \frac{-1}{6} < \left(\frac{-3}{4}\right)^n$$

As n is odd, then $\left(\frac{-3}{4}\right)^n = -\frac{3^n}{4}$

So $\frac{-1}{6} < -\left(\frac{3}{4}\right)^n \Rightarrow \frac{1}{6} > \left(\frac{3}{4}\right)^n$

$\log\left(\frac{1}{6}\right) = n\log\left(\frac{3}{4}\right) \Rightarrow 6.228 < n$

Hence, n should be 7.

73. **(d)** $\because a, b, c$ are in A.P. then

$a + c = 2b$

also it is given that,

$a + b + c = \frac{3}{4}$ ...(i)

$\Rightarrow 2b + b = \frac{3}{4} \Rightarrow b = \frac{1}{4}$ ...(ii)

Again it is given that, a^2, b^2, c^2 are in G.P. then

$(b^2)^2 = a^2c^2 \Rightarrow ac = \pm\frac{1}{16}$ ...(iii)

From (i), (ii) and (iii), we get;

$a \pm \frac{1}{16a} = \frac{1}{2} \qquad \Rightarrow 16a^2 - 8a \pm 1 = 0$

Case I: $16a^2 - 8a + 1 = 0$

$\Rightarrow a = \frac{1}{4}$ (not possible as $a < b$)

Case II: $16a^2 - 8a - 1 = 0 \Rightarrow a = \frac{8 \pm \sqrt{128}}{32}$

$\Rightarrow a = \frac{1}{4} \pm \frac{1}{2\sqrt{2}}$

$\therefore a = \frac{1}{4} - \frac{1}{2\sqrt{2}} \qquad (\because a < b)$

74. **(d)** Let the GP be a, ar and ar^2 then a = A + d; ar= A + 4d; $ar^2 = A + 8d$

$\Rightarrow \frac{ar^2 - ar}{ar - a} = \frac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}$

$r = \frac{4}{3}$

75. **(b)** $z = 1 + ai$

$z^2 = 1 - a^2 + 2ai$

$z^2.z = \{(1 - a^2) + 2ai\} \qquad \{1 + ai\}$

$= (1 - a^2) + 2ai + (1 - a^2) \qquad ai - 2a^2$

$\because z^3$ is real $\Rightarrow 2a + (1 - a^2)\,a = 0$

$a\,(3 - a^2) = 0 \Rightarrow a = \sqrt{3}\ (a > 0)$

$1 + z + z^2 .... z^{11} = \frac{z^{12}-1}{z-1} = \frac{(1+\sqrt{3}i)^{12}-1}{1+\sqrt{3}i-1}$

$= \frac{(1+\sqrt{3}i)^{12}-1}{\sqrt{3}i}$

$(1+\sqrt{3}i)^{12} = 2^{12}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{12}$

$= 2^{12}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{12} = 2^{12}(\cos 4\pi + i\sin 4\pi) = 2^{12}$

$\Rightarrow \frac{2^{12}-1}{\sqrt{3}i} = \frac{4095}{\sqrt{3}i} = -\frac{4095}{3}\sqrt{3}i = -1365\sqrt{3}i$

76. **(d)** $m = \frac{l+n}{2}$ and common ratio of G.P. = $r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$

$\therefore G_1 = l^{3/4}n^{1/4}, G_2 = l^{1/2}n^{1/2}, G_3 = l^{1/4}n^{3/4}$

$G_1^4 + 2G_2^4 + G_3^4 = l^3n + 2l^2n^2 + ln^3$

$= ln\,(l+n)^2 = ln \times (2m)^2 = 4lm^2n$

77. **(d)** Let a, ar and ar^2 be the first three terms of G.P

According to the question

$a\,(ar)\,(ar^2) = 1000 \Rightarrow (ar)^3 = 1000 \Rightarrow ar = 10$

and $ar^2 + ar^3 = 60 \Rightarrow ar\,(r + r^2) = 60$

$\Rightarrow r^2 + r - 6 = 0$

$\Rightarrow r = 2, -3$

$a = 5, a = -\frac{10}{3}$ (reject)

Hence, $T_7 = ar^6 = 5(2)^6 = 5 \times 64 = 320.$

78. **(b)** Let a, ar, ar^2 are in G.P.

According to the question

$a, 2ar, ar^2$ are in A.P.

$\Rightarrow 2 \times 2ar = a + ar^2$

$\Rightarrow 4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

Since $r > 1$

$\therefore r = 2 - \sqrt{3}$ is rejected

Hence, $r = 2 + \sqrt{3}$

79. **(b)** $1 - \frac{2}{3} - \frac{2}{3^2} .... \frac{2}{3^{n-1}} < \frac{1}{100}$

$\Rightarrow 1 - \frac{2}{3}\left[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ... \frac{1}{3^{n-1}}\right] < \frac{1}{100}$

$\Rightarrow \frac{1 - 2\left[\frac{1}{3}\left(\frac{1}{3^n} - 1\right)\right]}{\frac{1}{3} - 1} < \frac{1}{100}$

$$\Rightarrow 1-2\left[\frac{3^n-1}{2.3^n}\right]<\frac{1}{100}$$

$$\Rightarrow 1-\left[\frac{3^n-1}{3^n}\right]<\frac{1}{100}$$

$$\Rightarrow 1-1+\frac{1}{3^n}<\frac{1}{100} \Rightarrow 100 < 3^n$$

Thus, least value of n is 5

80. **(c)** According to Question

$\Rightarrow \frac{S_5}{S_5'} = 49$ {here, S_5 = Sum of first 5 terms and S_5 = Sum of their reciprocals)

$$\Rightarrow \frac{\frac{a(r^5-1)}{(r-1)}}{\frac{a^{-1}(r^{-5}-1)}{(r^{-1}-1)}} = 49$$

$$\Rightarrow \frac{a(r^5-1)\times(r^{-1}-1)}{a^{-1}(r^{-5}-1)\times(r-1)} = 49$$

or $\frac{a^2(\cancel{1-r^5})\times(\cancel{1-r})\times r^5}{(\cancel{1-r^5})\times(\cancel{1-r})\times r} = 49$

$\Rightarrow a^2r^4 = 49 \Rightarrow a^2r^4 = 7^2$

$\Rightarrow \boxed{ar^2 = 7}$ ...(i)

Also, given, $S_1 + S_3 = 35$

$a + ar^2 = 35$ ...(ii)

Now substituting the value of eq. (i) in eq. (ii)

$a + 7 = 35$

$\boxed{a = 28}$

81. **(d)** Let given expansion be

$S = (1 + x)^{1000} + x(1 + x)^{999} + x^2(1 + x)^{998} + ... + ... + x^{1000}$

Put $1 + x = t$

$S = t^{1000} + xt^{999} + x^2(t)^{998} + ... + x^{1000}$

This is a G.P with common ratio $\frac{x}{t}$

$$S = \frac{t^{1000}\left[1-\left(\frac{x}{t}\right)^{1001}\right]}{1-\frac{x}{t}}$$

$$= \frac{(1+x)^{1000}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]}{1-\frac{x}{1+x}}$$

$$= \frac{(1+x)^{1001}\left[(1+x)^{1001}-x^{1001}\right]}{(1+x)^{1001}}$$

$$= [(1+x)^{1001}-x^{1001}]$$

Now coeff of x^{50} in above expansion is equal to coeff of x^{50} in $(1 + x)^{1001}$ which is ${}^{1001}C_{50}$

$$= \frac{(1001)!}{50!(951)!}$$

82. **(b)** Let a, b, c, d be four numbers of the sequence.

Now, according to the question $b^2 = ac$ and $c - b = 6$ and $a - c = 6$

Also, given $\boxed{a = d}$

$\therefore b^2 = ac \Rightarrow b^2 = a\left[\frac{a+b}{2}\right]$ $(\because 2c = a + b)$

$\Rightarrow a^2 - 2b^2 + ab = 0$

Now, $c - b = 6$ and $a - c = 6$,

gives $a - b = 12$

$\Rightarrow b = a - 12$

$\therefore a^2 - 2b^2 + ab = 0$

$\Rightarrow a^2 - 2(a - 12)^2 + a(a - 12) = 0$

$\Rightarrow a^2 - 2a^2 - 288 + 48a + a^2 - 12a = 0$

$\Rightarrow 36a = 288 \Rightarrow a = 8$

Hence, last term is $d = a = 8$.

83. **(d)** The given relation can be written as

$(a^2p^2 - 2abp + b^2) + (b^2p^2 + c^2 - 2bpc) + (c^2p^2 + d^2 - 2pcd) \le 0$

or $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$ ...(i)

Since a, b, c, d and p are all real, the inequality (i) is possible only when each of factor is zero.

i.e., $ap - b = 0$, $bp - c = 0$ and $cp - d = 0$

or $p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

or a, b, c, d are in G.P.

84. **(c)** Let $a, ar, ar^2, ar^3, ar^4, ar^5$ be six terms of a G.P. where 'a' is first term and r is common ratio.

According to given conditions, we have

$ar^3 - a = 5 \Rightarrow a(r^3 - 1) = 52$ ...(i)

and $a + ar + ar^2 = 26$

$\Rightarrow a(1 + r + r^2) = 26$ ...(ii)

To find: $a(1 + r + r^2 + r^3 + r^4 + r^5)$

Consider

$a[1 + r + r^2 + r^3 + r^4 + r^5]$

$= a[1 + r + r^2 + r^3(1 + r + r^2)]$

$= a[1+r+r^2][1+r^3]$ ...(iii)

Divide (i) by (ii), we get

$\frac{r^3-1}{1+r+r^2} = 2$,

we know $r^3-1=(r-1)(1+r+r^2)$

$\therefore\ r-1=2 \Rightarrow r=3$ and $a=2$

$\therefore\ a(1+r+r^2+r^3+r^4+r^5) = a(1+r+r^2)(1+r^3)$

$= 2(1+3+9)(1+27) = 26\times 28 = 728$

85. **(b)** ATQ,

$a + ar = 12$ ...(i)

$ar^2 + ar^3 = 48$ ...(ii)

$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$

($\because$ terms are alternately + ve and –ve)

$\Rightarrow a = -12$

86. **(b)** Let the series a, ar, ar^2, ... are in geometric progression.

Given that, $a = ar + ar^2$

$\Rightarrow 1 = r + r^2 \quad \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1\pm\sqrt{1-4\times -1}}{2} = \frac{-1\pm\sqrt{5}}{2}$

$\Rightarrow r = \frac{\sqrt{5}-1}{2}$ [$\because$ terms of G.P. are positive

$\therefore$ r should be positive]

87. **(d)** $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$

$= i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11} - i\sin\frac{2k\pi}{11}\right)$ [$\because e^{i\theta} = \cos\theta + i\sin\theta$]

$= i\sum_{k=1}^{10} e^{-\frac{2k\pi}{11}i} = i\left\{\sum_{k=0}^{10} e^{-\frac{2k\pi}{11}i} - 1\right\}$

$= i\left[1 + e^{-\frac{2\pi}{11}i} + e^{-\frac{4\pi}{11}i} + ...11 \text{ terms}\right] - i$

$= i\left[\frac{1-\left(e^{-\frac{2\pi}{11}}\right)^{11}}{1-e^{-\frac{2\pi}{11}i}}\right] - i = i\left[\frac{1-e^{-2\pi i}}{1-e^{-\frac{2\pi}{11}i}}\right] - i$

$= i\times 0 - i$ [$\because e^{-2\pi i} = 1$]

$= -i$

88. **(d)** $(1-ax)^{-1}(1-bx)^{-1}$

$= (1+ax+a^2x^2+...)(1+bx+b^2x^2+...)$

$\therefore$ Coefficient of x^n

$x^n = b^n + ab^{n-1} + a^2b^{n-2} + ... + a^{n-1}b + a^n$

{which is a G.P. with $r = \frac{a}{b}$

$\therefore$ Its sum is $= \frac{b^n\left[1-\left(\frac{a}{b}\right)^{n+1}\right]}{1-\frac{a}{b}}$}

$= \frac{b^{n+1}-a^{n+1}}{b-a} \qquad \therefore\ a_n = \frac{b^{n+1}-a^{n+1}}{b-a}$

89. **(b)** Let two numbers be a and b then $\frac{a+b}{2} = 9$

$\Rightarrow a+b = 18$ and $\sqrt{ab} = 4 \Rightarrow ab = 16$

$\therefore$ Equation with roots a and b is

$x^2 - (a+b)x + ab = 0 \Rightarrow x^2 - 18x + 16 = 0$

90. **(b)** Let a = first term of G.P. and r = common ratio of G.P.; Then G.P. is a, ar, ar^2

Given $S_\infty = 20 \Rightarrow \frac{a}{1-r} = 20$

$\Rightarrow a = 20(1-r)$ ...(i)

Also $a^2 + a^2r^2 + a^2r^4 + ...$ to $\infty = 100$

$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow \frac{[20(1-r)]^2}{1-r^2} = 100$ [from (i)]

$\Rightarrow \frac{400(1-r)^2}{(1-r)(1+r)} = 100 \Rightarrow 4(1-r) = 1+r$

$\Rightarrow 1+r = 4-4r \Rightarrow 5r = 3 \Rightarrow r = 3/5.$

91. **(b)** $\because a_4 = 2 \Rightarrow ar^4 = 2$

Now, $a\times ar\times ar^2\times ar^3\times ar^4\times ar^5\times ar^6\times ar^7\times ar^8$

$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

92. **(c)** $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow \cos^2 2\theta(1-\cos^2 2\theta) = \frac{1}{4}$ ...(i)

$\because$ G.M. $\le$ A.M.

$\therefore (\cos^2 2\theta)(1-\cos^2 2\theta) \le \left(\frac{\cos^2 2\theta + (1-\cos^2 2\theta)}{2}\right)^2$

$= \frac{1}{4}$ ...(ii)

So, from equation (i) and (ii), we get.

G.M. = A.M.

It is possible only if,

$\cos^2 2\theta = 1 - \cos^2 2\theta$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} \Rightarrow \cos 2\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \quad \therefore \text{ Sum} = \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$

93. (39)

Let m arithmetic mean be $A_1, A_2 \ldots A_m$ and G_1, G_2, G_3 be geometric mean.

The A.P. formed by arithmetic mean is,

$3, A_1, A_2, A_3, \ldots\ldots .A_m, 243$

$\therefore\ d = \frac{243-3}{m+1} = \frac{240}{m+1}$

The G.P. formed by geometric mean

$3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$

$\because A_4 = G_2$

$\Rightarrow 3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$

$\Rightarrow 3 + \frac{960}{m+1} = 27 \Rightarrow m+1 = 40 \Rightarrow m = 39.$

94. (d) A.T.Q.,

A.M. = 5 G.M.

$\frac{a+b}{2} = 5\sqrt{ab}$

$\frac{a+b}{\sqrt{ab}} = 10$

$\therefore\ \frac{a}{b} = \frac{10+\sqrt{96}}{10-\sqrt{96}} = \frac{10+4\sqrt{6}}{10-4\sqrt{6}}$

Use Componendo and Dividendo

$\frac{a+b}{a-b} = \frac{20}{8\sqrt{6}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

95. (b) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{1 - \tan A \tan B}$ where $y = \tan A + \tan B$

$\Rightarrow \tan A \tan B = 1 - \sqrt{3}y$

Also AM $\geq$ GM

$\Rightarrow \frac{\tan A + \tan B}{2} \geq \sqrt{\tan A \tan B}$

$\Rightarrow y \geq 2\sqrt{1-\sqrt{3}y}$

$\Rightarrow y^2 \geq 4 - 4\sqrt{3}y$

$\Rightarrow y^2 + 4\sqrt{3}y - 4 \geq 0$

$\Rightarrow y \leq -2\sqrt{3} - 4$ or $y \geq -2\sqrt{3} + 4$

($y \leq -2\sqrt{3} - 4$ is not possible as $\tan A \tan B > 0$)

96. (b) $x + y + z = 12$

AM $\geq$ GM

$$\frac{3\left(\frac{x}{3}\right) + 4\left(\frac{y}{4}\right) + 5\left(\frac{z}{5}\right)}{12} \geq \sqrt[12]{\left(\frac{x}{3}\right)^3 \left(\frac{y}{4}\right)^4 \left(\frac{z}{5}\right)^5}$$

$\frac{x^3 y^4 z^5}{3^3 4^4 5^5} \leq 1$

$x^3 y^4 z^5 \leq 3^3 .4^4 .5^5$

$x^3 y^4 z^5 \leq (0.1)(600)^3$

But, given $x^3 y^4 z^5 = (0.1)(600)^3$

$\therefore$ all the number are equal

$\therefore\ \frac{x}{3} = \frac{y}{4} = \frac{z}{5}\ (= k)$

$x = 3k; y = 4k; z = 5k$

$x + y + z = 12$

$3k + 4k + 5k = 12$

$k = 1$

$\therefore\ x = 3; y = 4; z = 5$

$\therefore\ x^3 + y^3 + z^3 = 216$

97. (a) $G = \sqrt{ab}$

$M = \frac{\frac{1}{a} + \frac{1}{b}}{2}$

$M = \frac{a+b}{2ab}$

Given that $\frac{1}{M} : G = 4 : 5$

$\frac{2ab}{(a+b)\sqrt{ab}} = \frac{4}{5}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4}$

{Using Componendo & Dividendo}

$$\Rightarrow \frac{(\sqrt{a})^2+(\sqrt{b})^2+2\sqrt{ab}}{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{ab}} = \frac{9}{1}$$

$$\Rightarrow \left(\frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}-\sqrt{a}}\right)^2 = \frac{9}{1} \Rightarrow \frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}-\sqrt{a}} = \frac{3}{1}$$

$$\Rightarrow \frac{\sqrt{b}+\sqrt{a}+\sqrt{b}-\sqrt{a}}{\sqrt{b}+\sqrt{a}-\sqrt{b}+\sqrt{a}} = \frac{3+1}{3-1}$$ {Using Componendo & Dividendo}

$$\sqrt{\frac{b}{a}} = \frac{4}{2} = 2$$

$$\frac{b}{a} = \frac{4}{1}$$

$$\frac{a}{b} = \frac{1}{4} \Rightarrow a : b = 1 : 4$$

98. **(d)** $\because a_1, a_2, a_3.....a_n$ are in H.P.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} .... \frac{1}{a_n}$ are in A.P.

$$\therefore \frac{1}{a_2}-\frac{1}{a_1} = \frac{1}{a_3}-\frac{1}{a_2} = ... = \frac{1}{a_n}-\frac{1}{a_{n-1}} = d \text{ (say)}$$

Then $a_1a_2 = \frac{a_1-a_2}{d}$, $a_2a_3 = \frac{a_2-a_3}{d}$,

..........., $a_{n-1}a_n = \frac{a_{n-1}-a_n}{d}$

Adding all equations, we get

$$\therefore a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$$

$$= \frac{a_1-a_2}{d} + \frac{a_2-a_3}{d} + ... + \frac{a_{n-1}-a_n}{d}$$

$$= \frac{1}{d}[a_1-a_2+a_2-a_3+....+a_{n-1}-a_n] = \frac{a_1-a_n}{d}$$

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$$\Rightarrow \frac{a_1-a_n}{a_1a_n} = (n-1)d$$

$$\Rightarrow \frac{a_1-a_n}{d} = (n-1)a_1a_n$$

Which is the required result.

99. **(d)** $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow a = 1-\frac{1}{x}$

$$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \Rightarrow b = 1-\frac{1}{y}$$

$$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \Rightarrow c = 1-\frac{1}{z}$$

a, b, c are in A.P. $\Rightarrow$ $2b = a+c$

$$2\left(1-\frac{1}{y}\right) = 1-\frac{1}{x}+1-\frac{1}{z}$$

$\frac{2}{y} = \frac{1}{x}+\frac{1}{z}$ $\Rightarrow$ x, y, z are in H.P.

100. **(d)** $ax^2+bx+c=0$, $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

ATQ, $\alpha+\beta = \frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$\alpha+\beta = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} \Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2+bc^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a}+\frac{b}{c}$ [Divide both side by abc]

$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P.

$\therefore \frac{a}{c}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

101. **(b)** The given series is

$$1+(1-2^2\cdot 1)+(1-4^2\cdot 3)+(1-6^2\cdot 5)+...(1-20^2\cdot 19)$$

$$S = 1+\sum_{r=1}^{10}[1-(2r)^2(2r-1)]$$

$$= 1+\sum_{r=1}^{10}(1-8r^3+4r^2) = 1+10-\sum_{r=1}^{10}(8r^3-4r^2)$$

$$= 11-8\left(\frac{10\times 11}{2}\right)^2+4\times\left(\frac{10\times 11\times 21}{6}\right)$$

$$= 11-2\times(110)^2+4\times 55\times 7$$

$$= 11-220(110-7)$$

$$= 11-220\times 103 = \alpha-220\beta$$

$$\Rightarrow \alpha = 11, \beta = 103$$

$$\therefore (\alpha, \beta) = (11, 103)$$

102. **(a)** Given : $f(x+y)=f(x)+f(y), \forall x, y \in R, f(1)=2$

$\Rightarrow f(2)=f(1)+f(1)=2+2=4$

$f(3)=f(1)+f(2)=2+4=6$

$f(n-1)=2(n-1)$

Now, $g(n)=\sum_{k=1}^{n-1} f(k)$

$=f(1)+f(2)+f(3)+\ldots .f(n-1)$

$=2+4+6+\ldots . .+2(n-1)$

$=2[1+2+3+\ldots . .+(n-1)]$

$=2 \times \frac{(n-1)(n)}{2}=n^2-n$

$\because g(n)=20$ (given)

So, $n^2-n=20$

$\Rightarrow n^2-n-20=0$

$\Rightarrow(n-5)(n+4)=0$

$\Rightarrow n=5$ or $n=-4$ (not possible)

103. **(504)** $\left[\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}\right] \frac{1}{4}\left[\sum_{n=1}^{7}(2n^3+3n^2+n)\right]$

$=\frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2+3\left(\frac{7.8.15}{6}\right)+\frac{7.8}{2}\right]$

$\Rightarrow \frac{1}{4}[2 \times 49 \times 16+28 \times 15+28]$

$=\frac{1}{4}[1568+420+28]=504$

104. **(1540)** Given series can be written as

$\sum_{k=1}^{20} \frac{k(k+1)}{2}=\frac{1}{2} \sum_{k=1}^{20}(k^2+k)$

$=\frac{1}{2}\left[\frac{20(21)(41)}{6}+\frac{20(21)}{2}\right]$

$=\frac{1}{2}\left[\frac{420 \times 41}{6}+\frac{20 \times 21}{2}\right]=\frac{1}{2}[2870+210]=1540$

105. **(a)** $S=\underbrace{3+4}+\underbrace{8+9}+\underbrace{13+14}+\underbrace{18+19} \ldots . .40$ terms

$S=7+17+27+37+47+\ldots . .20$ terms

$S_{40}=\frac{20}{2}[2 \times 7+(19) 10]=10[14+190]$

$=10[2040]=(102)(20)$

$\Rightarrow m=20$

106. **(b)** $\because [x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right] \ldots .\left[x+\frac{n-1}{n}\right]=[nx]$

and $[x]+[-x]=-1 \ (x \notin z)$

$\therefore \left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-100\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]$

$=-100-\left\{\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{100}\right]+\ldots\left[\frac{1}{3}+\frac{99}{100}\right]\right\}$

$=-100-\left[\frac{100}{3}\right]=-133$

107. **(c)** r^{th} term of the series,

$T_r=\frac{(2r+1)(1^3+2^3+3^3+\ldots+r^3)}{1^2+2^2+3^2+\ldots+r^2}$

$T_r=(2r+1)\left(\frac{r(r+1)}{2}\right)^2 \times \frac{6}{r(r+1)(2r+1)}=\frac{3r(r+1)}{2}$

$\therefore$ sum of 10 terms is $=S=\sum_{r=1}^{10} T_r=\frac{3}{2} \sum_{r=1}^{10}(r^2+r)$

$=\frac{3}{2}\left\{\frac{10 \times(10+1)(2 \times 10+1)}{6}+\frac{10 \times 11}{2}\right\}$

$=\frac{3}{2} \times 5 \times 11 \times 8=660$

108. **(a)** Let, $S=1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\ldots 15$ terms

$T_n=\frac{1^3+2^3+\ldots n^3}{1+2+\ldots n}=\frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)}{2}}=\frac{n(n+1)}{2}$

Now, $S=\frac{1}{2}\left(\sum_{n=1}^{15} n^2+\sum_{n=1}^{15} n\right)=\frac{1}{2}\left(\frac{15(16)(31)}{6}+\frac{15(16)}{2}\right)$

$=680$

$\therefore$ required sum is, $680-\frac{1}{2} \frac{15(16)}{2}=680-60=620$

109. **(b)** $1+2.3+3.5+4.7+\ldots \ldots$

Let, $S=(2.3+3.5+4.7+\ldots \ldots)$

Now, $S_{10}=\sum_{n=1}^{10}(n+1)(2n+1)=\sum_{n=1}^{10}\left(2n^2+3n+1\right)$

$$= \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$$

Put $n = 10$

$$= \frac{2.10.11.21}{6} + \frac{3.10.11}{2} + 10 = 945$$

Hence required sum of the series = 1 + 945 = 946

110. **(d)** Number of balls used in equilateral triangle

$$= \frac{n(n+1)}{2}$$

$\because$ side of equilateral triangle has n-balls

$\therefore$ no. of balls in each side of square is $= (n-2)$

According to the question,

$$\frac{n(n+1)}{2} + 99 = (n-2)^2$$

$\Rightarrow n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow n^2 - 9n - 190 = 0 \Rightarrow (n-19)(n+10) = 0$

$\Rightarrow n = 19$

Number of balls used to form triangle

$$= \frac{n(n+1)}{2} = \frac{19\times 20}{2} = 190$$

111. **(c)** Let, $S = \sum_{k=1}^{20} k.\frac{1}{2^k}$

$$S = \frac{1}{2} + 2.\frac{1}{2^2} + 3.\frac{1}{2^3} + ... + 20.\frac{1}{2^{20}} \quad ...(i)$$

$$\frac{1}{2}S = \frac{1}{2^2} + 2.\frac{1}{2^3} + ... + 19\frac{1}{2^{20}} + 20\frac{1}{2^{21}} \quad ...(ii)$$

On subtracting equations (ii) by (i),

$$\frac{S}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{20}}\right) - 20\frac{1}{2^{21}}$$

$$= \frac{\frac{1}{2}\left(1 - \frac{1}{2^{20}}\right)}{1 - \frac{1}{2}} - 20.\frac{1}{2^{21}} = 1 - \frac{1}{2^{20}} - 10.\frac{1}{2^{20}}$$

$$\frac{S}{2} = 1 - 11.\frac{1}{2^{20}} \Rightarrow S = 2 - 11.\frac{1}{2^{19}} = 2 - \frac{11}{2^{19}}$$

112. **(c)** $\because 1 + 2 + 3 + ... + k = \frac{k(k+1)}{2}$

$$\therefore S_k = \frac{k(k+1)}{2k} = \frac{k+1}{2}$$

$$\Rightarrow \frac{5}{12}A = \frac{1}{4}[2^2 + 3^3 + ... + 11^2]$$

$$= \frac{1}{4}[1^2 + 2^2 + ... + 11^2 - 1]$$

$$= \frac{1}{4}\left[\frac{11(11+1)(2\times 11+1)}{6} - 1\right]$$

$$\frac{1}{4}\left[\frac{11\times 12\times 23}{6} - 1\right]$$

$$= \frac{1}{4}[505]$$

$$A = \frac{505}{4} \times \frac{12}{5} = 303$$

113. **(b)** $S = \left(\frac{3}{4}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{9}{4}\right)^3 + (3)^3 + ...$

$$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + ...$$

Let the general term of S be

$T_r = \left(\frac{3r}{4}\right)^3$, then

$$255K = \sum_{r=1}^{15} T_r = \left(\frac{3}{4}\right)^3 \sum_{r=1}^{15} r^3$$

$$255K = \frac{27}{64} \times \left(\frac{15\times 16}{2}\right)^2$$

$\Rightarrow K = 27$

114. **(d)** $S = 1 + 6 + \frac{9(1^2+2^2+3^2)}{7} + \frac{12(1^2+2^2+3^2+4^2)}{9} + \frac{15(1^2+2^2+3^2+4^2+5^2)}{11} + \ldots$

$$S = \frac{3\cdot(1)^2}{3} + \frac{6\cdot(1^2+2^2)}{5} + \frac{9\cdot(1^2+2^2+3^2)}{7} + \frac{12\cdot(1^2+2^2+3^2+4^2)}{9} + \ldots$$

Now, n^{th} term of the series,

$$t_n = \frac{3n\cdot(1^2+2^2+\ldots+n^2)}{(2n+1)}$$

$$\Rightarrow t_n = \frac{3n\cdot n(n+1)(2n+1)}{6(2n+1)} = \frac{n^3+n^2}{2}$$

$$\therefore S_n = \Sigma t_n = \frac{1}{2}\left\{\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}\right\}$$

$$=\frac{n(n+1)}{4}\left(\frac{n(n+1)}{2}+\frac{2n+1}{3}\right)$$

Hence, sum of the series upto 15 terms is,

$$S_{15}=\frac{15\times 16}{4}\left\{\frac{15\cdot 16}{2}+\frac{31}{3}\right\}$$

$$=60\times 120+60\times\frac{31}{3}$$

$$=7200+620=7820$$

115. (d) The general term of the given series $=\frac{2\times 2^r-1}{2^r}$, where $r\geq 0$

$$\therefore \text{ req. sum}=1+\sum_{r=1}^{19}\frac{2\times 2^r-1}{2^r}$$

Now, $\sum_{r=1}^{19}\left(\frac{2\times 2^r-1}{2^r}\right)=\sum_{r=1}^{19}\left(2-\frac{1}{2^r}\right)$

$$=2(19)-\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{19}\right)}{1-\frac{1}{2}}=38+\frac{\left(\frac{1}{2}\right)^{19}-1}{1}$$

$$=38+\left(\frac{1}{2}\right)^{19}-1=37+\left(\frac{1}{2}\right)^{19}$$

$$\therefore \text{ req. sum}=1+37+\left(\frac{1}{2}\right)^{19}=38+\left(\frac{1}{2}\right)^{19}$$

116. (a) Here, $B-2A$

$$=\sum_{n=1}^{40}a_n-2\sum_{n=1}^{20}a_n=\sum_{n=21}^{40}a_n-2\sum_{n=1}^{20}a_n$$

$$B-2A=(21^2+2.22^2+23^2+2.24^2+\ldots..+40^2)$$
$$-(1^2+2.2^2+3^2+2.4^2\ldots..+20^2)$$

$$=20\,[22+2.24+26+2.28+\ldots..+60]$$

$$=20\left[\underbrace{(22+24+26\ldots.60)}_{20\text{ terms}}+\underbrace{(24+28+\ldots.+60)}_{10\text{ terms}}\right]$$

$$20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]$$

$$=10[20.82+10.84]$$

$$=100[164+84]=100.248$$

117. (b) $f(x)=ax^2+bx+c$

$f(1)=a+b+c=3\Rightarrow f(1)=3$

Now $f(x+y)=f(x)+f(y)+xy$ ...(i)

Put $x=y=1$ in eqn (i)

$f(2)=f(1)+f(1)+1=2f(1)+1$

$f(2)=7$

$\Rightarrow f(3)=12$

$S_n=3+7+12+\ldots\ldots..+t_n$

$S_n=\quad 3+7+12+\ldots\ldots..+t_{n-1}+t_n$

$-\quad -\quad -\quad -\quad -\quad -$

$0=3+4+5\ldots\ldots..\text{to n term}-t_n$

$t_n=3+4+5+\ldots.\text{ upto n terms}$

$$t_n=\frac{(n^2+5n)}{2}$$

$$S_n=\sum t_n=\sum\frac{(n^2+5n)}{2}$$

$$S_n=\frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}+\frac{5n(n+1)}{2}\right]$$

$$=\frac{n(n+1)(n+8)}{6}$$

$$S_{10}=\frac{10\times 11\times 18}{6}=330$$

118. (a) $T_n=\dfrac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2}$

$$\Rightarrow T_n=\frac{2}{n(n+1)}$$

$$\Rightarrow S_n=\sum T_n=2\sum_{n=1}^{n}\left(\frac{1}{n}-\frac{1}{n+1}\right)=2\left\{1-\frac{1}{n+1}\right\}$$

$$\Rightarrow \boxed{S_n=\frac{2n}{n+1}}$$

$\because\ 100\,S_n=n$

$$\Rightarrow 100\times\frac{2n}{n+1}=n$$

$\Rightarrow n+1=200$

$\Rightarrow n=199$

119. (b) $\because$

$$\sqrt{3}\left[1+\sqrt{25}+\sqrt{81}+\sqrt{69}+\ldots\ldots\right]=435\sqrt{3}$$

$$\Rightarrow \sqrt{3}\,[1+5+9+13+\ldots..+T_n]=435\sqrt{3}$$

$$\Rightarrow \sqrt{3}\times\frac{n}{2}\left[2+(n-1)_4\right]=435\sqrt{3}$$

$\Rightarrow 2n+4n^2-4n=870$

$\Rightarrow 4n^2-2n-870=0$

$\Rightarrow\ 2n^2 - n - 435 = 0$

$n = \dfrac{1 \pm \sqrt{1+4\times2\times435}}{4} = \dfrac{1\pm59}{4}$

$\therefore\ n = \dfrac{1+59}{4} = 15;$ or $n = \dfrac{1-59}{4} = 14.5$

120. (d) $\left(\dfrac{8}{5}\right)^2 + \left(\dfrac{12}{5}\right)^2 + \left(\dfrac{16}{5}\right)^2 + \left(\dfrac{20}{5}\right)^2 \ldots + \left(\dfrac{44}{5}\right)^2$

$S = \dfrac{16}{25}\left(2^2+3^2+4^2+\ldots+11^2\right)$

$= \dfrac{16}{25}\left(\dfrac{11(11+1)(22+1)}{6} - 1\right) \qquad = \dfrac{16}{25}\times505 = \dfrac{16}{5}\times101$

$\Rightarrow \dfrac{16}{5}m = \dfrac{16}{5}\times101$

$\Rightarrow m = 101.$

121. (a) $S = (1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots + x^{2015}(1+x) + x^{2016}$ ...(i)

$\left(\dfrac{x}{1+x}\right) S = x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots +$

$x^{2016} + \dfrac{x^{2017}}{1+x}$ ...(ii)

Subtracting (i) from (ii)

$\dfrac{S}{1+x} = (1+x)^{2016} - \dfrac{x^{2017}}{1+x}$

$\therefore\ S = (1+x)^{2017} - x^{2017}$

$a_{17} =$ coefficient of $x^{17} = {}^{2017}C_{17} = \dfrac{2017!}{17!2000!}$

122. (d) n^{th} term of series $= \dfrac{\left[\dfrac{n(n+1)}{2}\right]^2}{n^2} = \dfrac{1}{4}(n+1)^2$

Sum of n term $= \Sigma\dfrac{1}{4}(n+1)^2 = \dfrac{1}{4}\left[\Sigma n^2 + 2\Sigma n + n\right]$

$= \dfrac{1}{4}\left[\dfrac{n(n+1)(2n+1)}{6} + \dfrac{2n(n+1)}{2} + n\right]$

Sum of 9 terms

$= \dfrac{1}{4}\left[\dfrac{9\times10\times19}{6} + \dfrac{18\times10}{2} + 9\right] = \dfrac{384}{4} = 96$

123. (c) General term of given expression can be written as

$T_r = \dfrac{1}{3}\left[\dfrac{1}{n(n+1)(n+2)} - \dfrac{1}{(n+1)(n+2)(n+3)}\right]$

on taking summation both the side, we get

$\displaystyle\sum_{r=1}^{5} T_r = \dfrac{1}{3}\left[\dfrac{1}{6} - \dfrac{1}{6.7.8}\right] = \dfrac{k}{3}$

$\Rightarrow \dfrac{1}{3}\times\dfrac{1}{6}\left(1-\dfrac{1}{56}\right) = \dfrac{k}{3} \Rightarrow \dfrac{1}{3}\times\dfrac{1}{6}\times\dfrac{55}{56} = \dfrac{k}{3}$

$\Rightarrow k = \dfrac{55}{336}$

124. (d) $\displaystyle\sum_{r=16}^{20}\left(r^2 - r - 6\right) = 7780$

125. (a) Given that $10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$

Let $x = 10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9$ ...(i)

Multiplied by $\dfrac{11}{10}$ on both the sides

$\dfrac{11}{10}x = 11.10^8 + 2.(11)^2.(10)^7 + \ldots + 9(11)^9 + 11^{10}$ ...(ii)

Subtract (ii) from (i), we get

$x\left(1-\dfrac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2\times(10)^7 + \ldots + 11^9 - 11^{10}$

$\Rightarrow\ -\dfrac{x}{10} = 10^9\left[\dfrac{\left(\dfrac{11}{10}\right)^{10} - 1}{\dfrac{11}{10} - 1}\right] - 11^{10}$

$\Rightarrow\ -\dfrac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$

$\Rightarrow\ x = 10^{11} = k.10^9$ Given

$\Rightarrow\ k = 100$

126. (b) Let a, d and $2n$ be the first term, common difference and total number of terms of an A.P. respectively i.e. $a + (a+d) + (a+2d) + \ldots + (a+(2n-1)d)$

No. of even terms $= n$, No. of odd terms $= n$

Sum of odd terms :

$S_o = \dfrac{n}{2}[2a + (n-1)(2d)] = 24$

$\Rightarrow n\ [a + (n-1)d] = 24$ ...(i)

Sum of even terms :

$S_e = \dfrac{n}{2}[2(a+d) + (n-1)2d] = 30$

$\Rightarrow n\ [a + d + (n-1)\ d] = 30$ ...(ii)

Subtracting equation (i) from (ii), we get

$nd = 6$ ...(iii)

Also, given that last term exceeds the first term by $\dfrac{21}{2}$

$a + (2n-1)\ d = a + \dfrac{21}{2}$

$2nd - d = \frac{21}{2}$

$\Rightarrow 2 \times 6 - \frac{21}{2} = d \qquad (\because nd = 6)$

$d = \frac{3}{2}$

Putting value of d in equation (3)

$n = \frac{6 \times 2}{3} = 4$

Total no. of terms = $2n = 2 \times 4 = 8$

127. (a) n^{th} term of given series is

$$\frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)}$$

Let n^{th} term, $a_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$

Sum of 20 terms, $S_{20} = a_1 + a_2 + a_3 + ... + a_{20}$

$$S_{20} = 6\left(\frac{1}{1} - \frac{1}{2}\right) + 6\left(\frac{1}{2} - \frac{1}{3}\right) + 6\left(\frac{1}{3} - \frac{1}{4}\right) + ...$$
$$+6\left(\frac{1}{18} - \frac{1}{19}\right) + 6\left(\frac{1}{19} - \frac{1}{20}\right) + 6\left(\frac{1}{20} - \frac{1}{21}\right)$$

$$S_{20} = \left[\left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + ... + \left(\cancel{\frac{1}{18}} - \cancel{\frac{1}{19}}\right) + \left(\cancel{\frac{1}{19}} - \cancel{\frac{1}{20}}\right) + \left(\cancel{\frac{1}{20}} - \frac{1}{21}\right)\right]$$

$S_{20} = 6\left(1 - \frac{1}{21}\right) = \frac{120}{21}$ .....(i)

Given that $S_{20} = \frac{k}{21}$ .....(ii)

On comparing (i) and (ii), we get

$k = 120$

128. (c) Let $S = \frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + ... +$ up to 20 terms

$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + ... + \text{up to 20 terms}\right]$

Multiply and divide by 9

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...\text{up to 20 terms}\right]$

$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + ...\text{up to 20 terms}\right]$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^{20}\right)}{1 - \frac{1}{10}}\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}[179 + (10)^{-20}]$$

129. (a) Consider $1^2 + 3^2 + 5^2 + ... + 25^2$

n^{th} term $T_n = (2n-1)^2, n = 1, ... 13$

Now, $S_n = \sum_{n=1}^{13} T_n = \sum_{n=1}^{13} (2n-1)^2$

$= \sum_{n=1}^{13} 4n^2 + \sum_{n=1}^{13} 1 - \sum_{n=1}^{13} 4n = 4\sum n^2 + 13 - 4\sum n$

$= 4\left[\frac{n(n+1)(2n+1)}{6}\right] + 13 - 4\frac{n(n+1)}{2}$

Put $n = 13$, we get

$S_n = 26 \times 14 \times 9 + 13 - 26 \times 14$

$= 3276 + 13 - 364 = 2925.$

130. (c) $2^2 + 2(4)^2 + 3(6)^2 + ...$ upto 10 terms

$= 2^2 [1^3 + 2^3 + 3^3 + ...$ upto 10 terms]

$= 4.\left(\frac{10 \times 11}{2}\right)^2 = 12100$

131. (c) Given sum is

$\frac{3}{12} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + ...$

nth term = T_n

$$= \frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)}$$

or $T_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$\therefore \quad S_n = \sum T_n = 6\sum\frac{1}{n} - 6\sum\frac{1}{n+1} = \frac{6n}{n} - \frac{6}{n+1}$

$= 6 - \frac{6}{n+1} = \frac{6n}{n+1}$

So, sum upto 11 terms means

$S_{11} = \frac{6 \times 11}{11+1} = \frac{66}{12} = \frac{33}{6} = \frac{11}{2}$

132. (c) $T_r = \dfrac{1}{1+2+3+...+r} = \dfrac{2}{r(r+1)}$

$$S_{10} = 2\sum_{r=1}^{10} \frac{1}{r(r+1)} = 2\sum_{r=1}^{10}\left[\frac{r+1}{r(r+1)} - \frac{r}{r(r+1)}\right]$$

$$= 2\sum_{r=1}^{10}\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

$$= 2\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{10}-\frac{1}{11}\right)\right]$$

$$= 2\left[1-\frac{1}{11}\right] = 2\times\frac{10}{11} = \frac{20}{11}$$

133. (b) nth term of the given series

$$= T_n = (n-1)^2 + (n-1)n + n^2$$

$$= \frac{((n-1)^3 - n^3)}{(n-1)-n} = n^3 - (n-1)^3$$

$$\Rightarrow S_n = \sum_{k=1}^{n}\left[k^3 - (k-1)^3\right] \Rightarrow 8000 = n^3$$

$\Rightarrow$ $n = 20$ which is a natural number.

Hence, both the given statements are true.

and statement 2 is correct explanation for statement 1.

134. (c) If n is odd, the required sum is

$1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + 2(n-1)^2 + n^2$

$$= \frac{(n-1)(n-1+1)^2}{2} + n^2 \quad (\because n-1 \text{ is even})$$

$$= \left(\frac{n-1}{2}+1\right)n^2 = \frac{n^2(n+1)}{2}$$

135. (c) Given series is $1 + \dfrac{4}{3} + \dfrac{10}{9} + \dfrac{28}{27} + ...$ n terms

$$= 1 + \left(1+\frac{1}{3}\right) + \left(1+\frac{1}{9}\right) + \left(1+\frac{1}{27}\right) + ... \ n \text{ terms}$$

$= (1 + 1 + 1 + ... + n \text{ terms})$

$$+\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+....n \text{ terms}\right)$$

$$= n + \frac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}} = n + \frac{1}{3}\times\frac{3}{2}[1-3^{-n}]$$

$$= n + \frac{1}{2}[1-3^{-n}] = n + \frac{1}{2} - \frac{1}{2.3^n}$$

136. (c) Given series is $\dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + ...$

n^{th} term $= \dfrac{1}{\sqrt{n}+\sqrt{n+1}}$

$\therefore$ 15^{th} term $= \dfrac{1}{\sqrt{15}+\sqrt{16}}$

Thus, given series upto 15 terms is

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + ... + \frac{1}{\sqrt{15}+\sqrt{16}}$$

This can be re-written as

$$\frac{1-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \frac{\sqrt{3}-\sqrt{4}}{-1} + ... + \frac{\sqrt{15}-\sqrt{16}}{-1}$$

(By rationalization)

$$= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + \sqrt{4} + ... - \sqrt{14} + \sqrt{15} - \sqrt{15} + \sqrt{16}$$

$$= -1 + \sqrt{16} = -1 + 4 = 3$$

Hence, the required sum = 3

137. (a) The sum of the given series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ... + 2(2m)^2$ is $\dfrac{2m(2m+1)^2}{2} = m(2m+1)^2$

138. (a) Let $S = 1 + \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + \dfrac{14}{3^4} + ...\infty$ ....(i)

Multiplying both sides by $\dfrac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...\infty \quad ....(i)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3}\times\frac{3}{2} \Rightarrow S = 3$$

139. (d) We know that $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + ...\infty$

Put $x = -1$

$$\therefore \quad e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}........\infty$$

$$\therefore \quad e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}........\infty$$

140. (d) We know that

$$\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}............$$

Putting $x = \dfrac{1}{2}$, we get

$$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+......\infty=\frac{e^{\frac{1}{2}}+e^{\frac{-1}{2}}}{2}$$

$$=\frac{\sqrt{e}+\frac{1}{\sqrt{e}}}{2}=\frac{e+1}{2\sqrt{e}}$$

141. **(b)** We know that

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}.....\infty$$

$$\therefore\ e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.......$$

and $e^{-1}=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+.......$

$$\therefore e+e^{-1}=2\left[1+\frac{1}{2!}+\frac{1}{4!}+....\right]$$

$$\therefore \frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+......=\frac{e+e^{-1}}{2}-1$$

$$=\frac{e^2+1-2e}{2e}=\frac{(e-1)^2}{2e}$$

142. **(b)** If n is odd, the required sum is

$$1^2+2.2^2+3^2+2.4^2+......+2.(n-1)^2+n^2$$

$$=\frac{(n-1)(n-1+1)^2}{2}+n^2$$

[$\because$ $(n-1)$ is even
$\therefore$ using given formula for the sum of $(n-1)$ terms.]

$$=\left(\frac{n-1}{2}+1\right)n^2=\frac{n^2(n+1)}{2}$$

143. **(d)** $S_n=\frac{1}{^nC_0}+\frac{1}{^nC_1}+\frac{1}{^nC_2}+....+\frac{1}{^nC_n}$

$$t_n=\frac{0}{^nC_0}+\frac{1}{^nC_1}+\frac{2}{^nC_2}+....+\frac{n}{^nC_n} \quad ...(i)$$

$$t_n=\frac{n}{^nC_n}+\frac{n-1}{^nC_{n-1}}+\frac{n-2}{^nC_{n-2}}+....+\frac{0}{^nC_0} \quad ...(ii)$$

Adding (i) and (ii), we get,

$$2t_n=(n)\left[\frac{1}{^nC_0}+\frac{1}{^nC_1}+....\frac{1}{^nC_n}\right]=nS_n$$

$$\because\ ^nC_r={}^nC_{n-r}$$

$$\therefore \frac{t_n}{S_n}=\frac{n}{2}$$

144. **(a)** Let $S=\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}..............\infty$

$$T_n=\frac{1}{n(n+1)}=\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$\therefore\ S=\left(\frac{1}{1}-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{4}-\frac{1}{5}\right).....$$

$$=1-2\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}...............\infty\right]$$

$$\left[\because \log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.......\infty\right]$$

$$=1-2[-\log(1+1)+1]=2\log 2-1=\log\left(\frac{4}{e}\right).$$

145. **(a)** $1^3-2^3+3^3-4^3+........+9^3$

$$=1^3+2^3+3^3+.......+9^3-2(2^3+4^3+6^3+8^3)$$

$$\left[\because \Sigma n^3=\left(\frac{n(n+1)}{2}\right)^2\right]$$

$$=\left[\frac{9\times 10}{2}\right]^2-2.2^3\left[1^3+2^3+3^3+4^3\right]$$

$$=(45)^2-16.\left[\frac{4\times 5}{2}\right]^2=2025-1600=425$$

146. **(b)** Let $P=2^{1/4}.2^{2/8}.2^{3/16}.........\infty$

$$=2^{1/4+2/8+3/16+.........\infty}$$

Now, let $S=\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+........\infty \quad ......(i)$

$$\frac{1}{2}S=\frac{1}{8}+\frac{2}{16}+........\infty \quad ......(ii)$$

Subtracting (ii) from (i)

$$\Rightarrow\ \frac{1}{2}S=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+........\infty$$

or $\frac{1}{2}S=\frac{a}{1-r}=\frac{1/4}{1-1/2}=\frac{1}{2}\Rightarrow S=1$

$$\therefore\ P=2^S=2$$

10

Straight Lines and Pair of Straight Lines

TOPIC 1 Distance Formula, Section Formula, Results of Triangle, Locus, Equation of Locus, Slope of a Straight Line, Slope of a line joining two points, Parallel and Perpendicular Lines

1. A triangle ABC lying in the first quadrant has two vertices as $A(1,2)$ and $B(3,1)$. If $\angle BAC = 90°$, and ar $(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is: **[Sep. 04, 2020 (I)]**

   (a) $1+\sqrt{5}$ (b) $1+2\sqrt{5}$ (c) $2+\sqrt{5}$ (d) $2\sqrt{5}-1$

2. If the perpendicular bisector of the line segment joining the points $P(1, 4)$ and $Q(k, 3)$ has y-intercept equal to -4, then a value of k is : **[Sep. 04, 2020 (II)]**

   (a) -2 (b) -4 (c) $\sqrt{14}$ (d) $\sqrt{15}$

3. If a ΔABC has vertices $A(-1, 7)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocentre has coordinates : **[Sep. 03, 2020 (II)]**

   (a) $\left(-\frac{3}{5}, \frac{3}{5}\right)$ (b) $(-3, 3)$

   (c) $\left(\frac{3}{5}, -\frac{3}{5}\right)$ (d) $(3, -3)$

4. Let $A(1, 0)$, $B(6, 2)$ and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is———. **[NA Jan. 7, 2020 (I)]**

5. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2, 3). Then the centroid of this triangle is : **[April 12, 2019 (II)]**

   (a) $\left(1, \frac{7}{3}\right)$ (b) $\left(\frac{1}{3}, 2\right)$ (c) $\left(\frac{1}{3}, 1\right)$ (d) $\left(\frac{1}{3}, \frac{5}{3}\right)$

6. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is : **[April 8, 2019 (I)]**

   (a) $8x^2 - 9y^2 + 9y = 18$ (b) $9x^2 - 8y^2 + 8y = 16$
   (c) $9x^2 + 8y^2 - 8y = 16$ (d) $8x^2 + 9y^2 - 9y = 18$

7. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant? **[Jan. 10, 2019 (II)]**

   (a) third (b) second
   (c) first (d) fourth

8. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is : **[2018]**

   (a) $2\sqrt{10}$ (b) $3\sqrt{\frac{5}{2}}$ (c) $\frac{3\sqrt{5}}{2}$ (d) $\sqrt{10}$

9. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is : **[Online April 9, 2017]**

   (a) $2\sqrt{3}-1$ (b) $2\sqrt{3}-2$ (c) $\sqrt{3}-2$ (d) $\sqrt{3}-1$

10. A ray of light is incident along a line which meets another line, $7x - y + 1 = 0$, at the point (0, 1). The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is : **[Online April 10, 2016]**

    (a) $41x - 25y + 25 = 0$ (b) $41x + 25y - 25 = 0$
    (c) $41x - 38y + 38 = 0$ (d) $41x + 38y - 38 = 0$

11. Let L be the line passing through the point P(1, 2) such that its intercepted segment between the co-ordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point (–2, 1), then the point of intersection of L and L_1 is : **[Online April 10, 2015]**

(a) $\left(\frac{4}{5}, \frac{12}{5}\right)$ (b) $\left(\frac{3}{5}, \frac{23}{10}\right)$

(c) $\left(\frac{11}{20}, \frac{29}{10}\right)$ (d) $\left(\frac{3}{10}, \frac{17}{5}\right)$

12. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) : **[Online April 10, 2015]**

(a) form an acute angled triangle.
(b) form a right angled triangle.
(c) lie on a straight line.
(d) form an obtuse angled triangle.

13. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[2013]**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $1+\sqrt{2}$ (d) $1-\sqrt{2}$

14. A light ray emerging from the point source placed at P(l, 3) is reflected at a point Q in the axis of x. If the reflected ray passes through the point R (6, 7), then the abscissa of Q is: **[Online April 9, 2013]**

(a) 1 (b) 3 (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

15. Let A (h, k), B(1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1square unit, then the set of values which 'k' can take is given by **[2007]**

(a) $\{-1, 3\}$ (b) $\{-3, -2\}$ (c) $\{1, 3\}$ (d) $\{0, 2\}$

16. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2) then the centroid of the triangle is **[2005]**

(a) $\left(-1, \frac{7}{3}\right)$ (b) $\left(\frac{-1}{3}, \frac{7}{3}\right)$ (c) $\left(1, \frac{7}{3}\right)$ (d) $\left(\frac{1}{3}, \frac{7}{3}\right)$

17. If the equation of the locus of a point equidistant from the point (a_1, b_1) and (a_2, b_2) is $(a_1 - b_2)x + (a_1 - b_2)y + c = 0$, then the value of `c' is **[2003]**

(a) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$ (b) $\frac{1}{2}a_2^2 + b_2^2 - a_1^2 - b_1^2$

(c) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (d) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

18. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and (1, 0), where t is a parameter, is **[2003]**

(a) $(3x+1)^2 + (3y)^2 = a^2 - b^2$

(b) $(3x-1)^2 + (3y)^2 = a^2 - b^2$

(c) $(3x-1)^2 + (3y)^2 = a^2 + b^2$

(d) $(3x+1)^2 + (3y)^2 = a^2 + b^2$

19. A triangle with vertices (4, 0), (–1, –1), (3, 5) is **[2002]**

(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles

TOPIC 2 Various Forms of Equation of a Line

20. Let f: R $\to$ R be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is ______. **[NA Sep. 06, 2020 (I)]**

21. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point: **[Jan. 9, 2020 (I)]**

(a) (–9, –6) (b) (9, 7) (c) (7, 6) (d) (–9, –7)

22. Slope of a line passing through P(2, 3) and intersecting the line x + y = 7 at a distance of 4 units from P, is: **[April 9, 2019 (I)]**

(a) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (b) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (c) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

23. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in : **[April 8, 2019 (I)]**

(a) 4th quadrant (b) 1st quadrant
(c) 1st and 2nd quadrants (d) 1st, 2nd and 4th quadrants

24. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is : **[April 08, 2019 (II)]**

(a) 15 (b) 18 (c) 12 (d) 16

25. If a straight line passing through the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is : **[Jan. 12, 2019 (II)]**

(a) $3x - 4y + 25 = 0$ (b) $4x - 3y + 24 = 0$
(c) $x - y + 7 = 0$ (d) $4x + 3y = 0$

26. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is : **[Jan. 11, 2019 (II)]**

(a) $5x-3y+1=0$ (b) $5x+3y-11=0$
(c) $3x-5y+7=0$ (d) $3x+5y-13=0$

27. A point P moves on the line $2x-3y+4=0$. If Q(1, 4) and R(3, –2) are fixed points, then the locus of the centroid of ΔPQR is a line: **[Jan. 10, 2019 (I)]**

(a) with slope $\frac{3}{2}$ (b) parallel to x-axis
(c) with slope $\frac{2}{3}$ (d) parallel to y-axis

28. If the line $3x+4y-24=0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is: **[Jan. 10, 2019 (I)]**

(a) (3,4) (b) (2,2) (c) (4,3) (d) (4,4)

29. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is : **[2018]**

(a) $2x+3y=xy$ (b) $3x+2y=xy$
(c) $3x+2y=6xy$ (d) $3x+2y=6$

30. In a triangle ABC, coordianates of A are (1, 2) and the equations of the medians through B and C are $x+y=5$ and $x=4$ respectively. Then area of ΔABC (in sq. units) is **[Online April 15, 2018]**

(a) 5 (b) 9 (c) 12 (d) 4

31. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus? **[2016]**

(a) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (b) $\left(-\frac{10}{3},-\frac{7}{3}\right)$
(c) (–3, –9) (d) (–3, –8)

32. A straight line through origin O meets the lines $3y=10-4x$ and $8x+6y+5=0$ at points A and B respectively. Then O divides the segment AB in the ratio : **[Online April 10, 2016]**

(a) 2 : 3 (b) 1 : 2 (c) 4 : 1 (d) 3 : 4

33. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is : **[Online April 9, 2016]**

(a) $7xy=6(x+y)$
(b) $4(x+y)^2-28(x+y)+49=0$
(c) $6xy=7(x+y)$
(d) $14(x+y)^2-97(x+y)+168=0$

34. The point (2, 1) is translated parallel to the line L : $x-y=4$ by $2\sqrt{3}$ units. If the new points Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is : **[Online April 9, 2016]**

(a) $x+y=2-\sqrt{6}$ (b) $2x+2y=1-\sqrt{6}$
(c) $x+y=3-3\sqrt{6}$ (d) $x+y=3-2\sqrt{6}$

35. A straight line L through the point (3, – 2) is inclined at an angle of 60° to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, then the equation of L is : **[Online April 11, 2015]**

(a) $y+\sqrt{3}x+2-3\sqrt{3}=0$
(b) $\sqrt{3}y+x-3+2\sqrt{3}=0$
(c) $y-\sqrt{3}x+2+3\sqrt{3}=0$
(d) $\sqrt{3}y-x+3+2\sqrt{3}=0$

36. The circumcentre of a triangle lies at the origin and its centroid is the mid point of the line segment joining the points (a^2+1, a^2+1) and $(2a, -2a)$, $a \neq 0$. Then for any a, the orthocentre of this triangle lies on the line: **[Online April 19, 2014]**

(a) $y-2ax=0$
(b) $y-(a^2+1)x=0$
(c) $y+x=0$
(d) $(a-1)^2x-(a+1)^2y=0$

37. If a line intercepted between the coordinate axes is trisected at a point A(4, 3), which is nearer to x-axis, then its equation is: **[Online April 12, 2014]**

(a) $4x-3y=7$ (b) $3x+2y=18$
(c) $3x+8y=36$ (d) $x+3y=13$

38. Given three points P, Q, R with P(5, 3) and R lies on the x-axis. If equation of RQ is $x-2y=2$ and PQ is parallel to the x-axis, then the centroid of ΔPQR lies on the line: **[Online April 9, 2014]**

(a) $2x+y-9=0$ (b) $x-2y+1=0$
(c) $5x-2y=0$ (d) $2x-5y=0$

39. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[2013]**

(a) $y=x+\sqrt{3}$ (b) $\sqrt{3}y=x-\sqrt{3}$
(c) $y=\sqrt{3}x-\sqrt{3}$ (d) $\sqrt{3}y=x-1$

40. Let A (–3, 2) and B (–2, 1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x+4y+2=0$, then the vertex C lies on the line : **[Online April 25, 2013]**

(a) $4x+3y+5=0$ (b) $3x+4y+3=0$
(c) $4x+3y+3=0$ (d) $3x+4y+5=0$

41. If the extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is $x = 2a$, then the area of the triangle, in square units, is : **[Online April 23, 2013]**

(a) $\frac{5}{4}a^2$ (b) $\frac{5}{2}a^2$ (c) $\frac{25a^2}{4}$ (d) $5a^2$

42. If the x-intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is : **[Online April 22, 2013]**

(a) -3 (b) $-3/8$ (c) $-3/2$ (d) $-3/16$

43. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3 :2, then k equals : **[2012]**

(a) $\frac{29}{5}$ (b) 5 (c) 6 (d) $\frac{11}{5}$

44. The line parallel to x-axis and passing through the point of intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is **[Online May 26, 2012]**

(a) above x-axis at a distance 2/3 from it
(b) above x-axis at a distance 3/2 from it
(c) below x-axis at a distance 3/2 from it
(d) below x-axis at a distance 2/3 from it

45. If the point $(1, a)$ lies between the straight lines $x + y = 1$ and $2(x + y) = 3$ then a lies in interval **[Online May 12, 2012]**

(a) $\left(\frac{3}{2}, \infty\right)$ (b) $\left(1, \frac{3}{2}\right)$ (c) $(-\infty, 0)$ (d) $\left(0, \frac{1}{2}\right)$

46. If the straight lines $x + 3y = 4$, $3x + y = 4$ and $x + y = 0$ form a triangle, then the triangle is **[Online May 7, 2012]**

(a) scalene
(b) equilateral triangle
(c) isosceles
(d) right angled isosceles

47. If A $(2, -3)$ and B $(-2, 1)$ are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is : **[2011RS]**

(a) $x - y = 1$ (b) $2x + 3y = 1$
(c) $2x + 3y = 3$ (d) $2x - 3y = 1$

48. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belong to **[2006]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$ (c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$

49. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is **[2006]**

(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$
(c) $4x + 3y = 24$ (d) $3x + 4y = 25$

50. The line parallel to the x- axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is **[2005]**

(a) below the x - axis at a distance of $\frac{3}{2}$ from it
(b) below the x - axis at a distance of $\frac{2}{3}$ from it
(c) above the x - axis at a distance of $\frac{3}{2}$ from it
(d) above the x - axis at a distance of $\frac{2}{3}$ from it

51. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is -1 is **[2004]**

(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

52. Let $A(2, -3)$ and $B(-2, 3)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line **[2004]**

(a) $3x - 2y = 3$ (b) $2x - 3y = 7$
(c) $3x + 2y = 5$ (d) $2x + 3y = 9$

53. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$ whre p is constant is **[2002]**

(a) $x^2 + y^2 = \frac{4}{p^2}$ (b) $x^2 + y^2 = 4p^2$
(c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$ (d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

TOPIC 3 Distance Between two Lines, Angle Between two Lines and Bisector of the Angle Between the two Lines, Perpendicular Distance of a Point from a Line, Foot of the Perpendicular, Position of a Point with Respect to a Line, Pedal Points, Condition for Concurrency of Three Lines

54. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point $(-1, -4)$ in this line is: **[Sep. 06, 2020 (II)]**

(a) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (b) $\left(\frac{29}{5}, \frac{8}{5}\right)$

(c) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (d) $\left(\frac{29}{5}, \frac{11}{5}\right)$

55. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible value of α and β is ______. **[NA Sep. 05, 2020 (I)]**

56. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is: **[Jan. 7, 2020 (II)]**

(a) $2x - 3y = 0$ (b) $5x - 7y = 0$

(c) $3x - 2y = 0$ (d) $7x - 5y = 0$

57. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is: **[April 12, 2019 (II)]**

(a) $x + \sqrt{3}y = 8$

(b) $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

(c) $\sqrt{3}x + y = 8$

(d) None of these

58. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines ? **[April 10, 2019 (II)]**

(a) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (b) $\left(\frac{1}{4}, -\frac{1}{3}\right)$

(c) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (d) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

59. If the two lines $x + (a-1)y = 1$ and $2x + a^2y = 1$ $(a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is: **[April 09, 2019 (II)]**

(a) $\sqrt{\frac{2}{5}}$ (b) $\frac{2}{5}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{2}}{5}$

60. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are $(-8, 5)$ and $(6, 5)$, then the area of the rectangle (in sq. units) is: **[April 09, 2019 (II)]**

(a) 84 (b) 98 (c) 72 (d) 56

61. Suppose that the points (h, k), $(1, 2)$ and $(-3, 4)$ lie on the line L_1. If a line L_2 passing through the points (h, k) and $(4, 3)$ is perpendicular on L_1, then equals : **[April 08, 2019 (II)]**

(a) $\frac{1}{3}$ (b) 0 (c) 3 (d) $-\frac{1}{7}$

62. If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7, 17)$ and $(15, \beta)$, then β equals : **[Jan. 12, 2019 (I)]**

(a) $\frac{35}{3}$ (b) -5 (c) $-\frac{35}{3}$ (d) 5

63. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at $(2, 4)$, then one of its vertex is: **[Jan. 10, 2019 (II)]**

(a) $(3, 5)$ (b) $(2, 1)$ (c) $(2, 6)$ (d) $(3, 6)$

64. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true? **[Jan. 9, 2019 (I)]**

(a) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

(b) Each line passes through the origin.

(c) The lines are all parallel.

(d) The lines are not concurrent.

65. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at $(1, 1)$, then the equation of its third side is: **[Jan. 09, 2019 (II)]**

(a) $122y - 26x - 1675 = 0$

(b) $122y + 26x + 1675 = 0$

(c) $26x + 61y + 1675 = 0$

(d) $26x - 122y - 1675 = 0$

66. The foot of the perpendicular drawn from the origin, on the line, $3x + y = \lambda (\lambda \neq 0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio $BP : PA$ is **[Online April 15, 2018]**

(a) $9 : 1$ (b) $1 : 3$ (c) $1 : 9$ (d) $3 : 1$

67. The sides of a rhombus $ABCD$ are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1, 2)$ and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is **[Online April 15, 2018]**

(a) 2 (b) $\frac{7}{4}$ (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

68. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then **[2014]**

(a) $3bc - 2ad = 0$ (b) $3bc + 2ad = 0$
(c) $2bc - 3ad = 0$ (d) $2bc + 3ad = 0$

69. Let PS be the median of the triangle vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is: **[2014]**

(a) $4x + 7y + 3 = 0$ (b) $2x - 9y - 11 = 0$
(c) $4x - 7y - 11 = 0$ (d) $2x + 9y + 7 = 0$

70. If a line L is perpendicular to the line $5x - y = 1$, and the area of the triangle formed by the line L and the coordinate axes is 5, then the distance of line L from the line $x + 5y = 0$ is: **[Online April 19, 2014]**

(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{5}{\sqrt{13}}$ (c) $\frac{7}{\sqrt{13}}$ (d) $\frac{5}{\sqrt{7}}$

71. If the three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a: **[Online April 12, 2014]**

(a) circle (b) hyperbola
(c) straight line (d) parabola

72. The base of an equilateral triangle is along the line given by $3x + 4y = 9$. If a vertex of the triangle is (1, 2), then the length of a side of the triangle is: **[Online April 11, 2014]**

(a) $\frac{2\sqrt{3}}{15}$ (b) $\frac{4\sqrt{3}}{15}$ (c) $\frac{4\sqrt{3}}{5}$ (d) $\frac{2\sqrt{3}}{5}$

73. If the image of point P(2, 3) in a line L is Q(4, 5), then the image of point R(0, 0) in the same line is: **[Online April 25, 2013]**

(a) $(2, 2)$ (b) $(4, 5)$ (c) $(3, 4)$ (d) $(7, 7)$

74. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers :

Statement-1: If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.
Statement-2: $\theta_1 = \theta_2$ for all c_2 and c_3. **[Online April 23, 2013]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation of Statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 is true; Statement-2 is false.

75. If the three lines $x - 3y = p$, $ax + 2y = q$ and $ax + y = r$ form a right-angled triangle then : **[Online April 9, 2013]**

(a) $a^2 - 9a + 18 = 0$ (b) $a^2 - 6a - 12 = 0$
(c) $a^2 - 6a - 18 = 0$ (d) $a^2 - 9a + 12 = 0$

76. Consider the straight lines

$L_1 : x - y = 1$
$L_2 : x + y = 1$
$L_3 : 2x + 2y = 5$
$L_4 : 2x - 2y = 7$

The correct statement is **[Online May 26, 2012]**

(a) $L_1 \parallel L_4$, $L_2 \parallel L_3$, L_1 intersect L_4.
(b) $L_1 \perp L_2$, $L_1 \parallel L_3$, L_1 intersect L_2.
(c) $L_1 \perp L_2$, $L_2 \parallel L_3$, L_1 intersect L_4.
(d) $L_1 \perp L_2$, $L_1 \perp L_3$, L_2 intersect L_4.

77. If $a, b, c \in$ R and 1 is a root of equation $ax^2 + bx + c = 0$, then the curve $y = 4ax^2 + 3bx + 2c$, $a \neq 0$ intersect x-axis at **[Online May 26, 2012]**

(a) two distinct points whose coordinates are always rational numbers
(b) no point
(c) exactly two distinct points
(d) exactly one point

78. Let L be the line $y = 2x$, in the two dimensional plane. **[Online May 19, 2012]**

Statement 1: The image of the point (0, 1) in L is the point $\left(\frac{4}{5}, \frac{3}{5}\right)$.

Statement 2: The points (0, 1) and $\left(\frac{4}{5}, \frac{3}{5}\right)$ lie on opposite sides of the line L and are at equal distance from it.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(d) Statement 1 is false, Statement 2 is true.

79. If two vertices of a triangle are (5, –1) and (–2, 3) and its orthocentre is at (0, 0), then the third vertex is **[Online May 12, 2012]**

(a) $(4, -7)$ (b) $(-4, -7)$ (c) $(-4, 7)$ (d) $(4, 7)$

80. If two vertical poles 20 m and 80 m high stand apart on a horizontal plane, then the height (in m) of the point of intersection of the lines joining the top of each pole to the foot of other is **[Online May 7, 2012]**

(a) 16 (b) 18 (c) 50 (d) 15

81. The point of intersection of the lines
$(a^3 + 3)x + ay + a - 3 = 0$ and
$(a^5 + 2)x + (a + 2)y + 2a + 3 = 0$ (a real) lies on the y-axis for **[Online May 7, 2012]**

(a) no value of a (b) more than two values of a
(c) exactly one value of a (d) exactly two values of a

82. The lines $x+y=|a|$ and $ax-y=1$ intersect each other in the first quadrant. Then the set of all possible values of a in the interval : **[2011RS]**

(a) $(0,\infty)$ (b) $[1,\infty)$ (c) $(-1,\infty)$ (d) $(-1,1)$

83. The lines $L_1 : y-x=0$ and $L_2 : 2x+y=0$ intersect the line $L_3 : y+2=0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[2011]**

Statement-1: The ratio $PR : RQ$ equals $2\sqrt{2}:\sqrt{5}$

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

84. The lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line for : **[2009]**

(a) exactly one values of p

(b) exactly two values of p

(c) more than two values of p

(d) no value of p

85. The shortest distance between the line $y-x=1$ and the curve $x=y^2$ is : **[2009]**

(a) $\frac{2\sqrt{3}}{8}$ (b) $\frac{3\sqrt{2}}{5}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$

86. The perpendicular bisector of the line segment joining P (1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is **[2008]**

(a) 1 (b) 2 (c) –2 (d) – 4

87. Let $P=(-1,0)$, $Q=(0,0)$ and $R=(3,3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is **[2007]**

(a) $\frac{\sqrt{3}}{2}x+y=0$ (b) $x+\sqrt{3}y=0$

(c) $\sqrt{3}x+y=0$ (d) $x+\frac{\sqrt{3}}{2}y=0$

88. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) **[2003]**

(a) are vertices of a triangle

(b) lie on a straight line

(c) lie on an ellipse

(d) lie on a circle.

89. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is **[2003]**

(a) $y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$

(b) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$

(c) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$

(d) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$

TOPIC 4 Pair of Straight Lines

90. The equation $y=\sin x\sin(x+2)-\sin^2(x+1)$ represents a straight line lying in : **[April 12, 2019 (I)]**

(a) second and third quadrants only

(b) first, second and fourth quadrant

(c) first, third and fourth quadrants

(d) third and fourth quadrants only

91. If one of the lines of $my^2+(1-m^2)xy-mx^2=0$ is a bisector of the angle between the lines $xy=0$, then m is **[2007]**

(a) 1 (b) 2 (c) –1/2 (d) –2

92. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals **[2004]**

(a) –3 (b) 1 (c) 3 (d) 1

93. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product c has the value **[2004]**

(a) –2 (b) –1 (c) 2 (d) 1

94. If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angle between the other pair, then **[2003]**

(a) $pq=-1$ (b) $p=q$ (c) $p=-q$ (d) $pq=1$.

95. The pair of lines represented by $3ax^2+5xy+(a^2-2)y^2=0$ are perpendicular to each other for **[2002]**

(a) two values of a (b) $\forall\ a$

(c) for one value of a (d) for no values of a

96. If the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersect on the y-axis then **[2002]**

(a) $2fgh=bg^2+ch^2$ (b) $bg^2\neq ch^2$

(c) $abc=2fgh$ (d) none of these

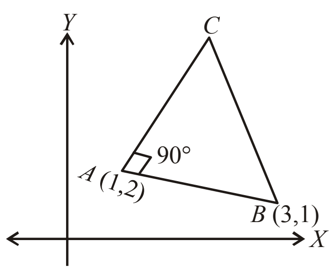
Hints & Solutions

1. **(b)** Let ΔABC be in the first quadrant

Slope of line $AB = -\frac{1}{2}$

Slope of line $AC = 2$

Length of $AB = \sqrt{5}$



It is given that ar$(\Delta ABC) = 5\sqrt{5}$

$\therefore \frac{1}{2} AB \cdot AC = 5\sqrt{5} \Rightarrow AC = 10$

$\therefore$ Coordinate of vertex $C = (1 + 10\cos\theta,\ 2 + 10\sin\theta)$

$\because \tan\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}},\ \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore$ Coordinate of C $= (1 + 2\sqrt{5},\ 2 + 4\sqrt{5})$

$\therefore$ Abscissa of vertex C is $1 + 2\sqrt{5}$.

2. **(b)** Mid point of line segment PQ be $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Slope of perpendicular line passing through

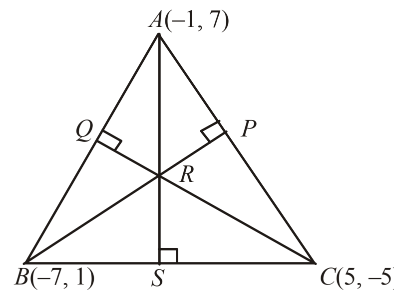
$(0, -4)$ and $\left(\frac{k+1}{2}, \frac{7}{2}\right) = \dfrac{\frac{7}{2}+4}{\frac{k+1}{2}-0} = \frac{15}{k+1}$

Slope of $PQ = \frac{4-3}{1-k} = \frac{1}{1-k}$

$\therefore \frac{15}{1+k} \times \frac{1}{1-k} = -1$

$1 - k^2 = -15 \Rightarrow k = \pm 4.$

3. **(b)**



$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$

$\therefore$ Equation of AS is $y - 7 = 2(x+1)$

$y = 2x + 9$ ...(i)

$m_{AC} = \frac{12}{-6} = -2$

$\therefore$ Equation of BP is $y - 1 = \frac{1}{2}(x+7)$

$y = \frac{x}{2} + \frac{9}{2}$ ...(ii)

From equs. (i) and (ii),

$2x + 9 = \frac{x+9}{2}$

$\Rightarrow 4x + 18 = x + 9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

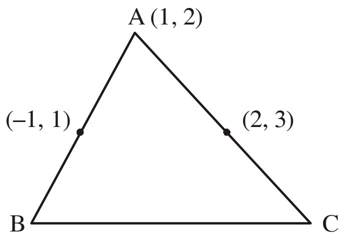
$\therefore y = 3$

4. **(5)** P will be centroid of ΔABC

$P\left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{(4)^2 + (3)^2} = 5$

5. **(b)** From the mid-point formula co-ordinates of vertex B and C are $B(-3, 0)$ and $C(3, 4)$.
Now, centroid of the triangle

$G \equiv \left(\frac{3-3+1}{3}, \frac{0+4+2}{3}\right) \Rightarrow G \equiv \left(\frac{1}{3}, 2\right)$

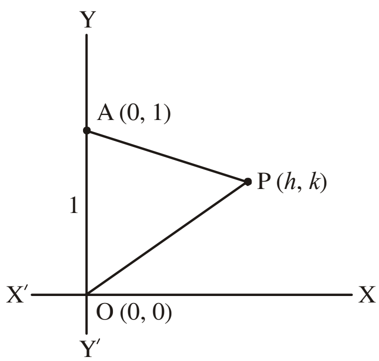


6. **(c)** Let point $P(h, k)$

$\because \; OA = 1$

So, $OP + AP = 3$

$\Rightarrow \sqrt{h^2+k^2} + \sqrt{h^2+(k-1)^2} = 3$



$\Rightarrow h^2+(k-1)^2 = 9 + h^2 + k^2 - 6\sqrt{h^2+k^2}$

$\Rightarrow 6\sqrt{h^2+k^2} = 2k + 8$

$\Rightarrow 9h^2 + 8k^2 - 8k - 16 = 0$

Hence, locus of point P is

$9x^2 + 8y^2 - 8y - 16 = 0$

7. **(b)** Since, $m_{QR} \times m_{PH} = -1$

$\Rightarrow m_{QR} = -\frac{1}{m_{PH}}$

$\Rightarrow m_{QR} = \frac{y-3}{x-4} = 0$

$\Rightarrow y = 3$

$m_{PQ} \times m_{RH} = -1$

$\Rightarrow \frac{1}{4} \times \frac{y}{x} = -1$

$\Rightarrow y = -4x$

$\Rightarrow x = -\frac{3}{4}$

Vertex R is $\left(\frac{-3}{4}, 3\right)$

Hence, vertex R lies in second quadrant.

8. **(b)** Since Orthocentre of the triangle is A(–3, 5) and centriod of the triangle is B(3, 3), then

$AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2 : 1

$\therefore$ AB : BC = 2 : 1

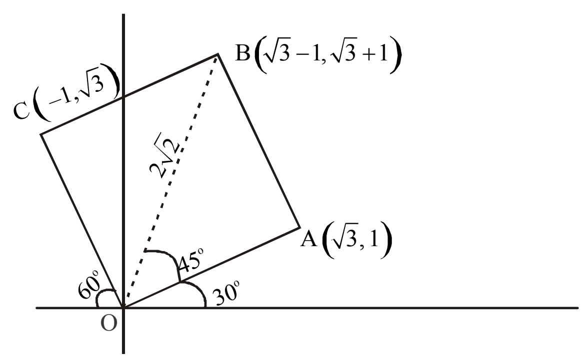
Now, $AB = \frac{2}{3}AC$

$\Rightarrow AC = \frac{3}{2}AB = \frac{3}{2}(2\sqrt{10}) \quad \Rightarrow AC = 3\sqrt{10}$

$\therefore$ Radius of circle with AC as diametre

$= \frac{AC}{2} = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$

9. **(b)**



For A;

$\frac{x}{\cos 30^\circ} = \frac{y}{\sin 30^\circ} = 2$

$\Rightarrow x = \sqrt{3}$ and $y = 1$

For C,

$\frac{x}{\cos 120^\circ} = \frac{y}{\sin 120^\circ} = 2$

$\Rightarrow x = -1, y = \sqrt{3}$

For B,

$\frac{x}{\cos 75^\circ} = \frac{y}{\sin 75^\circ} = 2\sqrt{2}$

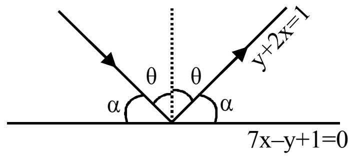
$\Rightarrow x = \sqrt{3} - 1$

and $y = \sqrt{3} + 1$

$\therefore$ Sum $= 2\sqrt{3} - 2$

10. **(c)** Let slope of incident ray be m.

$\therefore$ angle of incidence = angle of reflection



$\therefore \left|\frac{m-7}{1+7m}\right| = \left|\frac{-2-7}{1-14}\right| = \frac{9}{13}$

$\Rightarrow \frac{m-7}{1+7m} = \frac{9}{13}$ or $\frac{m-7}{1+7m} = -\frac{9}{13}$

$\Rightarrow 13m - 91 = 9 + 63m$ or $13m - 91 = -9 - 63m$

$\Rightarrow 50m = -100$ or $76m = 82$

$\Rightarrow m = -\frac{1}{2}$ or $m = \frac{41}{38}$

$\Rightarrow y - 1 = -\frac{1}{2}(x-0)$ or $y - 1 = \frac{41}{38}(x-0)$

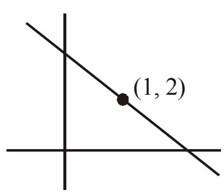
i.e $x + 2y - 2 = 0$ or $38y - 38 - 41x = 0$

$\Rightarrow 41x - 38y + 38 = 0$

11. (a) Equation of line L

$\frac{x}{2} + \frac{y}{4} = 1$

$2x + y = 4$ ...(i)



For line

$x - 2y = -4$ ...(ii)

solving equation (i) and (ii); we get point of intersection

$\left(4/5, \frac{12}{5}\right)$

12. (c) $A\left(0, \frac{8}{3}\right) B(1,3) C(89,30)$

Slope of AB $= \frac{1}{3}$

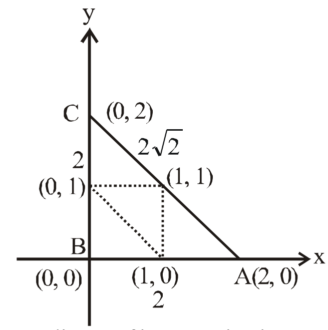
Slope of BC $= \frac{1}{3}$

So, lies on same line

13. (b) From the figure, we have

$a = 2, b = 2\sqrt{2}, c = 2$

$x_1 = 0, x_2 = 0, x_3 = 2$



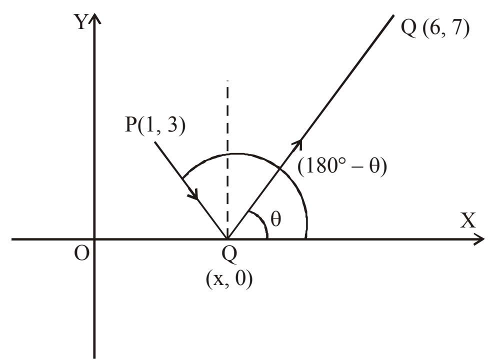
Now, x-co-ordinate of incentre is given as

$\frac{ax_1 + bx_2 + cx_3}{a+b+c}$

$\Rightarrow$ x-coordinate of incentre

$= \frac{2 \times 0 + 2\sqrt{2}.0 + 2.2}{2 + 2 + 2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$

14. (d) Let abcissa of Q $= x$



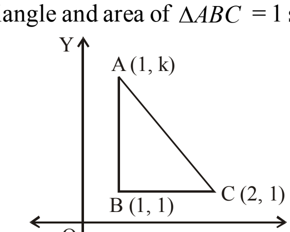
$\therefore$ Q $= (x, 0)$

$\tan\theta = \frac{0-7}{x-6}$, $\tan(180° - \theta) = \frac{0-3}{x-1}$

Now, $\tan(180° - \theta) = -\tan\theta$

$\therefore \frac{-3}{x-1} = \frac{-7}{x-6} \Rightarrow x = \frac{5}{2}$

15. (a) Given : A(l, k), B(1, 1) and C(2, 1) are vertices of a right angled triangle and area of $\Delta ABC = 1$ square unit

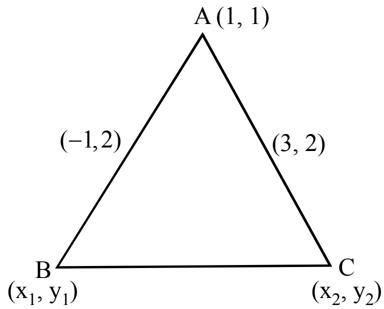


We know that, area of right angled triangle

$= \frac{1}{2} \times BC \times AB = 1 = \frac{1}{2}(a)|(k-1)|$

$\Rightarrow \pm(k-1) = 2 \Rightarrow k = -1, 3$

16. **(c)** Vertex of triangle is (1, 1) and midpoint of sides through this vertex is (– 1, 2) and (3, 2)



$$\frac{1+x_1}{2}=-1, \frac{1+y_1}{2}=2$$

$\Rightarrow$ B (–3, 3)

$$\frac{1+x_2}{2}=3, \frac{1+y_2}{2}=2$$

$\Rightarrow$ C (–5, 3)

$\therefore$ Centroid is $\frac{1-3+5}{3}, \frac{1+3+3}{3}$

$\Rightarrow \left(1, \frac{7}{3}\right)$

17. **(b)** $(x-a_1)^2+(y-b_1)^2 = (x-a_2)^2+(y-b_2)^2$

$(a_1-a_2)x+(b_1-b_2)y+\frac{1}{2}(a_2{}^2+b_2{}^2-a_1{}^2-b_1{}^2)=0$

Comparing with given eqn. we get

$c=\frac{1}{2}(a_2{}^2+b_2{}^2-a_1{}^2-b_1{}^2)$

18. **(c)** We know that centroid

$$(x, y)=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$x=\frac{a\cos t+b\sin t+1}{3}$$

$\Rightarrow a\cos t+b\sin t=3x-1$

$$y=\frac{a\sin t-b\cos t}{3}$$

$\Rightarrow a\sin t-b\cos t=3y$

Squaring and adding,

$(3x-1)^2+(3y)^2=a^2+b^2$

19. **(a)** $AB=\sqrt{(4+1)^2+(0+1)^2}=\sqrt{26}$;

$BC=\sqrt{(3+1)^2+(5+1)^2}=\sqrt{52}$

$CA=\sqrt{(4-3)^2+(0-5)^2}=\sqrt{26}$;

$\because AB=CA$

$\therefore$ Isosceles triangle

$\because (\sqrt{26})^2+(\sqrt{26})^2=52$

$BC^2=AB^2+AC^2$

$\therefore$ right angled triangle,

So, the given triangle is isosceles right angled .

20. **(5)**

$$f'(x)=\begin{cases} 5x^4\cdot\sin\left(\frac{1}{x}\right)-x^3\cos\left(\frac{1}{x}\right)+10x, & x<0 \\ 0, & x=0 \\ 5x^4\cos\left(\frac{1}{x}\right)+x^3\sin\left(\frac{1}{x}\right)+2\lambda x, & x>0 \end{cases}$$

$$f''(x)=\begin{cases} (20x^3-x)\sin\left(\frac{1}{x}\right)-8x^2\cos\left(\frac{1}{x}\right)+10, & x<0 \\ 0, & x=0 \\ (20x^3-x)\cos\left(\frac{1}{x}\right)+8x^2\sin\left(\frac{1}{x}\right)+2\lambda, & x>0 \end{cases}$$

Now, $f''(0^+)=f''(0^-)\Rightarrow 2\lambda=10\Rightarrow\lambda=5$

21. **(a)** Coordinates of centroides

$$C=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right)=(2,2)$$

The given equation of lines are

$x+3y-1=0$ ...(i)

$3x-y+1=0$ ...(ii)

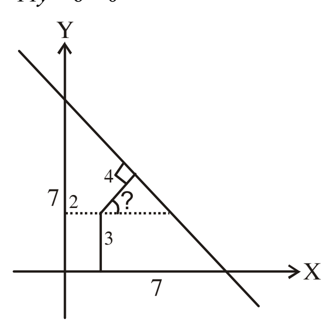
Then, from (i) and (ii)

point of intersection $P\left(-\frac{1}{5}, \frac{2}{5}\right)$

equation of line DP

$8x-11y+6=0$

22. **(b)**



Since point at 4 units from P (2, 3) will be

A (4 cosθ + 2, 4 sinθ + 3) and this point will satisfy the equation of line x + y = 7

$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$

On squaring

$$\Rightarrow \sin 2\theta - \frac{3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$$

$$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$$

$$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} \qquad \text{(ignoring –ve sign)}$$

$$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$$

23. **(c)** A point which is equidistant from both the axes lies on either $y = x$ and $y = -x$.

Since, point lies on the line $3x + 5y = 15$

Then the required point

$$\begin{array}{r} 3x + 5y = 15 \\ x + y = 0 \\ \hline x = -\frac{15}{2} \end{array}$$

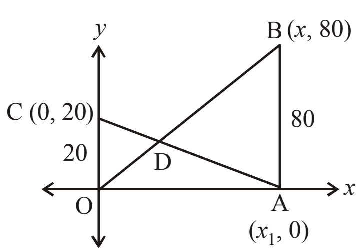
$$y = \frac{15}{2} \Rightarrow (x, y) = \left(-\frac{15}{2}, \frac{15}{2}\right) \ \{2^{nd}\text{ quadrant}\}$$

$$\text{or} \quad \begin{array}{r} 3x + 5y = 15 \\ x - y = 0 \\ \hline x = \frac{15}{8} \end{array}$$

$$y = \frac{15}{8} \Rightarrow (x, y) = \left(\frac{15}{8}, \frac{15}{8}\right) \ \{1^{st}\text{ quadrant}\}$$

Hence, the required point lies in 1^{st} and 2^{nd} quadrant.

24. **(d)**



Equations of lines OB and AC are respectively

$$y = \frac{80}{x_1}x \qquad \text{...(i)}$$

$$\frac{x}{x_1} + \frac{y}{20} = 1 \qquad \text{...(ii)}$$

$\because$ equations (i) and (ii) intersect each other

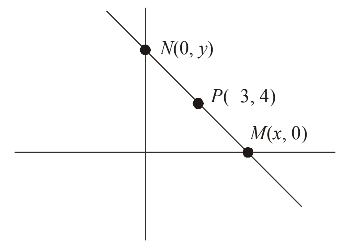
$\therefore$ substitute the value of x from equation (i) to equation (ii), we get

$$\frac{y}{80} + \frac{y}{20} = 1$$

$$\Rightarrow y + 4y = 80 \Rightarrow y = 16 \text{ m}$$

Hence, height of intersection point is 16 m.

25. **(b)** Since, P is mid point of MN



Then, $\frac{0+x}{2} = -3$

$\Rightarrow x = -3 \times 2 \Rightarrow x = -6$

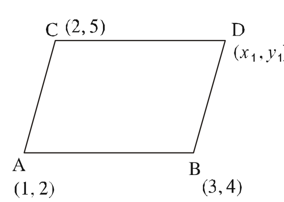
and $\frac{y+0}{2} = 4 \Rightarrow y + 0 = 2 \times 4 \Rightarrow y = 8$

Hence required equation of straight line MN is

$$\frac{x}{-6} + \frac{y}{8} = 1 \Rightarrow 4x - 3y + 24 = 0$$

26. **(a)** Since, in parallelogram mid points of both diagonals coinsides.

$\therefore$ mid-point of AD = mid-point of BC



$$\left(\frac{x_1+1}{2}, \frac{y_1+2}{2}\right) = \left(\frac{3+2}{2}, \frac{4+5}{2}\right)$$

$\therefore \quad (x_1, y_1) = (4, 7)$

Then, equation of AD is,

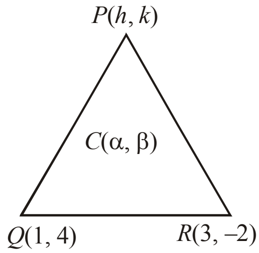
$$y - 7 = \frac{2-7}{1-4}(x-4)$$

$$y - 7 = \frac{5}{3}(x-4)$$

$$3y - 21 = 5x - 20$$

$$5x - 3y + 1 = 0$$

27. **(c)**



Let centroid C be (α, β)

we have $\alpha = \dfrac{1+3+h}{3} \Rightarrow h = 3\alpha - 4$

$\beta = \dfrac{4-2+k}{3} \Rightarrow k = 3\beta - 2$

but $P(h, k)$ lies on $2x - 3y + 4 = 0$

$\Rightarrow\ 2(3\alpha - 4) - 3(3\beta - 2) + 4 = 0$

$\Rightarrow\ 6\alpha - 9\beta - 8 + 6 + 4 = 0$

$\Rightarrow\ 6\alpha - 9\beta + 2 = 0$

Locus: $6x - 9y + 2 = 0$

$\Rightarrow\ y = \dfrac{6}{9}x + \dfrac{2}{9}\ \therefore$ its slope $= \dfrac{6}{9} = \dfrac{2}{3}$

28. **(b)** Equation of the line is:

$3x + 4y = 24$

or $\dfrac{x}{8} + \dfrac{y}{6} = 1$

$\therefore$ coordinates of A, B & O are (8, 0), (0, 6) & (0, 0) respectively.

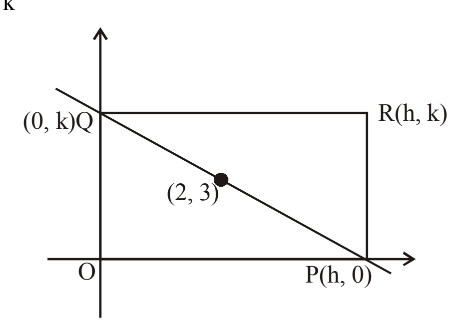
$\Rightarrow\ OA = 8, OB = 6\ \&\ AB = 10.$

$\therefore$ Incentre of ΔOAB is given as:

$$I \equiv \left(\frac{8\times0+6\times8+10\times0}{8+6+10}, \frac{8\times6+6\times0+10\times0}{8+6+10}\right) \equiv (2,2).$$

29. **(b)** Equation of PQ is

$\dfrac{x}{h} + \dfrac{y}{k} = 1$ ...(i)



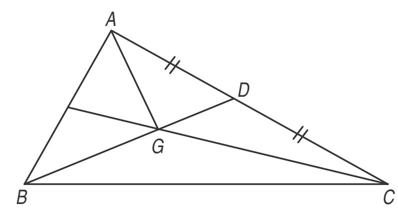
Since, (i) passes through the fixed point (2, 3) Then,

$\dfrac{2}{h} + \dfrac{3}{k} = 1$

Then, the locus of R is $\dfrac{2}{x} + \dfrac{3}{y} = 1$ or $3x + 2y = xy$.

30. **(b)** Median through C is $x = 4$

So the x coordianate of C is 4. let $C \equiv (4, y)$, then the midpoint of A (1, 2) and C (4, y) is D which lies on the median through B.



$\therefore\ D \equiv \left(\dfrac{1+4}{2}, \dfrac{2+y}{2}\right)$

Now, $\dfrac{1+4+2+y}{2} = 5 \Rightarrow y = 3.$

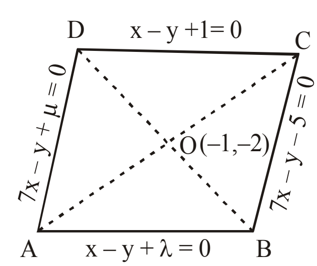
So, $C \equiv (4, 3)$.

The centroid of the triangle is the intersection of the mesians. Here the medians $x = 4$ and $x + 4$ and $x + y = 5$ intersect at G (4, 1).

The area of triangle $\Delta ABC = 3 \times \Delta AGC$

$= 3 \times \dfrac{1}{2}\ [1\,(1-3) + 4\,(3-2) + 4(2-1)] = 9.$

31. **(a)**



Let other two sides of rhombus are

$x - y + \lambda = 0$

and $7x - y + \mu = 0$

then O is equidistant from AB and DC and from AD and BC

$\therefore\ |-1+2+1| = |-1+2+\lambda| \Rightarrow \lambda = -3$

and $|-7+2-5| = |-7+2+\mu| \Rightarrow \mu = 15$

$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$

$\therefore$ On solving the eq^{n}s of sides pairwise, we get the vertices as

$\left(\dfrac{1}{3}, \dfrac{-8}{3}\right), (1,2), \left(\dfrac{-7}{3}, \dfrac{-4}{3}\right), (-3,-6)$

32. **(c)** Length of $\perp$ to $4x + 3y = 10$ from origin (0, 0)

$$P_1 = \frac{10}{5} = 2$$

Length of $\perp$ to $8x + 6y + 5 = 0$ from origin (0, 0)

$$P_2 = \frac{5}{10} = \frac{1}{2}$$

$\because$ Lines are parallel to each other $\Rightarrow$ ratio will be 4 : 1 or 1 : 4

33. **(a)** $L_1 : 4x + 3y - 12 = 0$

$L_2 : 3x + 4y - 12 = 0$

$L_1 + \lambda L_2 = 0$

$(4x + 3y - 12) + \lambda\,(3x + 4y - 12) = 0$

$x\,(4 + 3\lambda) + y\,(3 + 4\lambda) - 12\,(1 + \lambda) = 0$

Point A $\left(\frac{12(1+\lambda)}{4+3\lambda}, 0\right)$

Point B $\left(0, \frac{12(1+\lambda)}{3+4\lambda}\right)$

mid point $\Rightarrow h = \frac{6\,(1+\lambda)}{4+3\lambda}$ ..... (i)

$k = \frac{6\,(1+\lambda)}{3+4\lambda}$ ..... (ii)

Eliminate λ from (i) and (ii), then

$6(h + k) = 7hk$

$6(x + y) = 7xy$

34. **(d)** $x - y = 4$

To find equation of R

slope of L = 0 is 1

$\Rightarrow$ slope of QR = – 1

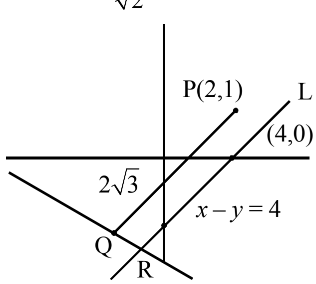
Let QR is $y = mx + c$

$y = -x + c$

$x + y - c = 0$

distance of QR from (2, 1) is $2\sqrt{3}$

$$2\sqrt{3} = \frac{|2 + 1 - c|}{\sqrt{2}}$$



$2\sqrt{6} = |3 - c|$

$c - 3 = \pm 2\sqrt{6}$ $c = 3 \pm 2\sqrt{6}$

Line can be $x + y = 3 \pm 2\sqrt{6}$

$x + y = 3 - 2\sqrt{6}$

35. **(c)** Given eqn of line is $y + \sqrt{3}x - 1 = 0$

$\Rightarrow y = -\sqrt{3}x + 1$

$\Rightarrow$ (slope) $m_2 = -\sqrt{3}$

Let the other slope be m_1

$$\therefore \tan 60° = \left|\frac{m_1 - (-\sqrt{3})}{1 + (-\sqrt{3}m_1)}\right|$$

$\Rightarrow m_1 = 0, m_2 = \sqrt{3}$

Since line L is passing through (3, –2)

$\therefore y - (-2) = +\sqrt{3}(x - 3)$

$\Rightarrow y + 2 = \sqrt{3}(x - 3)$

$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

36. **(d)** Circumcentre = (0, 0)

$$\text{Centroid} = \left(\frac{(a+1)^2}{2}, \frac{(a-1)^2}{2}\right)$$

We know the circumcentre (O),

Centroid (G) and orthocentre (H) of a triangle lie on the line joining the O and G.

Also, $\frac{HG}{GO} = \frac{2}{1}$

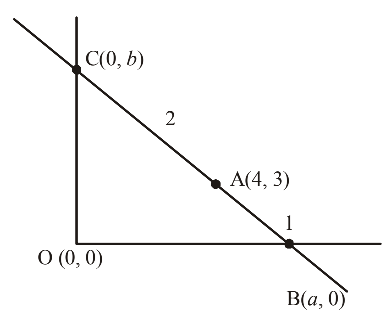
$\Rightarrow$ Coordinate of orthocentre $= \frac{3(a+1)^2}{2}, \frac{3(a-1)^2}{2}$

Now, these coordinates satisfies eqn given in option (d)

Hence, required eqn of line is

$(a - 1)^2\,x - (a + 1)^2\,y = 0$

37. **(b)**



A divides CB in 2 : 1

$\Rightarrow 4 = \left(\frac{1 \times 0 + 2 \times a}{1 + 2}\right) = \frac{2a}{3}$

$\Rightarrow a = 6 \Rightarrow$ coordinate of B is B (6, 0)

$3 = \left(\frac{1 \times b + 2 \times 0}{1 + 2}\right) = \frac{b}{3}$

$\Rightarrow b = 9$ and C (0, 9)

Slope of line passing through (6, 0), (0, 9)

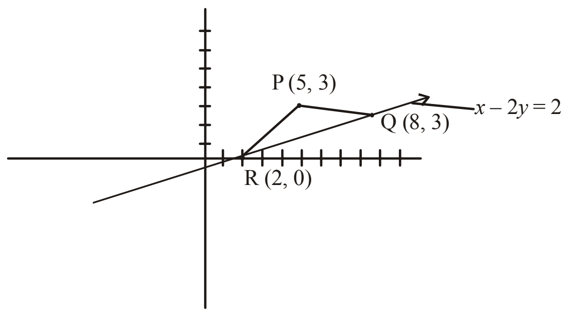
slope, $m = \frac{9}{-6} = -\frac{3}{2}$

Equation of line $y - 0 = \frac{-3}{2}(x-6)$

$2y = -3x + 18$

$3x + 2y = 18$

38. **(d)**



Equation of RQ is $x - 2y = 2$ ...(i)

at $y = 0$, $x = 2$ [R (2, 0)]

as PQ is parallel to x, y-coordinates of Q is also 3

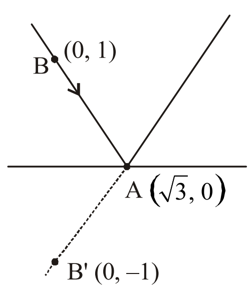
Putting value of y in equation (i), we get

Q (8, 3)

Centroid of $\Delta PQR = \left(\frac{8+5+2}{3}, \frac{3+3}{3}\right) = (5, 2)$

Only $(2x - 5y = 0)$ satisfy the given co-ordinates.

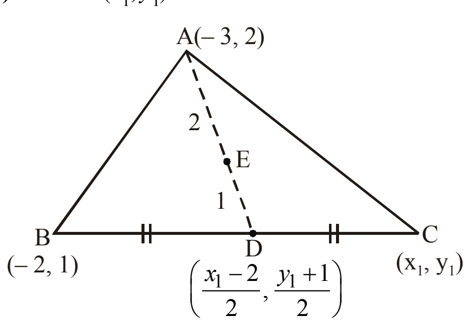
39. **(b)** Suppose B(0, 1) be any point on given line and co-ordinate of A is $(\sqrt{3}, 0)$. So, equation of



Reflected ray is $\frac{-1-0}{0-\sqrt{3}} = \frac{y-0}{x-\sqrt{3}}$

$\Rightarrow \sqrt{3}y = x - \sqrt{3}$

40. **(b)** Let $C = (x_1, y_1)$



Centroid, $E = \left(\frac{x_1-5}{3}, \frac{y_1+3}{3}\right)$

Since centroid lies on the line

$3x + 4y + 2 = 0$

$\therefore \quad 3\left(\frac{x_1-5}{3}\right) + 4\left(\frac{y_1+3}{3}\right) + 2 = 0$

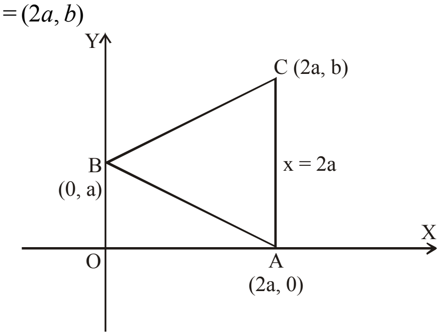
$\Rightarrow \; 3x_1 + 4y_1 + 3 = 0$

Hence vertex (x_1, y_1) lies on the line

$3x + 4y + 3 = 0$

41. **(b)** Let y-coordinate of C = b

$\therefore \; C = (2a, b)$



$AB = \sqrt{4a^2 + a^2} = \sqrt{5}a$

Now, $AC = BC \Rightarrow \; b = \sqrt{4a^2 + (b-a)^2}$

$\Rightarrow b^2 = 4a^2 + b^2 + a^2 - 2ab$

$\Rightarrow 2ab = 5a^2 \Rightarrow b = \frac{5a}{2}$

$\therefore \; C = \left(2a, \frac{5a}{2}\right)$

Hence area of the triangle

$$= \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 2a & \frac{5a}{2} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 0 & \frac{5a}{2} & 0 \end{vmatrix}$$

$$= \frac{1}{2} \times 2a\left(-\frac{5a}{2}\right) = -\frac{5a^2}{2}$$

Since area is always +ve, hence area

$= \frac{5a^2}{2}$ sq. unit

42. **(d)** Given line $3x + 4y = 12$ can be rewritten as

$\frac{3x}{12} + \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

$\Rightarrow x$-intercept = 4 and y-intercept = 3

Let the required line be

L : $\frac{x}{a}+\frac{y}{b}=1$ where

$a = x$-intercept and $b = y$-intercept

According to the question

$a = 4 \times 2 = 8$ and $b = 3/2$

$\therefore$ Required line is $\frac{x}{8}+\frac{2y}{3}=1$

$\Rightarrow 3x + 16y = 24$

$\Rightarrow y = \frac{-3}{16}x + \frac{24}{16}$

Hence, required slope = $\frac{-3}{16}$.

43. **(c)** Let the points be $A(1,1)$ and $B(2,4)$.
Let point C divides line AB in the ratio 3 : 2.
So, by section formula we have

$$C = \left(\frac{3\times2+2\times1}{3+2}, \frac{3\times4+2\times1}{3+2}\right) = \left(\frac{8}{5}, \frac{14}{5}\right)$$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5}, \frac{14}{5}\right)$

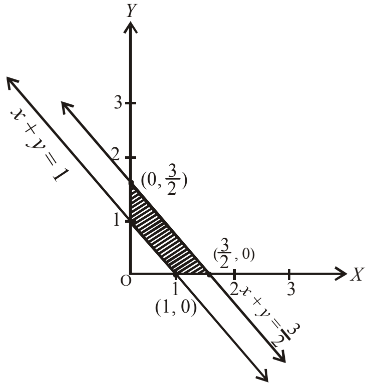
$\Rightarrow \frac{2\times8}{5}+\frac{14}{5}=k \Rightarrow k = 6$

44. **(c)** Given lines are
$ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$
Since, required line is || to x-axis
$\therefore x = 0$
We put $x = 0$ in given equation, we get

$2by = -3b \Rightarrow y = -\frac{3}{2}$

This shows that the required line is below x-axis at a distance of $\frac{3}{2}$ from it.

45. **(d)**



Since, $(1, a)$ lies between $x + y = 1$
and $2(x + y) = 3$
$\therefore$ Put $x = 1$ in $2(x + y) = 3$.
We get the range of y. Thus,

$2(1+y)=3 \Rightarrow y = \frac{3}{2}-1=\frac{1}{2}$

Thus 'a' lies in $\left(0, \frac{1}{2}\right)$

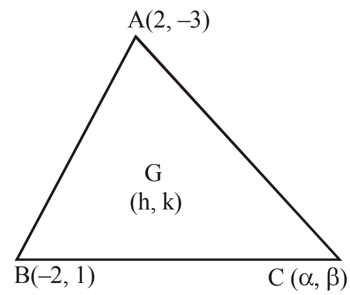
46. **(c)** Let equation of $AB : x + 3y = 4$
Let equation of $BC : 3x + y = 4$
Let equation of $CA : x + y = 0$
Now, By solving these equations we get
$A = (-2, 2), B = (1, 1)$ and $C = (2, -2)$
Now, $AB = \sqrt{9+1} = \sqrt{10}$,

$BC = \sqrt{1+9} = \sqrt{10}$

and $CA = \sqrt{16+16} = \sqrt{32}$

Since, length of AB and BC are same therefore triangle is isosceles.

47. **(b)**



Centroid $(h,k) = \left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right)$

$\therefore \quad \alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

Third vertex (α, β) lies on the line

$2x + 3y = 9$

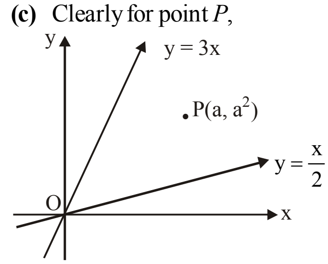
$2\alpha + 3\beta = 9$

$2(3h) + 3(3k+2) = 9$

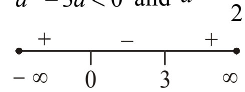
$2h + 3k = 1$

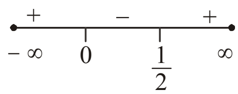
$2x + 3y = 1$

48. **(c)** Clearly for point P,



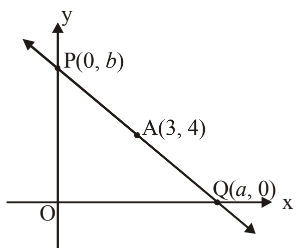
$a^2 - 3a < 0$ and $a^2 - \frac{a}{2} > 0$





$\Rightarrow \frac{1}{2} < a < 3$

49. **(c)**



$\because$ A is the mid point of PQ,

$\therefore \frac{a+0}{2} = 3, \frac{0+b}{2} = 4 \Rightarrow a = 6, b = 8$

$\therefore$ Equation of line is $\frac{x}{6} + \frac{y}{8} = 1$

or $4x + 3y = 24$

50. **(a)** The eqn. of line passing through the intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

Required line is parallel to x-axis.

$\therefore a + b\lambda = 0 \Rightarrow \lambda = -a/b$

$\Rightarrow ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$

$\Rightarrow ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$

$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$

$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$

$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$

So it is 3/2 units below x-axis.

51. **(a)** Let the required line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

then $a + b = -1 \Rightarrow b = -a - 1$ ....(ii)

(i) passes through (4, 3), $\Rightarrow \frac{4}{a} + \frac{3}{b} = 1$

$\Rightarrow 4b + 3a = ab$ ....(iii)

Putting value of b from (ii) in (iii), we get

$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1

$\therefore$ Equations of straight lines are

$\frac{x}{2} + \frac{y}{-3} = 1$ or $\frac{x}{-2} + \frac{y}{1} = 1$

52. **(d)** Let the vertex C be (h, k), then the centroid of ΔABC is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{2 - 2 + h}{3}, \frac{-3 + 1 + k}{3}\right)$

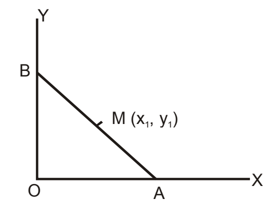
$= \left(\frac{h}{3}, \frac{-2 + k}{3}\right)$. It lies on $2x + 3y = 1$

$\Rightarrow \frac{2h}{3} - 2 + k = 1 \Rightarrow 2h + 3k = 9$

$\Rightarrow$ Locus of C is $2x + 3y = 9$

53. **(d)** Equation of AB is

$x \cos\alpha + y \sin\alpha = p$;



$\Rightarrow \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$;

$\Rightarrow \frac{x}{p/\cos\alpha} + \frac{y}{p/\sin\alpha} = 1$

So, co-ordinates of A and B are

$\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$;

So, coordinates of midpoint of AB are

$M(x_1, y_1) = \left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right)$

$x_1 = \frac{p}{2\cos\alpha}$ & $y_1 = \frac{p}{2\sin\alpha}$;

$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = p/2y_1$;

$\because \cos^2\alpha + \sin^2\alpha = 1$

Locus of (x_1, y_1) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

54. **(a)** The line in xy-plane is,

$\frac{x}{3} + y = 1 \Rightarrow x + 3y - 3 = 0$

Let image of the point $(-1, -4)$ be (α, β), then

$\frac{\alpha + 1}{1} = \frac{\beta + y}{3} = -\frac{2(-1 - 12 - 3)}{10}$

$\Rightarrow \alpha + 1 = \frac{\beta + 4}{3} = \frac{16}{5}$

$\Rightarrow \alpha = \frac{11}{5}, \beta = \frac{28}{5}$

55. **(30)**

$L_1 : 2x - y + 3 = 0$

$L_1 : 4x - 2y + \alpha = 0 \Rightarrow 2x - y + \frac{\alpha}{2} = 0$

$L_1 : 6x - 3y + \beta = 0 \Rightarrow 2x - y + \frac{\beta}{3} = 0$

Distance between L_1 and L_2 ;

$$\left|\frac{\alpha-6}{2\sqrt{5}}\right|=\frac{1}{\sqrt{5}} \Rightarrow |\alpha-6|=2$$

$\Rightarrow \alpha = 4, 8$

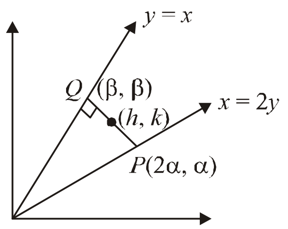
Distance between L_1 and L_3 :

$$\left|\frac{\beta-9}{3\sqrt{5}}\right|=\frac{2}{\sqrt{5}} \Rightarrow |\beta-9|=6$$

$\Rightarrow \beta = 15, 3$

Sum of all values $= 4 + 8 + 15 + 3 = 30$.

56. **(b)**



Since, slope of $PQ = \dfrac{k-\alpha}{h-2\alpha} = -1$

$\Rightarrow\ k-\alpha=-h+2\alpha$

$\Rightarrow\ \alpha=\dfrac{h+k}{3}$ ...(i)

Also, $2h = 2\alpha + \beta$ and

$2k = \alpha + \beta$

$\Rightarrow\ 2h = \alpha + 2k$

$\Rightarrow\ \alpha = 2h - 2k$ ...(ii)

From (i) and (ii), we have

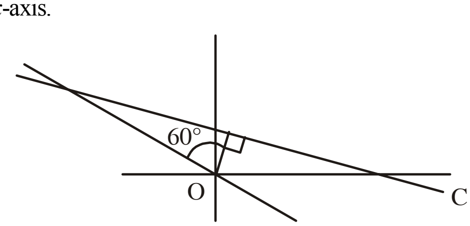
$$\frac{h+k}{3}=2(h-k)$$

So, locus is $6x - 6y = x + y$

$\Rightarrow\ 5x = 7y \Rightarrow 5x - 7y = 0$

57. **(b)** $\because$ perpendicular makes an angle of 60° with the line $x + y = 0$.

$\therefore$ the perpendicular makes an angle of 15° or 75° with x-axis.



Hence, the equation of line will be

$x \cos 75° + y \sin 75° = 4$

or $x \cos 15° + y \sin 15° = 4$

$(\sqrt{3}-1)x+(\sqrt{3}+1)y=8\sqrt{2}$

or $(\sqrt{3}+1)x+(\sqrt{3}-1)y=8\sqrt{2}$

58. **(a)** Let straight line be $4x - 3y + \alpha = 0$

$\because$ distance from origin $=\dfrac{3}{5}$

$\therefore \dfrac{3}{5}=\left|\dfrac{\alpha}{5}\right| \Rightarrow \alpha=\pm 3$

Hence, line is $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$

Clearly $\left(-\dfrac{1}{4},\dfrac{2}{3}\right)$ satisfies $4x - 3y + 3 = 0$

59. **(a)** $\because$ two lines are perpendicular $\Rightarrow m_1m_2 = -1$

$\Rightarrow \left(\dfrac{-1}{a-1}\right)\left(\dfrac{-2}{a^2}\right)=-1$

$\Rightarrow 2 = a^2(1-a) \Rightarrow a^3 - a^2 + 2 = 0$

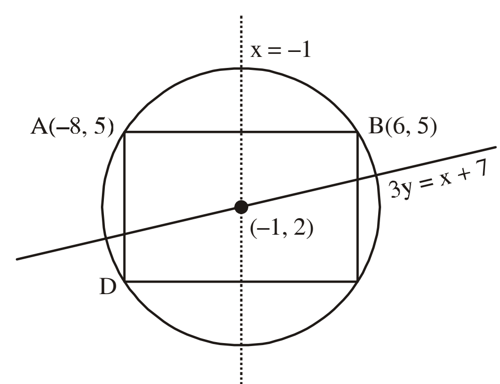
$\Rightarrow (a+1)(a^2+2a+2)=0 \Rightarrow a=-1$

Hence equations of lines are $x - 2y = 1$ and $2x + y = 1$

$\therefore$ intersection point is $\left(\dfrac{3}{5},\dfrac{-1}{5}\right)$

Now, distance from origin $=\sqrt{\dfrac{9}{25}+\dfrac{1}{25}}=\sqrt{\dfrac{10}{25}}=\sqrt{\dfrac{2}{5}}$

60. **(a)** Given situation



$\because$ perpendicular bisector of AB will pass from centre.

$\therefore$ equation of perpendicular bisector $x = -1$

Hence centre of the circle is $(-1, 2)$

Let co-ordinate of D is (α, β)

$\Rightarrow \dfrac{\alpha+6}{2}=-1$ and $\dfrac{\beta+5}{2}=2$

$\Rightarrow \alpha = -8$ and $\beta = -1$, $\therefore$ D $\equiv (-8, -1)$

$|AD| = 6$ and $|AB| = 14$

Area $= 6 \times 14 = 84$

61. **(a)** $\because (h, k), (1, 2)$ and $(-3, 4)$ are collinear

$$\therefore \begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0 \Rightarrow -2h - 4k + 10 = 0$$

$\Rightarrow h + 2k = 5$ ...(i)

Now, $m_{L_1} = \dfrac{4-2}{-3-1} = -\dfrac{1}{2} \Rightarrow m_{L_2} = 2 \quad [\because L_1 \perp L_2]$

By the given points (h, k) and $(4, 3)$,

$$m_{L_2} = \frac{k-3}{h-4} \Rightarrow \frac{k-3}{h-4} = 2 \Rightarrow k - 3 = 2h - 8$$

$2h - k = 5$ ...(ii)

From (i) and (ii)

$$h = 3, k = 1 \Rightarrow \frac{k}{h} = \frac{1}{3}$$

62. **(d)** $\because$ Equation of straight line can be rewritten as,

$$y = \frac{2}{3}x + \frac{17}{3}.$$

$\therefore$ Slope of straight line $= \dfrac{2}{3}$

Slope of line passing through the points $(7, 17)$ and $(15, \beta)$

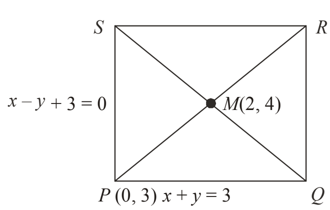
$$= \frac{\beta - 17}{15 - 7} = \frac{\beta - 17}{8}$$

Since, lines are perpendicular to each other.

Hence, $m_1 m_2 = -1$

$$\Rightarrow \left(\frac{2}{3}\right)\left(\frac{\beta - 17}{8}\right) = -1 \Rightarrow \beta = 5$$

63. **(d)**



Since, $x - y + 3 = 0$ and $x + y = 3$ are perpendicular lines and intersection point of $x - y + 3 = 0$ and $x + y = 3$ is $P(0, 3)$.

$\Rightarrow$ M is mid-point of $PR \Rightarrow R(4, 5)$

Let $S(x_1, x_1 + 3)$ and $Q(x_2, 3 - x_2)$

M is mid-point of SQ

$\Rightarrow x_1 + x_2 = 4, x_1 + 3 + 3 - x_2 = 8$

$\Rightarrow x_1 = 3, x_2 = 1$

Then, the vertex D is $(3, 6)$.

64. **(a)** The given equations of the set of all lines

$px + qy + r = 0$ ...(i)

and given condition is :

$3p + 2q + 4r = 0$

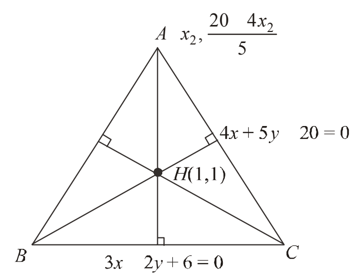
$\Rightarrow \dfrac{3}{4}p + \dfrac{2}{4}q + r = 0$ ...(ii)

From (i) & (ii) we get :

$$\therefore x = \frac{3}{4}, y = \frac{1}{2}$$

Hence the set of lines are concurrent and passing through the fixed point $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$

65. **(d)**



$$\left(x_1, \frac{3x_1 + 6}{2}\right)$$

Since, AH is perpendicular to BC

Hence, $m_{AH} \cdot m_{BC} = -1$

$$\left(\frac{\frac{20 - 4x_2}{5} - 1}{x_2 - 1}\right) \times \frac{3}{2} = -1$$

$$\frac{15 - 4x_2}{5(x_2 - 1)} = -\frac{2}{3}$$

$45 - 12x_2 = -10x_2 + 10$

$$2x_2 = 35 \Rightarrow x_2 = \frac{35}{2}$$

$$\Rightarrow A\left(\frac{35}{2}, -10\right)$$

Since, BH is perpendicular to CA.

Hence, $m_{BH} \times m_{CA} = -1$

$$\left(\frac{\frac{3x_1}{2} + 3 - 1}{x_1 - 1}\right)\left(-\frac{4}{5}\right) = -1$$

$$\frac{(3x_1 + 4)}{2(x_1 - 1)} \times 4 = 5$$

$\Rightarrow\ 6x_1 + 8 = 5x_1 - 5 \Rightarrow x_1 = -13 \Rightarrow \left(-13, \frac{-33}{2}\right)$

$\Rightarrow$ Equation of line AB is

$$y + 10 = \left(\frac{-\frac{33}{2} + 10}{-13 - 35}\right)\left(x - \frac{35}{2}\right)$$

$\Rightarrow\ -61y - 610 = -13x + \frac{455}{2}$

$\Rightarrow\ -122y - 1220 = -26x + 455$

$\Rightarrow\ 26x - 122y - 1675 = 0$

66. **(d)** Equation of the line, which is perpendicular to the line, $3x + y = \lambda(\lambda \neq 0)$ and passing through origin, is given by

$$\frac{x-0}{3} = \frac{y-0}{1} = r$$

For foot of perpendicular

$$r = \frac{-\left((3\times 0) + (1\times 0) - \lambda\right)}{3^2 + 1^2} = \frac{\lambda}{10}$$

So, foot of perpendicular $P = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Given the line meets X-axis at $A = \left(\frac{\lambda}{3}, 0\right)$ and meets Y-axis at $B = (0, \lambda)$

So, $BP = \sqrt{\left(\frac{3\lambda}{10}\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2} \Rightarrow BP = \sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}$

$$\Rightarrow BP = \sqrt{\frac{90\lambda^2}{100}}$$

Now, $PA = \sqrt{\left(\frac{\lambda}{3} - \frac{3\lambda}{10}\right)^2 + \left(0 - \frac{\lambda}{10}\right)^2}$

$\Rightarrow PA \sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}} \Rightarrow PA = \sqrt{\frac{10\lambda^2}{900}}$

Therefore $BP : PA = 3 : 1$

67. **(d)** Let the coordinate A be $(0, c)$

Equations of the given lines are

$x - y + 2 = 0$ and

$7x - y + 3 = 0$

We know that the diagonals of the rhombus will be parallel to the angle bisectors of the two given lines; $y = x + 2$ and $y = 7x + 3$

$\therefore$ equation of angle bisectors is given as:

$$\frac{x - y + 2}{\sqrt{2}} = \pm \frac{7x - y + 3}{5\sqrt{2}}$$

$5x - 5y + 10 = \pm\ (7x - y + 3)$

$\therefore$ Parallel equations of the diagonals are $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$\therefore$ slopes of diagonals are $\frac{-1}{2}$ and 2.

Now, slope of the diagonal from $A(0, c)$ and passing through $P(1, 2)$ is $(2 - c)$

$\therefore\ 2 - c = 2\ \Rightarrow\ c = 0$ (not possible)

$\therefore\ 2 - c = \frac{-1}{2} \Rightarrow c = \frac{5}{2}$

$\therefore$ ordinate of A is $\frac{5}{2}$.

68. **(a)** Given lines are

$4ax + 2ay + c = 0$

$5bx + 2by + d = 0$

The point of intersection will be

$$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$$

$\Rightarrow\ x = \frac{2(ad - bc)}{-2ab} = \frac{bc - ad}{ab}$

$\Rightarrow\ y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$

$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

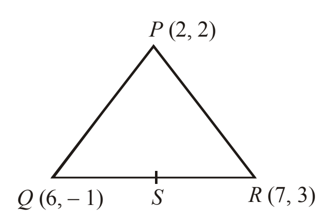
Also distance from axes is same

So $x = -y$

($\because$ distance from x-axis is $-y$ as y is negative)

$$\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab} \Rightarrow 3bc - 2ad = 0$$

69. **(d)** Let P, Q, R, be the vertices of ΔPQR



Since PS is the median

S is mid-point of QR

So, $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Now, slope of $PS = \frac{2-1}{2 - \frac{13}{2}} = -\frac{2}{9}$

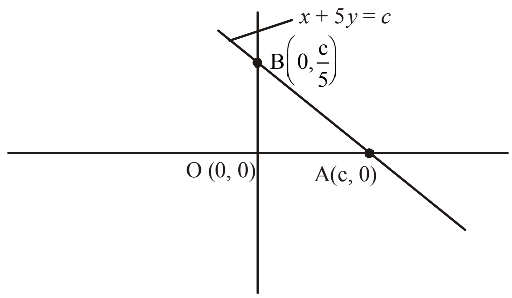
Since, required line is parallel to PS therefore slope of required line = slope of PS

Now, eqn. of line passing through $(1, -1)$ and having slope $-\frac{2}{9}$ is

$$y-(-1)=-\frac{2}{9}(x-1)$$

$$9y+9=-2x+2 \Rightarrow 2x+9y+7=0$$

70. **(b)** Let equation of line L, perpendicular to $5x - y = 1$ be $x + 5y = c$



Given that area of ΔAOB is 5.
We know

$$\left\{\text{area, } A=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]\right\}$$

$$\Rightarrow 5 = \frac{1}{2}\left[c\left(\frac{c}{5}\right)\right]$$

$$\left(\begin{array}{l}\because (x_1, y_1)=(10,0),(x_3,y_3)=\left(0,\frac{c}{5}\right)\\ (x_2,y_2)=(c,0)\end{array}\right)$$

$$\Rightarrow c = \pm\sqrt{50}$$

$\therefore$ Equation of line L is $x + 5y = \pm\sqrt{50}$
Distance between L and line $x + 5y = 0$ is

$$d = \left|\frac{\pm\sqrt{50}-0}{\sqrt{1^2+5^2}}\right| = \frac{\sqrt{50}}{\sqrt{26}} = \frac{5}{\sqrt{13}}$$

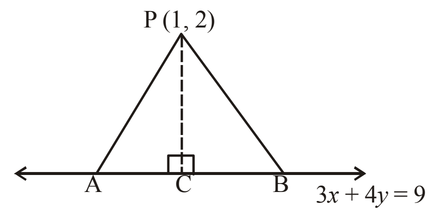
71. **(c)** $x+2ay+a = 0$ ...(i)

$x+3by+b = 0$ ...(ii)

$x+4ay+a = 0$ ...(iii)

Subtracting equation (iii) from (i)
$-2ay = 0$
$ay = 0$ χ $y = 0$
Putting value of y in equation (i), we get
$x + 0 + a = 0$
$x = -a$
Putting value of x and y in equation (ii), we get
$-a + b = 0 \Rightarrow a = b$
Thus, (a, b) lies on a straight line

72. **(b)**



Shortest distance of a point (x_1, y_1) from line

$ax + by = c$ is $d = \left|\frac{ax_1+by_1-c}{\sqrt{a^2+b^2}}\right|$

Now shortest distance of P (1, 2) from $3x + 4y = 9$ is

$$PC = d = \left|\frac{3(1)+4(2)-9}{\sqrt{3^2+4^2}}\right| = \frac{2}{5}$$

Given that ΔAPB is an equilateral triangle
Let 'a' be its side

then PB = a, CB = $\frac{a}{2}$
Now, In ΔPCB, $(PB)^2 = (PC)^2 + (CB)^2$
(By Pythagoras theoresm)

$$a^2 = \left(\frac{2}{5}\right)^2 + \frac{a^2}{4}$$

$$a^2-\frac{a^4}{4} = \frac{4}{25} \Rightarrow \frac{3a^2}{4} = \frac{4}{25}$$

$$a^2 = \frac{16}{75} \Rightarrow a = \sqrt{\frac{16}{75}} = \frac{4}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{15}$$

$\therefore$ Length of Equilateral triangle $(a) = \frac{4\sqrt{3}}{15}$

73. **(d)** Mid-point of P(2, 3) and Q(4, 5) = (3, 4)
Slope of PQ = 1
Slope of the line L = – 1
Mid-point (3, 4) lies on the line L.
Equation of line L,
$y-4=-1(x-3) \Rightarrow x+y-7=0$ ...(i)
Let image of point R(0, 0) be S(x_1, y_1)

Mid-point of RS = $\left(\frac{x_1}{2},\frac{y_1}{2}\right)$

Mid-point $\left(\frac{x_1}{2},\frac{y_1}{2}\right)$ lies on the line (i)

$\therefore \quad x_1+y_1=14$ ...(ii)

Slope of RS = $\frac{y_1}{x_1}$

Since RS $\perp$ line L

$\therefore \quad \frac{y_1}{x_1} \times (-1) = -1$

$\therefore \quad x_1 = y_1$ ...(iii)

From (ii) and (iii),

$x_1 = y_1 = 7$

Hence the image of R = (7, 7)

74. **(a)** Two lines $-x + 5y + c_2 = 0$ and $-x + 5y + c_3 = 0$ are parallel to each other. Hence statement-1 is true, statement-2 is true and statement-2 is the correct explanation of statement-1.

75. **(a)** Since three lines x – 3y = p,

ax + 2y = q and ax + y = r

form a right angled triangle

$\therefore$ product of slopes of any two lines = –1

Suppose ax + 2y = q and x – 3y = p are $\perp$ to each other.

$\therefore \quad \frac{-a}{2} \times \frac{1}{3} = -1 \Rightarrow a = 6$

Now, consider option one by one

$a = 6$ satisfies only option (a)

$\therefore$ Required answer is $a^2 - 9a + 18 = 0$

76. **(d)** Consider the lines

$L_1 : x - y = 1$

$L_2 : x + y = 1$

$L_3 : 2x + 2y = 5$

$L_4 : 2x - 2y = 7$

$L_1 \perp L_2$ is correct statement

($\because$ Product of their slopes = – 1)

$L_1 \perp L_3$ is also correct statement

($\because$ Product of their slopes = – 1)

Now, $L_2 : x + y = 1$

$L_4 : 2x - 2y = 7$

$\Rightarrow 2x - 2(1 - x) = 7$

$\Rightarrow 2x - 2 + 2x = 7$

$\Rightarrow x = \frac{9}{4}$ and $y = \frac{-5}{4}$

Hence, L_2 intersects L_4.

77. **(d)** Given $ax^2 + bx + c = 0$

$\Rightarrow ax^2 = -bx - c$

Now, consider

$y = 4ax^2 + 3bx + 2c$

$= 4[-bx - c] + 3bx + 2c$

$= -4bx - 4c + 3bx + 2c = -bx - 2c$

Since, this curve intersects x-axis

$\therefore$ put $y = 0$, we get

$-bx - 2c = 0 \Rightarrow -bx = 2c$

$\Rightarrow x = \frac{-2c}{b}$

Thus, given curve intersects x-axis at exactly one point.

78. **(c)** Statement - 1

Let $P'(x_1, y_1)$ be the image of (0, 1) with respect to the line $2x - y = 0$ then

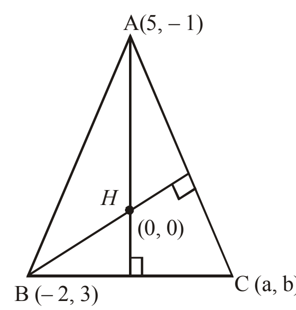
$\frac{x_1}{2} = \frac{y_1 - 1}{-1} = \frac{-4(0) + 2(1)}{5}$

$\Rightarrow x_1 = \frac{4}{5}, y_1 = \frac{3}{5}$

Thus, statement-1 is true.

Also, statement-2 is true and correct explanation for statement-1.

79. **(b)**



Let the third vertex of ΔABC be (a, b).

Orthocentre = $H(0, 0)$

Let A (5, – 1) and B (– 2, 3) be other two vertices of ΔABC.

Now, (Slope of AH) × (Slope of BC) = – 1

$\Rightarrow \left(\frac{-1-0}{5-0}\right)\left(\frac{b-3}{a+2}\right) = -1$

$\Rightarrow b - 3 = 5(a + 2)$ ...(i)

Similarly,

(Slope of BH) × (Slope of AC) = – 1

$\Rightarrow -\left(\frac{3}{2}\right) \times \left(\frac{b+1}{a-5}\right) = -1$

$\Rightarrow 3b + 3 = 2a - 10$

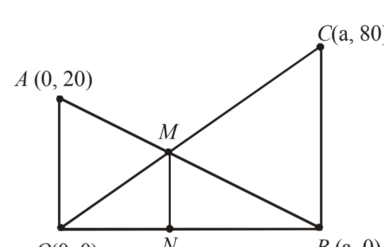
$\Rightarrow 3b - 2a + 13 = 0$ ...(ii)

On solving equations (i) and (ii) we get

$a = -4, b = -7$

Hence, third vertex is (– 4, – 7).

80. **(a)**



We put one pole at origin.

$BC = 80\,\text{m}, OA = 20\,\text{m}$

Line OC and AB intersect at M.

To find: Length of MN.

Eqn of OC: $y = \left(\frac{80-0}{a-0}\right)x$

$\Rightarrow \quad y = \frac{80}{a}x$ ...(i)

Eqn of AB: $y = \left(\frac{20-0}{0-a}\right)(x-a)$

$\Rightarrow y = \frac{-20}{a}(x-a)$ ...(ii)

At M: (i) = (ii)

$\Rightarrow \quad \frac{80}{a}x = \frac{-20}{a}(x-a)$

$\Rightarrow \quad \frac{80}{a}x = \frac{-20}{a}x + 20 \Rightarrow x = \frac{a}{5}$

$\therefore \quad y = \frac{80}{a} \times \frac{a}{5} = 16$

81. **(a)** Given equation of lines are

$(a^3+3)x + ay + a - 3 = 0$ and

$(a^5+2)x + (a+2)y + 2a + 3 = 0$ (a real)

Since point of intersection of lines lies on y-axis.

$\therefore$ Put $x = 0$ in each equation, we get

$ay + a - 3 = 0$ and

$(a+2)y + 2a + 3 = 0$

On solving these we get

$(a+2)(a-3) - a(2a+3) = 0$

$\Rightarrow \quad a^2 - a - 6 - 2a^2 - 3a = 0$

$\Rightarrow \quad -a^2 - 4a - 6 = 0 \Rightarrow a^2 + 4a + 6 = 0$

$\Rightarrow \quad a = \frac{-4 \pm \sqrt{16-24}}{2} = \frac{-4 \pm \sqrt{-8}}{2}$

(not real)

This shows that the point of intersection of the lines lies on the y-axis for no value of 'a'.

82. **(b)** Given that $x + y = |a|$

and $ax - y = 1$

Case I : If $a > 0$

$x + y = a$ .... (i)

$ax - y = 1$ .... (ii)

On adding equations (i) and (ii), we get

$x(1+a) = 1 + a \Rightarrow x = 1$

$y = a - 1$

Since given that intersection point lies in first quadrant

So, $\quad a - 1 \geq 0$

$\Rightarrow \quad a \geq 1$

$\Rightarrow \quad a \in [1, \infty)$

Case II : If $a < 0$

$x + y = -a$ .... (iii)

$ax - y = 1$ .... (iv)

On adding equations (iii) and (iv), we get

$x(1+a) = 1 - a$

$x = \frac{1-a}{1+a} > 0 \Rightarrow \frac{a-1}{a+1} < 0$

Since $a - 1 < 0$

$\therefore \quad a + 1 > 0$

$\Rightarrow \quad a > -1$ .... (v)

$y = -a - \frac{1-a}{1+a} > 0 \quad = \frac{-a-a^2-1+a}{1+a} > 0$

$\Rightarrow \quad -\left(\frac{a^2+1}{a+1}\right) > 0 \Rightarrow \frac{a^2+1}{a+1} < 0$

Since $a^2 + 1 > 0$

$\therefore \quad a + 1 < 0$

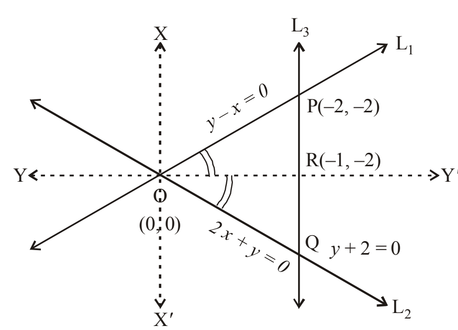
$\Rightarrow \quad a < -1$ .... (vi)

From (v) and (vi), $a \in \phi$

Hence, Case-II is not possible.

So, correct answer is $a \in [1, \infty)$

83. **(b)**



$L_1 : y - x = 0$

$L_2 : 2x + y = 0$

$L_3 : y + 2 = 0$

On solving the equation of line L_1 and L_2 we get their point of intersection (0, 0) i.e., origin O.

On solving the equation of line L_1 and L_3,

we get $P = (-2, -2)$.

Similarly, solving equation of line L_2 and L_3, we get

$Q = (-1, -2)$

We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$\therefore \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$

$\therefore$ Statement 1 is true but $\angle OPR \neq \angle OQR$

So ΔOPR and ΔOQR not similar

$\therefore$ Statement 2 is false.

84. **(a)** Given that the lines $p(p^2+1)x - y + q = 0$ and $(p^2+1)^2 x + (p^2+1)y + 2q = 0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore \quad m_1 = m_2$

$$\Rightarrow -\frac{p(p^2+1)}{-1} = -\frac{(p^2+1)^2}{p^2+1}$$

$\Rightarrow (p^2+1)^2\ (p+1) = 0$

$\Rightarrow p = -1$

$\therefore$ p can have exactly one value.

85. **(d)** Let (a^2, a) be the point of shortest distance on $x = y^2$

Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right| = \frac{|a^2 - a + 1|}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left|(a - \frac{1}{2})^2 + \frac{3}{4}\right|$$

It is min when $a = \frac{1}{2}$ and

$$D_{min} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

86. **(d)** Slope of $PQ = \frac{3-4}{k-1} = \frac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of $PQ = (k-1)$

Also, mid point of PQ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of perpendicular bisector of PQ is

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

$\Rightarrow\ 2y - 7 = 2(k-1)x - (k^2-1)$

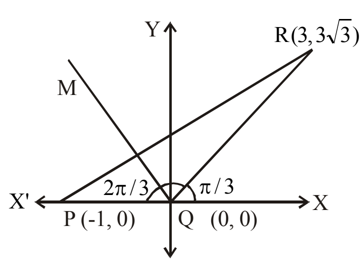
$\Rightarrow\ 2(k-1)x - 2y + (8 - k^2) = 0$

Given that y-intercept

$$= \frac{8-k^2}{2} = -4$$

$\Rightarrow\ 8 - k^2 = -8$ or $k^2 = 16 \Rightarrow k = \pm 4$

87. **(c)** **Given :** The coordinates of points P, Q, R are $(-1, 0)$, $(0, 0)$, $(3, 3\sqrt{3})$ respectively.



Slope of QR $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3\sqrt{3}}{3}$

$\Rightarrow\ \tan\theta = \sqrt{3} \Rightarrow\ \theta = \frac{\pi}{3}$

$\Rightarrow\ \angle RQX = \frac{\pi}{3}$

$\therefore\ \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Let QM bisects the $\angle PQR$,

$\therefore\ \angle MQR = \frac{\pi}{3} \Rightarrow \angle MQX = \frac{2\pi}{3}$

$\therefore$ Slope of the line QM $= \tan\frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y - 0) = -\sqrt{3}\,(x - 0)$

$\Rightarrow\ y = -\sqrt{3}\,x \Rightarrow\ \sqrt{3}\,x + y = 0$

88. **(b)** Taking co-ordinates as

$A\left(\frac{x}{r}, \frac{y}{r}\right)$; $B(x, y)$ and $C(xr, yr)$.

Then slope of line joining

$$A\left(\frac{x}{r}, \frac{y}{r}\right), B(x, y) = \frac{y\left(1 - \frac{1}{r}\right)}{x\left(1 - \frac{1}{r}\right)} = \frac{y}{x}$$

and slope of line joining $B(x, y)$ and $C(xr, yr)$

$$= \frac{y(r-1)}{x(r-1)} = \frac{y}{x}$$

$\therefore\ m_1 = m_2$

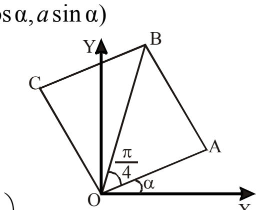
$\because$ Slope of AB and BC are same and one point B common.

$\Rightarrow$ Points lie on the straight line.

89. **(a)** Co-ordinates of $A = (a\cos\alpha, a\sin\alpha)$

Equation of OB,

$$y = \tan\left(\frac{\pi}{4} + \alpha\right)x$$



$CA \perp^r$ to OB

$\therefore$ Slope of $CA = -\cot\left(\frac{\pi}{4} + \alpha\right)$

Equation of CA

$$y - a\sin\alpha = -\cot\left(\frac{\pi}{4} + \alpha\right)(x - a\cos\alpha)$$

$$\Rightarrow (y - a\sin\alpha)\left(\tan\left(\frac{\pi}{4} + \alpha\right)\right) = (a\cos\alpha - x)$$

$$\Rightarrow (y - a\sin\alpha)\left(\frac{\tan\frac{\pi}{4} + \tan\alpha}{1 - \tan\frac{\pi}{4}\tan\alpha}\right) = (a\cos\alpha - x)$$

$\Rightarrow (y - a\sin\alpha)(1 + \tan\alpha) = (a\cos\alpha - x)(1 - \tan\alpha)$

$\Rightarrow (y - a\sin\alpha)(\cos\alpha + \sin\alpha)$

$= (a\cos\alpha - x)(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos + \sin\alpha) - a\sin\alpha\cos\alpha - a\sin^2\alpha$

$= a\cos^2\alpha - a\cos\alpha\sin\alpha - x(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

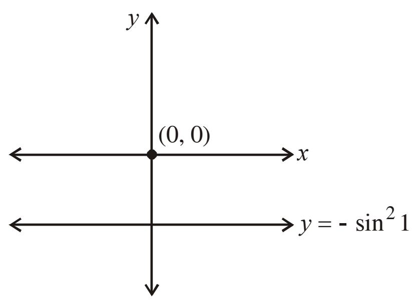
$y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a.$

90. **(d)** Consider the equation,

$y = \sin x.\sin(x+2) - \sin^2(x+1)$

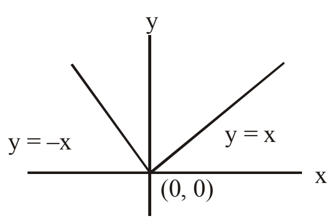
$$= \frac{1}{2}\cos(-2) - \frac{\cos(2x+2)}{2} - \left[\frac{1-\cos(2x+2)}{2}\right]$$

$$= \frac{(\cos 2) - 1}{2} = -\sin^2 1$$



By the graph y lies in III and IV quadrant.

91 **(a)** From figure equation of bisectors of lines, $xy = 0$ are $y = \pm x$



$\therefore$ Put $y = \pm x$ in the given equation

$my^2 + (1 - m^2)xy - mx^2 = 0$

$\therefore$ $mx^2 \pm (1 - m^2)x^2 - mx^2 = 0$

$\Rightarrow 1 - m^2 = 0 \Rightarrow m = \pm 1$

92. **(a)** $3x + 4y = 0$ is one of the line of the pair equations. of lines

$6x^2 - xy + 4cy^2 = 0$, Put $y = -\frac{3}{4}x$,

we get, $6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0$

$\Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$

93. **(c)** Let the lines be $y = m_1x$ and $y = m_2x$ then

$m_1 + m_2 = -\frac{2c}{7}$ and $m_1m_2 = -\frac{1}{7}$

Given that $m_1 + m_2 = 4\ m_1m_2$

$\Rightarrow -\frac{2c}{7} = -\frac{4}{7} \Rightarrow c = 2$

94. **(a)** Equation of bisectors of second pair of straight lines is,

$qx^2 + 2xy - qy^2 = 0$ ....(i)

It must be identical to the first pair

$x^2 - 2pxy - y^2 = 0$ ....(ii)

from (i) and (ii)

$\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$.

95. **(a)** We know that pair of straighty lines

$ax^2 + 2hxy + by^2 = 0$ are perpendicular when $a + b = 0$

$3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$.;

$\Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

96. **(a)** Put $x = 0$ in the given equation

$\Rightarrow by^2 + 2fy + c = 0.$

For unique point of intersection, $f^2 - bc = 0$

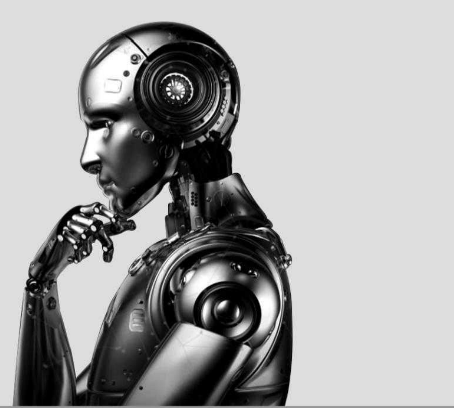
$\Rightarrow af^2 - abc = 0.$

We know that for pair of straight line

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow 2fgh - bg^2 - ch^2 = 0$

11



Conic Sections

TOPIC 1 Circles

1. If the length of the chord of the circle, $x^2 + y^2 = r^2 (r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to : **[Sep. 05, 2020 (II)]**

   (a) $\frac{9}{5}$ (b) 12 (c) $\frac{24}{5}$ (d) $\frac{12}{5}$

2. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point: **[Sep. 04, 2020 (II)]**

   (a) $(-1, 3)$ (b) $(-3, 6)$ (c) $(-3, 1)$ (d) $(1, -3)$

3. Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ______________. **[NA Sep. 04, 2020 (II)]**

4. The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is __________. **[NA Sep. 03, 2020 (I)]**

5. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ________. **[NA Sep. 02, 2020 (I)]**

6. A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle? **[Jan. 9, 2020 (I)]**

   (a) $4x - 3y + 17 = 0$ (b) $3x - 4y - 24 = 0$
   (c) $3x + 4y - 6 = 0$ (d) $4x + 3y - 8 = 0$

7. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ______. **[NA Jan. 9, 2020 (II)]**

8. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then: **[Jan. 8, 2020 (II)]**

   (a) $c^2 - 7c + 6 = 0$ (b) $c^2 + 7c + 6 = 0$
   (c) $c^2 + 6c + 7 = 0$ (d) $c^2 - 6c + 7 = 0$

9. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to: **[Jan. 7, 2020 (II)]**

   (a) $\frac{52}{5}$ (b) $\frac{56}{5}$ (c) $\frac{64}{5}$ (d) $\frac{32}{5}$

10. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is : **[April 12, 2019 (I)]**

    (a) $\frac{13}{5}$ (b) $\frac{120}{13}$ (c) $\frac{60}{13}$ (d) $\frac{13}{2}$

11. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point : **[April 12, 2019 (II)]**

    (a) (3, 10) (b) (3, 5)
    (c) (2, 3) (d) (1, 5)

12. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in \mathbf{R})$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for: **[April 10, 2019 (I)]**

    (a) infinitely many values of K
    (b) no value of K.
    (c) exactly two values of K
    (d) exactly one value of K

13. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is: **[April 10, 2019 (I)]**

    (a) 3 (b) $2\sqrt{2}$ (c) 2 (d) $3\sqrt{2}$

14. The locus of the centres of the circles, which touch the circle, $x^2+y^2=1$ externally, also touch the y-axis and lie in the first quadrant, is: **[April 10, 2019 (II)]**

(a) $x=\sqrt{1+4y}, y\geq 0$ (b) $y=\sqrt{1+2x}, x\geq 0$

(c) $y=\sqrt{1+4x}, x\geq 0$ (d) $x=\sqrt{1+2y}, y\geq 0$

15. All the points in the set $S=\left\{\frac{\alpha+i}{\alpha-1}: \propto\in R\right\}(i=\sqrt{-1})$ lie on a: **[April 09, 2019 (I)]**

(a) straight line whose slope is 1.

(b) circle whose radius is 1.

(c) circle whose radius is $\sqrt{2}$.

(d) straight line whose slope is –1.

16. If a tangent to the circle $x^2+y^2=1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is: **[April 09, 2019 (I)]**

(a) $x^2+y^2-4x^2y^2=0$ (b) $x^2+y^2-2xy=0$

(c) $x^2+y^2-16x^2y^2=0$ (d) $x^2+y^2-2x^2y^2=0$

17. The common tangent to the circles $x^2+y^2=4$ and $x^2+y^2+6x+8y-24=0$ also passes through the point: **[April 09, 2019 (II)]**

(a) $(4,-2)$ (b) $(-6,4)$ (c) $(6,-2)$ (d) $(-4,6)$

18. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2+y^2=16$, by the lines, $x+y=n$, $n\in N$, where N is the set of all natural numbers, is : **[April 08, 2019 (I)]**

(a) 320 (b) 105 (c) 160 (d) 210

19. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is : **[Jan. 12, 2019 (II)]**

(a) $\left(x^2+y^2\right)^2=4R^2x^2y^2$

(b) $\left(x^2+y^2\right)^3=4R^2x^2y^2$

(c) $\left(x^2+y^2\right)^2=4Rx^2y^2$

(d) $\left(x^2+y^2\right)(x+y)=R^2xy$

20. Let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2=0$ and $x^2+y^2-6x-6y+14=0$ respectively. If P and Q are the points of intersection of these circles then, the area (in sq. units) of the quadrilateral PC_1QC_2 is : **[Jan. 12, 2019 (I)]**

(a) 8 (b) 6 (c) 9 (d) 4

21. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ are on its opposite sides, then the set of all values of λ is the interval : **[Jan. 12, 2019 (I)]**

(a) $(2,17)$ (b) $[13,23]$

(c) $[12,21]$ (d) $(23,31)$

22. A square is inscribed in the circle $x^2+y^2-6x+8y-103=0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is : **[Jan. 11, 2019 (I)]**

(a) 6 (b) $\sqrt{137}$ (c) $\sqrt{41}$ (d) 13

23. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is : **[Jan. 11, 2019 (I)]**

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $\sqrt{2}$

24. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is : **[Jan. 11, 2019 (II)]**

(a) a hyperbola (b) an ellipse

(c) a straight line (d) a parabola

25. If a circle C passing through the point (4, 0) touches the circle $x^2+y^2+4x-6y=12$ externally at the point $(1,-1)$, then the radius of C is: **[Jan 10, 2019 (I)]**

(a) $2\sqrt{5}$ (b) 4 (c) 5 (d) $\sqrt{57}$

26. If the area of an equilateral triangle inscribed in the circle, $x^2+y^2+10x+12y+c=0$ is $27\sqrt{3}$ sq. units then c is equal to: **[Jan. 10, 2019 (II)]**

(a) 13 (b) 20 (c) -25 (d) 25

27. Three circles of radii a, b, c ($a<b<c$) touch each other externally. If they have x-axis as a common tangent, then: **[Jan 09, 2019 (I)]**

(a) $\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}$ (b) $\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}$

(c) a, b, c are in A.P (d) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

28. If the circles $x^2+y^2-16x-20y+164=r^2$ and $(x-4)^2+(y-7)^2=36$ intersect at two distinct points, then: **[Jan. 09, 2019 (II)]**

(a) $r>11$ (b) $0<r<1$

(c) $r=11$ (d) $1<r<11$

29. The straight line $x+2y=1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is : **[Jan. 11, 2019 (I)]**

(a) $\frac{\sqrt{5}}{2}$ (b) $2\sqrt{5}$ (c) $\frac{\sqrt{5}}{4}$ (d) $4\sqrt{5}$

30. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is : **[2018]**

(a) 185 (b) 85 (c) 95 (d) 195

31. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle c, on the x-axis is equal to **[Online April 16, 2018]**

(a) $\sqrt{5}$ (b) $2\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{5}$

32. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to **[Online April 15, 2018]**

(a) $\sqrt{5}$ (b) 1 (c) $\sqrt{2}$ (d) 2

33. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. if the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is? **[Online April 15, 2018]**

(a) $3(x+y)+4=0$ (b) $8(2x+y)+3=0$

(c) $4(x+y)+3=0$ (d) $x+2y+3=0$

34. The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose centre is (3, – 2). The radius of C_2 is **[Online April 15, 2018]**

(a) $\sqrt{6}$ (b) 2 (c) $\sqrt{2}$ (d) 3

35. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is : **[2017]**

(a) $4(\sqrt{2}+1)$ (b) $2(\sqrt{2}+1)$

(c) $2(\sqrt{2}-1)$ (d) $4(\sqrt{2}-1)$

36. The equation

$\text{Im}\left(\frac{iz-2}{z-i}\right)+1=0, z \in C, z \neq i$ represents a part of a circle having radius equal to : **[Online April 9, 2017]**

(a) 2 (b) 1 (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

37. A line drawn through the point P(4, 7) cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then PA·PB is equal to : **[Online April 9, 2017]**

(a) 53 (b) 56 (c) 74 (d) 65

38. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is : **[Online April 9, 2017]**

(a) 12.5 (b) 13.2 (c) 12 (d) 13

39. Let $z \in C$, the set of complex numbers. Then the equation, $2|z+3i| - |z-i| = 0$ represents : **[Online April 8, 2017]**

(a) a circle with radius $\frac{8}{3}$.

(b) a circle with diameter $\frac{10}{3}$.

(c) an ellipse with length of major axis $\frac{16}{3}$.

(d) an ellipse with length of minor axis $\frac{16}{9}$

40. If a point P has co–ordinates (0, –2) and Q is any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is : **[Online April 8, 2017]**

(a) $\frac{25+\sqrt{6}}{2}$ (b) $14+5\sqrt{3}$

(c) $\frac{47+10\sqrt{6}}{2}$ (d) $8+5\sqrt{3}$

41. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is : **[Online April 8, 2017]**

(a) $\frac{4}{\sqrt{7}}$ (b) $\frac{8}{\sqrt{7}}$ (c) $\frac{8}{7}$ (d) $\frac{16}{7}$

42. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is: **[2016]**

(a) 5 (b) 10 (c) $5\sqrt{2}$ (d) $5\sqrt{3}$

43. Equation of the tangent to the circle, at the point (1, –1) whose centre is the point of intersection of the straight lines $x - y = 1$ and $2x + y = 3$ is : **[Online April 10, 2016]**

(a) $x + 4y + 3 = 0$ (b) $3x - y - 4 = 0$

(c) $x - 3y - 4 = 0$ (d) $4x + y - 3 = 0$

44. A circle passes through (–2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle ? **[Online April 9, 2016]**

(a) $2x - 3y + 10 = 0$ (b) $3x + 4y - 3 = 0$

(c) $4x + 5y - 6 = 0$ (d) $5x + 2y + 4 = 0$

45. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0, k \in \mathbf{R}$, is a : **[2015]**

(a) circle of radius $\sqrt{2}$.

(b) circle of radius $\sqrt{3}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis

46. The number of common tangents to the circles $x^2 + y^2 - 4x - 6x - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is: **[2015]**

(a) 3 (b) 4 (c) 1 (d) 2

47. If the incentre of an equilateral triangle is (1, 1) and the equation of its one side is $3x + 4y + 3 = 0$, then the equation of the circumcircle of this triangle is : **[Online April 11, 2015]**

(a) $x^2 + y^2 - 2x - 2y - 14 = 0$
(b) $x^2 + y^2 - 2x - 2y - 2 = 0$
(c) $x^2 + y^2 - 2x - 2y + 2 = 0$
(d) $x^2 + y^2 - 2x - 2y - 7 = 0$

48. If a circle passing through the point (–1, 0) touches y-axis at (0, 2), then the length of the chord of the circle along the x-axis is : **[Online April 11, 2015]**

(a) $\frac{3}{2}$ (b) 3 (c) $\frac{5}{2}$ (d) 5

49. Let the tangents drawn to the circle, $x^2 + y^2 = 16$ from the point P(0, h) meet the x-axis at point A and B. If the area of ΔAPB is minimum, then h is equal to : **[Online April 10, 2015]**

(a) $4\sqrt{2}$ (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $4\sqrt{3}$

50. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is : **[Online April 10, 2015]**

(a) $x^2+y^2+3x+9y=0$ (b) $x^2+y^2+3x-9y=0$
(c) $x^2+y^2-3x-9y=0$ (d) $x^2+y^2-3x+9y=0$

51. The largest value of r for which the region represented by the set $\{\omega \in C | |\omega - 4 - i| \le r\}$ is contained in the region represented by the set $(z \in c / |z-1| \le |z+i|)$, is equal to: **[Online April 10, 2015]**

(a) $\frac{5}{2}\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{3}{2}\sqrt{2}$ (d) $\sqrt{17}$

52. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to **[2014]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

53. The equation of circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter is: **[Online April 19, 2014]**

(a) $x^2+y^2+3x+y-11=0$
(b) $x^2+y^2+3x+y+1=0$
(c) $x^2+y^2+3x+y-2=0$
(d) $x^2+y^2+3x+y-22=0$

54. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$, there is/are **[Online April 12, 2014]**

(a) one pair of common tangents
(b) two pair of common tangents
(c) three pair of common tangents
(d) no common tangent

55. The set of all real values of λ for which exactly two common tangents can be drawn to the circles
$x^2 + y^2 - 4x - 4y + 6 = 0$ and
$x^2 + y^2 - 10x - 10y + \lambda = 0$ is the interval: **[Online April 11, 2014]**

(a) (12, 32) (b) (18, 42)
(c) (12, 24) (d) (18, 48)

56. If the point (1, 4) lies inside the circle $x^2 + y^2 - 6x - 10y + P = 0$ and the circle does not touch or intersect the coordinate axes, then the set of all possible values of P is the interval: **[Online April 9, 2014]**

(a) (0, 25) (b) (25, 39)
(c) (9, 25) (d) (25, 29)

57. Let a and b be any two numbers satisfying $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$.

Then, the foot of perpendicular from the origin on the variable line, $\frac{x}{a}+\frac{y}{b}=1$, lies on: **[Online April 9, 2014]**

(a) a hyperbola with each semi-axis = $\sqrt{2}$
(b) a hyperbola with each semi-axis = 2
(c) a circle of radius = 2
(d) a circle of radius = $\sqrt{2}$

58. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[2013]**

(a) (–5, 2) (b) (2, –5) (c) (5, –2) (d) (–2, 5)

59. If a circle of unit radius is divided into two parts by an arc of another circle subtending an angle 60° on the circumference of the first circle, then the radius of the arc is: **[Online April 25, 2013]**

(a) $\sqrt{3}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

60. **Statement 1**: The only circle having radius $\sqrt{10}$ and a diameter along line $2x + y = 5$ is $x^2 + y^2 - 6x + 2y = 0$.

Statement 2 : $2x + y = 5$ is a normal to the circle $x^2 + y^2 - 6x + 2y = 0$. **[Online April 25, 2013]**

(a) Statement 1 is false; Statement 2 is true.
(b) Statement 1 is true; Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is true; Statement 2 is false.
(d) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

61. If the circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x, then a equals. **[Online April 23, 2013]**

(a) 0 (b) ±4 (c) ±2 (d) ±3

62. If a circle C passing through (4, 0) touches the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ externally at a point (1, –1), then the radius of the circle C is : **[Online April 22, 2013]**

(a) 5 (b) $2\sqrt{5}$ (c) 4 (d) $\sqrt{57}$

63. If two vertices of an equilateral triangle are A $(-a, 0)$ and $B(a, 0)$, $a > 0$, and the third vertex C lies above x-axis then the equation of the circumcircle of ΔABC is : **[Online April 22, 2013]**

(a) $3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$

(b) $3x^2 + 3y^2 - 2ay = 3a^2$

(c) $x^2 + y^2 - 2ay = a^2$

(d) $x^2 + y^2 - \sqrt{3}ay = a^2$

64. If each of the lines $5x + 8y = 13$ and $4x - y = 3$ contains a diameter of the circle
$x^2 + y^2 - 2(a^2 - 7a + 11)\, x - 2(a^2 - 6a + 6)\, y + b^3 + 1 = 0$, then **[Online April 9, 2013]**

(a) $a = 5$ and $b \notin (-1,1)$ (b) $a = 1$ and $b \notin (-1,1)$

(c) $a = 2$ and $b \notin (-\infty,1)$ (d) $a = 5$ and $b \in (-\infty,1)$

65. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is: **[2012]**

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$ (c) $\frac{6}{5}$ (d) $\frac{5}{3}$

66. The number of common tangents of the circles given by $x^2 + y^2 - 8x - 2y + 1 = 0$ and $x^2 + y^2 + 6x + 8y = 0$ is **[Online May 26, 2012]**

(a) one (b) four (c) two (d) three

67. If the line $y = mx + 1$ meets the circle $x^2 + y^2 + 3x = 0$ in two points equidistant from and on opposite sides of x-axis, then **[Online May 19, 2012]**

(a) $3m + 2 = 0$ (b) $3m - 2 = 0$

(c) $2m + 3 = 0$ (d) $2m - 3 = 0$

68. If three distinct points A, B, C are given in the 2-dimensional coordinate plane such that the ratio of the distance of each one of them from the point (1, 0) to the distance from $(-1, 0)$ is equal to $\frac{1}{2}$, then the circumcentre of the triangle ABC is at the point **[Online May 19, 2012]**

(a) $\left(\frac{5}{3}, 0\right)$ (b) $(0, 0)$

(c) $\left(\frac{1}{3}, 0\right)$ (d) $(3, 0)$

69. The equation of the circle passing through the point (1, 2) and through the points of intersection of $x^2 + y^2 - 4x - 6y - 21 = 0$ and $3x + 4y + 5 = 0$ is given by **[Online May 7, 2012]**

(a) $x^2 + y^2 + 2x + 2y + 11 = 0$

(b) $x^2 + y^2 - 2x + 2y - 7 = 0$

(c) $x^2 + y^2 + 2x - 2y - 3 = 0$

(d) $x^2 + y^2 + 2x + 2y - 11 = 0$

70. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is - **[2011 RS]**

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$

(b) $x^2 + y^2 - x - y = 0$

(c) $x^2 + y^2 + 2x + 2y - 7 = 0$

(d) $x^2 + y^2 + x + y - 2 = 0$

71. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other if **[2011]**

(a) $|a| = c$ (b) $a = 2c$

(c) $|a| = 2c$ (d) $2|a| = c$

72. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[2010]**

(a) $-35 < m < 15$ (b) $15 < m < 65$

(c) $35 < m < 85$ (d) $-85 < m < -35$

73. If P and Q are the points of intersection of the circles
$x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and
$x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and (1, 1) for: **[2009]**

(a) all except one value of p

(b) all except two values of p

(c) exactly one value of p

(d) all values of p

74. Three distinct points A, B and C are given in the 2-dimensional coordinates plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point $(-1, 0)$ is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point: **[2009]**

(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$

(c) $\left(\frac{5}{3}, 0\right)$ (d) $(0, 0)$

75. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is **[2008]**

(a) $(3, -4)$ (b) $(-3, 4)$

(c) $(-3, -4)$ (d) $(3, 4)$

76. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval **[2007]**

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$

(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

77. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is

(a) $x^2+y^2=\frac{3}{2}$ (b) $x^2+y^2=1$ **[2006]**

(c) $x^2+y^2=\frac{27}{4}$ (d) $x^2+y^2=\frac{9}{4}$

78. If the lines $3x-4y-7=0$ and $2x-3y-5=0$ are two diameters of a circle of area 49π square units, the equation of the circle is **[2006]**

(a) $x^2+y^2+2x-2y-47=0$

(b) $x^2+y^2+2x-2y-62=0$

(c) $x^2+y^2-2x+2y-62=0$

(d) $x^2+y^2-2x+2y-47=0$

79. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is **[2005]**

(a) $x^2+y^2-3ax-4by+(a^2+b^2-p^2)=0$

(b) $2ax+2by-(a^2-b^2+p^2)=0$

(c) $x^2+y^2-2ax-3by+(a^2-b^2-p^2)=0$

(d) $2ax+2by-(a^2+b^2+p^2)=0$

80. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then **[2005]**

(a) $3a^2-10ab+3b^2=0$

(b) $3a^2-2ab+3b^2=0$

(c) $3a^2+10ab+3b^2=0$

(d) $3a^2+2ab+3b^2=0$

81. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a=0$ passes through P and Q for **[2005]**

(a) exactly one value of a
(b) no value of a
(c) infinitely many values of a
(d) exactly two values of a

82. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is **[2004]**

(a) $2ax-2by-(a^2+b^2+4)=0$

(b) $2ax+2by-(a^2+b^2+4)=0$

(c) $2ax-2by+(a^2+b^2+4)=0$

(d) $2ax+2by+(a^2+b^2+4)=0$

83. A variable circle passes through the fixed point $A(p,q)$ and touches x-axis. The locus of the other end of the diameter through A is **[2004]**

(a) $(y-q)^2=4px$ (b) $(x-q)^2=4py$

(c) $(y-p)^2=4qx$ (d) $(x-p)^2=4qy$

84. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is **[2004]**

(a) $x^2+y^2+2x-2y-23=0$

(b) $x^2+y^2-2x-2y-23=0$

(c) $x^2+y^2+2x+2y-23=0$

(d) $x^2+y^2-2x+2y-23=0$

85. Intercept on the line $y=x$ by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is **[2004]**

(a) $x^2+y^2+x-y=0$ (b) $x^2+y^2-x+y=0$

(c) $x^2+y^2+x+y=0$ (d) $x^2+y^2-x-y=0$

86. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct point, then **[2003]**

(a) $r>2$ (b) $2<r<8$
(c) $r<2$ (d) $r=2$.

87. The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle having area as 154 sq.units.Then the equation of the circle is **[2003]**

(a) $x^2+y^2-2x+2y=62$

(b) $x^2+y^2+2x-2y=62$

(c) $x^2+y^2+2x-2y=47$

(d) $x^2+y^2-2x+2y=47$.

88. If the chord $y = mx + 1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45° at the major segment of the circle then value of m is **[2002]**

(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$

(c) $-1 \pm \sqrt{2}$ (d) none of these

89. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is **[2002]**

(a) $4 \le x^2+y^2 \le 64$ (b) $x^2+y^2 \le 25$

(c) $x^2+y^2 \ge 25$ (d) $3 \le x^2+y^2 \le 9$

90. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$ is **[2002]**

(a) $\left(\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},-\sqrt{2}\right)$ (c) $\left(\frac{3}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\frac{3}{2}\right)$

91. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is **[2002]**

(a) $x^2+y^2=9a^2$ (b) $x^2+y^2=16a^2$

(c) $x^2+y^2=4a^2$ (d) $x^2+y^2=a^2$

TOPIC 2 Parabola

92. Let L_1 be a tangent to the parabola $y^2 = 4(x+1)$ and L_2 be a tangent to the parabola $y^2 = 8(x+2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line : **[Sep. 06, 2020 (I)]**

(a) $x+3=0$ (b) $2x+1=0$

(c) $x+2=0$ (d) $x+2y=0$

93. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2,4) is: **[Sep. 06, 2020 (II)]**

(a) $\left(\frac{-53}{10},\frac{16}{5}\right)$ (b) $\left(\frac{6}{5},\frac{53}{10}\right)$

(c) $\left(\frac{3}{10},\frac{16}{5}\right)$ (d) $\left(\frac{-16}{5},\frac{53}{10}\right)$

94. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2+y^2=c^2$, then c is equal to: **[Sep. 05, 2020 (I)]**

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

95. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then : **[Sep. 03, 2020 (I)]**

(a) $PN=4$ (b) $MQ=\frac{1}{3}$

(c) $MQ=\frac{1}{4}$ (d) $PN=3$

96. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is : **[Sep. 03, 2020 (II)]**

(a) $8\sqrt{5}$ (b) 8 (c) 12 (d) $4\sqrt{5}$

97. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is : **[Sep. 02, 2020 (II)]**

(a) $64\sqrt{3}$ (b) $256\sqrt{3}$

(c) $192\sqrt{3}$ (d) $128\sqrt{3}$

98. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{\sqrt{2}},-2\right)$, then the equation of the tangent to it at B is: **[Jan. 9, 2020 (II)]**

(a) $2x+y-24=0$ (b) $x-2y+8=0$

(c) $x+2y+8=0$ (d) $2x-y-24=0$

99. The locus of a point which divides the line segment joining the point (0, –1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is: **[Jan. 8, 2020 (I)]**

(a) $9x^2-12y=8$ (b) $9x^2-3y=2$

(c) $x^2-3y=2$ (d) $4x^2-3y=2$

100. Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q, If area $(\Delta OPQ) = 4$ sq. units, then m is equal to _______. **[NA Jan. 8, 2020 (II)]**

101. If $y = mx + 4$ is a tangent to both the parabolas, $y^2=4x$ and $x^2=2by$, then b is equal to: **[Jan. 7, 2020 (I)]**

(a) –32 (b) –64 (c) –128 (d) 128

102. The tangents to the curve $y = (x-2)^2-1$ at its points of intersection with the line $x - y = 3$, intersect at the point : **[April 12, 2019 (II)]**

(a) $\left(\frac{5}{2},1\right)$ (b) $\left(-\frac{5}{2},-1\right)$ (c) $\left(\frac{5}{2},-1\right)$ (d) $\left(-\frac{5}{2},1\right)$

103. If the line $ax + y = c$, touches both the curves $x^2+y^2=1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to **[April 10, 2019 (II)]**

(a) 2 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$

104. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is: **[April 09, 2019 (II)]**

(a) $8\pi(2-\sqrt{2})$ (b) $4\pi(2-\sqrt{2})$

(c) $4\pi(3+\sqrt{2})$ (d) $8\pi(3-2\sqrt{2})$

105. If one end of a focal chord of the parabola, $y^2 = 16x$ is at $(1, 4)$, then the length of this focal chord is: **[April 09, 2019 (I)]**

(a) 25 (b) 22 (c) 24 (d) 20

106. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is : **[April 08, 2019 (I)]**

(a) 2 (b) $\frac{7}{8}$ (c) $\frac{7}{4\sqrt{2}}$ (d) $\frac{11}{4\sqrt{2}}$

107. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (a, b) are perpendicular to each other, then a^2 is equal to : **[April 08, 2019 (I)]**

(a) $\frac{128}{17}$ (b) $\frac{64}{17}$ (c) $\frac{4}{17}$ (d) $\frac{2}{17}$

108. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point : **[April 08, 2019 (II)]**

(a) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (b) $\left(\frac{1}{4}, \frac{3}{4}\right)$

(c) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (d) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

109. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is : **[Jan. 12, 2019 (II)]**

(a) $y = x\tan\theta + 2\cot\theta$ (b) $y = x\tan\theta - 2\cot\theta$

(c) $x = y\cot\theta + 2\tan\theta$ (d) $x = y\cot\theta - 2\tan\theta$

110. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is : **[Jan. 11, 2019 (I)]**

(a) $x + y + 1 = 0$ (b) $x - 2y + 4 = 0$

(c) $x + 2y + 4 = 0$ (d) $4x + 2y + 1 = 0$

111. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is : **[Jan. 11, 2019 (II)]**

(a) $5\sqrt{5}$ (b) $5(2^{1/3})$ (c) $(10)^{2/3}$ (d) 5

112. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c)? **[Jan 10, 2019 (I)]**

(a) $\left(\frac{1}{2}, 2, 3\right)$ (b) $(1, 1, 3)$

(c) $\left(\frac{1}{2}, 2, 0\right)$ (d) $(1, 1, 0)$

113. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is: **[Jan. 10, 2019 (II)]**

(a) $3\sqrt{2}$ (b) $2\sqrt{11}$ (c) $8\sqrt{2}$ (d) $6\sqrt{3}$

114. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it? **[Jan 09, 2019 (I)]**

(a) $(5, 2\sqrt{6})$ (b) $(8, 6)$

(c) $(6, 4\sqrt{2})$ (d) $(4, -4)$

115. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is : **[Jan 09, 2019 (I)]**

(a) $2\sqrt{3}y = 12x + 1$ (b) $\sqrt{3}y = x + 3$

(c) $2\sqrt{3}y = -x - 12$ (d) $\sqrt{3}y = 3x + 1$

116. Let A $(4, -4)$ and B $(9, 6)$ be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is: **[Jan. 09, 2019 (II)]**

(a) $31\frac{1}{4}$ (b) $30\frac{1}{2}$ (c) 32 (d) $31\frac{3}{4}$

117. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is: **[2018]**

(a) 2 (b) 3 (c) $\frac{4}{3}$ (d) $\frac{1}{2}$

118. Tangents drawn from the point $(-8, 0)$ to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to **[Online April 15, 2018]**

(a) 48 (b) 32 (c) 24 (d) 64

119. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to : **[Online April 9, 2017]**

(a) $2\sqrt{3}$ (b) $8\sqrt{3}$ (c) $10\sqrt{3}$ (d) $16\sqrt{3}$

120. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is : **[Online April 8, 2017]**

(a) $\sqrt{2} + 1$ (b) $2(3 + 2\sqrt{2})$

(c) $2(\sqrt{2} + 1)$ (d) $3 + 2\sqrt{2}$

121. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is: **[2016]**

(a) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(b) $x^2 + y^2 - 4x + 9y + 18 = 0$

(c) $x^2 + y^2 - 4x + 8y + 12 = 0$

(d) $x^2 + y^2 - x + 4y - 12 = 0$

122. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is : **[Online April 10, 2016]**

(a) 8 (b) 4 (c) 6 (d) 2

123. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then locus of P is : **[2015]**

(a) $y^2 = 2x$ (b) $x^2 = 2y$
(c) $x^2 = y$ (d) $y^2 = x$

124. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant. If R divides PQ in the ratio 2 : 1 then the locus of R is : **[Online April 11, 2015]**

(a) $3y^2 = -2x$ (b) $3y^2 = 2x$
(c) $9y^2 = 4x$ (d) $9y^2 = -4x$

125. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is **[2014]**

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

126. A chord is drawn through the focus of the parabola $y^2 = 6x$ such that its distance from the vertex of this parabola is $\frac{\sqrt{5}}{2}$, then its slope can be: **[Online April 19, 2014]**

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{3}}$

127. Two tangents are drawn from a point $(-2, -1)$ to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan \alpha|$ is equal to: **[Online April 12, 2014]**

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 3

128. Let L_1 be the length of the common chord of the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, and L_2 be the length of the latus rectum of $y^2 = 8x$, then: **[Online April 11, 2014]**

(a) $L_1 > L_2$ (b) $L_1 = L_2$
(c) $L_1 < L_2$ (d) $\frac{L_1}{L_2} = \sqrt{2}$

129. **Given :** A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-2 : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

130. The point of intersection of the normals to the parabola $y^2 = 4x$ at the ends of its latus rectum is : **[Online April 23, 2013]**

(a) $(0, 2)$ (b) $(3, 0)$ (c) $(0, 3)$ (d) $(2, 0)$

131. **Statement-1:** The line $x - 2y = 2$ meets the parabola, $y^2 + 2x = 0$ only at the point $(-2, -2)$.

Statement-2: The line $y = mx - \frac{1}{2m} (m \neq 0)$ is tangent to the parabola, $y^2 = -2x$ at the point $\left(-\frac{1}{2m^2}, -\frac{1}{m}\right)$.

[Online April 22, 2013]

(a) Statement-1 is true; Statement-2 is false.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 a true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.

132. The normal at $\left(2, \frac{3}{2}\right)$ to the ellipse, $\frac{x^2}{16} + \frac{y^2}{3} = 1$ touches a parabola, whose equation is **[Online May 26, 2012]**

(a) $y^2 = -104x$ (b) $y^2 = 14x$
(c) $y^2 = 26x$ (d) $y^2 = -14x$

133. The chord PQ of the parabola $y^2 = x$, where one end P of the chord is at point $(4, -2)$, is perpendicular to the axis of the parabola. Then the slope of the normal at Q is **[Online May 26, 2012]**

(a) -4 (b) $-\frac{1}{4}$ (c) 4 (d) $\frac{1}{4}$

134. **Statement 1:** $y = mx - \frac{1}{m}$ is always a tangent to the parabola, $y^2 = -4x$ for all non-zero values of m.

Statement 2: Every tangent to the parabola, $y^2 = -4x$ will meet its axis at a point whose abscissa is non-negative. **[Online May 7, 2012]**

(a) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

135. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is **[2011]**

(a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{8}{3\sqrt{2}}$ (c) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

136. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is **[2010]**
(a) $2x+1=0$ (b) $x=-1$
(c) $2x-1=0$ (d) $x=1$

137. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at **[2008]**
(a) $(0,2)$ (b) $(1,0)$
(c) $(0,1)$ (d) $(2,0)$

138. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is **[2007]**
(a) $(2,4)$ (b) $(-2,0)$ (c) $(-1,1)$ (d) $(0,2)$

139. The locus of the vertices of the family of parabolas $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$ is **[2006]**
(a) $xy=\frac{105}{64}$ (b) $xy=\frac{3}{4}$
(c) $xy=\frac{35}{16}$ (d) $xy=\frac{64}{105}$

140. Let P be the point $(1, 0)$ and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is **[2005]**
(a) $y^2-4x+2=0$ (b) $y^2+4x+2=0$
(c) $x^2+4y+2=0$ (d) $x^2-4y+2=0$

141. A circle touches the x- axis and also touches the circle with centre at $(0,3)$ and radius 2. The locus of the centre of the circle is **[2005]**
(a) an ellipse (b) a circle
(c) a hyperbola (d) a parabola

142. If $a \neq 0$ and the line $2bx+3cy+4d=0$ passes through the points of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then **[2004]**
(a) $d^2+(3b-2c)^2=0$ (b) $d^2+(3b+2c)^2=0$
(c) $d^2+(2b-3c)^2=0$ (d) $d^2+(2b+3c)^2=0$

143. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then **[2003]**
(a) $t_2=t_1+\frac{2}{t_1}$ (b) $t_2=-t_1-\frac{2}{t_1}$
(c) $t_2=-t_1+\frac{2}{t_1}$ (d) $t_2=t_1-\frac{2}{t_1}$

144. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are **[2002]**
(a) $x=\pm(y+2a)$ (b) $y=\pm(x+2a)$
(c) $x=\pm(y+a)$ (d) $y=\pm(x+a)$

TOPIC 3 Ellipse

145. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci? **[Sep. 06, 2020 (I)]**
(a) $(-2,\sqrt{3})$ (b) $(-1,\sqrt{2})$
(c) $(-1,\sqrt{3})$ (d) $(1,2)$

146. If the normal at an end of a latus rectum of an ellipse passes through an extermity of the minor axis, then the eccentricity e of the ellipse satisfies: **[Sep. 06, 2020 (II)]**
(a) $e^4+2e^2-1=0$ (b) $e^2+e-1=0$
(c) $e^4+e^2-1=0$ (d) $e^2+2e-1=0$

147. If the co-ordinates of two points A and B are $(\sqrt{7},0)$ and $(-\sqrt{7},0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to : **[Sep. 05, 2020 (I)]**
(a) 16 (b) 8 (c) 6 (d) 9

148. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, – 4), then PQ^2 is equals to : **[Sep. 05, 2020 (I)]**
(a) 36 (b) 48 (c) 21 (d) 29

149. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t)=\frac{5}{12}+t-t^2$, then $a^2 + b^2$ is equal to: **[Sep. 04, 2020 (I)]**
(a) 145 (b) 116 (c) 126 (d) 135

150. Let $x = 4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta), \beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is : **[Sep. 04, 2020 (II)]**
(a) $4x-3y=2$ (b) $8x-2y=5$
(c) $7x-4y=1$ (d) $4x-2y=1$

151. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does **not** pass through which of the following points? **[Sep. 03, 2020 (I)]**
(a) $\left(\frac{1}{\sqrt{2}},0\right)$ (b) $\left(-\sqrt{\frac{3}{2}},1\right)$
(c) $\left(1,-\frac{1}{\sqrt{2}}\right)$ (d) $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$

152. Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is : **[Sep. 02, 2020 (I)]**

(a) $6(\pi-2)$ (b) $3(\pi-2)$
(c) $3(4-\pi)$ (d) $6(4-\pi)$

153. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola, $\frac{x^2}{9}-\frac{y^2}{4}=1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2+3y^2=k$, then k is equal to **[Jan. 9, 2020 (I)]**

(a) 16 (b) 17 (c) 15 (d) 14

154. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x+6y=8$; then its eccentricity is: **[Jan. 9, 2020 (II)]**

(a) $\frac{1}{2}\sqrt{\frac{11}{3}}$ (b) $\sqrt{\frac{5}{6}}$ (c) $\frac{1}{2}\sqrt{\frac{5}{3}}$ (d) $\frac{1}{3}\sqrt{\frac{11}{3}}$

155. Let the line $y=mx$ and the ellipse $2x^2+y^2=1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}},0\right)$ and $(0, \beta)$, then β is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\frac{\sqrt{2}}{3}$

156. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is: **[Jan. 7, 2020 (I)]**

(a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\frac{3}{\sqrt{2}}$ (d) $2\sqrt{3}$

157. If $3x+4y=12\sqrt{2}$ is *a* tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some $a \in R$, then the distance between the foci of the ellipse is: **[Jan. 7, 2020 (II)]**

(a) $2\sqrt{7}$ (b) 4 (c) $2\sqrt{5}$ (d) $2\sqrt{2}$

158. If the normal to the ellipse $3x^2+4y^2=12$ at a point P on it is parallel to the line, $2x+y=4$ and the tangent to the ellipse at P passes through Q (4,4) then PQ is equal to : **[April 12, 2019 (I)]**

(a) $\frac{5\sqrt{5}}{2}$ (b) $\frac{\sqrt{61}}{2}$ (c) $\frac{\sqrt{221}}{2}$ (d) $\frac{\sqrt{157}}{2}$

159. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points ? **[April 12, 2019 (II)]**

(a) $(\sqrt{2},2)$ (b) $(2,\sqrt{2})$
(c) $(2,2\sqrt{2})$ (d) $(1,2\sqrt{2})$

160. If the line $x-2y=12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,\frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is : **[April 10, 2019 (I)]**

(a) 9 (b) $12\sqrt{2}$ (c) 5 (d) $8\sqrt{3}$

161. The tangent and normal to the ellipse $3x^2+5y^2=32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is : **[April 10, 2019 (II)]**

(a) $\frac{34}{15}$ (b) $\frac{14}{3}$ (c) $\frac{16}{3}$ (d) $\frac{68}{15}$

162. If the tangent to the parabola $y^2=x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2+2y^2=1$, then α is equal to: **[April 09, 2019 (II)]**

(a) $\sqrt{2}-1$ (b) $2\sqrt{2}-1$
(c) $2\sqrt{2}+1$ (d) $\sqrt{2}+1$

163. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is: **[April 08, 2019 (II)]**

(a) 10 (b) 5 (c) 8 (d) 6

164. Let S and S′ be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS)=8$ sq. units, hen the length of a latus rectum of the ellipse is : **[Jan. 12, 2019 (II)]**

(a) 4 (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 2

165. If tangents are drawn to the ellipse $x^2+2y^2=2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve : **[Jan. 11, 2019 (I)]**

(a) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (b) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(c) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (d) $\frac{x^2}{2}+\frac{y^2}{4}=1$

166. Two sets A and B are as under :

$A = \{(a, b) \in R \times R : |a-5| < 1 \text{ and } |b-5| < 1\};$

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \le 36\}$. Then : **[2018]**

(a) $A \subset B$

(b) $A \cap B = \phi$ (an empty set)

(c) neither $A \subset B$ nor $B \subset A$

(d) $B \subset A$

167. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is? **[Online April 16, 2018]**

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

168. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, –1) and (–2, 2) is : **[Online April 9, 2017]**

(a) $\frac{1}{2}$ (b) $\frac{2}{\sqrt{5}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$

169. Consider an ellipse, whose centre is at the origin and its major axis is along the x–axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is : **[Online April 8, 2017]**

(a) 8 (b) 32 (c) 80 (d) 40

170. If the tangent at a point on the ellipse $\frac{x^2}{27} + \frac{y^2}{3} = 1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is : **[Online April 9, 2016]**

(a) $3\sqrt{3}$ (b) $\frac{9}{2}$ (c) 9 (d) $\frac{9}{\sqrt{3}}$

171. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is : **[2015]**

(a) $\frac{27}{2}$ (b) 27 (c) $\frac{27}{4}$ (d) 18

172. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is: **[Online April 11, 2015]**

(a) $\frac{2\sqrt{2}-1}{2}$ (b) $\sqrt{2}-1$

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{2}-1}{2}$

173. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is **[2014]**

(a) $\left(x^2+y^2\right)^2 = 6x^2 + 2y^2$

(b) $\left(x^2+y^2\right)^2 = 6x^2 - 2y^2$

(c) $\left(x^2-y^2\right)^2 = 6x^2 + 2y^2$

(d) $\left(x^2-y^2\right)^2 = 6x^2 - 2y^2$

174. A stair-case of length l rests against a vertical wall and a floor of a room. Let P be a point on the stair-case, nearer to its end on the wall, that divides its length in the ratio 1 : 2. If the stair-case begins to slide on the floor, then the locus of P is: **[Online April 11, 2014]**

(a) an ellipse of eccentricity $\frac{1}{2}$

(b) an ellipse of eccentricity $\frac{\sqrt{3}}{2}$

(c) a circle of radius $\frac{1}{2}$

(d) a circle of radius $\frac{\sqrt{3}}{2}l$

175. If OB is the semi-minor axis of an ellipse, F_1 and F_2 are its foci and the angle between F_1B and F_2B is a right angle, then the square of the eccentricity of the ellipse is: **[Online April 9, 2014]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4}$

176. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is **[2013]**

(a) $x^2 + y^2 - 6y - 7 = 0$

(b) $x^2 + y^2 - 6y + 7 = 0$

(c) $x^2 + y^2 - 6y - 5 = 0$

(d) $x^2 + y^2 - 6y + 5 = 0$

177. A point on the ellipse, $4x^2+9y^2=36$, where the normal is parallel to the line, $4x-2y-5=0$, is : **[Online April 25, 2013]**

(a) $\left(\frac{9}{5},\frac{8}{5}\right)$ (b) $\left(\frac{8}{5},-\frac{9}{5}\right)$

(c) $\left(-\frac{9}{5},\frac{8}{5}\right)$ (d) $\left(\frac{8}{5},\frac{9}{5}\right)$

178. Let the equations of two ellipses be

$E_1:\frac{x^2}{3}+\frac{y^2}{2}=1$ and $E_2:\frac{x^2}{16}+\frac{y^2}{b^2}=1$,

If the product of their eccentricities is $\frac{1}{2}$, then the length of the minor axis of ellipse E_2 is : **[Online April 22, 2013]**

(a) 8 (b) 9 (c) 4 (d) 2

179. Equation of the line passing through the points of intersection of the parabola $x^2=8y$ and the ellipse $\frac{x^2}{3}+y^2=1$ is : **[Online April 9, 2013]**

(a) $y-3=0$ (b) $y+3=0$
(c) $3y+1=0$ (d) $3y-1=0$

180. If P_1 and P_2 are two points on the ellipse $\frac{x^2}{4}+y^2=1$ at which the tangents are parallel to the chord joining the points (0, 1) and (2, 0), then the distance between P_1 and P_2 is **[Online May 12, 2012]**

(a) $2\sqrt{2}$ (b) $\sqrt{5}$ (c) $2\sqrt{3}$ (d) $\sqrt{10}$

181. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is **[2011]**

(a) $5x^2+3y^2-48=0$ (b) $3x^2+5y^2-15=0$
(c) $5x^2+3y^2-32=0$ (d) $3x^2+5y^2-32=0$

182. The ellipse $x^2+4y^2=4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is : **[2009]**

(a) $x^2+12y^2=16$ (b) $4x^2+48y^2=48$

(c) $4x^2+64y^2=48$ (d) $x^2+16y^2=16$

183. A focus of an ellipse is at the origin. The directrix is the line $x=4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is **[2008]**

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

184. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is **[2006]**

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

185. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is **[2005]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

186. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x=4$, then the equation of the ellipse is: **[2004]**

(a) $4x^2+3y^2=1$ (b) $3x^2+4y^2=12$

(c) $4x^2+3y^2=12$ (d) $3x^2+4y^2=1$

TOPIC 4 Hyperbola

187. If the line $y=mx+c$ is a common tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{64}=1$ and the circle $x^2+y^2=36$, then which one of the following is true? **[Sep. 05, 2020 (II)]**

(a) $c^2=369$ (b) $5m=4$
(c) $4c^2=369$ (d) $8m+5=0$

188. Let $P(3,3)$ be a point on the hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to : **[Sep. 04, 2020 (I)]**

(a) $\left(\frac{9}{2},3\right)$ (b) $\left(\frac{3}{2},2\right)$ (c) $\left(\frac{9}{2},2\right)$ (d) $(9,3)$

189. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1\ (b<5)$ and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2=1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α,β) is equal to : **[Sep. 03, 2020 (II)]**

(a) $(8,12)$ (b) $\left(\frac{20}{3},12\right)$

(c) $\left(\frac{24}{5},10\right)$ (d) $(8,10)$

190. A line parallel to the straight line $2x-y=0$ is tangent to the hyperbola $\frac{x^2}{4}-\frac{y^2}{2}=1$ at the point (x_1, y_1). Then $x_1^2+5y_1^2$ is equal to : **[Sep. 02, 2020 (I)]**

(a) 6 (b) 8 (c) 10 (d) 5

191. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2-y^2\sec^2\theta=10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta+y^2=5$, then the length of the latus rectum of the ellipse, is : **[Sep. 02, 2020 (II)]**

(a) $2\sqrt{6}$ (b) $\sqrt{30}$

(c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{4\sqrt{5}}{3}$

192. If a hyperbola passes through the point $P(10,16)$ and it has vertices at $(\pm 6,0)$, then the equation of the normal to it at P is: **[Jan. 8, 2020 (II)]**

(a) $3x+4y=94$ (b) $2x+5y=100$

(c) $x+2y=42$ (d) $x+3y=58$

193. Let P be the point of intersection of the common tangents to the parabola $y^2=12x$ and hyperbola $8x^2-y^2=8$. If S and S′ denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS′ in a ratio : **[April 12, 2019 (I)]**

(a) 13 : 11 (b) 14 : 13 (c) 5 : 4 (d) 2 : 1

194. The equation of a common tangent to the curves, $y^2=16x$ and $xy=-4$, is : **[April 12, 2019 (II)]**

(a) $x-y+4=0$ (b) $x+y+4=0$

(c) $x-2y+16=0$ (d) $2x-y+2=0$

195. If a directrix of a hyperbola centred at the origin and passing through the point $(4,-2\sqrt{3})$ is $5x=4\sqrt{5}$ and its eccentricity is e, then : **[April 10, 2019 (I)]**

(a) $4e^4-24e^2+27=0$ (b) $4e^4-12e^2-27=0$

(c) $4e^4-24e^2+35=0$ (d) $4e^4+8e^2-35=0$

196. If $5x+9=0$ is the directrix of the hyperbola $16x^2-9y^2=144$, then its corresponding focus is : **[April 10, 2019 (II)]**

(a) $(5,0)$ (b) $\left(-\frac{5}{3},0\right)$ (c) $\left(\frac{5}{3},0\right)$ (d) $(-5,0)$

197. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$, then a value of m is : **[April 09, 2019 (I)]**

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{15}}{2}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{3}{\sqrt{5}}$

198. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is : **[April. 08, 2019 (II)]**

(a) $x-2y+8=0$ (b) $2x-3y+10=0$

(c) $2x-y-2=0$ (d) $3x-2y=0$

199. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola? **[Jan. 12, 2019 (I)]**

(a) $\left(-6,2\sqrt{10}\right)$ (b) $\left(2\sqrt{6},5\right)$

(c) $\left(4,\sqrt{15}\right)$ (d) $\left(6,5\sqrt{2}\right)$

200. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is : **[Jan. 11, 2019 (II)]**

(a) $\frac{13}{12}$ (b) 2 (c) $\frac{13}{6}$ (d) $\frac{13}{8}$

201. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it? **[Jan. 11, 2019 (II)]**

(a) $\left(4\sqrt{2},2\sqrt{2}\right)$ (b) $\left(4\sqrt{3},2\sqrt{2}\right)$

(c) $\left(4\sqrt{3},2\sqrt{3}\right)$ (d) $\left(4\sqrt{2},2\sqrt{3}\right)$

202. The equation of a tangent to the hyperbola $4x^2-5y^2=20$ parallel to the line $x-y=2$ is: **[Jan 10, 2019 (I)]**

(a) $x-y+1=0$ (b) $x-y+7=0$

(c) $x-y+9=0$ (d) $x-y-3=0$

203. Let

$$S=\left\{(x, y)\in \mathbf{R}^2 : \frac{y^2}{1+r}-\frac{x^2}{1-r}=1\right\},$$

where $r \neq \pm 1$ Then S represents: **[Jan. 10, 2019 (II)]**

(a) a hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0<r<1$

(b) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r>1$

(c) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0<r<1$

(d) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r>1$

204. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval: **[Jan 09, 2019 (I)]**

(a) $(3, \infty)$ (b) $(3/2, 2]$ (c) $(2, 3]$ (d) $(1, 3/2]$

205. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is : **[Jan. 09, 2019 (II)]**

(a) $\frac{3}{2}$ (b) $\sqrt{3}$ (c) 2 (d) $\frac{2}{\sqrt{3}}$

206. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is : **[2018]**

(a) $54\sqrt{3}$ (b) $60\sqrt{3}$ (c) $36\sqrt{5}$ (d) $45\sqrt{5}$

207. The locus of the point of intersection of the lines, $\sqrt{2}x - y + 4\sqrt{2}k = 0$ and $\sqrt{2}kx + ky - 4\sqrt{2} = 0$ (k is any non-zero real parameter) is. **[Online April 16, 2018]**

(a) A hyperbola with length of its transverse axis $8\sqrt{2}$

(b) An ellipse with length of its major axis $8\sqrt{2}$

(c) An ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$

(d) A hyperbola whose eccentricity is $\sqrt{3}$

208. A hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point : **[2017]**

(a) $\left(-\sqrt{2}, -\sqrt{3}\right)$ (b) $\left(3\sqrt{2}, 2\sqrt{3}\right)$

(c) $\left(2\sqrt{2}, 3\sqrt{3}\right)$ (d) $\left(\sqrt{3}, \sqrt{2}\right)$

209. The locus of the point of intersection of the straight lines,
$tx - 2y - 3t = 0$
$x - 2ty + 3 = 0$ ($t \in R$), is : **[Online April 8, 2017]**

(a) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

(b) an ellipse with the length of major axis 6

(c) a hyperbola with eccentricity $\sqrt{5}$

(d) a hyperbola with the length of conjugate axis 3

210. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[2016]**

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{4}{3}$ (d) $\frac{4}{\sqrt{3}}$

211. A hyperbola whose transverse axis is along the major axis of the conic, $\frac{x^2}{3} + \frac{y^2}{4} = 4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it ? **[Online April 10, 2016]**

(a) $(\sqrt{5}, 2\sqrt{2})$ (b) $(0, 2)$

(c) $(5, 2\sqrt{3})$ (d) $(\sqrt{10}, 2\sqrt{3})$

212. Let a and b respectively be the semitransverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2 - 18e + 5 = 0$. If S(5, 0) is a focus and $5x = 9$ is the corresponding directrix of this hyperbola, then $a^2 - b^2$ is equal to : **[Online April 9, 2016]**

(a) –7 (b) –5 (c) 5 (d) 7

113. An ellipse passes through the foci of the hyperbola, $9x^2 - 4y^2 = 36$ and its major and minor axes lie along the transverse and conjugate axes of the hyperbola respectively. If the product of eccentricities of the two conics is $\frac{1}{2}$, then which of the following points **does not** lie on the ellipse? **[Online April 10, 2015]**

(a) $\left(\sqrt{\frac{13}{2}}, \sqrt{6}\right)$ (b) $\left(\frac{\sqrt{39}}{2}, \sqrt{3}\right)$

(c) $\left(\frac{1}{2}\sqrt{13}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(\sqrt{13}, 0\right)$

114. The tangent at an extremity (in the first quadrant) of latus rectum of the hyperbola $\frac{x^2}{4} - \frac{y^2}{5} = 1$, meet x-axis and y-axis at A and B respectively. Then $(OA)^2 - (OB)^2$, where O is the origin, equals: **[Online April 19, 2014]**

(a) $-\frac{20}{9}$ (b) $\frac{16}{9}$ (c) 4 (d) $-\frac{4}{3}$

115. Let P $(3 \sec\theta, 2\tan\theta)$ and Q $(3\sec\phi, 2\tan\phi)$ where $\theta + \phi = \frac{\pi}{2}$, be two distinct points on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$. Then the ordinate of the point of intersection of the normals at P and Q is: **[Online April 11, 2014]**

(a) $\frac{11}{3}$ (b) $-\frac{11}{3}$ (c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

216. A common tangent to the conics $x^2 = 6y$ and $2x^2 - 4y^2 = 9$ is: **[Online April 25, 2013]**

(a) $x - y = \frac{3}{2}$ (b) $x + y = 1$

(c) $x + y = \frac{9}{2}$ (d) $x - y = 1$

217. A tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ meets x-axis at P and y-axis at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is the origin). Then R lies on: **[Online April 23, 2013]**

(a) $\frac{4}{x^2} + \frac{2}{y^2} = 1$ (b) $\frac{2}{x^2} - \frac{4}{y^2} = 1$

(c) $\frac{2}{x^2} + \frac{4}{y^2} = 1$ (d) $\frac{4}{x^2} - \frac{2}{y^2} = 1$

218. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ coincide with the foci of the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$, then b^2 is equal to **[Online May 19, 2012]**

(a) 8 (b) 10 (c) 7 (d) 9

219. If the eccentricity of a hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$, which passes through $(k, 2)$, is $\frac{\sqrt{13}}{3}$, then the value of k^2 is **[Online May 7, 2012]**

(a) 18 (b) 8 (c) 1 (d) 2

220. The equation of the hyperbola whose foci are $(-2, 0)$ and $(2, 0)$ and eccentricity is 2 is given by : **[2011RS]**

(a) $x^2 - 3y^2 = 3$ (b) $3x^2 - y^2 = 3$

(c) $-x^2 + 3y^2 = 3$ (d) $-3x^2 + y^2 = 3$

221. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies = ? **[2007]**

(a) abscissae of vertices (b) abscissae of foci

(c) eccentricity (d) directrix.

222. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is **[2005]**

(a) an ellipse (b) a circle

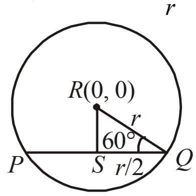
(c) a parabola (d) a hyperbola

223. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is **[2003]**

(a) 9 (b) 1 (c) 5 (d) 7

Hints & Solutions

1. **(d)** In right ΔRSQ, $\sin 60° = \frac{RS}{r}$



$\Rightarrow RS = r \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}r}{2}$

Now equation of PQ is $y - 2x - 3 = 0$

$\therefore \frac{\sqrt{3}r}{2} = \frac{|0+0-3|}{\sqrt{5}}$

$\Rightarrow \frac{\sqrt{3}r}{2} = \frac{3}{\sqrt{5}} \Rightarrow r = \frac{2\sqrt{3}}{5} \Rightarrow r^2 = \frac{12}{5}$

2. **(b)** We know family of circle be $S_1 + \lambda S_2 = 0$

$x^2 + y^2 - 6x + \lambda(x^2 + y^2 - 4y) = 0$

$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 - 6x - 4\lambda y = 0$ ...(i)

Centre $(-g, -f) = \left(\frac{3}{1+\lambda}, \frac{2\lambda}{\lambda+1}\right)$

Centre lies on $2x - 3y + 12 = 0$, then

$\frac{6}{\lambda+1} - \frac{6\lambda}{\lambda+1} + 12 = 0 \Rightarrow \lambda = -3$

Equation of circle (i),

$-2x^2 - 2y^2 - 6x + 12y = 0$

$\Rightarrow x^2 + y^2 + 3x - 6y = 0$ ...(ii)

Only $(-3, 6)$ satisfy equation (ii).

3. **(7)**

Let $P(3\cos\theta, 3\sin\theta)$, $Q(-3\cos\theta, -3\sin\theta)$

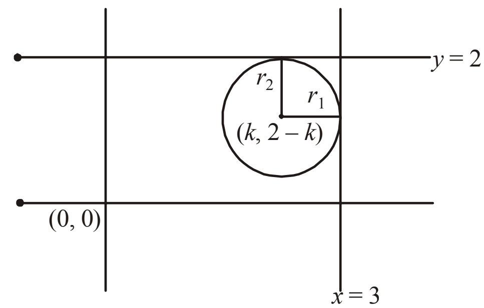
$\alpha = \left|\frac{3\cos\theta + 3\sin\theta - 2}{\sqrt{2}}\right|$, $\beta = \left|\frac{-3\cos\theta - 3\sin\theta - 2}{\sqrt{2}}\right|$

$\alpha\beta = \left|\frac{(3\cos\theta + 3\sin\theta)^2 - 4}{2}\right| = \left|\frac{5 + 9\sin 2\theta}{2}\right|$

$\alpha\beta$ is max. when $\sin 2\theta = 1$

$\therefore \alpha\beta|_{max} = \frac{5+9}{2} = 7.$

4. **(3)**



$\Rightarrow$ Radius $(r_1) = 3 - k$

$\because$ Centre lies on $x + y = 2$

Let $x = k$

$\therefore y = 2 - k$

$\Rightarrow$ Centre $= (k, 2-k)$

Also, radius $(r_2) = 2 - (2 - k)$

$\therefore 3 - k = 2 - (2 - k)$

$\Rightarrow k = \frac{3}{2}$

$r = 3 - \frac{3}{2} = \frac{3}{2}$

Hence, diameter = 3.

5. **(9)**

The given circle is $x^2 + y^2 - 2x - 4y + 4 = 0$

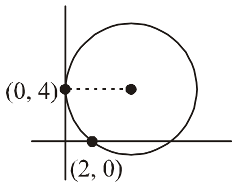
$\therefore$ Centre of circle $(1, 2)$, $r = 1$.

If line cuts circle then $p < r$, where $p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

$\Rightarrow \left|\frac{3 + 8 - k}{5}\right| < 1 \Rightarrow k \in (6, 16)$

$k = 7, 8, 9, 10, 11, 12, 13, 14, 15$

6. **(d)**



Equation of family of circle

$(x-0)^2+(y-4)^2+\lambda x=0$

Passes through the point (2, 0) then

$4+16+2\lambda=0 \Rightarrow \lambda=-10$

Hence, the equation of circle

$x^2+y^2-10x-8y+16=0$

$\Rightarrow (x-5)^2+(y-4)^2=25$

Centre (5, 4).

$$R=\sqrt{\frac{1}{2}\text{coeff. of } x+\frac{1}{2}\text{coeff. of } y-\text{constant}}$$

$$=\sqrt{25+16-16}=5$$

Perpendicular distance of $4x+3y-8=0$ from the centre of circle

$$=\left|\frac{20+16-8}{\sqrt{16+9}}\right|=\frac{28}{5}\neq 5$$

Hence, $4x+3y-8=0$ can not be tangent to the circle.

7. **(36)** The given equation of circle

$x^2-6x+y^2+8=0$

$(x-3)^2+y^2=1$ ...(i)

So, centre of circle (i) is $C_1(3, 0)$ and radius $r_1=1$.

And the second equation of circle

$x^2-8y+y^2+16-k=0\ (k>0)$

$x^2+(y-4)^2=\left(\sqrt{k}\right)^2$ ...(ii)

So, centre of circle (ii) is $C_2(0, 4)$ and radius $r_2=\sqrt{k}$

Two circles touches each other when

$C_1C_2=|r_1\pm r_2| \Rightarrow 5=\left|1\pm\sqrt{k}\right|$

Distance between $C_2(3, 0)$ and $C_1(0, 4)$ is

either $\sqrt{k}+1$ or $\left|\sqrt{k}-1\right|$ $(C_1C_2=5)$

$\Rightarrow \sqrt{k}+1=5$ or $\left|\sqrt{k}-1\right|=5$

$\Rightarrow k=16$ or $k=36$

Hence, maximum value of k is 36

The given equation of circles

$x^2-6x+y^2+8=0$

$\Rightarrow (x-3)^2+y^2=1$

8. **(c)** Slope of tangent of $x^2+y^2=1$ at $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$$\frac{1}{\sqrt{2}}x+\frac{1}{\sqrt{2}}y-1=0$$

$x+y\sqrt{2}=0$, which is perpendicular to $x-y+c=0$

At $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ which is tangent of $(x-3)^2+y^2=1$

So, $m=1 \Rightarrow y=x+c$

Now, distance of (3, 0) from $y=x+c$ is

$$\left|\frac{c+3}{\sqrt{2}}\right|=1$$

$\Rightarrow c=-3\pm\sqrt{2}$

$\Rightarrow (c+3)^2=2$

$\Rightarrow c^2+6c+9=2$

$\therefore c^2+6c+7=0$

9. **(c)** $L=\sqrt{S_1}=\sqrt{16}=4$

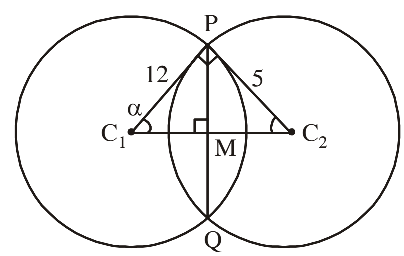
$R=\sqrt{16+4-16}=2$

Length of chord of contact

$$=\frac{2LR}{\sqrt{L^2+R^2}}=\frac{2\times4\times2}{\sqrt{16+4}}=\frac{16}{\sqrt{20}}$$

Square of length of chord of contact $=\frac{64}{5}$

10. **(b)**



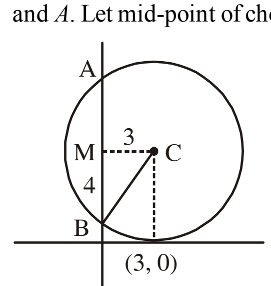
According to the diagram,

In ΔPC_1C_2, $\tan\alpha=\frac{5}{12}\Rightarrow\sin\alpha=\frac{5}{13}$

In ΔPC_1M, $\sin\alpha=\frac{PM}{12}\Rightarrow\frac{5}{13}=\frac{PM}{12}\Rightarrow PM=\frac{60}{13}$

Hence, length of common chord $(PQ)=\frac{120}{13}$

11. **(a)** Let centre of circle is C and circle cuts the y-axis at B and A. Let mid-point of chord BA is M.



$CB = \sqrt{MC^2 + MB^2}$

$\sqrt{3^2+4^2} = 5 =$ radius of circle

$\therefore$ equation of circle is,

$(x-3)^2 + (y-5)^2 = 5^2$

(3, 10) satisfies this equation.

Although there will be another circle satisfying the same conditions that will lie below the x-axis having equation $(x-3)^2 + (y-5)^2 = 5^2$

12. **(b)** $S_1 \equiv x^2 + y^2 + 5Kx + 2y + K = 0$

$S_2 \equiv x^2 + y^2 + Kx + \frac{3}{2}y - \frac{1}{2} = 0$

Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow 4Kx + \frac{y}{2} + K + \frac{1}{2} = 0$ ...(i)

Equation of the line passing through the intersection points P & Q is,

$4x + 5y - K = 0$ ...(ii)

Comparing (i) and (ii),

$\frac{4K}{4} = \frac{1}{10} = \frac{2K+1}{-2K}$ ...(iii)

$\Rightarrow K = \frac{1}{10}$ and $-2K = 20K + 10$

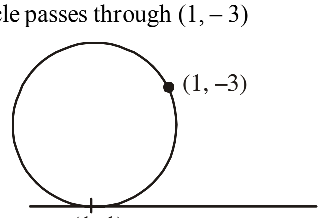
$\Rightarrow 22K = -10 \Rightarrow K = \frac{-5}{11}$

$\because K = \frac{1}{10}$ or $\frac{-5}{11}$ is not satisfying equation (3)

$\therefore$ No value of K exists.

13. **(b)** Equation of circle which touches the line $y = x$ at (1, 1) is, $(x-1)^2 + (y-1)^2 + \lambda(y-x) = 0$

This circle passes through (1, – 3)



$\therefore 0 + 16 + \lambda(-3-1) = 0$

$\Rightarrow 16 + \lambda(-4) = 0 \Rightarrow \lambda = 4$

Hence, equation of circle will be,

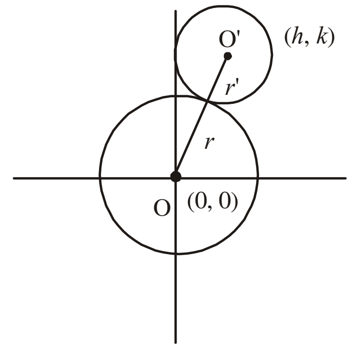
$(x-1)^2 + (y-1)^2 + 4y - 4x = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\therefore$ Radius $= \sqrt{9+1-2} = 2\sqrt{2}$

14. **(b)** Let centre of required circle is (h, k).

$\therefore$ OO' $= r + r'$ [By the diagram]



$\Rightarrow \sqrt{h^2+k^2} = 1 + h$

$\Rightarrow h^2 + k^2 = 1 + h^2 + 2h$

$\Rightarrow k^2 = 1 + 2h$

$\therefore$ locus is $y = \sqrt{1+2x}$, $x \geq 0$

15. **(b)** Let z∈S then $z = \frac{\alpha+i}{\alpha-i}$

Since, z is a complex number and let z = x + iy

Then, $x + iy = \frac{(\alpha+i)^2}{\alpha^2+1}$ (by rationalisation)

$\Rightarrow x + iy = \frac{(\alpha^2-1)}{\alpha^2+1} + \frac{i(2\alpha)}{\alpha^2+1}$

Then compare both sides

$x = \frac{\alpha^2-1}{\alpha^2+1}$ ...(i)

$y = \frac{2\alpha}{\alpha^2+1}$ ...(ii)

Now squaring and adding equations (i) and (ii)

$\Rightarrow x^2 + y^2 = \frac{(\alpha^2-1)^2}{(\alpha^2+1)^2} + \frac{4\alpha^2}{(\alpha^2+1)^2} = 1$

16. **(a)** Let any tangent to circle $x^2 + y^2 = 1$ is

$x\cos\theta + y\sin\theta = 1$

Since, P and Q are the point of intersection on the co-ordinate axes.

Then $P \equiv \left(\frac{1}{\cos\theta}, 0\right)$ & $Q \equiv \left(0, \frac{1}{\sin\theta}\right)$

$\therefore$ mid-point of PQ be $M \equiv \left(\frac{1}{2\cos\theta}, \frac{1}{2\sin\theta}\right) \equiv (h, k)$

$\Rightarrow \cos\theta = \dfrac{1}{2h}$ ...(i)

$\sin\theta = \dfrac{1}{2k}$ ...(ii)

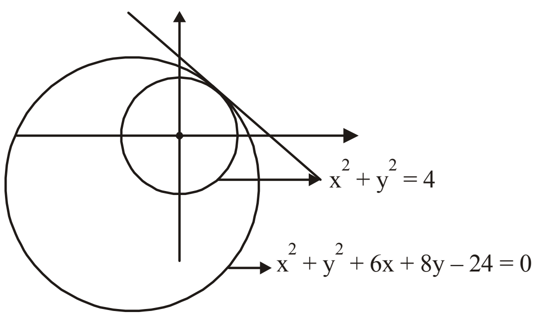
Now squaring and adding equation (i) and (ii)

$\dfrac{1}{h^2} + \dfrac{1}{k^2} = 4$

$\therefore$ locus of M is : $x^2 + y^2 = 4x^2y^2$

$\Rightarrow h^2 + k^2 = 4h^2k^2$

17. **(c)** By the diagram, $d_{c_1c_2} = |r_1 - r_2|$



Equation of common tangent is,

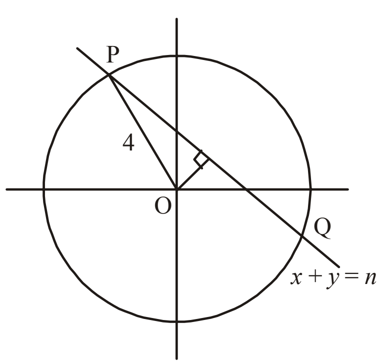
$S_1 - S_2 = 0$

$6x + 8y - 20 = 0 \Rightarrow 3x + 4y - 10 = 0$

Hence $(6, -2)$ lies on it.

18. **(d)** Let the chord $x + y = n$ cuts the circle $x^2 + y^2 = 16$ at P and Q Length of perpendicular from O on PQ

$= \left|\dfrac{0+0-n}{\sqrt{1^2+1^2}}\right| = \dfrac{n}{\sqrt{2}}$



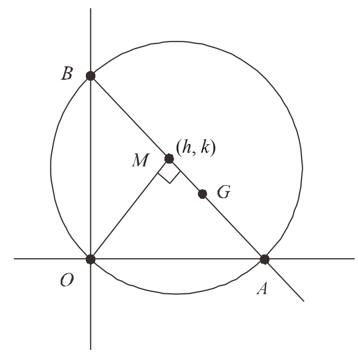
Then, length of chord $PQ = 2\sqrt{4^2 - \left(\dfrac{n}{\sqrt{2}}\right)^2} = 2\sqrt{16 - \dfrac{n^2}{2}}$

Thus only possible values of n are 1, 2, 3, 4, 5.

Hence, the sum of squares of lengths of chords

$= \sum_{n=1}^{5} 4\left(16 - \dfrac{n^2}{2}\right) = 64 \times 5 - 2.\dfrac{5\times6\times11}{6} = 210$

19. **(b)** As $\angle AOB = 90°$



Let AB diameter and $M(h, k)$ be foot of perpendicular, then

$M_{AB} = \dfrac{-h}{k}$

Then, equation of AB

$(y - k) = \dfrac{-h}{k}(x - h)$

$\Rightarrow \; hx + ky = h^2 + k^2$

Then, $A\left(\dfrac{h^2+k^2}{h}, 0\right)$ and $B\left(0, \dfrac{h^2+k^2}{k}\right)$

$\because AB$ is the diameter, then

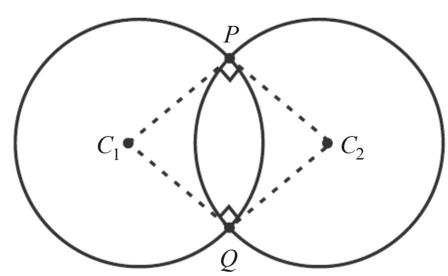
$AB = 2R$

$\Rightarrow \; AB^2 = 4R^2$

$\Rightarrow \; \left(\dfrac{h^2+k^2}{h}\right)^2 + \left(\dfrac{h^2+k^2}{k}\right) = 4R^2$

Hence, required locus is $(x^2 + y^2)^3 = 4R^2x^2y^2$

20. **(d)**



$2g_1g_2 + 2f_1f_2 = 2(-1)(-3) + 2(-1)(-3) = 12$

$c_1 + c_2 = 14 - 2 = 12$

Since, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Hence, circles intersect orthogonally

$\therefore$ Area of the quadrilateral PC_1QC_1

$= 2\left(\frac{1}{2}(C_1P)(C_2P)\right)$

$= 2\times\frac{1}{2}r_1r_2 = (2)(2) = 4$ sq, units

21. **(c)** **Condition 1:** The centre of the two circles are (1, 1) and (9, 1). The circles are on opposite sides of the line $3x + 4y - \lambda = 0$.

Put $x = 1, y = 1$ in the equation of line,

$3(1) + 4(1) - \lambda = 0 \Rightarrow 7 - \lambda = 0$

Now, put $x = 9, y = 1$ in the equation of line,

$3(9) + 4(1) - \lambda = 0$

Then, $(7 - \lambda)(27 + 4 - \lambda) < 0$

$\Rightarrow (\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$ ...(i)

Condition 2: Perpendicular distance from centre on line $\geq$ radius of circle.

For $x^2 + y^2 - 2x - 2y = 1$,

$\Rightarrow \frac{|3+4-\lambda|}{5} \geq 1$

$\Rightarrow |\lambda - 7| \geq 5$

$\Rightarrow \lambda \geq 12$ or $\lambda \Rightarrow 2$ ...(ii)

For $x^2 + y^2 - 18x - 2y + 78 = 0$

$\frac{|27+4-\lambda|}{5} \geq 2$

$\Rightarrow \lambda \geq 41$ or $\lambda \leq 21$ ...(iii)

Intersection of (1), (2) and (3) gives $\lambda \in [12, 21]$.

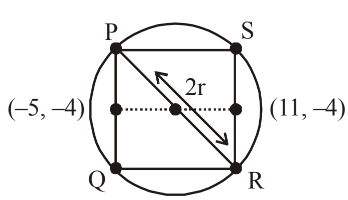
22. **(c)** The equation of circle is,

$x^2 + y^2 - 6x + 8y - 103 = 0$

$\Rightarrow (x-3)^2 + (y+4)^2 = (8\sqrt{2})^2$

$C(3, -4), r = 8\sqrt{2}$

$\Rightarrow$ Length of side of square $= \sqrt{2}r = 16$



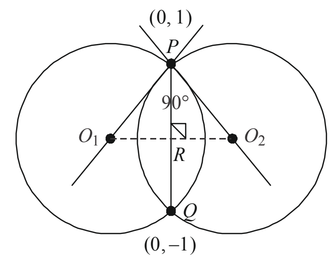
$\Rightarrow P(-5, 4), Q(-5, -12)$

$R(11, -12), S(11, 4)$

$\Rightarrow$ Required distance $= OP$

$= \sqrt{(-5-0)^2 + (-4-0)^2} = \sqrt{25+16} = \sqrt{41}$

23. **(b)** $\because$ Two circles of equal radii intersect each other orthogonally. Then R is mid point of PQ.

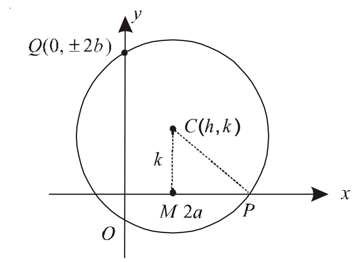


and $PR = O_1R = O_2R$

$PR = \frac{1}{2}\sqrt{(0-0)^2 + (1+1)^2} = 1$

$\therefore$ Distance between centres $= 1 + 1 = 2$.

24. **(d)**



Let centre be $C(h, k)$

$CQ = CP = r$

$\Rightarrow CQ^2 = CP^2$

$(h-0)^2 + (k \pm 0)^2 = CM^2 + MP^2$

$h^2 + (k \pm 2b)^2 = k^2 + 4a^2$

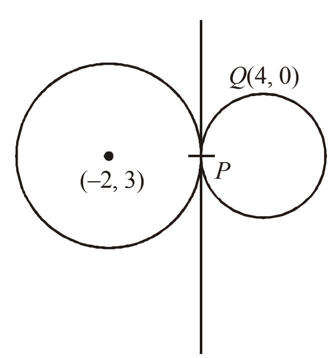
$h^2 + k^2 + 4b^2 \pm 4bk = k^2 + 4a^2$

Then, the locus of centre $C(h, k)$

$x^2 + 4b^2 \pm 4by = 4a^2$

Hence, the above locus of the centre of circle is a parabola.

25. **(c)** The equation of circle $x^2 + y^2 + 4x - 6y = 12$ can be written as $(x+2)^2 + (y-3)^2 = 25$



Let $P = (1, -1)$ & $Q = (4, 0)$

Equation of tangent at $P(1, -1)$ to the given circle :

$x(1) + y(-1) + 2(x + 1) - 3(y - 1) - 12 = 0$

$3x - 4y - 7 = 0$ ...(i)

The required circle is tangent to (1) at $(1, -1)$.

$\therefore \quad (x - 1)^2 + (y + 1)^2 + \lambda(3x - 4y - 7) = 0$ ...(ii)

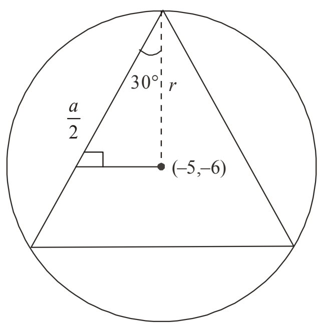
Equation (ii) passes through $Q(4, 0)$

$\Rightarrow \quad 3^2 + 1^2 + \lambda(12 - 7) = 0 \Rightarrow 5\lambda + 10 = 0 \Rightarrow \lambda = -2$

Equation (2) becomes $x^2 + y^2 - 8x + 10y + 16 = 0$

radius $= \sqrt{(-4)^2 + (5)^2 - 16} = 5$

26. (d)



Let the sides of equilateral Δ inscribed in the circle be a,

then $\cos 30° = \dfrac{a}{2r}$

$\dfrac{\sqrt{3}}{2} = \dfrac{a}{2r} \Rightarrow a = \sqrt{3}r$

Then, area of the equilateral triangle $= \dfrac{\sqrt{3}}{4}a^2$

$= \dfrac{\sqrt{3}}{4}\left(\sqrt{3}r\right)^2 = \dfrac{3\sqrt{3}}{4}r^2$

But it is given that area of equilateral triangle $= 27\sqrt{3}$

Then, $27\sqrt{3} = \dfrac{3\sqrt{3}}{4}r^2$

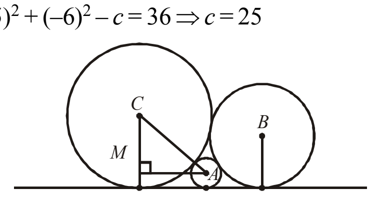
$r^2 = 36 \Rightarrow r = 6$

But $\left(-\dfrac{1}{2}\text{ coeff. of } x\right)^2 + \left(-\dfrac{1}{2}\text{ coeff. of } y\right)^2$

$-$ constant term $= r^2$

$(-5)^2 + (-6)^2 - c = 36 \Rightarrow c = 25$

27. (a)



$AM^2 = AC^2 - MC^2$

$= \quad (a + c)^2 - (a - c)^2 = 4ac$

$\Rightarrow \quad AM^2 = XY^2 = 4ac$

$\Rightarrow \quad XY = 2\sqrt{ac}$

Similarly, $YZ = 2\sqrt{ba}$ and $XZ = 2\sqrt{bc}$

Then, $XZ = XY + YZ$

$\Rightarrow \quad 2\sqrt{bc} = 2\sqrt{ac} + 2\sqrt{ba}$

$\Rightarrow \quad \dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{b}} + \dfrac{1}{\sqrt{c}}$

28. (d) Consider the equation of circles as,

$x^2 + y^2 - 16x - 20y + 164 = r^2$

i.e. $(x - 8)^2 + (y - 10)^2 = r^2$ ...(i)

and $(x - 4)^2 + (y - 7)^2 = 36$ ...(ii)

Both the circles intersect each other at two distinct points.

Distance between centres

$= \sqrt{(8 - 4)^2 + (10 - 7)^2} = 5$

$\therefore \quad |r - 6| < 5 < |r + 6|$

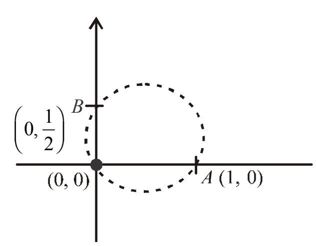
$\therefore \quad$ If $|r - 6| < 5 \Rightarrow r \in (1, 11)$ ...(iii)

and $|r + 6| > 5 \Rightarrow r \in (-\infty, -11) \cup (-1, \infty)$ ...(iv)

From (iii) and (iv),

$r \in (1, 11)$

29. (a)



Let equation of circle be $x^2 + y^2 + 2gx + 2fy = 0$

As length of intercept on x axis is $1 = 2\sqrt{g^2 - c}$

$\Rightarrow \quad |g| = \dfrac{1}{2}$

length of intercept on y-axis $= \dfrac{1}{2} = 2\sqrt{f^2 - c}$

$\Rightarrow \quad |f| = \dfrac{1}{4}$

Equation of circle that passes through given points is

$x^2 + y^2 - x - \dfrac{y}{2} = 0$

Tangent at $(0, 0)$ is,

$(y-0)=\left(\dfrac{dy}{dx}\right)_{(0,0)}^{(x-0)}\cdot(x-0)$

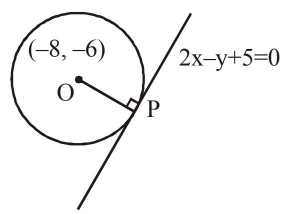
$\Rightarrow\ 2x+y=0$

Perpendicular distance from $B(1, 0)$ on the tangent to the circle $=\dfrac{\frac{1}{2}}{\sqrt{5}}$

Perpendicular distance from $B\left(0,\dfrac{1}{2}\right)$ on the tangent to the circle $=\dfrac{2}{\sqrt{5}}$

Sum of perpendicular distance $=\dfrac{\frac{1}{2}+2}{\sqrt{5}}=\dfrac{\sqrt{5}}{2}.$

30. **(c)** Equation of tangent at (1, 7) to $x^2 = y-6$ is $2x-y+5=0$.



Now, perpendicular from centre O(–8, –6) to $2x-y+5=0$ should be equal to radius of the circle

$\therefore\ \left|\dfrac{-16+6+5}{\sqrt{5}}\right|=\sqrt{64+36-C}$

$\Rightarrow\ \sqrt{5}=\sqrt{100-c}\ \Rightarrow c=95$

31. **(d)** Given circle is:

$x^2+y^2+2x-4y-4=0$

$\therefore$ its centre is $(-1, 2)$ and radius is 3 units.

Let $A=(x, y)$ be the centre of the circle C

$\therefore\ \dfrac{x-1}{2}=2\ \Rightarrow\ x=5$ and $\dfrac{y+2}{2}=2\ \Rightarrow\ y=2$

So the centre of C is (5, 2) and its radius is 3

$\therefore$ equation of centre C is:

$x^2+y^2-10x-4y+20=0$

$\therefore$ The length of the intercept it cuts on the x-axis

$=2\sqrt{g^2-c}=2\sqrt{25-20}=2\sqrt{5}$

32. **(c)** Equation of the line passing through the points (2, 3) and (4, 5) is

$y-3=\left(\dfrac{5-3}{4-2}\right)x-2\Rightarrow\ x-y+1=0$ ..... (i)

Equation of the perpendicular line passing through the midpoint (3, 4) is $x+y-7=0$ ..... (ii)

Lines (1) and (2) intersect at the center of the circle. So, the center of the circle is (3, 4)

Therefore, the radius is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(2-3)^2+(3-4)^2}=\sqrt{2}$ units.

33. **(c)** As origin is the only common point to x-axis and y-axis, so, origin is the common vertex

Let the equation of two of parabolas be $y^2 = 4ax$ and $x^2 = 4by$

Now latus rectum of both parabolas = 3

$\therefore\ 4a=4b=3\Rightarrow a=b=\dfrac{3}{4}$

$\therefore$ Two parabolas are $y^2 = 3x$ and $x^2 = 3y$

Suppose $y = mx + c$ is the common tangent.

$\therefore\ y^2=3x\Rightarrow(mx+c)^2=3x\Rightarrow m^2x^2+(2mc-3)x+c^2=0$

As, the tangent touches at one point only

So, $b^2-4ac=0$

$\Rightarrow(2mc-3)^2-4m^2c^2=0$

$\Rightarrow\ 4m^2c^2+9-12mc-4m^2c^2=0$

$\Rightarrow c=\dfrac{9}{12m}=\dfrac{3}{4m}$ .... (i)

$\therefore\ x^2=3y\Rightarrow x^2=3\,(mx+c)\Rightarrow x^2-3mx-3c=0$

Again, $b^2-4ac=0$

$\Rightarrow 9m^2-4\,(1)\,(-3c)=0$

$\Rightarrow 9m^2=-12c$ ..... (ii)

Form (i) and (ii)

$m^2=\dfrac{-4c}{3}=\dfrac{-4}{3}\left(\dfrac{3}{4m}\right)$

$\Rightarrow m^3=-1\Rightarrow m=-1\Rightarrow c=\dfrac{-3}{4}$

Hence, $y=mx+c=-x-\dfrac{3}{4}$

$\Rightarrow 4\,(x+y)+3=0$

34. **(a)** Here, equation of tangent on C_1 at (2, 1) is:

$2x+y-(x+2)-1=0$

Or $x+y=3$

If it cuts off the chord of the circle C_2 then the equation of the chord is:

$x+y=3$

$\therefore$ distance of the chord from $(3, -2)$ is :

$d=\left|\dfrac{3-2-3}{\sqrt{2}}\right|=\sqrt{2}$

Also, length of the chord is $l=4$

$\therefore$ radius of $C_2=r=\sqrt{\left(\dfrac{l}{2}\right)^2+d^2}$

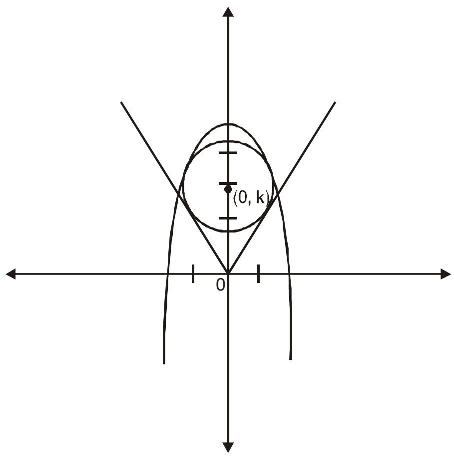
$=\sqrt{(2)^2+(\sqrt{2})^2}=\sqrt{6}$

35. **(None)**

(Let the equation of circle be

$x^2+(y-k)^2=r^2$

It touches $x-y=0$



$\Rightarrow \left|\dfrac{0-k}{\sqrt{2}}\right|=r$

$\Rightarrow k=r\sqrt{2}$

$\therefore$ Equation of circle becomes

$x^2+(y-k)^2=\dfrac{k^2}{2}$ ...(i)

It touches $y=4-x^2$ as well

$\therefore$ Solving the two equations

$\Rightarrow 4-y+(y-k)^2=\dfrac{k^2}{2}$

$\Rightarrow 1y^2-y(2k+1)+\dfrac{k^2}{2}+4=0$

It will give equal roots $\therefore D=0$

$\Rightarrow (2k+1)^2=4\left(\dfrac{k^2}{2}+4\right)$

$\Rightarrow 2k^2+4k-15=0$

$\Rightarrow k=\dfrac{-2+\sqrt{34}}{2}$

$\therefore r=\dfrac{k}{\sqrt{2}}=\dfrac{-2+\sqrt{34}}{2\sqrt{2}}$

Which is not matching with any of the option given here.

36. **(c)** Let $z=x+y$

$$\text{Im}\left[\left(\frac{ix-y-2}{x+(y-1)i}\right)\left(\frac{x-(y-1)i}{x-(y-1)i}\right)\right]+1=0$$

On solving, we get:

$2x^2+2y^2-y-1=0$

$\Rightarrow x^2+y^2-1/2y-1/2=0$

$\Rightarrow x^2+\left(y-\dfrac{1}{4}\right)^2=\dfrac{9}{16}$

$\Rightarrow r=\dfrac{3}{4}$

37. **(b)** P (4, 7). Here, $x=4$, $y=7$

$x-y=-3$

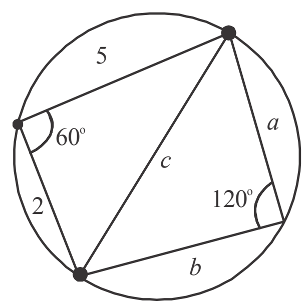
$\therefore PA\times PB=PT^2$

Also; $PT=\sqrt{x^2+y^2-(x-y)^2}$

$\Rightarrow PT=\sqrt{16+49-9}=\sqrt{56}$

$\Rightarrow PT^2=56 \therefore PA\times PB=56$

38. **(c)**



Here; $\cos\theta=\dfrac{a^2+b^2-c^2}{2ab}$

and $\theta=60^o$

$\Rightarrow \cos 60^o=\dfrac{4+25-c^2}{2.2.5}$

$\Rightarrow 10=29-c^2$

$\Rightarrow c^2=19$

$\Rightarrow \boxed{c=\sqrt{19}}$

also; $\cos\theta=\dfrac{a^2+b^2-c^2}{2ab}$

and $\theta=120^o$

$\Rightarrow -\dfrac{1}{2}=\dfrac{a^2+b^2-19}{2ab}$

$\Rightarrow a^2+b^2-19=-ab$

$\Rightarrow a^2+b^2+ab=19$

$\therefore \text{Area}=\dfrac{1}{2}\times 2\times 5\sin 60+\dfrac{1}{2}ab\sin 120^o=4\sqrt{3}$

$\Rightarrow \dfrac{5\sqrt{3}}{2}+\dfrac{ab\sqrt{3}}{4}=4\sqrt{3}$

$\Rightarrow \frac{ab}{4} = 4 - \frac{5}{2} = \frac{3}{2}$

$\Rightarrow \boxed{ab = 6}$

$\therefore a^2 + b^2 = 13$

$\Rightarrow a = 2, b = 3$

Perimeter = Sum of all sides = 2 + 5 + 2 + 3 = 12

39. (a) Let $z = x + iy$

$\Rightarrow 2|x + i(y+3)| = |x + i(y-1)|$

$\Rightarrow 2\sqrt{x^2 + (y+3)^2} = \sqrt{x^2 + (y-1)^2}$

$\Rightarrow 4x^2 + 4(y+3)^2 = x^2 + (y-1)^2$

$\Rightarrow 3x^2 = y^2 - 2y + 1 - 4y^2 - 24y - 36$

$\Rightarrow 3x^2 + 3y^2 + 26y + 35 = 0$ (which is a circle)

$\Rightarrow x^2 + y^2 + \frac{26}{3}y + \frac{35}{3} = 0$

$\Rightarrow r = \sqrt{0 + \frac{169}{9} - \frac{35}{3}}$

$\Rightarrow r = \sqrt{\frac{64}{9}} = \frac{8}{3}$

40. (b) Given that $x^2 + y^2 - 5x - y + 5 = 0$

$\Rightarrow (x - 5/2)^2 - \frac{25}{4} + (y - 1/2)^2 - 1/4 = 0$

$\Rightarrow (x - 5/2)^2 + (y - 1/2)^2 = 3/2$

on circle $\left[Q \equiv \left(5/2 + \sqrt{3/2}\cos Q, \frac{1}{2} + \sqrt{3/2}\sin Q\right)\right]$

$\Rightarrow PQ^2 = \left(\frac{5}{2} + \sqrt{3/2}\cos Q\right)^2 + \left(\frac{5}{2} + \sqrt{3/2}\sin Q\right)^2$

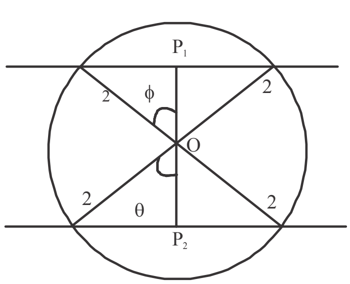
$\Rightarrow PQ^2 = \frac{25}{2} + \frac{3}{2} + 5\sqrt{3/2}(\cos Q + \sin Q)$

$= 14 + 5\sqrt{3/2}\ (\cos Q + \sin Q)$

$\therefore$ Maximum value of PQ^2

$= 14 + 5\sqrt{3/2} \times \sqrt{2} = 14 + 5\sqrt{3}$

41. (b)



Since $\cos 2\theta = 1/7 \Rightarrow 2\cos^2 Q - 1 = 1/7$

$\Rightarrow 2\cos^2\theta = 8/7$

$\Rightarrow \cos^2\theta = 4/7$

$\Rightarrow \cos^2\theta = \frac{4}{7}$

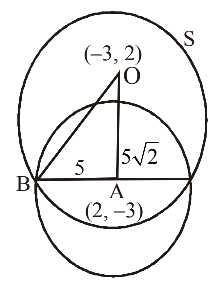
$\Rightarrow \cos^2\phi = \frac{2}{\sqrt{7}}$

Also, $\sec^2\phi = 7 = \frac{1}{2\cos^2\phi - 1} = 7$

$= \cos^2\phi - 1 = \frac{1}{7} = 2\cos^2\phi = \frac{8}{7} = \cos\phi = \frac{2}{\sqrt{7}}$

$P_1P_2 = r\cos\theta + r\cos\phi = \frac{4}{\sqrt{7}} + \frac{4}{\sqrt{7}} = \frac{8}{\sqrt{7}}$

42. (d)



Given, centre of S is O (–3, 2) and centre of given circle is A(2, –3) and radius is 5.

$OA = 5\sqrt{2}$

Also AB = 5 ($\because$ AB = radius of the given circle)

Using pythagoras theorem in $\triangle OAB$

$r = 5\sqrt{3}$

43. (a) Point of intersection of lines

$x - y = 1$ and $2x + y = 3$ is $\left(\frac{4}{3}, \frac{1}{3}\right)$

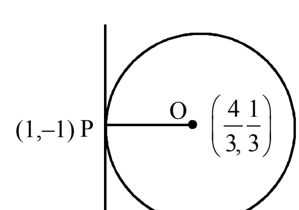
Slope of OP $= \frac{\frac{1}{3} + 1}{\frac{4}{3} - 1} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$

Slope of tangent $= -\frac{1}{4}$

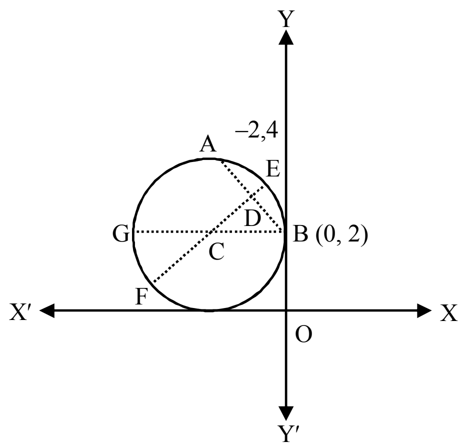
Equation of tangent $\quad y + 1 = -\frac{1}{4}(x - 1)$

$4y + 4 = -x + 1$

$x + 4y + 3 = 0$



44. (a)



EF = perpendicular bisector of chord AB

BG = perpendicular to y-axis

Here C = centre of the circle

mid-point of chord AB, D = (– 1, 3)

slope of AB $= \dfrac{4-2}{-2-0} = -1$

$\because$ EF $\perp$ AB

$\therefore$ Slope of EF = 1

Equation of EF, $y - 3 = 1(x + 1)$

$\Rightarrow \quad y = x + 4$ ...(i)

Equation of BG

$y = 2$ ...(ii)

From equations (i) and (ii)

$x = -2, y = 2$

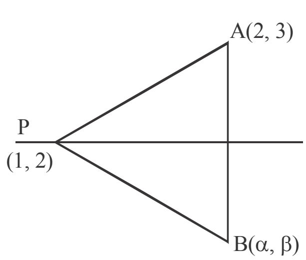
since C be the point of intersection of EF and BG, therefore centre, C = (–2, 2)

Now coordinates of centre C satisfy the equation

$2x - 3y + 10 = 0$

Hence $2x - 3y + 10 = 0$ is the equation of the diameter

45. (a) Intersection point of $2x - 3y + 4 = 0$ and $x - 2y + 3 = 0$ is (1, 2)



Let image of $A(2, 3)$ is $B(\alpha, \beta)$.

Since, P is the fixed point for given family of lines

So, PB = PA

$(\alpha - 1)^2 + (\beta - 2)^2 = (2-1)^2 + (3-2)^2$

$(\alpha - 1)^2 + (\beta - 2)^2 = 1 + 1 = 2$

$(x-1)^2 + (y-2)^2 = (\sqrt{2})^2$

Compare with

$(x - a)^2 + (y - b)^2 = r^2$

Therefore, given locus is a circle with centre (1, 2) and radius $\sqrt{2}$.

46. (a) $x^2 + y^2 - 4x - 6y - 12 = 0$ ...(i)

Centre, $C_1 = (2, 3)$

Radius, $r_1 = 5$ units

$x^2 + y^2 + 6x + 18y + 26 = 0$ ...(ii)

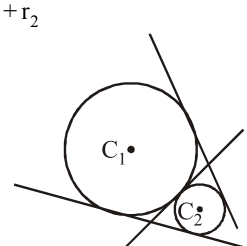
Centre, $C_2 = (-3, -9)$

Radius, $r_2 = 8$ units

$C_1C_2 = \sqrt{(2+3)^2 + (3+9)^2} = 13$ units

$r_1 + r_2 = 5 + 8 = 13$

$\therefore C_1 C_2 = r_1 + r_2$



Therefore there are three common tangents.

47. (a) Let radius of circumcircle be

According to the question,

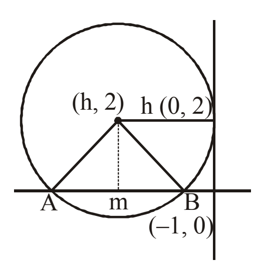
$\dfrac{r}{2} = \dfrac{10}{5} \Rightarrow r = 4$

So equation of required circle is

$(x-1)^2 + (y-1)^2 = 16$

$\Rightarrow x^2 + y^2 - 2x - 2y - 14 = 0$

48. (b) Let 'h' be the radius of the circle and since circle touches y–axis at (0, 2) therefore centre = (h, 2)



Now, eqn of circle is

$(h + 1)^2 + 2^2 = h^2$

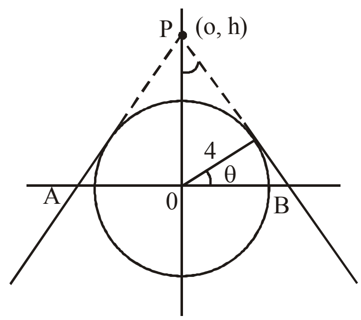
$\Rightarrow 2h + 5 = 0$

$h = -\dfrac{5}{2}$

From the figure, it is clear that AB is the chord along x–axis

$\therefore$ AB $= 2$ (AM) $= 2\sqrt{\dfrac{25}{4}-4} = 2\left(\dfrac{3}{2}\right) = 3$

49. **(a)** $OP = \dfrac{4}{\sin\theta}$



$OB = \dfrac{4}{\cos\theta}$

Area $= OP \times OB = \dfrac{16}{\sin\theta\cos\theta} = \dfrac{32}{\sin 2\theta}$

least value $\sin 2\theta = 1$; $\theta = 45°$

So, $h = \dfrac{4}{\sin 45°} = 4\sqrt{2}$

50. **(d)** Given that $y + 3x = 0$ is the equation of a chord of the circle then

$y = -3x$ ....(i)

$(x^2) + (-3x)^2 - 30x = 0$

$10x^2 - 30x = 0$

$10x(x-3) = 0$

$x = 0, y = 0$

so the equation of the circle is

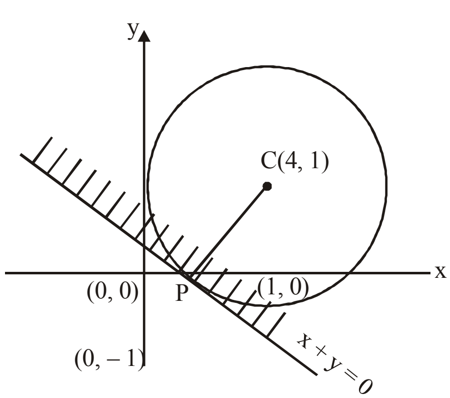
$(x-3)(x-0) + (y+9)(y-0) = 0$

$x^2 - 3x + y^2 + 9y = 0$

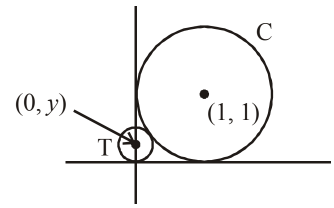
$x^2 + y^2 - 3x + 9y = 0$

51. **(a)** Radius

$CP = \dfrac{4+1}{\sqrt{2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5}{2}\sqrt{2}$



52. **(b)**



Equation of circle

$C \equiv (x-1)^2 + (y-1)^2 = 1$

Radius of T $= |y|$

T touches C externally therefore,

Distance between the centres = sum of their radii

$\Rightarrow \sqrt{(0-1)^2 + (y-1)^2} = 1 + |y|$

$\Rightarrow (0-1)^2 + (y-1)^2 = (1+|y|)^2$

$\Rightarrow 1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$

$2|y| = 1 - 2y$

If $y > 0$ then $2y = 1 - 2y \Rightarrow y = \dfrac{1}{4}$

If $y < 0$ then $-2y = 1 - 2y \Rightarrow 0 = 1$ (not possible)

$\therefore \quad y = \dfrac{1}{4}$

53. **(a)** Given circle is $x^2 + y^2 - 16 = 0$

Eqn of chord say AB of given circle is $3x + y + 5 = 0$.

Equation of required circle is

$x^2 + y^2 - 16 + \lambda(3x + y + 5) = 0$

$\Rightarrow x^2 + y^2 + (3\lambda)x + (\lambda)y + 5\lambda - 16 = 0$ ...(1)

Centre C $= \left(\dfrac{-3\lambda}{2}, \dfrac{-\lambda}{2}\right)$.

If line AB is the diameter of circle (1), then

$C\left(\dfrac{-3\lambda}{2}, \dfrac{-\lambda}{2}\right)$ will lie on line AB.

i.e. $3\left(\dfrac{-3\lambda}{2}\right) + \left(\dfrac{-\lambda}{2}\right) + 5 = 0$

$\Rightarrow -\dfrac{9\lambda - \lambda}{2} + 5 = 0 \Rightarrow \lambda = 1$

Hence, required eqn of circle is

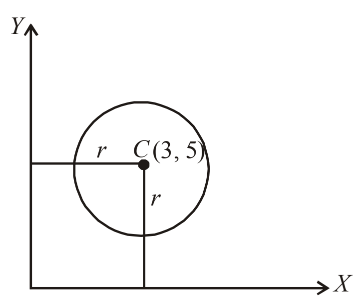
$x^2 + y^2 + 3x + y + 5 - 16 = 0$

$\Rightarrow x^2 + y^2 + 3x + y - 11 = 0$

54. **(d)** Let, $x^2 + y^2 = 16$ or $x^2 + y^2 = 4^2$
radius of circle $r_1 = 4$, centre C_1 (0, 0)
we have, $x^2 + y^2 - 2y = 0$
$\Rightarrow x^2 + (y^2 - 2y + 1) - 1 = 0$ or $x^2 + (y - 1)^2 = 1^2$
Radius 1, centre C_2 (0, 1)
$|C_1C_2| = 1$
$|r_2 - r_1| = |4 - 1| = 3$
$|C_1C_2| < |r_2 - r_1|$

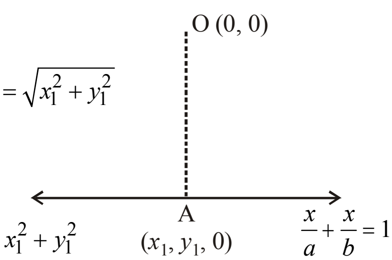
55. **(b)** The equations of the circles are
$x^2 + y^2 - 10x - 10y + \lambda = 0$ ...(1)
and $x^2 + y^2 - 4x - 4y + 6 = 0$ ...(2)
C_1 = centre of (1) = (5, 5)
C_2 = centre of (2) = (2, 2)
d = distance between centres
$= C_1C_2 = \sqrt{9+9} = \sqrt{18}$
$r_1 = \sqrt{50-\lambda}$, $r_2 = \sqrt{2}$
For exactly two common tangents we have
$r_1 - r_2 < C_1C_2 < r_1 + r_2$
$\Rightarrow \sqrt{50-\lambda} - \sqrt{2} < 3\sqrt{2} < \sqrt{50-\lambda} + \sqrt{2}$
$\Rightarrow \sqrt{50-\lambda} - \sqrt{2} < 3\sqrt{2}$ or $3\sqrt{2} < \sqrt{50-\lambda} + \sqrt{2}$
$\Rightarrow \sqrt{50-\lambda} < 4\sqrt{2}$ or $2\sqrt{2} < \sqrt{50-\lambda}$
$\Rightarrow 50 - \lambda < 32$ or $8 < 50 - \lambda$
$\Rightarrow \lambda > 18$ or $\lambda < 42$
Required interval is (18, 42)

56. **(d)**



The equation of circle is
$x^2 + y^2 - 6x - 10y + P = 0$ ...(i)
$(x-3)^2 + (y-5)^2 = (\sqrt{34-P})^2$
Centre (3, 5) and radius $'r' = \sqrt{34-P}$
If circle does not touch or intersect the x-axis then radius $x < y$ - coordiante of centre C
or $\sqrt{34-P} < 5$
$\Rightarrow 34 - P < 25$
$\Rightarrow P > 9$ ...(ii)
Also if the circle does not touch or intersect x-axis the radius $r < x$-coordinate of centre C.
or $\sqrt{34-P} < 3 \Rightarrow 34 - P < 9 \Rightarrow P > 25$ ... (iii)
If the point (1, 4) is inside the circle, then its distance from centre C < r.
or $\sqrt{[(3-1)^2 + (5-4)^2]} < \sqrt{34-P}$
$\Rightarrow 5 < 34 - K$
$\Rightarrow P < 29$ ... (iv)
Now all the conditions (ii), (iii) and (iv) are satisfied if 25 < P < 29 which is required value of P.

57. **(c)** Let the foot of the perpendicular from (0, 0) on the variable line $\frac{x}{a} + \frac{y}{b} = 1$ is $(x_1 > y_1)$
Hence, perpendicular distance of the variable line $\frac{x}{a} + \frac{y}{b} = 1$ from the point O (0, 0) = OA

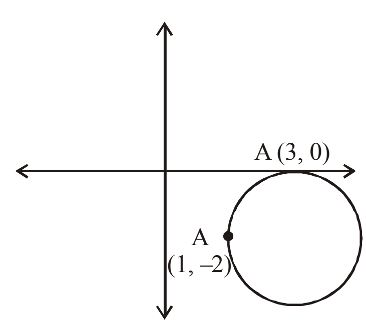


$\Rightarrow \frac{|-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \sqrt{x_1^2 + y_1^2}$

$\Rightarrow \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}} = x_1^2 + y_1^2$

$\Rightarrow 4 = x_1^2 + y_1^2 \quad \left[\because \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}\right]$,
which is equation of a circle with radius 2.
Hence (x_1, y_1) i.e., the foot of the perpendicular from the point (0, 0) to the variable line $\frac{x}{a} + \frac{x}{b} = 1$ is lies on a circle with radius = 2

58. **(c)** Since circle touches x-axis at (3, 0)
$\therefore$ The equation of circle be
$(x-3)^2 + (y-0)^2 + \lambda y = 0$



As it passes through (1, –2)

$\therefore$ Put $x = 1, y = -2$

$\Rightarrow$ $(1-3)^2 + (-2)^2 + \lambda(-2) = 0$

$\Rightarrow$ $\lambda = 4$

$\therefore$ equation of circle is

$(x-3)^2 + y^2 - 8 = 0$

Now, from the options (5, –2) satisfies equation of circle.

59. **(*)** Given information is incomplete in the question.

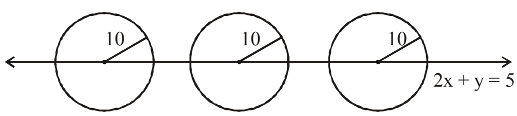
60. **(a)** Circle : $x^2 + y^2 - 6x + 2y = 0$ ...(i)

Line : $2x + y = 5$ ...(ii)

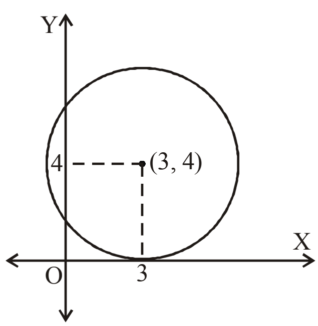
Centre = (3, – 1)

Now, $2 \times 3 - 1 = 5$, hence centre lies on the given line. Therefore line passes through the centre. The given line is normal to the circle.

Thus statement-2 is true, but statement-1 is not true as there are infinite circle according to the given conditions.



61. **(b)**

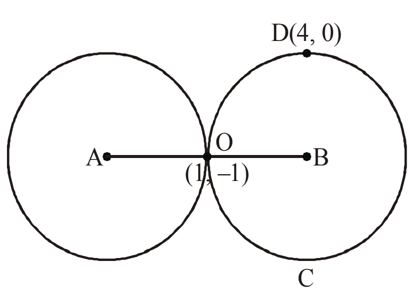


$x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$

Radius = $4 = \sqrt{9 + 16 + (25 - a^2)}$

$\Rightarrow$ $a = \pm 4$

62. **(a)** Let A be the centre of given circle and B be the centre of circle C.



$x^2 + y^2 + 4x - 6y - 12 = 0$

$\therefore$ A = (– 2, 3) and B = (g, f)

Now, from the figure, we have

$\frac{-2+g}{2} = 1$ and $\frac{3+f}{2} = -1$ (By mid point formula)

$\Rightarrow$ $g = 4$ and $f = -5$

63. **(a)** Let C = (x, y)

Now, $CA^2 = CB^2 = AB^2$

$\Rightarrow$ $(x+a)^2 + y^2 = (x-a)^2 + y^2 = (2a)^2$

$\Rightarrow$ $x^2 + 2ax + a^2 + y^2 = 4a^2$ ...(i)

and $x^2 - 2ax + a^2 + y^2 = 4a^2$ ...(ii)

From (i) and (ii), $x = 0$ and $y = \pm\sqrt{3}a$

Since point C(x, y) lies above the x-axis and $a > 0$, hence $y = \sqrt{3}a$

$\therefore$ C = $(0, \sqrt{3}a)$

Let the equation of circumcircle be

$x^2 + y^2 + 2gx + 2fy + C = 0$

Since points A$(-a, 0)$, B$(a, 0)$ and C$(0, \sqrt{3}a)$ lie on the circle, therefore

$a^2 - 2ga + C = 0$ ...(iii)

$a^2 + 2ga + C = 0$ ...(iv)

and $3a^2 + 2\sqrt{3}af + C = 0$ ...(v)

From (iii), (iv), and (v)

$g = 0, c = -a^2, f = -\frac{a}{\sqrt{3}}$

Hence equation of the circumcircle is

$x^2 + y^2 - \frac{2a}{\sqrt{3}}y - a^2 = 0$

$\Rightarrow$ $x^2 + y^2 - \frac{2\sqrt{3}ay}{3} - a^2 = 0$

$\Rightarrow$ $3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$

64. **(d)** Point of intersection of two given lines is (1, 1). Since each of the two given lines contains a diameter of the given circle, therefore the point of intersection of the two given lines is the centre of the given circle.

Hence centre = (1, 1)

$\therefore\ a^2 - 7a + 11 = 1 \Rightarrow a = 2, 5$ ...(i)

and $a^2 - 6a + 6 = 1 \Rightarrow a = 1, 5$ ...(ii)

From both (i) and (ii), $a = 5$

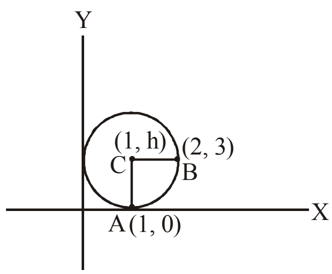
Now on replacing each of $(a^2 - 7a + 11)$ and $(a^2 - 6a + 6)$ by 1, the equation of the given circle is $x^2 + y^2 - 2x - 2y + b^3 + 1 = 0$

$\Rightarrow (x-1)^2 + (y-1)^2 + b^3 = 1$

$\Rightarrow b^3 = 1 - [(x-1)^2 + (y-1)^2]$

$\therefore\ b \in (-\infty, 1)$

65. **(a)** Since, circle touches , the x-axis at (1, 0). So, let centre of the circle be (1, h)



Given that circle passes through the point B (2,3)

$\therefore\ CA = CB$ (radius)

$\Rightarrow CA^2 = CB^2$

$\Rightarrow (1-1)^2 + (h-0)^2 = (1-2)^2 + (h-3)^2$

$\Rightarrow h^2 = 1 + h^2 + 9 - 6h$

$\Rightarrow h = \frac{10}{6} = \frac{5}{3}$

$\therefore$ Length of the diameter $= \frac{10}{3}$

66. **(c)** Given circles are

$x^2 + y^2 - 8x - 2y + 1 = 0$

and $x^2 + y^2 + 6x + 8y = 0$

Their centres and radius are

$C_1(4, 1), r_1 = \sqrt{16} = 4$

$C_2(-3, -4), r_2 = \sqrt{25} = 5$

Now, $C_1C_2 = \sqrt{49 + 25} = \sqrt{74}$

$r_1 - r_2 = -1, r_1 + r_2 = 9$

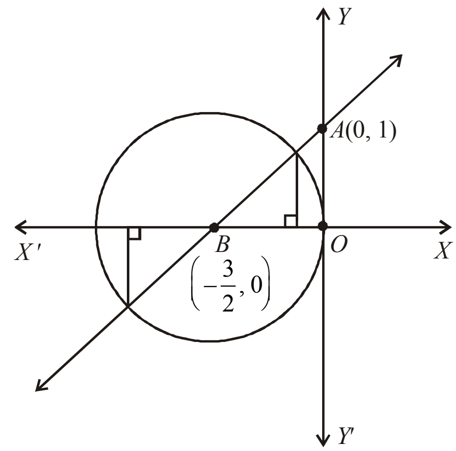
Since, $r_1 - r_2 < C_1C_2 < r_1 + r_2$

$\therefore$ Number of common tangents = 2

67. **(b)** Circle : $x^2 + y^2 + 3x = 0$

Centre, $B = \left(-\frac{3}{2}, 0\right)$

Radius $= \frac{3}{2}$ units.



Line : $y = mx + 1$

y-intercept of the line = 1

$\therefore$ A = (0, 1)

Slope of line, $m = \tan\theta = \frac{OA}{OB}$

$\Rightarrow m = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$\Rightarrow 3m - 2 = 0$

68. **(a)** Let $P(1, 0)$ and $Q(-1, 0)$, $A(x, y)$

Given: $\frac{AP}{AQ} = \frac{BP}{BQ} = \frac{CP}{CQ} = \frac{1}{2}$

$\Rightarrow 2AP = AQ$

$\Rightarrow 4(AP)^2 = AQ^2$

$\Rightarrow 4[(x-1)^2 + y^2] = (x+1)^2 + y^2$

$\Rightarrow 4(x^2 + 1 - 2x) + 4y^2 = x^2 + 1 + 2x + y^2$

$\Rightarrow 3x^2 + 3y^2 - 8x - 2x + 4 - 1 = 0$

$\Rightarrow 3x^2 + 3y^2 - 10x + 3 = 0$

$\Rightarrow x^2 + y^2 - \frac{10}{3}x + 1 = 0$ ...(1)

$\therefore$ A lies on the circle given by (1). As B and C also follow the same condition.

$\therefore$ Centre of circumcircle of ΔABC = centre of circle given by (1) $= \left(\frac{5}{3}, 0\right)$.

69. **(d)** Point (1, 2) lies on the circle $x^2 + y^2 + 2x + 2y - 11 = 0$, because coordinates of point (1, 2) satisfy the equation $x^2 + y^2 + 2x + 2y - 11 = 0$

Now, $x^2 + y^2 - 4x - 6y - 21 = 0$ ...(i)

$x^2 + y^2 + 2x + 2y - 11 = 0$ ...(ii)

$3x + 4y + 5 = 0$ ...(iii)

From (i) and (iii),

$$x^2+\left(-\frac{3x+5}{4}\right)^2-4x-6\left(-\frac{3x+5}{4}\right)-21=0$$

$\Rightarrow\ 16x^2+9x^2+30x+25-64x+72x+120-336=0$

$\Rightarrow\ 25x^2+38x-191=0$ ...(iv)

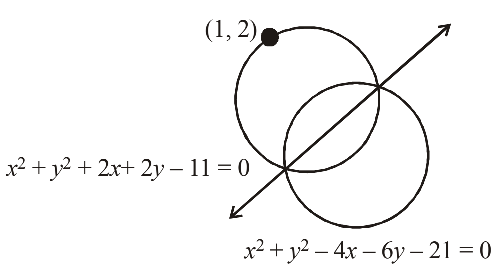
From (ii) and (iii),

$$x^2+\left(-\frac{3x+5}{4}\right)^2+2x+2\left(-\frac{3x+5}{4}\right)-11=0$$

$\Rightarrow\ 16x^2+9x^2+30x+25+32x-24x-40-176=0$

$\Rightarrow\ 25x^2+38x-191=0$ ...(v)

Thus we get the same equation from (ii) and (iii) as we get from equation (i) and (iii). Hence the point of intersections of (ii) and (iii) will be same as the point of intersections of (i) and (iii). Therefore the circle (ii) passing through the point of intersection of circle(i) and point (1, 2) also as shown in the figure.



Hence equation(ii) i.e.

$x^2+y^2+2x+2y-11=0$ is the equation of required circle.

70. **(b)** Given circle whose diametric end points are (1,0) and (0,1) will be of smallest radius. Equation of this smallest circle is

$(x-1)(x-0)+(y-0)(y-1)=0$

$\Rightarrow\ x^2+y^2-x-y=0$

71. **(a)** If the two circles touch each other and centre (0, 0) of $x^2+y^2=c^2$ is lies on circle $x^2+y^2=ax$ then they must touch each other internally.

So, $\frac{|a|}{2}=c-\frac{|a|}{2}\ \Rightarrow |a|=c$

72. **(a)** Given equation of circle is

$x^2+y^2-4x-8y-5=0$

Centre = (2, 4), Radius = $\sqrt{4+16+5}=5$

Given circle is intersecting the line $3x-4y=m,$ at two distinct points.

$\Rightarrow$ length of perpendicular from centre to the line < radius

$\Rightarrow \frac{|6-16-m|}{5}<5\quad \Rightarrow |10+m|<25$

$\Rightarrow -25<m+10<25 \Rightarrow -35<m<15$

73. **(a)** The given circles are

$S_1\equiv x^2+y^2+3x+7y+2p-5=0$....(1)

$S_2\equiv x^2+y^2+2x+2y-p^2=0$ ....(2)

$\therefore$ Equation of common chord PQ is

$S_1-S_2=0$ [From (i) and (ii)]

$\Rightarrow\ L\equiv x+5y+p^2+2p-5=0$

$\Rightarrow$ Equation of circle passing through P and Q is

$S_1+\lambda\ L=0$

$\Rightarrow (x^2+y^2+3x+7y+2p-5)$

$+\lambda\ (x+5y+p^2+2p-5)=0$

Given that it passes through (1, 1), therefore

$(7+2p)+\lambda\ (2p+p^2+1)=0$

$\Rightarrow \lambda=-\frac{2p+7}{(p+1)^2}$

which does not exist for p = – 1

74. **(a)** Given that $P(1,0), Q(-1,0)$

and $\frac{AP}{AQ}=\frac{BP}{BQ}=\frac{CP}{CQ}=\frac{1}{3}$

$\Rightarrow 3AP=AQ$

Let $A=(x,y)$ then

$3AP=AQ \Rightarrow 9AP^2=AQ^2$

$\Rightarrow 9(x-1)^2+9y^2=(x+1)^2+y^2$

$\Rightarrow 9x^2-18x+9+9y^2=x^2+2x+1+y^2$

$\Rightarrow 8x^2-20x+8y^2+8=0$

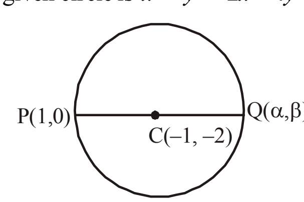
$\Rightarrow x^2+y^2-\frac{5}{3}x+1=0$ ....(1)

$\therefore$ A lies on the circle given by eq (1). As B and C also follow the same condition, they must lie on the same circle.

$\therefore$ Centre of circumcircle of ΔABC

= Centre of circle given by (1) = $\left(\frac{5}{4},0\right)$

75. **(c)** The given circle is $x^2+y^2+2x+4y-3=0$



Centre $(-g,-f)=(-1,-2)$

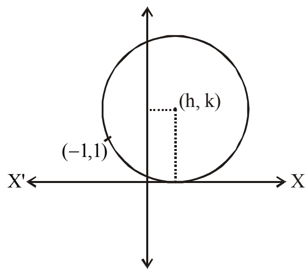
Let $Q\,(h,k)$ be the point diametrically opposite to the point $P(1,0)$,

then $\frac{1+h}{2}=-1$ and $\frac{0+k}{2}=-2$

$\Rightarrow$ h = –3, k = – 4

So, Q is (–3, –4)

76. **(d)** Equation of circle whose centre is (h, k) and touch the x-axis
i.e $(x-h)^2 + (y-k)^2 = k^2$



(radius of circle = k because circle is tangent to x-axis)

$\because$ Equation of circle passing through $(-1, 1)$

$\therefore (-1-h)^2 + (1-k)^2 = k^2$

$\Rightarrow 1 + h^2 + 2h + 1 + k^2 - 2k = k^2$

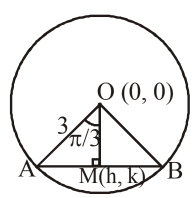
$\Rightarrow h^2 + 2h - 2k + 2 = 0$

$D \geq 0$

$\therefore (2)^2 - 4 \times 1.(-2k+2) \geq 0$

$\Rightarrow 4 - 4(-2k+2) \geq 0 \Rightarrow 1 + 2k - 2 \geq 0 \Rightarrow k \geq \dfrac{1}{2}$

77. **(d)** Given that centre of circle be $(0, 0)$ and radius is 3 unit
Let $M(h, k)$ be the mid point of chord AB where $\angle AOB = \dfrac{2\pi}{3}$



$\therefore \angle AOM = \dfrac{\pi}{3}$. Also $OM = 3\cos\dfrac{\pi}{3} = \dfrac{3}{2}$

$\Rightarrow \sqrt{h^2 + k^2} = \dfrac{3}{2} \Rightarrow h^2 + k^2 = \dfrac{9}{4}$

$\therefore$ Locus of (h, k) is $x^2 + y^2 = \dfrac{9}{4}$

78. **(d)** On solving we get point of intersection of $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ is $(1, -1)$ which is the centre of the circle

Area of circle $= \pi r^2 = 49\pi$

$\therefore$ radius = 7

$\therefore$ Equation is $(x-1)^2 + (y+1)^2 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$

79. **(d)** Let the centre variable circle be (α, β)

$\because$ It cuts the circle $x^2 + y^2 = p^2$ orthogonally

$\therefore$ Using $2g_1g_2 + 2f_1f_2 = c_1 + c_2$, we get

$2(-\alpha) \times 0 + 2(-\beta) \times 0 = c_1 - p^2$

$\Rightarrow c_1 = p^2$

Let equation of circle is $x^2 + y^2 - 2\alpha x - 2\beta y + p^2 = 0$

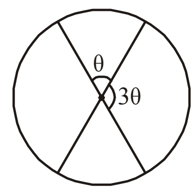
$\because$ It passes through (a, b)

$\Rightarrow a^2 + b^2 - 2\alpha a - 2\beta b + p^2 = 0$

$\therefore$ Locus of (α, β) is

$\therefore 2ax + 2by - (a^2 + b^2 + p^2) = 0$.

80. **(d)**



Given that area of one sector
= 3 × area of another sector

$\Rightarrow$ Angle at centre by one sector $= 3 \times$ angle at centre by another sector

Let one angle be θ then other $= 3\theta$

Clearly $\theta + 3\theta = 180 \Rightarrow \theta = 45^\circ$ (Linear pair)

$\therefore$ Angle between the diameters represented by pair of equation

$ax^2 + 2(a+b)xy + by^2 = 0$ is 45°

$\therefore$ Using $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$

we get, $\tan 45^\circ = \dfrac{2\sqrt{(a+b)^2 - ab}}{a+b}$

$\Rightarrow 1 = \dfrac{2\sqrt{a^2 + b^2 + ab}}{a+b}$

$\Rightarrow (a+b)^2 = 4\left(a^2 + b^2 + ab\right)$

$\Rightarrow a^2 + b^2 + 2ab = 4a^2 + 4b^2 + 4ab$

$\Rightarrow 3a^2 + 3b^2 + 2ab = 0$

81. **(b)** Given that

$s_1 = x^2 + y^2 + 2ax + cy + a = 0$

$s_2 = x^2 + y^2 - 3ax + dy - 1 = 0$

Equation of common chord PQ of circles s_1 and s_2 is given by $s_1 - s_2 = 0$

$\Rightarrow 5ax + (c-d)y + a + 1 = 0$

Given that $5x + by - a = 0$ passes through P and Q

$\therefore$ The two equations should represent the same line

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{a}{1} = \frac{c-d}{b} = \frac{a+1}{-a} \Rightarrow a+1 = -a^2$

$a^2 + a + 1 = 0$ $[\because D = -3]$

$\therefore$ No real value of a.

82. (b) Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ......(1)

It passes through (a, b)

$\therefore a^2 + b^2 + 2ga + 2fb + c = 0$ ......(2)

Circle (1) cuts $x^2 + y^2 = 4$ orthogonally

Two circles intersect orthogonally if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\therefore 2(g \times 0 + f \times 0) = c - 4 \Rightarrow c = 4$

$\therefore$ from (2) $a^2 + b^2 + 2ga + 2fb + 4 = 0$

$\therefore$ Locus of centre $(-g, -f)$ is

$a^2 + b^2 - 2ax - 2by + 4 = 0$

or $2ax + 2by = a^2 + b^2 + 4$

83. (d) Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(1)

Since it passes through (p, q)

$\therefore p^2 + q^2 + 2gp + 2fq + c = 0$ ....(2)

Circle (1) touches x-axis,

$\therefore g^2 - c = 0 \Rightarrow c = g^2$. From (2)

$p^2 + q^2 + 2gp + 2fq + g^2 = 0$ ....(3)

Let the other end of diameter through (p, q) be (h, k), then

$\frac{h+p}{2} = -g$ and $\frac{k+q}{2} = -f$

Putting value of g and f in (3), we get

$p^2 + q^2 + 2p\left(-\frac{h+p}{2}\right) + 2q\left(-\frac{k+q}{2}\right) + \left(\frac{h+p}{2}\right)^2 = 0$

$\Rightarrow h^2 + p^2 - 2hp - 4kq = 0$

$\therefore$ locus of (h, k) is

$x^2 + p^2 - 2xp - 4yq = 0$

$\Rightarrow (x-p)^2 = 4qy$

84. (d) Two diameters are along

$2x + 3y + 1 = 0$ and $3x - y - 4 = 0$

On solving we get centre $(1, -1)$

Circumference of circle $= 2\pi r = 10\pi$

$\therefore r = 5$.

Required circle is, $(x-1)^2 + (y+1)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 2x + 2y - 23 = 0$

85. (d) Solving $y = x$ and the circle

$x^2 + y^2 - 2x = 0$, we get

$x = 0, y = 0$ and $x = 1, y = 1$

$\therefore$ Extremities of diameter of the required circle are A (0, 0) and B (1, 1). Hence, the equation of circle is

$(x-0)(x-1) + (y-0)(y-1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

86. (b) $\because$ Given two circles intersect at two points

$\therefore |r_1 - r_2| < C_1C_2$

$\Rightarrow r - 3 < 5 \Rightarrow 0 < r < 8$ .... (1)

and $r_1 + r_2 > C_1C_2$, $r + 3 > 5 \Rightarrow r > 2$ .... (2)

From (1) and (2), $2 < r < 8$.

87. (d) Area of circle $= \pi r^2 = 154 \Rightarrow r = 7$

For centre, solving equation

$2x - 3y = 5$ & $3x - 4y = 7$ we get, $x = 1, y = -1$

$\therefore$ centre $= (1, -1)$

Equation of circle, $(x-1)^2 + (y+1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y = 47$

88. (c) Given equation of circle $x^2 + y^2 = 1 = (1)^2$

$\Rightarrow x^2 + y^2 = (y - mx)^2$

$\Rightarrow x^2 = m^2x^2 - 2\,mxy;$

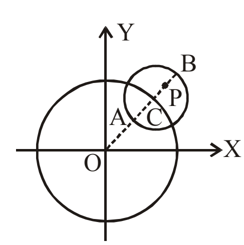
$\Rightarrow x^2(1 - m^2) + 2mxy = 0$. Which represents the pair of lines between which the angle is 45°.

$\therefore \tan 45 = \pm\frac{2\sqrt{m^2 - 0}}{1 - m^2} = \frac{\pm 2m}{1 - m^2};$

$\Rightarrow 1 - m^2 = \pm\, 2m \Rightarrow m^2 \pm 2m - 1 = 0$

$\Rightarrow m = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$.

89. (a) $\because$ The centre C of circle of radius 3 lies on circle of radius 5. Let $P(x, y)$ in the smaller circle.



we should have

$OA \le OP \le OB$

$\Rightarrow (5-3) \le \sqrt{x^2+y^2} \le 5+3$

$\Rightarrow 4 \le x^2+y^2 \le 64$

90. **(b)** Let the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since it passes through (0, 0) and (1, 0)

On putting these values, we get

$\Rightarrow c = 0$ and $g = -\frac{1}{2}$

Points (0, 0) and (1, 0) lie inside the circle $x^2+y^2=9$, so two circles touch internally

$\Rightarrow c_1c_2 = r_1 - r_2$

$\therefore \sqrt{g^2+f^2} = 3-\sqrt{g^2+f^2} \Rightarrow \sqrt{g^2+f^2} = \frac{3}{2}$

Squaring both side, we get

$\Rightarrow f^2 = \frac{9}{4}-\frac{1}{4} = 2 \qquad \therefore f = \pm\sqrt{2}$.

Hence, the centres of required circle are

$\left(\frac{1}{2}, \sqrt{2}\right)$ or $\left(\frac{1}{2}, -\sqrt{2}\right)$

91. **(c)** Let ABC be an equilateral triangle, whose median is AD.

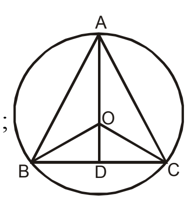
In equilateral triangle median is also altitude

So, $AD \perp BC$

Given $AD = 3a$.

Let $AB = BC = AC = x$.

In ΔABD, $AB^2 = AD^2 + BD^2$;



$\Rightarrow x^2 = 9a^2 + (x^2/4)$

$\frac{3}{4}x^2 = 9a^2 \Rightarrow x^2 = 12a^2$.

In ΔOBD, $OB^2 = OD^2 + BD^2$

$\Rightarrow r^2 = (3a-r)^2 + \frac{x^2}{4}$

$\Rightarrow r^2 = 9a^2 - 6ar + r^2 + 3a^2$

$\Rightarrow 6ar = 12a^2$

$\Rightarrow r = 2a$

So equation of circle is $x^2 + y^2 = 4a^2$

92. **(a)** $L_1 : y = m_1(x+1) + \frac{1}{m_1}$ [Tangent to $y^2 = 4(x+1)$]

$L_2 : y = m_2(x+2) + \frac{2}{m_2}$ [Tangent to $y^2 = 8(x+2)$]

$m_1^2(x+1) - ym_1 + 1 = 0 \qquad$ ...(i)

$m_2^2(x+2) - ym_2 + 2 = 0 \qquad$ ...(ii)

$\because m_2 = -\frac{1}{m_1} \qquad (\because L_1 \perp L_2)$

[From (ii)]

$\Rightarrow 2m_1^2 + ym_1 + (x+2) = 0 \qquad$ ...(iii)

From (i) and (iii),

$\frac{x+1}{2} = \frac{-y}{y} = \frac{1}{x+2} \Rightarrow x+3 = 0$

93. **(d)** Circle passes through $A(0, 1)$ and $B(2, 4)$. So its centre is the point of intersection of perpendicular bisector of AB and normal to the parabola at (2, 4).

Perpendicular bisector of AB;

$y - \frac{5}{2} = -\frac{2}{3}(x-1) \Rightarrow 4x + 6y = 19 \qquad$ ...(i)

Equation of normal to the parabola at (2, 4) is,

$y - 4 = -\frac{1}{4}(x-2) \Rightarrow x + 4y = 18 \qquad$ ...(ii)

$\therefore$ From (i) and (ii), $x = -\frac{16}{5}$, $y = \frac{53}{10}$

$\therefore$ Centre of the circle is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

94. **(b)** Equation tangent to parabola $y^2 = 4x$ with slope m be:

$y = mx + \frac{1}{m} \qquad$ ...(i)

$\because$ Equation of tangent to $x^2 = 4y$ with slope m be :

$y = mx - am^2 \qquad$ ...(ii)

From eq. (i) and (ii),

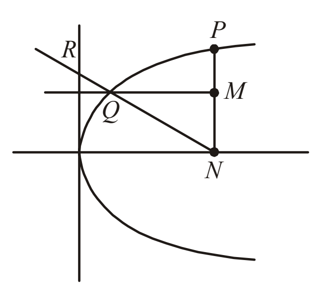
$\frac{1}{m} = -m^2 \Rightarrow m = -1$

$\therefore$ Equation tangent : $x + y + 1 = 0$

It is tangent to circle $x^2 + y^2 = c^2$

$\Rightarrow c = \frac{1}{\sqrt{2}}$

95. **(c)**



$\because y^2 = 12x$

$\therefore a = 3$

Let $P(at^2, 2at)$

$\Rightarrow N(at^2, 0) \Rightarrow M(at^2, at)$

$\because$ Equation of QM is $y = at$

So, $y^2 = 4ax \Rightarrow x = \dfrac{at^2}{4}$

$\Rightarrow Q\left(\dfrac{at^2}{4}, at\right)$

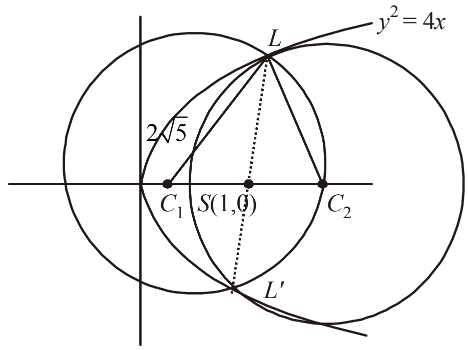
$\Rightarrow$ Equation of QN is $y = \dfrac{-4}{3t}(x - at^2)$

$\because$ QN passes through $\left(0, \dfrac{4}{3}\right)$, then

$\dfrac{4}{3} = -\dfrac{4}{3t}(-at^2) \Rightarrow at = 1 \Rightarrow t = \dfrac{1}{3}$

Now, $MQ = \dfrac{3}{4}at^2 = \dfrac{1}{4}$ and $PN = 2at = 2$

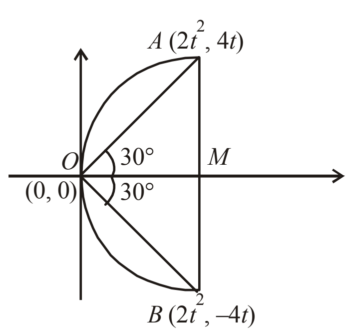
96. **(b)**



Distance between the centres

$= C_1C_2 = 2C_1S = 2\sqrt{20-4} = 8.$

97. **(c)** Let $A = (2t^2, 4t)$ and $B = (2t^2, -4t)$



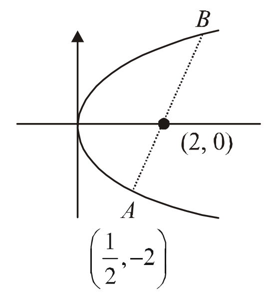
For equilateral triangle $(\angle AOM = 30°)$

$\tan 30° = \dfrac{4t}{2t^2} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{4t}{2t^2} \Rightarrow t = 2\sqrt{3}$

Area $= \dfrac{1}{2} \cdot 8(2\sqrt{3}) \cdot 2 \cdot 24 = 192\sqrt{3}.$

98. **(b)** Let parabola $y^2 = 8x$ at point $\left(\dfrac{1}{2}, -2\right)$ is $(2t^2, 4t)$

$\Rightarrow \quad t = \dfrac{-1}{2}$



Parameter of other end of focal chord is 2

So, coordinates of B is (8, 8)

$\Rightarrow$ Equation of tangent at B is $8y - 4(x + 8) = 0$

$\Rightarrow \quad 2y - x = 8$

$\Rightarrow \quad x - 2y + 8 = 0$

99. **(a)** Let point P be $(2t, t^2)$ and Q be (h, k)

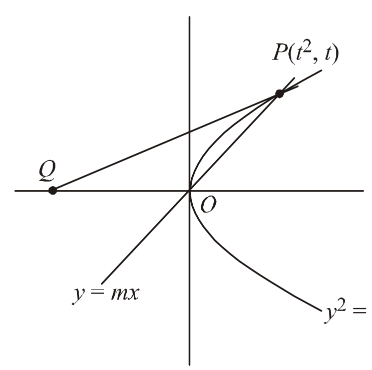
Using section formula,

$h = \dfrac{2t}{3}, k = \dfrac{-2 + t^2}{3}$

Hence, locus is $3k + 2 = \left(\dfrac{3h}{2}\right)^2$

$\Rightarrow \quad 9x^2 = 12y + 8$

100. **(0.5)** Let the coordinates of $P = P(t^2, t)$



Tangent at $P(t^2, t)$ is $ty = \dfrac{x + t^2}{2}$

$\Rightarrow \quad 2ty = x + t^2$

$Q(-t^2, 0), O(0, 0)$

$\therefore$ Area of $\Delta OPQ = \dfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = 4$

$\Rightarrow |t|^3 = 8$

$t = \pm 2 \ (t > 0)$

$\therefore$ $4y = x + 4$ is a tangent

$\therefore$ P is $(4, 2)$

Now, $y = mx \quad \therefore m = \dfrac{1}{2}$

101. **(c)** $y = mx + 4$ ...(i)

Tangent of $y^2 = 4x$ is

$\Rightarrow y = mx + \dfrac{1}{m}$ ...(ii)

[$\because$ Equation of tangent of $y^2 = 4ax$ is $y = mx + \dfrac{a}{m}$]

From (i) and (ii)

$4 = \dfrac{1}{m} \Rightarrow m = \dfrac{1}{4}$

So, line $y = \dfrac{1}{4}x + 4$ is also tangent to parabola

$x^2 = 2by$, so solve both equations.

$x^2 = 2b\left(\dfrac{x+16}{4}\right)$

$\Rightarrow 2x^2 - bx - 16b = 0$

$\Rightarrow D = 0$ [For tangent]

$\Rightarrow b^2 - 4 \times 2 \times (-16b) = 0$

$\Rightarrow b^2 + 32 \times 4b = 0$

$b = -128, b = 0$ (not possible)

102. **(c)** Tangent to the curve $y = (x-2)^2 - 1$ at any point (h, k) is,

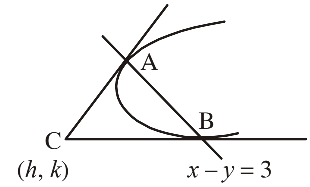
$\Rightarrow \dfrac{1}{2}(y + k) = (x-2)(h-2) - 1$

$\Rightarrow \dfrac{y+k}{2} = xh - 2x - 2h + 3$

$\Rightarrow (2h-4)x - y - 4h + 6 - k = 0$

Given line, $x - y - 3 = 0$

$\Rightarrow \dfrac{2h-4}{1} = \dfrac{4h-6+k}{3} = 1$



$\Rightarrow h = \dfrac{5}{2}, k = -1$

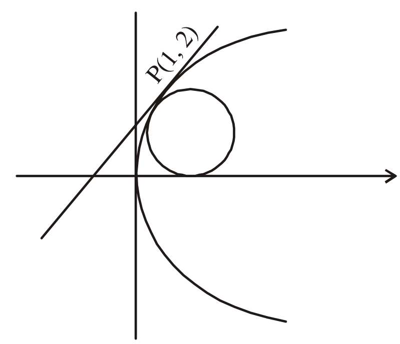
103. **(d)** Equation of tangent on $y^2 = 4\sqrt{2}x$ is $yt = x + \sqrt{2}t^2$

This is also tangent on circle

$\therefore \left|\dfrac{\sqrt{2}t^2}{\sqrt{1+t^2}} = 1\right| \Rightarrow 2t^4 = 1 + t^2 \Rightarrow t^2 = 1$

Hence, equation is $\pm y = x + \sqrt{2} \Rightarrow |c| = \sqrt{2}$

104. **(d)** The circle and parabola will have common tangent at P (1, 2).



So, equation of tangent to parabola is,

$y \times (2) = \dfrac{4(x+1)}{2} \Rightarrow 2y = 2x + 2 \Rightarrow y = x + 1$

Let equation of circle (by family of circles) is

$(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$

$\Rightarrow c \equiv (x-1)^2 + (y-2)^2 + \lambda(x - y + 1) = 0$

$\because$ circles touches x-axis.

$\therefore$ y-coordinate of centre = radius

$\Rightarrow c = x^2 + y^2 + (\lambda - 2)x + (-\lambda - 4)y + (\lambda + 5) = 0$

$\dfrac{\lambda+4}{2} = \sqrt{\left(\dfrac{\lambda-2}{2}\right)^2 + \left(\dfrac{-\lambda-4}{2}\right)^2 - (\lambda+5)}$

$\Rightarrow \dfrac{\lambda^2 - 4\lambda + 4}{4} = \lambda + 5 \Rightarrow \lambda^2 - 4\lambda + 4 = 4\lambda + 20$

$\Rightarrow \lambda^2 - 8\lambda - 16 = 0 \Rightarrow \lambda = 4 \pm 4\sqrt{2}$

$\Rightarrow \lambda = 4 - 4\sqrt{2}$ ($\because \lambda = 4 + 4\sqrt{2}$ forms bigger circle)

Hence, centre of circle $\left(2\sqrt{2} - 2, 4 - 2\sqrt{2}\right)$ and radius

$= 4 - 2\sqrt{2}$

$\therefore$ area $= \pi\left(4 - 2\sqrt{2}\right)^2 = 8\pi\left(3 - 2\sqrt{2}\right)$

105. **(a)** $\because$ $y^2 = 16x$

$\Rightarrow a = 4$

One end of focal chord of the parabola is at (1, 4)

y – coordinate of focal chord is 2at

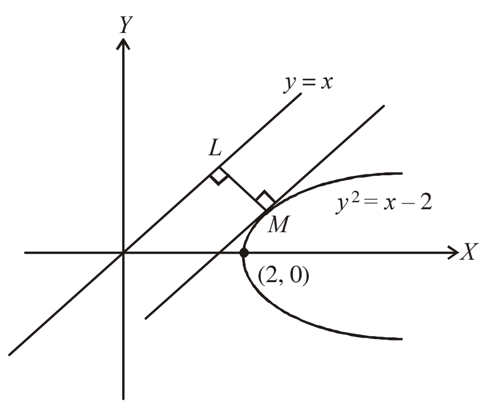
$\therefore 2at = 4$

$\Rightarrow t = \dfrac{1}{2}$

Hence, the required length of focal chord

$= a\left(t + \dfrac{1}{t}\right)^2 = 4 \times \left(2 + \dfrac{1}{2}\right)^2 = 25$

106. **(c)** The shortest distance between line $y = x$ and parabola = the distance LM between line $y = x$ and tangent of parabola having slope 1.



Let equation of tangent of parabola having slope 1 is,

$y = m(x - 2) + \dfrac{a}{m}$

Here $m = 1$ and $a = \dfrac{1}{4}$

$\therefore$ equation of tangent is: $y = x - \dfrac{7}{4}$

Distance between the line $y - x = 0$ and $y - x + \dfrac{7}{4} = 0$

$= \left|\dfrac{\frac{7}{4} - 0}{\sqrt{1^2 + 1^2}}\right| = \dfrac{7}{4\sqrt{2}}$

107. **(d)** Since (a, b) touches the given ellipse $4x^2 + y^2 = 8$

$\therefore$ $4a^2 + b^2 = 8$ ...(i)

Equation of tangent on the ellipse at the point A (1, 2) is:

$4x + 2y = 8 \Rightarrow 2x + y = 4 \Rightarrow y = -2x + 4$

But, also equation of tangent at $P(a, b)$ is:

$4ax + by = 8 \Rightarrow y = \dfrac{-4a}{b} + \dfrac{8}{b}$.

Since, tangents are perpendicular to each other.

$\Rightarrow \dfrac{-4a}{b} = \dfrac{-1}{2} \Rightarrow b = 8a$ ...(ii)

from (1) & (2) we get:

$\Rightarrow a = \pm \dfrac{2}{\sqrt{34}} \Rightarrow a^2 = \dfrac{2}{17}$

108. **(c)** To find intersection point of $x^2 + y^2 = 5$ and $y^2 = 4x$, substitute $y^2 = 4x$ in $x^2 + y^2 = 5$, we get

$x^2 + 4x - 5 = 0 \Rightarrow x^2 + 5x - x - 5 = 0$

$\Rightarrow x(x + 5) - 1(x + 5) = 0$

$\therefore x = 1, -5$

Intersection point in 1st quadrant be (1, 2).

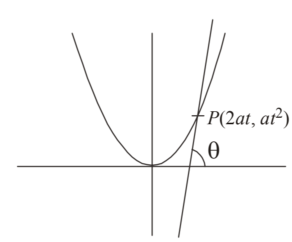
Now, equation of tangent to $y^2 = 4x$ at (1, 2) is

$y \times 2 = 2(x + 1) \Rightarrow y = x + 1$

$\Rightarrow x - y + 1 = 0$ ...(i)

Hence, $\left(\dfrac{3}{4}, \dfrac{7}{4}\right)$ lies on (i)

109. **(c)** $x^2 = 8y$



Then, equation of tangent at P

$tx = y + at^2$

$\Rightarrow y = tx - at^2$

Then, slope $t = \tan\theta$

Now, $y = \tan\theta x - 2\tan^2\theta$

$\Rightarrow \cot\theta y = x - 2\tan\theta$

$x = y\cot\theta + 2\tan\theta$

110. **(c)** Equation of a tangent to parabola $y^2 = 4x$ is:

$y = mx + \dfrac{1}{m}$

This line is a tangent to $xy = 2$

$\therefore \quad x\left(mx+\frac{1}{m}\right) = 2 \Rightarrow mx^2+\frac{1}{m}x-2 = 0$

$\because$ Tangent is common for parabola and hyperbola.

$\therefore \quad D=\left(\frac{1}{m}\right)^2 - 4\cdot m\cdot(-2) = 0$

$\frac{1}{m^2}+8m = 0$

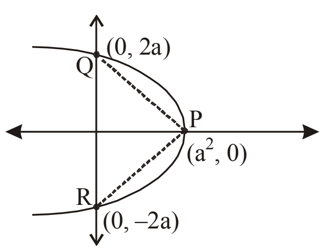
$1+8m^3=0$

$m^3 = -\frac{1}{8} \Rightarrow m = -\frac{1}{2}$

$\therefore$ Equation of common tangent: $y = -\frac{1}{2}x-2$

$\Rightarrow \quad 2y=-x-4 \Rightarrow x+2y+4=0$

111. **(d)** $y^2 = -4(x-a^2)$



Area $= \frac{1}{2}(4a)(a^2) = 2a^3$

Since $2a^3 = 250 \Rightarrow a = 5$

112. **(a, b, c, d)**

Normal to $y^2 = 8ax$ is

$y = mx - 4am - 2am^3$ ...(i)

and normal to $y^2 = 4b(x-c)$ with slope m is

$y = m(x-c) - 2bm - bm^3$ ...(ii)

Since, both parabolas have a common normal.

$\therefore \quad 4am + 2am^3 = cm + 2bm + bm^3$

$\Rightarrow \quad 4a + 2am^2 = c + 2b + bm^2$ or $m = 0$

$\Rightarrow \quad (4a - c - 2b) = (b - 2a)m^2$

or (X-axis is common normal always)

Since, x-axis is a common normal. Hence all the options are correct for $m = 0$.

113. **(d)** Let intersection points be $P(x_1, y_1)$ and $Q(x_2, y_2)$

The given equations

$x^2 = 4y$ ...(i)

$x - \sqrt{2}y + 4\sqrt{2} = 0$ ...(ii)

Use eqn (i) in eqn (ii)

$x - \sqrt{2}\frac{x^2}{4} + 4\sqrt{2} = 0$

$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$

$x_1 + x_2 = 2\sqrt{2}, x_1x_2 = -16, (x_1 - x_2)^2 = 8 + 64 = 72$

Since, points P and Q both satisfy the equations (ii), then

$x_1 - \sqrt{2}y_1 + 4\sqrt{2} = 0$

$x_1 - \sqrt{2}y_2 + 4\sqrt{2} = 0$

$(x_2 - x_1) = \sqrt{2}(y_2 - y_1) \Rightarrow (x_2 - x_1)^2 = 2(y_2 - y_1)^2$

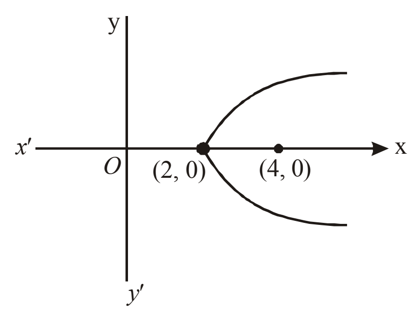
$\Rightarrow \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_2 - x_1)^2 + \frac{(x_2 - y_1)^2}{2}}$

$= |x_2 - x_1| \cdot \frac{\sqrt{3}}{\sqrt{2}} = 6\sqrt{2} \times \frac{\sqrt{3}}{\sqrt{2}} = 6\sqrt{3}$

Hence, length of chord $= 6\sqrt{3}$.

114. **(b)** Since, vertex and focus of given parabola is (2, 0) and (4, 0) respectively



Then, equation of parabola is

$(y-0)^2 = 4 \times 2(x-2)$

$\Rightarrow y^2 = 8x - 16$

Hence, the point (8, 6) does not lie on given parabola.

115. **(b)** Since, the equation of tangent to parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ...(i)

The line (i) is also the tangent to circle

$x^2 + y^2 - 6x = 0$

Then centre of circle = (3, 0)

radius of circle = 3

The perpendicular distance from centre to tangent is equal to the radius of circle

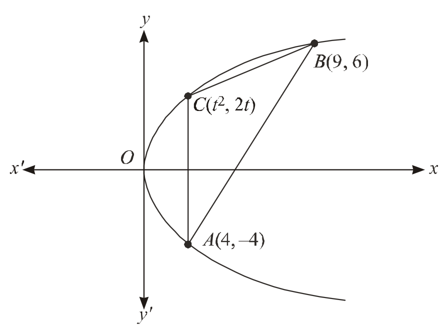
$\therefore \dfrac{\left|3m+\dfrac{1}{m}\right|}{\sqrt{1+m^2}}=3 \Rightarrow \left(3m+\dfrac{1}{m}\right)^2=9\left(1+m^2\right)$

$\Rightarrow m=\pm\dfrac{1}{\sqrt{3}}$

Then, from equation (i): $y=\pm\dfrac{1}{\sqrt{3}}x\pm\sqrt{3}$

Hence, $\sqrt{3}y=x+3$ is one of the required common tangent.

116. (a)



Let the coordinates of C is $(t^2, 2t)$.
Since, area of ΔACB

$$=\frac{1}{2}\left\|\begin{matrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{matrix}\right\|$$

$$=\frac{1}{2}|t^2(6+4)-2t(9-4)+1(-36-24)|$$

$$=\frac{1}{2}|10t^2-10t-60|$$

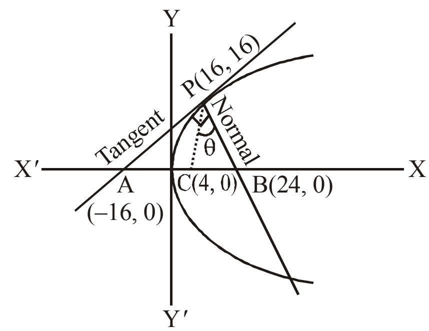
$$=5|t^2-t-6|$$

$$=5\left|\left(t-\frac{1}{2}\right)^2-\frac{25}{4}\right| \qquad [\text{Here, } t\in(0,3)]$$

For maximum area, $t=\dfrac{1}{2}$

Hence, maximum area $=\dfrac{125}{4}=31\dfrac{1}{4}$ sq. units

117. (a) Equation of tnagent at P(16, 16) is given as:
$x-2y+16=0$



Slope of PC $(m_1)=\dfrac{4}{3}$

Slope of PB $(m_2)=-2$

Hence, $\tan\theta=\left|\dfrac{m_1-m_2}{1+m_1\cdot m_2}\right|=\left|\dfrac{\dfrac{4}{3}+2}{1-\dfrac{4}{3}\cdot 2}\right|$

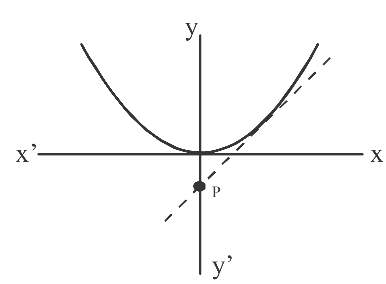
$\Rightarrow \tan\theta=2$

118. (a) Equation of the chord of contact PQ is given by:
$T=0$
or $T \equiv yy_1-4(x+x_1)$, where $(x_1, y_1)\equiv(-8, 0)$
$\therefore$ Equation becomes: $x=8$
& Chord of contact is $x=8$
$\therefore$ Coordinates of point P and Q are $(8, 8)$ and $(8, -8)$
and focus of the parabola is $F(2, 0)$

$\therefore$ Area of triangle $PQF=\dfrac{1}{2}\times(8-2)\times(8+8)=48$ sq. units

119. (c) $c=-29m-9m^3$
$a=2$
Given $(at^2-a)^2+4a^2t^2=64$
$\Rightarrow (a(t^2+1))=8$
$\Rightarrow t^2+1=4$
$\Rightarrow t^2=3$
$\Rightarrow t=\sqrt{3}$
$\therefore c=2at(2+t^2)=2\sqrt{3}(5)$
$|c|=10\sqrt{3}$

120. (c)



Tangent to $x^2+y^2=4$ is

$y=mx\pm 2\sqrt{1+m^2}$

Also, $x^2=4y$

$x^2=4mx+8\sqrt{1+m^2}$ or $x^2=4mx-8\sqrt{1+m^2}$

For D = 0

we have; $16m^2+4.8\sqrt{1+m^2}=0$

$\Rightarrow\ m^2+2\sqrt{1+m^2}=0$

$\Rightarrow\ m^2=-2\sqrt{1+m^2}$

$\Rightarrow\ m^4=4+4m^2$

$\Rightarrow\ m^4-4m^2-4=0$

$\Rightarrow\ m^2=\dfrac{4\pm\sqrt{16+16}}{2}$

$\Rightarrow\ m^2=\dfrac{4\pm4\sqrt{2}}{2}$

$\Rightarrow\ m^2=2+2\sqrt{2}$

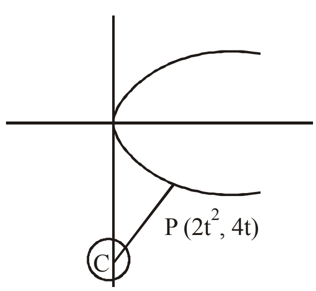
121. **(c)** Minimum distance $\Rightarrow$ perpendicular distance

Eq^n of normal at $p(2t^2, 4t)$

$y=-tx+4t+2t^3$

It passes through C(0, –6)

$t^3+2t+3=0\Rightarrow t=-1$



Centre of new circle $=P(2t^2, 4t)=P(2,-4)$

Radius = PC $=\sqrt{(2-0)^2+(-4+6)^2}=2\sqrt{2}$

$\therefore$ Equation of circle is :

$(x-2)^2+(y+4)=\left(2\sqrt{2}\right)^2$

$\Rightarrow\ x^2+y^2-4x+8y+12=0$

122. **(a)** $t_1=-t-\dfrac{2}{t}$

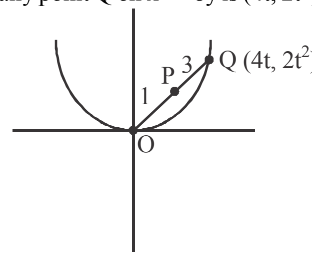
$t_1^2=t^2+\dfrac{4}{t^2}+4$

$t^2+\dfrac{4}{t^2}\geq 2\sqrt{t^2.\dfrac{4}{t^2}}=4$

Minimum value of $t_1^2=8$

123. **(b)** Let P(h, k) divides OQ in the ratio 1 : 3

Let any point Q on $x^2=8y$ is $(4t, 2t^2)$.



Then by section formula

$\Rightarrow\ k=\dfrac{t^2}{2}$ and $h=t$

$\Rightarrow\ 2k=h^2$

Required locus of P is $x^2=2y$

124. **(d)** Let $P(-at_1^2, 2at_1), Q(-at_1^2, -2at_1)$ and R (h, k)

By using section formula, we have

$h=-at_1^2,\ k=\dfrac{-2at_1}{3}$

$k=-\dfrac{2at_1}{3}$

$\Rightarrow\ 3k=-2at_1$

$\Rightarrow\ 9k^2=4a^2t_1^{\,2}=4a(-h)$

$\Rightarrow\ 9k^2=-4ah$

$\Rightarrow\ 9k^2=-4h\Rightarrow 9y^2=-4x$

125. **(c)** Given parabolas are

$y^2=4x$ ...(1)

$x^2=-32y$ ...(2)

Let m be slope of common tangent

Equation of tangent of parabola (1)

$y=mx+\dfrac{1}{m}$ ...(i)

Equation of tangent of parabola (2)

$y=mx+8m^2$ ...(ii)

(i) and (ii) are identical

$\Rightarrow\ \dfrac{1}{m}=8m^2\Rightarrow m^3=\dfrac{1}{8}\Rightarrow \boxed{m=\dfrac{1}{2}}$

ALTERNATIVE METHOD:

Let tangent to $y^2=4x$ be $y=mx+\dfrac{1}{m}$

Since this is also tangent to $x^2=-32y$

$\therefore\ x^2=-32\left(mx+\dfrac{1}{m}\right)$

$\Rightarrow\ x^2+32mx+\dfrac{32}{m}=0$

Now, D = 0

$(32)^2-4\left(\frac{32}{m}\right)=0$

$\Rightarrow m^3=\frac{4}{32} \quad \Rightarrow m=\frac{1}{2}$

126. (a) Equation of parabola, $y^2 = 6x$

$\Rightarrow y^2 = 4\times\frac{3}{2}x$

$\therefore$ Focus $= \left(\frac{3}{2},0\right)$

Let equation of chord passing through focus be

$ax+by+c = 0$ ...(1)

Since chord is passing through $\left(\frac{3}{2},0\right)$

$\therefore$ Put $x = \frac{3}{2}$, $y = 0$ in eqn (1), we get

$\frac{3}{2}a+c = 0$

$\Rightarrow c = -\frac{3}{2}a$ ...(2)

distance of chord from origin is $\frac{\sqrt{5}}{2}\ \frac{\sqrt{5}}{2}$

$= \left|\frac{a(0)+b(0)+c}{\sqrt{a^2+b^2}}\right| = \frac{c}{\sqrt{a^2+b^2}}$

Squaring both sides

$\frac{5}{4} = \frac{c^2}{a^2+b^2}$

$\Rightarrow a^2 + b^2 = \frac{4}{5}c^2$

Putting value of c from (2), we get

$a^2 + b^2 = \frac{4}{5}\times\frac{9}{4}a^2$

$b^2 = \frac{9}{5}a^2 - a^2 = \frac{4}{5}a^2$

$\frac{a^2}{b^2} = \frac{5}{4}, \frac{a}{b} = \pm\frac{\sqrt{5}}{2}$

Slope of chord, $\frac{dy}{dx} = -\frac{a}{b} = -\left(\frac{\pm\sqrt{5}}{2}\right) = \mp\frac{\sqrt{5}}{2}$

127. (d) The locus of the point of intersection of tangents to the parabola $y^2 = 4ax$ inclined at an angle α to each other is

$\tan^2\alpha\ (x + a)^2 = y^2 - 4ax$

Given equation of Parabola $y^2 = 4x$ $\{a = 1\}$

Point of intersection $(-2, -1)$

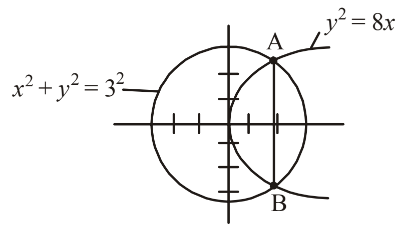
$\tan^2\alpha\ (-2 + 1)^2 = (-1)^2 - 4 \times 1 \times (-2)$

$\Rightarrow \tan^2\alpha = 9$

$\Rightarrow \tan\alpha = \pm 3$

$\Rightarrow |\tan\alpha| = 3$

128. (c)



We have

$x^2 + (8x) = 9$

$x^2 + 9x - x - 9 = 0$

$x(x + 9) - 1(x + 9) = 0$

$(x + 9)(x - 1) = 0$

$x = -9, 1$

for $x = 1$, $y = \pm 2\sqrt{2x} = \pm 2\sqrt{2}$

L_1 = Length of AB $= \sqrt{(2\sqrt{2}+2\sqrt{2})^2+(1-1)^2} = 4\sqrt{2}$

L_2 = Length of latus rectum $= 4a = 4 \times 2 = 8$

$L_1 < L_2$

129. (b) Let common tangent be

$y = mx + \frac{\sqrt{5}}{m}$

Since, perpendicular distance from centre of the circle to the common tangent is equal to radius of the circle, therefore

$\frac{\frac{\sqrt{5}}{m}}{\sqrt{1+m^2}} = \sqrt{\frac{5}{2}}$

On squaring both the side, we get

$m^2(1+m^2) = 2$

$\Rightarrow m^4+m^2-2=0$

$\Rightarrow (m^2+2)(m^2-1)=0$

$\Rightarrow m=\pm 1 \quad (\because m^2 \neq -2)$

$y = \pm\left(x+\sqrt{5}\right)$, both statements are correct as $m = \pm 1$ satisfies the given equation of statement-2.

130. (b) We know that point of intersection of the normal to the parabola $y^2 = 4ax$ at the ends of its latus rectum is $(3a, 0)$

Hence required point of intersection = (3, 0)

131. (b) Both statements are true and statement-2 is the correct explanation of statement-1

$\therefore$ The straight line $y = mx + \frac{a}{m}$ is always a tangent to the parabola $y^2 = 4ax$ for any value of m.

The co-ordinates of point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

Now, required radius $= OB = \sqrt{9+16} = \sqrt{25} = 5$

132. (a) Ellipse is $\frac{x^2}{16}+\frac{y^2}{3}=1$

Now, equation of normal at (2, 3/2) is

$$\frac{16x}{2}-\frac{3y}{3/2}=16-3$$

$\Rightarrow\ 8x-2y=13$

$\Rightarrow\ y=4x-\frac{13}{2}$

Let $y=4x-\frac{13}{2}$ touches a parabola

$y^2=4ax$.

We know, a straight line $y=mx+c$ touches a parabola $y^2=4ax$ if $a-mc=0$

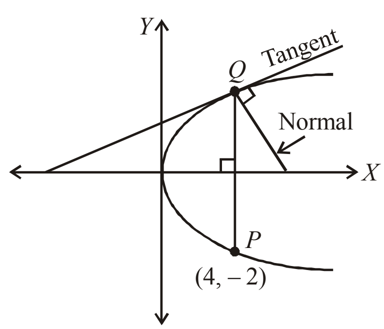
$\therefore\ a-(4)\left(-\frac{13}{2}\right)=0\Rightarrow a=-26$

Hence, required equation of parabola is

$y^2=4(-26)x=-104x$

133. (a) Point P is $(4, -2)$ and $PQ\perp x$-axis

So, $Q=(4, 2)$



Equation of tangent at (4, 2) is

$yy_1=\frac{1}{2}(x+x_1)$

$\Rightarrow 2y=\frac{1}{2}(x+2)\Rightarrow 4y=x+2$

$\Rightarrow y=\frac{x}{4}+\frac{1}{2}$

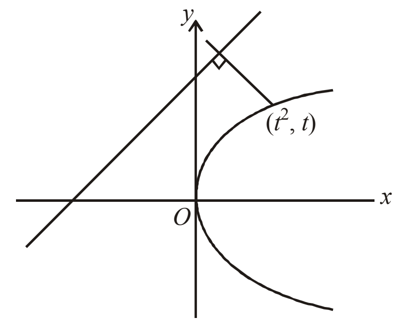
So, slope of tangent $=\frac{1}{4}$

$\therefore$ Slope of normal $=-4$

134. (d) Both the given statements are true.

Statement - 2 is not the correct explanation for statement - 1.

135. (a)



Let (t^2, t) be point on parabola from that line have shortest distance.

$\therefore$ Distance $=\left|\frac{t^2-t+1}{\sqrt{2}}\right|$

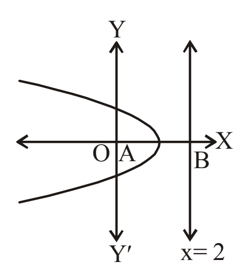
$=\frac{1}{\sqrt{2}}\left[\left(t-\frac{1}{2}\right)^2+\frac{3}{4}\right]$

Distance is minimum when $t-\frac{1}{2}=0$

$\therefore$ Shortest distance $=\frac{1}{\sqrt{2}}\left[0+\frac{3}{4}\right]=\frac{3\sqrt{2}}{8}$

136. (b) We know that the locus of perpendicular tangents is directrix i.e., $x=-a; x=-1$

137. (b) We know that vertex of a parabola is the mid point of focus and the point

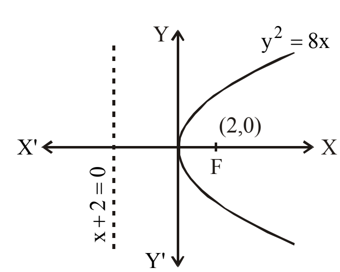


where directrix meets the axis of the parabola.

Given that focus is $O(0, 0)$ and directrix meets the axis at $B(2, 0)$

$\therefore$ Vertex of the parabola is $\left(\frac{0+2}{2},0\right)=(1, 0)$

138. (b) Given that parabola $y^2=8x$



We know that the locus of point of intersection of two perpendicular tangents to a parabola is its directrix.
Point must be on the directrix of parabola

$\because$ Equation of directrix $x+2=0$

$\Rightarrow x=-2$

Hence the point is (–2, 0)

139. (a) Given that family of parabolas is

$$y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$$

$$\Rightarrow y=\frac{a^3}{3}\left(x^2+\frac{3}{2a}x+\frac{9}{16a^2}\right)-\frac{3a}{16}-2a$$

$$\Rightarrow y+\frac{35a}{16}=\frac{a^3}{3}\left(x+\frac{3}{4a}\right)^2$$

$\therefore$ Vertex of parabola is $\left(\frac{-3}{4a},\frac{-35a}{16}\right)$

To find locus of this vertex,

$$x=\frac{-3}{4a} \text{ and } y=\frac{-35a}{16}$$

$$\Rightarrow a=\frac{-3}{4x} \text{ and } a=-\frac{16y}{35}$$

$$\Rightarrow \frac{-3}{4x}=\frac{-16y}{35} \quad \Rightarrow 64xy=105$$

$\Rightarrow xy=\frac{105}{64}$ which is the required equation of locus.

140. (a) Given $P=(1,0)$, let $Q=(h,k)$

Since Q lies on $y^2=8x$

$\therefore K^2=8h$ ...(i)

Let (α,β) be the midpoint of PQ

$$\therefore \alpha=\frac{h+1}{2},\ \beta=\frac{k+0}{2}$$

$2\alpha-1=h \qquad 2\beta=k.$

Putting value of h and k in (i)

$(2\beta)^2=8(2\alpha-1) \Rightarrow \beta^2=4\alpha-2$

$\Rightarrow y^2-4x+2=0$.

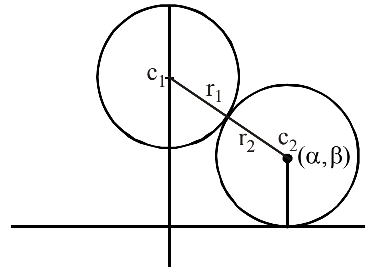
141. (d) Equation of circle with centre (0, 3) and radius 2 is

$x^2+(y-3)^2=4$

Let locus of the centre of the variable circle is (α,β)

$\because$ It touches x - axis.

$\therefore$ It's equation is $(x-\alpha)^2+(y-\beta)^2=\beta^2$



Circle touch externally $\Rightarrow c_1c_2=r_1+r_2$

$\therefore \sqrt{\alpha^2+(\beta-3)^2}=2+\beta$

$\alpha^2+(\beta-3)^2=\beta^2+4+4\beta$

$\alpha^2+\beta^2-6\beta+9=\beta^2+4+4\beta$

$\Rightarrow \alpha^2=10(\beta-1/2)$

$\therefore$ Locus is $x^2=10\left(y-\frac{1}{2}\right)$

Which is equation of parabola.

142. (d) Solving equations of parabolas

$y^2=4ax$ and $x^2=4ay$, we get (0, 0) and ($4a$, $4a$)

Putting in the given equation of line

$2bx+3cy+4d=0$, we get

$d=0$ and $2b+3c=0$

$\Rightarrow d^2+(2b+3c)^2=0$

143. (b) Equation of the normal to a parabola $y^2=4bx$ at point $\left(bt_1^2,2bt_1\right)$ is

$y=-t_1x+2bt_1+bt_1^3$

Given that, it also passes through $\left(bt_2^2,2bt_2\right)$ then

$2bt_2=-t_1bt_2^2+2bt_1+bt_1^3$

$\Rightarrow 2t_2-2t_1=-t_1\left(t_2^2-t_1^2\right)$

$\Rightarrow 2(t_2-t_1)=-t_1(t_2+t_1)(t_2-t_1)$

$$\Rightarrow 2=-t_1(t_2+t_1) \Rightarrow t_2+t_1=-\frac{2}{t_1}$$

$$\Rightarrow t_2=-t_1-\frac{2}{t_1}$$

144. (b) The equation of any tangent to the parabola $y^2=8ax$ is

$y=mx+\frac{2a}{m}$ ...(i)

If (i) is also a tangent to the circle, $x^2+y^2=2a^2$ then,

$$\sqrt{2}a=\pm\frac{2a}{m\sqrt{m^2+1}}$$

$\Rightarrow m^2(1+m^2)=2 \Rightarrow (m^2+2)(m^2-1)=0 \Rightarrow m=\pm 1.$

Putting the value of m in eqn (i), we get

$y=\pm(x+2a)$.

145. **(c)** We know that the locus of the feet of the perpendicular draw from foci to any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is the auxiliary circle $x^2+y^2=a^2$

$\therefore$ Auxiliary circle : $x^2+y^2=4$

$\therefore (-1, \sqrt{3})$ satisfies the given equation.

146. **(c)** Normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $\left(ae, \frac{b^2}{a}\right)$ is

$$\frac{a^2x}{ae}-\frac{b^2y}{b^2/a}=a^2-b^2$$

$$\Rightarrow x-ey=\frac{e(a^2-b^2)}{a} \quad ...(i)$$

$\because (0, -b)$ lies on equation (i), then

$$be=\frac{e(a^2-b^2)}{a}$$

$$\Rightarrow ab=a^2e^2 \Rightarrow b=ae^2 \Rightarrow \frac{b^2}{a^2}=e^4$$

$$\therefore 1-e^2=e^4 \Rightarrow e^4+e^2-1=0$$

147. **(b)** Ellipse : $\frac{x^2}{16}+\frac{y^2}{9}=1$,

$a=4, b=3, c=\sqrt{16-9}=\sqrt{7}$

$\therefore (\pm\sqrt{7}, 0)$ are the foci of given ellipse. So for any point P on it; $PA+PB=2a$

$\Rightarrow PA+PB=2(4)=8.$

148. **(a)** Ellipse $\equiv \frac{x^2}{5}+\frac{y^2}{4}=1$

Let a point on ellipse be $(\sqrt{5}\cos\theta, 2\sin\theta)$

$$\therefore PQ^2=(\sqrt{5}\cos\theta)^2+(-4-2\sin\theta)^2$$

$$=5\cos^2\theta+4\sin^2\theta+16+16\sin\theta$$

$$=21+16\sin\theta-\sin^2\theta$$

$$=21+64-(\sin\theta-8)^2=85-(\sin\theta-8)^2$$

PQ^2 to be maximum when $\sin\theta=1$

$\therefore PQ^2_{\max}=85-49=36.$

149. **(c)** The given ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$$

Length of latus rectum $=\frac{2b^2}{a}$

$$\Rightarrow \frac{2b^2}{a}=10 \Rightarrow b^2=5a \quad ...(i)$$

Now $\phi(t)=\frac{5}{12}+t-t^2$

$$\phi'(t)=1-2t=0 \Rightarrow t=\frac{1}{2}$$

$\phi''(t)=-2<0 \Rightarrow$ maximum

$$\Rightarrow \phi(t)_{\max}=\frac{5}{12}+\frac{1}{2}-\frac{1}{4}=\frac{8}{12}=\frac{2}{3}$$

Since, $\phi(t)_{\max.}=$ eccentricity

$$\Rightarrow e=\frac{2}{3}$$

Now, $b^2=a^2(1-e^2)$

$$5a=a^2\left(1-\frac{4}{9}\right) \Rightarrow 5a=\frac{5a^2}{9} \Rightarrow a^2-9a=0$$

$\Rightarrow a=9 \Rightarrow a^2=81$ and $b^2=45$

$\therefore a^2+b^2=81+45=126$

150. **(d)** $\frac{a}{e}=4 \Rightarrow a=4\times\frac{1}{2}=2$

Now, $b^2=a^2(1-e^2)$

$$\Rightarrow b^2=4\left(1-\frac{1}{4}\right)=4\times\frac{3}{4}=3$$

So, equation $\frac{x^2}{4}+\frac{y^2}{3}=1$

$$\Rightarrow 3x^2+4y^2=12 \quad ...(i)$$

Now, $P(1, \beta)$ lies on it

$$\Rightarrow 3+4\beta^2=12 \Rightarrow \beta=\frac{3}{2}$$

So, equation of normal at $P\left(1, \frac{3}{2}\right)$

$$\Rightarrow \frac{a^2x}{1}-\frac{b^2y}{3/2}=a^2-b^2 \Rightarrow 4x-2y=1$$

151. (d) The given ellipse :

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

$\because c=\sqrt{a^2-b^2}=\sqrt{4-3}=1$

$\therefore$ Foci $=(\pm 1, 0)$

Now for hyperbola :

Given : $2a=\sqrt{2} \Rightarrow a=\frac{1}{\sqrt{2}}$

$\because c^2=a^2+b^2 \Rightarrow 1=\frac{1}{2}+b^2 \Rightarrow b=\frac{1}{\sqrt{2}}$

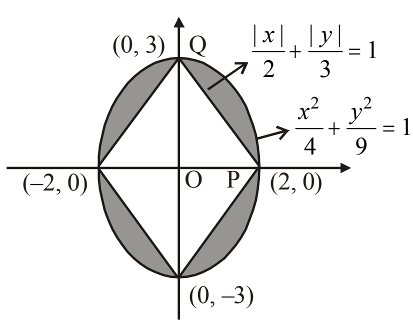
So, equation of hyperbola is

$$\frac{x^2}{\frac{1}{2}}-\frac{y^2}{\frac{1}{2}}=1$$

$\Rightarrow 2x^2-2y^2=1$

So, option (d) does not satisfy it.

152. (a)



$\because$ Area of ellipse $=\pi ab=\pi\times 2\times 3=6\pi$

$\therefore$ Required area = Area of ellipse -4 (Area of triangle OPQ)

$$=6\pi-4\left(\frac{1}{2}\times 2\times 3\right)$$

$$=6\pi-12=6(\pi-2) \text{ sq. units}$$

153. (a) Eccentricity of ellipse

$$e_1=\sqrt{1-\frac{4}{18}}=\sqrt{\frac{7}{9}}=\frac{\sqrt{7}}{3}$$

Eccentricity of hyperbola

$$e_2=\sqrt{1+\frac{4}{9}}=\sqrt{\frac{13}{9}}=\frac{\sqrt{13}}{3}$$

Since, the point (e_1, e_2) is on the ellipse

$15x^2+3y^2=k.$

Then, $15e_1^2+3e_2^2=k$

$$\Rightarrow k=15\left(\frac{7}{9}\right)+3\left(\frac{13}{9}\right)$$

$\Rightarrow k=16$

154. (a) Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1; a>b$

$$2b=\frac{4}{\sqrt{3}} \Rightarrow b=\frac{2}{\sqrt{3}}$$

Equation of tangent $\equiv y=mx\pm\sqrt{a^2m^2+b^2}$

Comparing with $\equiv y=\frac{-x}{6}+\frac{4}{3}$

$m=\frac{-1}{6}$ and $a^2m^2+b^2=\frac{16}{9}$

$$\Rightarrow \frac{a^2}{36}+\frac{4}{3}=\frac{16}{9} \Rightarrow \frac{a^2}{36}=\frac{16}{9}-\frac{4}{3}=\frac{4}{9}$$

$\Rightarrow a^2=16 \Rightarrow a=\pm 4$

Now, eccentricity of ellipse $(e)=\sqrt{1-\frac{b^2}{a^2}}$

$$\Rightarrow e=\sqrt{1-\frac{4}{3\times 16}}=\sqrt{\frac{11}{12}}=\frac{1}{2}\sqrt{\frac{11}{3}}$$

155. (d) Let P be (x_1, y_1).

So, equation of normal at P is

$$\frac{x}{2x_1}-\frac{y}{y_1}=-\frac{1}{2}$$

It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right)$

$$\Rightarrow \frac{-1}{6\sqrt{2}x_1}=-\frac{1}{2} \Rightarrow x_1=\frac{1}{3\sqrt{2}}$$

So, $y_1=\frac{2\sqrt{2}}{3}$ (as P lies in I[st] quadrant)

So, $\beta=\frac{y_1}{2}=\frac{\sqrt{2}}{3}$

156. (b) $2ae=6$ and $\frac{2a}{e}=12$

$\Rightarrow ae=3$ ...(i)

and $\frac{a}{e}=6 \Rightarrow e=\frac{a}{6}$ ...(ii)

$\Rightarrow a^2=18$ [From (i) and (ii)]

$\Rightarrow b^2=a^2-a^2e^2=18-9=9$

$\therefore$ Latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{3\sqrt{2}}=3\sqrt{2}$

157. (a) $3x+4y=12\sqrt{2}$

$\Rightarrow 4y=-3x+12\sqrt{2}$

$\Rightarrow y=-\frac{3}{4}x+3\sqrt{2}$

Now, condition of tangency, $c^2=a^2m^2+b^2$

$\therefore 18=a^2.\frac{9}{16}+9 \Rightarrow a^2.\frac{9}{16}=9$

$\Rightarrow a^2=16 \Rightarrow a=4$

Eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

$\therefore ae=\frac{\sqrt{7}}{4}.4=\sqrt{7}$

$\therefore$ Focus are $(\pm\sqrt{7},0)$

$\therefore$ Distance between foci of ellipse $=2\sqrt{7}$

158. (a) Slope of tangent on the line $2x+y=4$ at point P is $\frac{1}{2}$.

Given ellipse is,

$3x^2+4y^2=12 \Rightarrow \frac{x^2}{2^2}+\frac{y^2}{(\sqrt{3})^2}=1$

Let point $P(2\cos\theta, \sqrt{3}\sin\theta)$

$\therefore$ equation of tangent on the ellipse, at P is,

$\frac{x}{2}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$

$\Rightarrow m_T=-\frac{\sqrt{3}}{2}\cot\theta$

$\because$ both the tangents are parallel $\Rightarrow -\frac{\sqrt{3}}{2}\cot\theta=\frac{1}{2}$

$\Rightarrow \tan\theta=-\sqrt{3} \Rightarrow \theta=\pi-\frac{\pi}{3}$ or $\theta=2\pi-\frac{\pi}{3}$

Case-1: $\theta=\frac{2\pi}{3}$, then point $P\left(-1,\frac{3}{2}\right)$ and $PQ=\frac{5\sqrt{5}}{2}$

Case-2: $\theta=\frac{5\pi}{3}$, then tangent does not pass through $Q(4,4)$.

159. (a) Let the equation of ellipse :

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Given that length of minor axis is 4 i.e. $a=4$.

Also given be = 2

$\because a^2=b^2(1-e^2) \Rightarrow 4=b^2-4 \Rightarrow b=2\sqrt{2}$

Hence, equation of ellipse will be $\frac{x^2}{4}+\frac{y^2}{8}=1$

$\because (\sqrt{2},2)$ satisfies this equation.

$\therefore$ ellipse passes through $(\sqrt{2},2)$.

160. (a) Equation of tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $\left(3,-\frac{9}{2}\right)$ is,

$\frac{3x}{a^2}-\frac{9y}{2b^2}=1$

But given equation of tangent is, $x-2y=12$

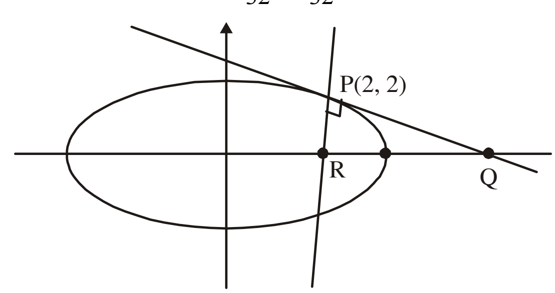
$\therefore \frac{3}{a^2}=\frac{-9}{2b^2.(-2)}=\frac{1}{12}$ (On comparing)

$\Rightarrow a^2=3\times 12$ and $b^2=\frac{9\times 12}{4}$

$\Rightarrow a=6$ and $b=3\sqrt{3}$

Therefore, latus rectum $=\frac{2b^2}{a}=\frac{2\times 27}{6}=9$

161. (d) $3x^2+5y^2=32 \Rightarrow \frac{3x^2}{32}+\frac{5y^2}{32}=1$



Tangent on the ellipse at P is

$\frac{3(2)x}{32}+\frac{5(2)y}{32}=1 \Rightarrow \frac{3x}{16}+\frac{5y}{16}=1$

$\therefore$ co-ordinates of Q will be $\left(\frac{16}{3},0\right)$

Now, normal at P is $\frac{32}{3(2)}-\frac{32y}{5(2)}=\frac{32}{3}-\frac{32}{5}$

$\therefore$ co-ordinates of R will be $\left(\frac{4}{5},0\right)$

Hence, area of $\Delta PQR=\frac{1}{2}(PQ)(PR)$

$=\frac{1}{2}\sqrt{\frac{136}{9}}.\sqrt{\frac{136}{25}}=\frac{68}{15}$

162. (d) Let tangent to parabola at point $\left(\frac{1}{4m^2},-\frac{1}{2m}\right)$ is

$y=mx+\frac{1}{4m}$

and tangent to ellipse is, $y=mx\pm\sqrt{m^2+\frac{1}{2}}$

Now, condition for common tangency,

$\frac{1}{4m}=\pm\sqrt{m^2+\frac{1}{2}} \Rightarrow \frac{1}{16m^2}=m^2+\frac{1}{2}$

$\Rightarrow 16m^4+8m^2-1=0 \Rightarrow m^2=\frac{-8\pm\sqrt{64+64}}{2(16)}$

$=\frac{-8\pm 8\sqrt{2}}{2(16)}=\frac{\sqrt{2}-1}{4}$

$\alpha=\frac{1}{4m^2}=\frac{1}{4\frac{\sqrt{2}-1}{4}}=\sqrt{2}+1$

163. (b) Given that focus is $(0,5\sqrt{3}) \Rightarrow |b|>|a|$

Let $b>a>0$ and foci is $(0,\pm be)$

$\because a^2=b^2-b^2e^2 \Rightarrow b^2e^2=b^2-a^2$

$be=\sqrt{b^2-a^2} \Rightarrow b^2-a^2=75$ ...(i)

$\because 2b-2a=10 \Rightarrow b-a=5$ ...(ii)

From (i) and (ii)

$b+a=15$ ...(iii)

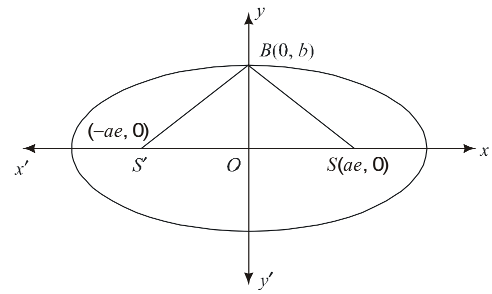
On solving (ii) and (iii), we get

$\Rightarrow b=10, a=5$

Now, length of latus rectum $=\frac{2a^2}{b}=\frac{50}{10}=5$

164. (a) $\because \Delta S'BS$ is right angled triangle, then

(Slope of BS) $\times$ (Slope of BS') $=-1$



$\frac{b}{-ae}\times\frac{b}{ae}=-1 \Rightarrow b^2=a^2e^2$ ...(i)

Since, area of $\Delta S'BS=8$

$\Rightarrow \frac{1}{2}\cdot 2ae\cdot b=8 \Rightarrow b^2=8$ ...(ii)

From eq^n (i)

$a^2e^2=8$

Also, $e^2=1-\frac{b^2}{a^2}$

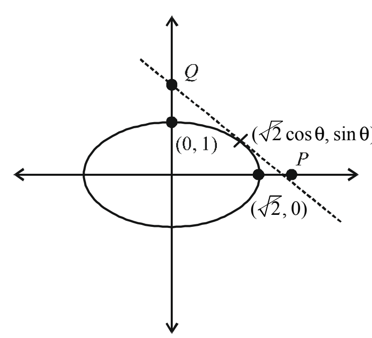
$\Rightarrow a^2e^2=a^2-b^2 \Rightarrow 8=a^2-8 \Rightarrow a^2=16$

Hence, required length of latus rectum $=\frac{2b^2}{a}=\frac{2(8)}{4}$

$=4$ units

165. (c) Given the equation of ellipse,

$\frac{x^2}{(\sqrt{2})^2}+y^2=1$



$\frac{\sqrt{2}\cos\theta\, x}{2}+y\sin\theta=1$

$P\left(\frac{\sqrt{2}}{\cos\theta},0\right)$ and $Q\left(0,\frac{1}{\sin\theta}\right)$

Let mid point be (h, k)

$\Rightarrow h=\frac{1}{\sqrt{2}\cos\theta}, k=\frac{1}{2\sin\theta}$

As $\cos^2\theta+\sin^2\theta=1$

$\therefore \quad \frac{1}{2h^2}+\frac{1}{4k^2}=1$

Locus is $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

166. (a) $A=\{(a,b)\in R\times R:|a-5|<1,|b-5|<1\}$

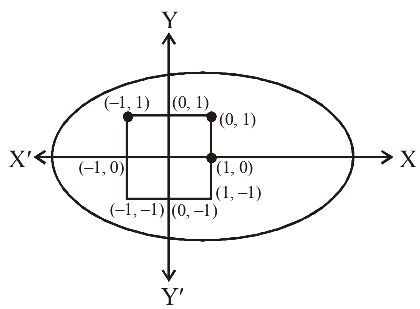
Let $a-5, = x, b-5=y$

Set A contains all points inside $|x|<1,|y|<1$

$B=\{(a,b)\in R\times R:4(a-6)^2+9(B-5)^2\le 36\}$

Set B contains all points inside or on

$\frac{(x-1)^2}{9}+\frac{y^2}{4}=1$



$\therefore \quad (\pm1,\pm1)$ lies inside the ellipse.

Hence, $A\subset B$.

167. (d) Let for ellipse coordinates of focus and vertex are $(ae, 0)$ and $(a, 0)$ respectively.

$\therefore$ Distance between focus and vertex $= a(1-e)=\frac{3}{2}$ (given)

$\Rightarrow a-\frac{3}{2}=ae$

$\Rightarrow a^2+\frac{9}{4}-3a=a^2e^2 \quad$ ...(i)

Length of latus rectum $=\frac{2b^2}{a}=4$

$\Rightarrow b^2=2a \quad$ ...(ii)

$e^2=1-\frac{b^2}{a^2}$

$\Rightarrow e^2=1-\frac{2a}{a^2} \quad$ (from (ii))

$\Rightarrow e^2=1-\frac{2}{a} \quad$ ... (iii)

Substituting the value of e^2 in eq. (i) we get;

$\Rightarrow a^2+\frac{9}{4}-3a=a^2\left(1-\frac{2}{a}\right)$

$\Rightarrow a=\frac{9}{4}$

$\therefore$ from eq. (iii) we get;

$e^2=1-\frac{2}{a}=1-\frac{8}{9}=\frac{1}{9}$

$\Rightarrow \ e=\frac{1}{3}$

168. (c) Centre at (0, 0)

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

at point (4, –1)

$\frac{16}{a^2}+\frac{1}{b^2}=1$

$\Rightarrow \ 16b^2+a^2=a^2b^2 \quad$ ...(i)

at point (–2, 2)

$\frac{4}{a^2}+\frac{4}{b^2}=1$

$\Rightarrow \ 4b^2+4a^2=a^2b^2 \quad$ ...(ii)

$\Rightarrow \ 16b^2+a^2=4a^2+4b^2$

From equations (i) and (ii)

$\Rightarrow \ 3a^2=12b^2 \ \Rightarrow \ \boxed{a^2=4b^2}$

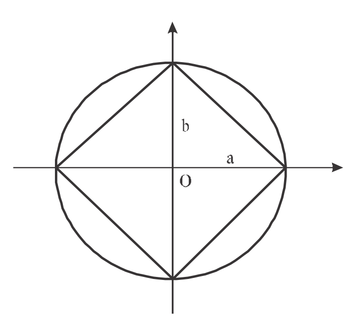
$b^2=a^2(1-e^2)$

$\Rightarrow \ e^2=\frac{3}{4} \ \Rightarrow \ e=\frac{\sqrt{3}}{2}$

169. (d) $e=3/5$ & $2ae=6\Rightarrow a=5$

$\because \ b^2=a^2(1-e^2)$

$\Rightarrow \ b^2=25(1-9/25)$



$\Rightarrow \ b=4$

$\therefore$ area of required quadrilateral $=4(1/2\ ab)=2ab=40$

170. (c) Equation of tangent to ellipse

$$\frac{x}{\sqrt{27}}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$$

Area bounded by line and co-ordinate axis

$$\Delta=\frac{1}{2}\cdot\frac{\sqrt{27}}{\cos\theta}\cdot\frac{\sqrt{3}}{\sin\theta}=\frac{9}{\sin 2\theta}$$

$\Delta=$ will be minimum when $\sin 2\theta=1$

$\Delta_{\min}=9$

171. (b) The end point of latus rectum of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in first quadrant is $\left(ae,\frac{b^2}{a}\right)$ and the tangent at this point intersects x-axis at $\left(\frac{a}{e},0\right)$ and y-axis at (0, a).

The given ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

Then $a^2=9, b^2=5$

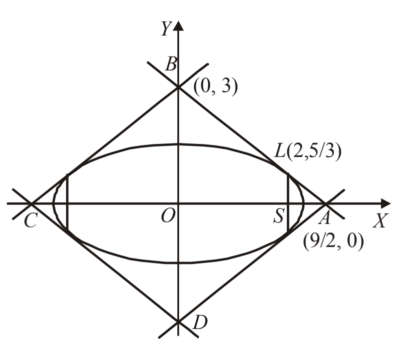
$\Rightarrow\ e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$

$\therefore$ End point of latus rectum in first quadrant is L (2, 5/3)

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$

[$\because$ It meets x-axis at A (9/2, 0) and y-axis at B (0, 3)]

$\therefore$ Area of ΔOAB $=\frac{1}{2}\times\frac{9}{2}\times 3=\frac{27}{4}$



By symmetry area of quadrilateral

$=4\times(\text{Area }\Delta OAB)=4\times\frac{27}{4}=27$ sq. units.

172. (b) Focus of an ellipse is given as (± ae, 0)

Distance between them = 2ae

According to the question, $2ae=\frac{b^2}{a}$

$\Rightarrow 2a^2e=b^2=a^2(1-e^2)$

$\Rightarrow 2e=1-e^2\Rightarrow(e+1)^2=2\Rightarrow e=\sqrt{2}-1$

173. (a) Given equation of ellipse can be written as

$$\frac{x^2}{6}+\frac{y^2}{2}=1$$

$\Rightarrow\ a^2=6, b^2=2$

Now, equation of any variable tangent is

$$y=mx\pm\sqrt{a^2m^2+b^2}\qquad\text{...(i)}$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y=\frac{-x}{m}\qquad\text{...(ii)}$$

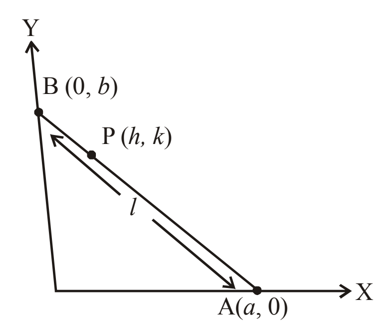
Eliminating m, we get

$$(x^4+y^4+2x^2y^2)=a^2x^2+b^2y^2$$

$$\Rightarrow\ (x^2+y^2)^2=a^2x^2+b^2y^2$$

$$\Rightarrow\ (x^2+y^2)^2=6x^2+2y^2$$

174. (b) Let point A (a, 0) is on x-axis and B (0, b) is on y-axis.



Let P (h, k) divides AB in the ratio 1 : 2.

So, by section formula

$$h=\frac{2(0)+1(a)}{1+2}=\frac{a}{3}$$

$$k=\frac{2(b)+1(0)}{3}=\frac{2b}{3}$$

$\Rightarrow a=3h$ and $b=\frac{3k}{2}$

Now, $a^2+b^2=l^2$

$$\Rightarrow 9h^2+\frac{9k^2}{4}=l^2$$

$$\Rightarrow \frac{h^2}{\left(\frac{l}{3}\right)^2}+\frac{k^2}{\left(\frac{2l}{3}\right)^2} = 1$$

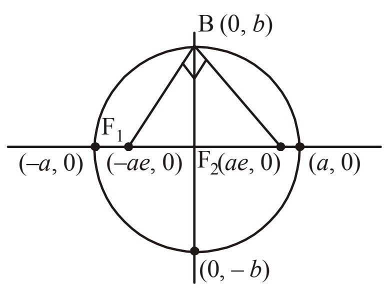
Now $e = \sqrt{1-\left(\frac{l^2}{9}\times\frac{9}{4l^2}\right)} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

Thus, required locus of P is an ellipse with eccentricity $\frac{\sqrt{3}}{2}$.

175. **(a)** Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be the equation of ellipse.

Given that F_1B and F_2B are perpendicular to each other.
Slope of F_1B × slope of F_2B = – 1

$$\left(\frac{0-b}{-ae-0}\right)\times\left(\frac{0-b}{ae-0}\right) = -1$$



$$\left(\frac{b}{ae}\right)\times\left(\frac{-b}{ae}\right) = -1$$

$b^2 = a^2e^2$

$e^2 = \frac{b^2}{a^2}$ $\qquad \left\{\because\ e^2 = 1-\frac{b^2}{a^2}\right\}$

$1-\frac{b^2}{a^2} = \frac{b^2}{a^2}$

$1 = 2\frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2}=\frac{1}{2}$

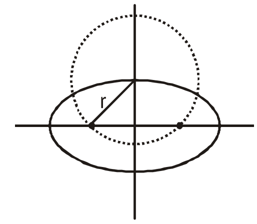
$e^2 = 1-\frac{b^2}{a^2}=1-\frac{1}{2}=\frac{1}{2}$

$e^2 = \frac{1}{2}$

No common tangents for these two circles.

176. **(a)** From the given equation of ellipse, we have

$a=4, b=3, e=\sqrt{1-\frac{9}{16}}$



$\Rightarrow e = \frac{\sqrt{7}}{4}$

Now, radius of this circle $= a^2 = 16$

$\Rightarrow$ Foci $= (\pm\sqrt{7}, 0)$

Now equation of circle is

$(x-0)^2+(y-3)^2=16$

$x^2+y^2-6y-7=0$

177. **(c)** Given ellipse is $4x^2+9y^2=36$

$\Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$

Normal at the point is parallel to the line

$4x-2y-5=0$

Slope of normal = 2

Slope of tangent $= \frac{-1}{2}$

Point of contact to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and line is $\left(\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b}{\sqrt{a^2m^2+b^2}}\right)$

Now, $a^2=9,\ b^2=4$

$\therefore$ Point $=\left(\frac{-9}{5},\frac{8}{5}\right)$

178. **(c)** Given equations of ellipses

$E_1: \frac{x^2}{3}+\frac{y^2}{2}=1$

$\Rightarrow e_1 = \sqrt{1-\frac{2}{3}} = \frac{1}{\sqrt{3}}$

and $E_2: \frac{x^2}{16}+\frac{y^2}{b^2}=1$

$\Rightarrow e_2 = \sqrt{\frac{1-b^2}{16}} = \sqrt{\frac{16-b^2}{4}}$

Also, given $e_1\times e_2 = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{3}}\times\sqrt{\frac{16-b^2}{4}} = \frac{1}{2} \Rightarrow 16-b^2 = 12$

$\Rightarrow \quad b^2 = 4$

$\therefore$ Length of minor axis of

$E_2 = 2b = 2 \times 2 = 4$

179. **(d)** $x^2 = 8y$ ...(i)

$\frac{x^2}{3} + y^2 = 1$ ...(ii)

From (i) and (ii),

$\frac{8y}{3} + y^2 = 1 \Rightarrow y = -3, \frac{1}{3}$

When $y = -3$, then $x^2 = -24$, which is not possible.

When $y = \frac{1}{3}$, then $x = \pm\frac{2\sqrt{6}}{3}$

Point of intersection are

$\left(\frac{2\sqrt{6}}{3}, \frac{1}{3}\right)$ and $\left(-\frac{2\sqrt{6}}{3}, \frac{1}{3}\right)$

Required equation of the line,

$y - \frac{1}{3} = 0 \Rightarrow 3y - 1 = 0$

180. **(d)** Any tangent on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by

$y = mx \pm \sqrt{a^2m^2 + b^2}$

Here $a = 2, b = 1$

$m = \frac{1-0}{0-2} = -\frac{1}{2}$

$c = \sqrt{4\left(-\frac{1}{2}\right)^2 + 1^2} = \sqrt{2}$

So, $y = -\frac{1}{2}x \pm \sqrt{2}$

For ellipse : $\frac{x^2}{4} + \frac{y^2}{1} = 1$

We put $y = -\frac{1}{2}x + \sqrt{2}$

$\therefore \frac{x^2}{4} + \left(-\frac{x}{2} + \sqrt{2}\right)^2 = 1$

$\frac{x^2}{4} + \left(\frac{x^2}{4} - 2\left(\frac{x}{2}\right)\sqrt{2} + 2\right) = 1$

$\Rightarrow x^2 + 2\sqrt{2}x + 2 = 0$

or $x^2 - 2\sqrt{2}x + 2 = 0$

$\Rightarrow x = \sqrt{2}$ or $-\sqrt{2}$

If $x = \sqrt{2}, y = \frac{1}{\sqrt{2}}$ and $x = -\sqrt{2}, y = -\frac{1}{\sqrt{2}}$

$\therefore$ Points are $\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right), \left(-\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

$\therefore P_1P_2 = \sqrt{\left\{\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right\}^2 + \left\{\sqrt{2} - \left(-\sqrt{2}\right)\right\}^2}$

$= \sqrt{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(2\sqrt{2}\right)^2} = \sqrt{2+8} = \sqrt{10}$

181. **(d)** Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given it passes through (–3, 1) so

$\frac{9}{a^2} + \frac{1}{b^2} = 1$ ...(i)

Also, we know that

$b^2 = a^2(1 - e^2) = a^2(1 - 2/5)$

$\Rightarrow 5b^2 = 3a^2$ ...(ii)

Solving (i) and (ii) we get $a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$

So, the equation of the ellipse is

$3x^2 + 5y^2 = 32$

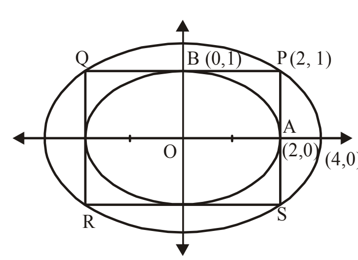
182. **(a)** The given equation of ellipse is

$\frac{x^2}{4} + \frac{y^2}{1} = 1$

So, $A = (2, 0)$ and $B = (0, 1)$

If $PQRS$ is the rectangle in which it is inscribed, then $P = (2, 1)$.

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the ellipse circumscribing the rectangle $PQRS$.



Then it passed through P (2,1)

$\therefore \frac{4}{a^2} + \frac{1}{b^2} = 1$ ....(i)

Also, given that, it passes through (4, 0)

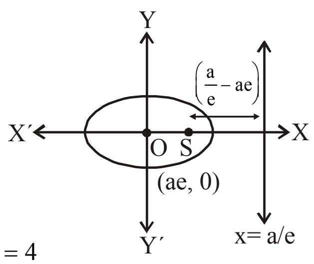
$\therefore \dfrac{16}{a^2} + 0 = 1 \Rightarrow a^2 = 16$

$\Rightarrow b^2 = 4/3$ [putting $a^2 = 16$ in eq^n (i)]

$\therefore$ The required equation of ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{4/3} = 1$

or $x^2 + 12y^2 = 16$

183. (a) Perpendicular distance of directrix $x = \pm\dfrac{a}{e}$ from focus $(\pm ae, 0)$



$= \dfrac{a}{e} - ae = 4$

$\Rightarrow a\left(2 - \dfrac{1}{2}\right) = 4$

$\Rightarrow a = \dfrac{8}{3}$

$\therefore$ Semi major axis $= 8/3$

184. (a) Given that distance between foci is $2ae = 6 \Rightarrow ae = 3$ and length of minor axis is $2b = 8 \Rightarrow b = 4$

we know that $b^2 = a^2(1 - e^2)$

$\Rightarrow 16 = a^2 - a^2e^2 \Rightarrow a^2 = 16 + 9 = 25 \Rightarrow a = 5$

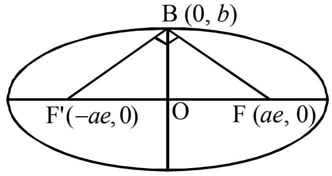
$\therefore e = \dfrac{3}{a} = \dfrac{3}{5}$

185. (a) Given that $\angle FBF' = 90°$

$\Rightarrow FB^2 + F'B^2 = FF'^2$

$\therefore \left(\sqrt{a^2e^2 + b^2}\right)^2 + \left(\sqrt{a^2e^2 + b^2}\right)^2 = (2ae)^2$

$\Rightarrow 2(a^2e^2 + b^2) = 4a^2e^2 \Rightarrow e^2 = \dfrac{b^2}{a^2}$ ...(i)



We know that $e^2 = 1 - b^2/a^2 = 1 - e^2$ [from (i)]

$\Rightarrow 2e^2 = 1,\ e = \dfrac{1}{\sqrt{2}}.$

186. (b) Given that $e = \dfrac{1}{2}$. Directrix, $x = \dfrac{a}{e} = 4$

$\therefore a = 4 \times \dfrac{1}{2} = 2 \quad \therefore b = 2\sqrt{1 - \dfrac{1}{4}} = \sqrt{3}$

Equation of ellipse is

$\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$

187. (c) General tangent to hyperbola in slope form is

$y = mx \pm \sqrt{100m^2 - 64}$

and the general tangent to the circle in slope form is

$y = mx \pm 6\sqrt{1 + m^2}$

For common tangent,

$36(1 + m^2) = 100m^2 - 64$

$\Rightarrow 100 = 64m^2 \Rightarrow m^2 = \dfrac{100}{64}$

$\therefore c^2 = 36\left(1 + \dfrac{100}{64}\right) = \dfrac{164 \times 36}{64} = \dfrac{369}{4}$

$\Rightarrow 4c^2 = 369$

188. (a) $\because$ The equation of hyperbola is

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\because$ Equation of hyperbola passes through (3, 3)

$\dfrac{1}{a^2} - \dfrac{1}{b^2} = \dfrac{1}{9}$ ...(i)

Equation of normal at point (3, 3) is :

$\dfrac{x - 3}{\frac{1}{a^2} \cdot 3} = \dfrac{y - 3}{-\frac{1}{b^2} \cdot 3}$

$\because$ It passes through (9, 0)

$\dfrac{6}{\frac{1}{a^2}} = \dfrac{-3}{-\frac{1}{b^2}}$

$\therefore \dfrac{1}{b^2} = \dfrac{1}{2a^2}$ ...(ii)

From equations (i) and (ii),

$a^2 = \dfrac{9}{2},\ b^2 = 9$

$\because$ Eccentricity $= e$, then $e^2 = 1 + \frac{b^2}{a^2} = 3$

$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$

189. **(d)** Equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$

Then, $e_1 = \sqrt{1 - \frac{b^2}{25}}$

The equation of hyperbola, $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

Then, $e_2 = \sqrt{1 + \frac{b^2}{16}}$

$e_1 e_2 = 1$

$\Rightarrow (e_1 e_2)^2 = 1 \Rightarrow \left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$

$\Rightarrow 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{25 \times 16} = 1$

$\Rightarrow \frac{9}{16 \cdot 25} b^2 - \frac{b^4}{25 \cdot 16} = 0 \Rightarrow b^2 = 9$

$\therefore e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

And, $e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Distance between focii of ellipse

$= \alpha = 2ae_1 = 2(5)(e_1) = 8$

Distance between focii of hyperbola

$= \beta = 2ae_2 = 2(4)(e_2) = 10$

$\therefore (\alpha, \beta) = (8, 10)$

190. **(a)** The tangent to the hyperbola at the point (x_1, y_1) is,

$xx_1 - 2yy_1 - 4 = 0$

The given equation of tangent is

$2x - y = 0$

$\Rightarrow \frac{x_1}{2y_1} = 2$

$\Rightarrow x_1 = 4y_1$ ...(i)

Since, point (x_1, y_1) lie on hyperbola.

$\therefore \frac{x_1^2}{4} - \frac{y_1^2}{2} - 1 = 0$ ...(ii)

On solving eqs. (i) and (ii)

$y_1^2 = \frac{2}{7}, x_1^2 = \frac{32}{7}$

$\therefore x_1^2 + 5y_1^2 = \frac{32}{7} + 5 \times \frac{2}{7} = 6$

191. **(d)** Hyperbola : $\frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1 \Rightarrow e_1 = \sqrt{1 + \cos^2\theta}$

and Ellipse : $\frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1$

$\Rightarrow e_2 = \sqrt{1 - \cos^2\theta} = \sin\theta$

According to the question, $e_1 = \sqrt{5} e_2$

$\Rightarrow 1 + \cos^2\theta = 5\sin^2\theta \Rightarrow \cos^2\theta = \frac{2}{3}$

Now length of latus rectum of ellipse

$= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$

192. **(b)** Let the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

If a hyperbola passes through vertices at ($\pm$ 6, 0), then

$\therefore a = 6$

As hyperbola passes through the point $P(10, 16)$

$\therefore \frac{100}{36} - \frac{256}{b^2} = 1 \Rightarrow b^2 = 144$

$\therefore$ Required hyperbola is $\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is $\frac{36x}{10} + \frac{144y}{16} = 36 + 144$

$\therefore$ At P(10, 16) normal is

$\frac{36x}{10} + \frac{144y}{16} = 36 + 144$

$\therefore 2x + 5y = 100.$

193. **(c)** Equation of tangent to $y^2 = 12x$ is $y = mx + \frac{3}{m}$

Equation of tangent to

$\frac{x^2}{1} - \frac{y^2}{8} = 1$ is $y = mx \pm\sqrt{m^2 - 8}$

$\because$ parabola and hyperbola have common tangent.

$\therefore \quad \frac{3}{m} = \pm\sqrt{m^2-8} \Rightarrow \frac{9}{m^2} = m^2 - 8$

Put $m^2 = u$

$u^2 - 8u - 9 = 0 \Rightarrow \quad u^2 - 9u + u - 9 = 0$

$\Rightarrow \; (u+1)(u-9) = 0$

$\because u = m^2 \geq 0 \Rightarrow u = m^2 = 9 \Rightarrow m = \pm 3$

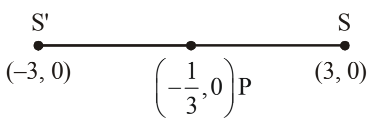
$\therefore$ equation of tangent is $y = 3x + 1$

or $y = -3x - 1$

$\therefore$ intersection point is $P\left(-\frac{1}{3}, 0\right)$.

$e = \sqrt{1+\frac{b^2}{a^2}} \Rightarrow e = \sqrt{1+\frac{8}{1}} \Rightarrow e = 3$

$\therefore$ foci $(\pm 3, 0)$



$$\frac{SP}{SP'} = \frac{3+\frac{1}{3}}{3-\frac{1}{3}} = \frac{10}{8} = \frac{5}{4}$$

194. **(a)** Given curves, $y^2 = 16x$ and $xy = -4$

Equation of tangent to the given parabola;

$y = mx + \frac{4}{m}$

$\because$ This is common tangent.

So, put $y = mx + \frac{4}{m}$ in $xy = -4$.

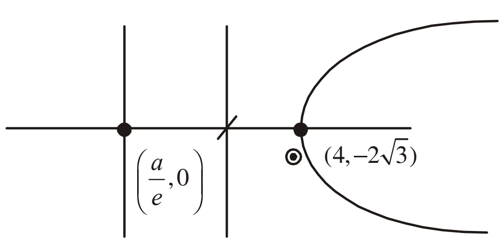
$x\left(mx + \frac{4}{m}\right) + 4 = 0 \Rightarrow mx^2 + \frac{4}{m}x + 4 = 0$

$D = 0 \Rightarrow \frac{16}{m^2} = 16m \Rightarrow m^3 = 1 \Rightarrow m = 1$

$\therefore$ equation of common tangent is $y = x + 4$

195. **(c)** $\because$ directrix of a hyperbola is,

$5x = 4\sqrt{5} \Rightarrow x = \frac{4}{\sqrt{5}} \Rightarrow \frac{a}{e} = \frac{4}{\sqrt{5}}$



Now, hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes throug $(4, -2\sqrt{3})$

$\therefore \frac{16}{a^2} - \frac{12}{a^2e^2 - a^2} = 1$

$$\left[\because e^2 = 1 + \frac{b^2}{a^2} \Rightarrow a^2e^2 - a^2 = b^2\right]$$

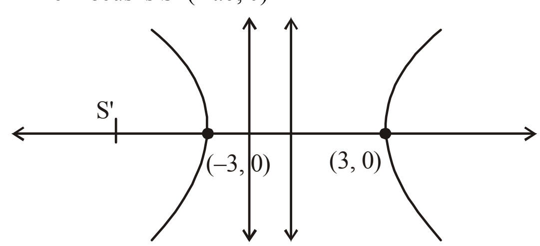
$\Rightarrow \frac{4}{a^2}\left[\frac{4}{1} - \frac{3}{e^2-1}\right] = 1 \Rightarrow 4e^2 - 4 - 3 = (e^2-1)\left(\frac{a^2}{4}\right)$

$\Rightarrow 4(4e^2 - 7) = (e^2 - 1)\left(\frac{4e}{\sqrt{5}}\right)^2$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

196. **(d)** $16x^2 - 9y^2 = 144 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

Then focus is S' $(-ae, 0)$



$a = 3, b = 4 \Rightarrow e^2 = 1 + \frac{16}{9} = \frac{25}{9} \quad \left[\because e = \sqrt{1+\frac{b^2}{a^2}}\right]$

$\therefore$ the focus $S' \equiv \left(3 - \times\frac{5}{3}, 0\right) \equiv (-5, 0)$

197. **(c)** Since, $lx + my + n = 0$ is a normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

then $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

but it is given that $mx - y + 7\sqrt{3}$ is normal to hyperbola

$\frac{x^2}{24} - \frac{y^2}{18} = 1$

then $\frac{24}{m^2} - \frac{18}{(-1)^2} = \frac{(24+18)^2}{(7\sqrt{3})^2} \Rightarrow m = \frac{2}{\sqrt{5}}$

198. (c) Let equation of hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad ...(i)$$

$$\because e=\sqrt{1+\frac{b^2}{a^2}} \Rightarrow b^2=a^2(e^2-1)$$

$$e=2 \Rightarrow b^2=3a^2 \quad ...(ii)$$

Equation (i) passes through (4, 6),

$$\therefore \frac{16}{a^2}-\frac{36}{b^2}=1 \quad ...(iii)$$

On solving (i) and (ii), we get

$a^2=4, b^2=12$

Now equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1$

Now equation of tangent to the hyperbola at (4, 6) is

$$\frac{4x}{4}-\frac{6y}{12}=1 \Rightarrow x-\frac{y}{2}=1 \Rightarrow 2x-y=2$$

199. (d) Let the points are,

$A(2, 0), A'(-2, 0)$ and $S(-3, 0)$

$\Rightarrow$ Centre of hyperbola is $O(0, 0)$

$AA'=2a \Rightarrow 4=2a \Rightarrow a=2$

$\because$ Distance between the centre and foci is ae.

$$\therefore OS=ae \Rightarrow 3=2e \Rightarrow e=\frac{3}{2}$$

$$\Rightarrow b^2=a^2(e^2-1)=a^2e^2-a^2=9-4=5$$

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{5}=1$ ...(i)

$\because$ (6, 552) does not satisfy eq (i).

$\therefore$ $(6, 5\sqrt{2})$ does not lie on this hyperbola.

200. (a) $\therefore$ Conjugate axis = 5

$\therefore$ $2b=5$

Distance between foci =13

$2ae=13$

Then, $b^2=a^2(e^2-1)$

$\Rightarrow$ $a^2=36$

$\therefore$ $a=6$

$$ae=\frac{13}{2} \Rightarrow e=\frac{13}{12}$$

201. (b) Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Then, $\frac{2b^2}{a}=8,\ 2ae=b^2$ and $b^2=a^2(1-e^2)$

$\Rightarrow$ $a=8, b^2=32$

Then, the equation of the ellipse

$$\frac{x^2}{64}+\frac{y^2}{32}=1$$

Hence, the point $(4\sqrt{3}, 2\sqrt{2})$ lies on the ellipse.

202. (a) Given, the equation of line,

$x-y=2 \Rightarrow y=x-2$

$\therefore$ its slope $=m=1$

Equation of hyperbola is:

$$\frac{x^2}{5}-\frac{y^2}{4}=1 \Rightarrow a^2=5, b^2=4$$

The equation of tangent to the hyperbola is,

$$y=mx\pm\sqrt{a^2m^2-b^2}$$

$$=x\pm\sqrt{5-4} \Rightarrow y=x\pm1$$

203. (b) Since, $r\neq\pm1$, then there are two cases, when $r>1$

$$\frac{x^2}{r-1}+\frac{y^2}{r+1}=1 \text{ (Ellipse)}$$

Then,

$$(r-1)=(r+1)(1-e^2) \Rightarrow 1-e^2=\frac{(r-1)}{(r+1)}$$

$$\Rightarrow e^2=1-\frac{(r-1)}{(r+1)}=\frac{2}{(r+1)}$$

$$\Rightarrow e=\sqrt{\frac{2}{(r+1)}}$$

When $0<r<1$, then

$$\frac{x^2}{1-r}-\frac{y^2}{1+r}=-1 \text{ (Hyperbola)}$$

Then,

$$(1-r)=(1+r)(e^2-1) \Rightarrow e^2=1+\frac{(r-1)}{(r+1)}=\frac{2r}{(r+1)}$$

$$\Rightarrow e=\sqrt{\frac{2r}{r+1}}$$

204. (a) $\because a^2=\cos^2\theta, b^2=\sin^2\theta$

and $e>2 \Rightarrow e^2>4 \Rightarrow 1+b^2/a^2>4$

$\Rightarrow$ $1+\tan^2\theta>4$

$$\Rightarrow \sec^2\theta>4 \Rightarrow \theta\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$$

Latus rectum,

$LR=\frac{2b^2}{a}=\frac{2\sin^2\theta}{\cos\theta}=2(\sec\theta-\cos\theta)$

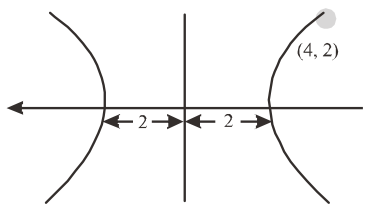
$\Rightarrow \frac{d(LR)}{d\theta}=2(\sec\theta\tan\theta+\sin\theta)>0\ \forall\theta\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$

$\therefore \min(LR)=2\left(\sec\frac{\pi}{3}-\cos\frac{\pi}{3}\right)=2\left(2-\frac{1}{2}\right)=3$

max (LR) tends to infinity as $\theta\to\frac{\pi}{2}$

Hence, length of latus rectum lies in the interval $(3,\infty)$

205. **(d)**



Consider equation of hyperbola

$\frac{x^2}{2^2}-\frac{y^2}{b^2}=1$

$\because$ (4, 2) lies on hyperbola

$\therefore \frac{16}{4}-\frac{4}{b^2}=1$

$\therefore b^2=\frac{4}{3}$

Since, eccentricity $=\sqrt{1+\frac{b^2}{a^2}}$

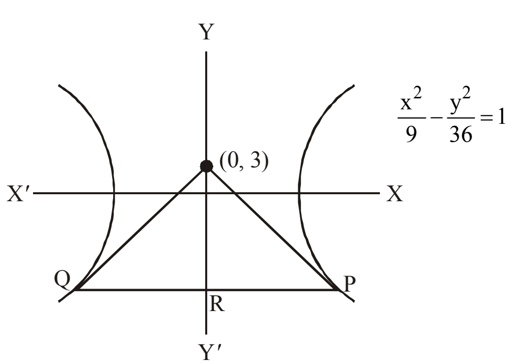
Hence, eccentricity $=\sqrt{1+\frac{\frac{4}{3}}{4}}=\sqrt{1+\frac{1}{3}}=\frac{2}{\sqrt{3}}$

206. **(d)** Here equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{36}=1$

Now, PQ is the chord of contant

$\therefore$ Equation of PQ is : $\frac{x(0)}{9}-\frac{y(3)}{36}=1$

$\Rightarrow y=-12$



$\therefore$ Area of $\Delta PQT=\frac{1}{2}\times TR\times PQ$

$\because\ P\equiv(3\sqrt{5},-12)\ \therefore\ TR=3+12=15,$

$\therefore$ Area of $\Delta PQT=\frac{1}{2}\times15\times6\sqrt{5}=45\sqrt{5}$ sq. units

207. **(a)** Here, lines are:

$\sqrt{2}x-y+4\sqrt{2}k=0$

$\Rightarrow \sqrt{2}x+4\sqrt{2}k=y$ ...(i)

and $\sqrt{2}kx+ky-4\sqrt{2}=0$ ...(ii)

Put the value of y from (i) in (ii) we get;

$\Rightarrow 2\sqrt{2}kx+4\sqrt{2}\left(k^2-1\right)=0$

$\Rightarrow x=\frac{2(1-k^2)}{k},\ y=\frac{2\sqrt{2}(1+k^2)}{k}$

$\therefore \left(\frac{y}{4\sqrt{2}}\right)^2-\left(\frac{x}{4}\right)^2=1$

$\therefore$ length of transverse axis

$2a=2\times4\sqrt{2}=8\sqrt{2}$

Hence, the locus is a hyperbola with length of its transverse axis equal to $8\sqrt{2}$

208. **(c)** Equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

foci is $(\pm2,0)\Rightarrow ae=\pm2\Rightarrow a^2e^2=4$

Since $b^2=a^2(e^2-1)$

$b^2=a^2e^2-a^2\ \therefore\ a^2+b^2=4$ ...(i)

Hyperbola passes through $(\sqrt{2},\sqrt{3})$

$\therefore \frac{2}{a^2}-\frac{3}{b^2}=1$ ...(ii)

$\frac{2}{4-b^2}-\frac{3}{b^2}=1$ [from (i)]

$\Rightarrow b^4+b^2-12=0$

$\Rightarrow (b^2-3)(b^2+4)=0$

$\Rightarrow b^2=3$

$b^2=-4$ (Not possible)

For $b^2=3$

$\Rightarrow a^2=1\ \therefore\ \frac{x^2}{1}-\frac{y^2}{3}=1$

Equation of tangent is $\frac{\sqrt{2}x}{1}-\frac{\sqrt{3}y}{3}=1$

Clearly $(2\sqrt{2},3\sqrt{3})$ satisfies it.

209. (d) Here, $tx-2y-3t=0$ & $x-2ty+3=0$
On solving, we get;

$y=\frac{6t}{2t^2-2}=\frac{3t}{t^2-1}$ & $x=\frac{3t^2+3}{t^2-1}$

Put $t=\tan\theta$

$\therefore$ $x=-3\sec 2\theta$ & $2y=3(-\tan 2\theta)$

$\because$ $\sec^2 2\theta-\tan^2 2\theta=1$

$\Rightarrow \frac{x^2}{9}-\frac{y^2}{9/4}=1$

which represents a hyperbola

$\therefore$ $a^2=9$ & $b^2=9/4$

λ(T.A.) = 6; $e^2=1+\frac{9/4}{9}=1+\frac{1}{4}\Rightarrow e=\frac{\sqrt{5}}{2}$

210. (a) $\frac{2b^2}{a}=8$ and $2b=\frac{1}{2}(2ae)$

$\Rightarrow 4b^2=a^2e^2 \Rightarrow 4a^2(e^2-1)=a^2e^2$

$\Rightarrow 3e^2=4 \Rightarrow e=\frac{2}{\sqrt{3}}$

211. (c) $\frac{x^2}{12}+\frac{y^2}{16}=1$

$e=\sqrt{1-\frac{12}{16}}=\frac{1}{2}$

Foci (0, 2) & (0, – 2)

So, transverse axis of hyperbola = $2b=4\Rightarrow b=2$

& $a^2=1^2(e^2-1)$

$\Rightarrow a^2=4\left(\frac{9}{4}-1\right)$

$\Rightarrow a^2=5$

$\therefore$ It's equation is $\frac{x^2}{5}-\frac{y^2}{4}=-1$

The point $(5, 2\sqrt{3})$ does not satisfy the above equation.

212. (a) $S(5, 0)$ is focus $\Rightarrow ae=5$ (focus) ——— (a)

$x=\frac{a}{5}\Rightarrow\frac{a}{e}=\frac{9}{5}$ (directrix) ——— (b)

(a) & (b) $\Rightarrow a^2=9$

(a) $\Rightarrow (e)=\frac{5}{3}$

$b^2=a^2(e^2-1)\Rightarrow b^2=16$

$a^2-b^2=9-16=-7$

213. (c) Equation of hyperbola is

$\frac{x^2}{4}-\frac{y^2}{9}=1$

Its Foci = $\left(\pm\sqrt{13},0\right)$

$e=\frac{\sqrt{13}}{2}$

If e, be the eccentricity of the ellipse, then

$e_1\times\frac{\sqrt{13}}{2}=\frac{1}{2} \Rightarrow e_1=\frac{1}{\sqrt{13}}$

Equation of ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Since ellipse passes through the foci $(\pm\sqrt{13}, 0)$ of the hyperbola, therefore

$a^2=13$

Now $\sqrt{a^2-b^2}=ae_1$

$\therefore 13-b^2=1$

$\Rightarrow b^2=12$

Hence, equation of ellipse is

$\frac{x^2}{13}+\frac{y^2}{12}=1$

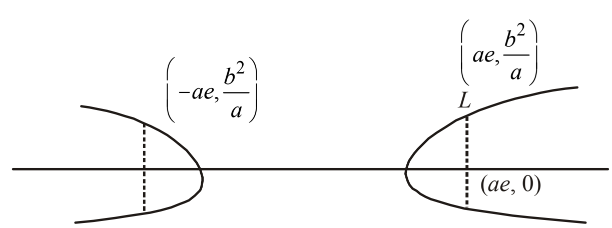
Now putting the coordinate of the point $\left(\frac{\sqrt{13}}{2},\frac{\sqrt{3}}{2}\right)$ in the equation of the ellipse, we get

$\frac{13}{4\times13}+\frac{3}{4\times12}=1$

$\Rightarrow \frac{1}{4}+\frac{1}{16}=1$, which is not true,

Hence the point $\left(\frac{\sqrt{13}}{2},\frac{\sqrt{3}}{2}\right)$ does not lie on the ellipse.

214. (a)



Given $\frac{x^2}{4}-\frac{y^2}{5}=1$

$\Rightarrow\ a^2=4,\ b^2=5$

$e=\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{4+5}{4}}=\frac{3}{2}$

$L=\left(2\times\frac{3}{2},\frac{5}{2}\right)=\left(3,\frac{5}{2}\right)$

Equation of tangent at (x_1, y_1) is

$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

Here $x_1=3,\ y_1=\frac{5}{2}$

$\Rightarrow \frac{3x}{4}-\frac{y}{2}=1 \Rightarrow \frac{x}{\frac{4}{3}}+\frac{y}{-2}=1$

x-intercept of the tangent, OA $=\frac{4}{3}$

y-intercept of the tangent, OB $=-2$

$OA^2-OB^2=\frac{16}{9}-4=-\frac{20}{9}$

215. (d) Let the coordinate at point of intersection of normals at P and Q be (h, k)

Since, equation of normals to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ At point (x_1, y_1) is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

therefore equation of normal to the hyperbola $\frac{x^2}{3^2}-\frac{y^2}{2^2}=1$ at point P $(3\sec\theta,\ 2\tan\theta)$ is

$\frac{3^2x}{3\sec\theta}+\frac{2^2y}{2\tan\theta}=3^2+2^2$

$\Rightarrow \boxed{3x\cos\theta+2y\cot\theta=3^2+2^2}$ ...(1)

Similarly, Equation of normal to the hyperbola $\frac{x^2}{3^2}-\frac{y^2}{2^2}$ at point Q $(3\sec\phi,\ 2\tan\phi)$ is

$\frac{3^2x}{3\sec\phi}+\frac{2^2y}{2\tan\phi}=3^2+2^2$

$\Rightarrow \boxed{3x\cos\phi+2y\cot\phi=3^2+2^2}$ ...(2)

Given $\theta+\phi=\frac{\pi}{2}\Rightarrow\phi=\frac{\pi}{2}-\theta$ and these passes through (h, k)

$\therefore$ From eq. (2)

$3x\cos\left(\frac{\pi}{2}-\theta\right)+2y\cot\left(\frac{\pi}{2}-\theta\right)=3^2+2^2$

$\Rightarrow \boxed{3h\sin\theta+2k\tan\theta=3^2+2^2}$ ...(3)

and $\boxed{3h\cos\theta+2k\cot\theta=3^2+2^2}$ ...(4)

Comparing equation (3) & (4), we get

$3h\cos\theta+2k\cot\theta=3h\sin\theta+2k\tan\theta$

$3h\cos\theta-3h\sin\theta=2k\tan\theta-2k\cot\theta$

$3h(\cos\theta-\sin\theta)=2k(\tan\theta-\cot\theta)$

$3h(\cos\theta-\sin\theta)=2k\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta\cos\theta}$

or, $3h=\frac{-2k(\sin\theta+\cos\theta)}{\sin\theta\cos\theta}$ ...(5)

Now, putting the value of equation (5) in eq. (3)

$\frac{-2k(\sin\theta+\cos\theta)\sin\theta}{\cancel{\sin}\theta\cos\theta}+2k\tan\theta=3^2+2^2$

$\Rightarrow 2k\cancel{\tan}\theta-2k+2k\cancel{\tan}\theta=13$

$-2k=13\Rightarrow k=\frac{-13}{2}$

Hence, ordinate of point of intersection of normals at P and Q is $\frac{-13}{2}$

216. (a) $x^2-6y=0$ ...(i)

$2x^2-4y^2=9$ ...(ii)

Consider the line,

$x-y=\frac{3}{2}$ ...(iii)

On solving (i) and (iii), we get only

$x=3, y=\frac{3}{2}$

Hence $\left(3,\frac{3}{2}\right)$ is the point of contact of conic (i), and line (iii)

On solving (ii) and (iii), we get only $x=3,\ y=\frac{3}{2}$

Hence $\left(3, \frac{3}{2}\right)$ is also the point of contact of conic (ii) and line (iii).

Hence line (iii) is the common tangent to both the given conics.

217. **(d)** Equation of the tangent at the point 'θ' is

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

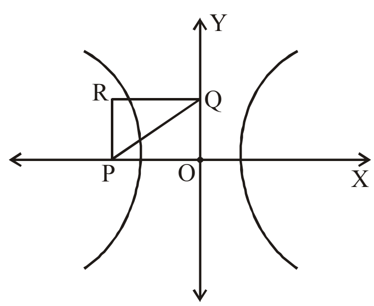
$\Rightarrow P = (a\cos\theta, 0)$ and $Q = (0, -b\cot\theta)$

Let R be (h, k) $\Rightarrow$ h = a cos θ, k = $-b$ cot θ

$$\Rightarrow \frac{k}{h} = \frac{-b}{a\sin\theta} \Rightarrow \sin\theta = \frac{-bh}{ak} \text{ and } \cos\theta = \frac{h}{a}$$

By squaring and adding,

$$\frac{b^2h^2}{a^2k^2} + \frac{h^2}{a^2} = 1$$



$$\Rightarrow \frac{b^2}{k^2} + 1 = \frac{a^2}{h^2} \quad \Rightarrow \quad \frac{a^2}{h^2} - \frac{b^2}{k^2} = 1$$

Now, given eq^n of hyperbola is $\frac{x^2}{4} - \frac{y^2}{2} = 1$

$\Rightarrow a^2 = 4, b^2 = 2$

$\therefore$ R lies on $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ i.e., $\frac{4}{x^2} - \frac{2}{y^2} = 1$

218. **(c)** Given equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$$

eccentricity $= e = \sqrt{1 - \frac{b^2}{16}}$

foci: $\pm ae = \pm 4\sqrt{1 - \frac{b^2}{16}}$

Equation of hyperbola is $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$\Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

eccentricity $= e = \sqrt{1 + \frac{81}{25} \times \frac{25}{144}} = \sqrt{1 + \frac{81}{144}}$

$$= \sqrt{\frac{225}{144}} = \frac{15}{12}$$

foci: $\pm ae = \pm \frac{12}{5} \times \frac{15}{12} = \pm 3$

Since, foci of ellipse and hyperbola coincide

$$\therefore \pm 4\sqrt{1 - \frac{b^2}{16}} = \pm 3 \Rightarrow b^2 = 7$$

$$\frac{K^2}{9} - \frac{4}{4} = 1 \qquad (\because b = \pm 2)$$

$\Rightarrow K^2 = 18$

219. **(a)** Given hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{b^2} = 1$$

Since this passes through $(K, 2)$, therefore

$$\frac{K^2}{9} - \frac{4}{b^2} = 1 \qquad ...(1)$$

Also, given $e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{\sqrt{13}}{3}$

$$\Rightarrow \sqrt{1 + \frac{b^2}{9}} = \frac{\sqrt{13}}{3} \Rightarrow 9 + b^2 = 13$$

$\Rightarrow b = \pm 2$

Now, from eq^n (1), we have

$$\frac{K^2}{9} - \frac{4}{4} = 1 \qquad (\because b = \pm 2)$$

$\Rightarrow K^2 = 18$

220. **(b)** Given that $ae = 2$ and $e = 2$

$\therefore \quad a = 1$

We know, $b^2 = a^2\left(e^2 - 1\right)$

$b^2 = 1(4-1)$

$b^2 = 3$

$\therefore$ Equation of hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1$

$3x^2 - y^2 = 3$

221. **(b)** Given, equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

Compare with equation of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get $a^2 = \cos^2\alpha$ and

$b^2 = \sin^2\alpha$

We know that, $b^2 = a^2(e^2 - 1)$

$\Rightarrow \sin^2\alpha = \cos^2\alpha(e^2 - 1)$

$\Rightarrow \sin^2\alpha + \cos^2\alpha = \cos^2\alpha . e^2$

$\Rightarrow e^2 = \sec^2\alpha$

$\Rightarrow e = \sec\alpha$

$\therefore \quad ae = \cos\alpha \,.\, \frac{1}{\cos\alpha} = 1$

Co-ordinates of foci are $(\pm ae, 0)$

i.e. $(\pm 1, 0)$

Hence, abscissae of foci remain constant when α varies.

222. **(d)** We know that tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Given that $y = \alpha x + \beta$ is the tangent of hyperbola.

$\Rightarrow m = \alpha$ and $a^2m^2 - b^2 = \beta^2$

$\therefore \quad a^2\alpha^2 - b^2 = \beta^2$

Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

223. **(d)** $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$a = \sqrt{\frac{144}{25}} = \frac{12}{5}, b = \sqrt{\frac{81}{25}} = \frac{9}{5},$

$e = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$

$\therefore$ Foci $= (\pm ae, 0) = (\pm 3, 0)$

$\therefore$ foci of ellipse = foci of hyperbola

$\therefore$ for ellipse $ae = 3$ but $a = 4$,

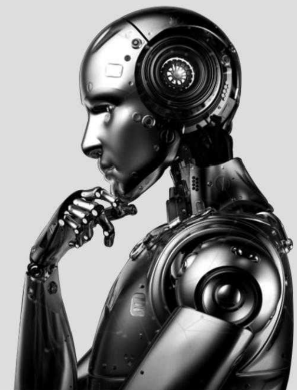
$\therefore \quad e = \frac{3}{4}$

Then, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 16\left(1 - \frac{9}{16}\right) = 7$

12

Limits and Derivatives



TOPIC 1 Limit of a Function, Left Hand & Right Hand limits, Existance of Limits, Sandwitch Theorem, Evaluation of Limits when X→ ∞, Limits by Factorisation, Substitution & Rationalisation

1. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim\limits_{x \to \alpha^+} \dfrac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to: **[Sep. 05, 2020 (I)]**

(a) $\dfrac{3}{2}$ (b) $\dfrac{3}{\sqrt{2}}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{1}{2}$

2. $\lim\limits_{x \to 0} \dfrac{x\left(e^{(\sqrt{1+x^2+x^4}-1)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$ **[Sep. 05, 2020 (II)]**

(a) is equal to $\sqrt{e}$ (b) is equal to 1
(c) is equal to 0 (d) does not exist

3. Let $[t]$ denote the greatest integer $\le t$. If for some $\lambda \in \mathbf{R} - \{0, 1\}$, $\lim\limits_{x \to 0} \left|\dfrac{1-x+|x|}{\lambda - x + [x]}\right| = L$, then L is equal to : **[Sep. 03, 2020 (I)]**

(a) 1 (b) 2 (c) $\dfrac{1}{2}$ (d) 0

4. If $\lim\limits_{x \to 0}\left\{\dfrac{1}{x^8}\left(1-\cos\dfrac{x^2}{2}-\cos\dfrac{x^2}{4}+\cos\dfrac{x^2}{2}\cos\dfrac{x^2}{4}\right)\right\} = 2^{-k}$, then the value of k is __________. **[NA Sep. 03, 2020 (I)]**

5. $\lim\limits_{x \to 2} \dfrac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to________. **[NA Jan. 7, 2020 (I)]**

6. Let $f(x) = 5 - |x-2|$ and $g(x) = |x+1|$, $x \in$ R. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim\limits_{x \to -\alpha\beta} \dfrac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to : **[April 12, 2019 (II)]**

(a) 1/2 (b) –3/2 (c) –1/2 (d) 3/2

7. $\lim\limits_{x \to 0} \dfrac{x + 2\sin x}{\sqrt{x^2+2\sin x+1} - \sqrt{\sin^2 x - x + 1}}$ is **[April 12, 2019 (II)]**

(a) 6 (b) 2 (c) 3 (d) 1

8. If $\lim\limits_{x \to 1} \dfrac{x^4-1}{x-1} = \lim\limits_{x \to k} \dfrac{x^3-k^3}{x^2-k^2}$, then k is: **[April 10, 2019 (I)]**

(a) $\dfrac{8}{3}$ (b) $\dfrac{3}{8}$ (c) $\dfrac{3}{2}$ (d) $\dfrac{4}{3}$

9. If $\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{x-1} = 5$, then $a + b$ is equal to : **[April 10, 2019 (II)]**

(a) –4 (b) 5 (c) –7 (d) 1

10. $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ equals : **[April 8, 2019 (I)]**

(a) $4\sqrt{2}$ (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) 4

11. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is: **[Jan. 12, 2019 (I)]**

(a) 4 (b) $4\sqrt{2}$ (c) $8\sqrt{2}$ (d) 8

12. $\lim\limits_{x\to 1^-}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to: **[Jan. 12, 2019 (II)]**

(a) $\dfrac{1}{\sqrt{2\pi}}$ (b) $\sqrt{\dfrac{2}{\pi}}$ (c) $\sqrt{\dfrac{\pi}{2}}$ (d) $\sqrt{\pi}$

13. Let [x] denote the greatest integer less than or equal to x. Then :

$$\lim_{x\to 0}\frac{\tan\left(\pi\sin^2 x\right)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}:$$

[Jan. 11, 2019 (I)]

(a) does not exist (b) equals π
(c) equals $\pi+1$ (d) equals 0

14. $\lim\limits_{x\to 0}\dfrac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to : **[Jan. 11, 2019 (II)]**

(a) 0 (b) 2 (c) 4 (d) 1

15. For each $t\in\mathbf{R}$, let [t] be the greatest integer less than or equal to t. Then, **[Jan. 10, 2019 (I)]**

$$\lim_{x\to 1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(a) equals 1 (b) equals 0
(c) equals – 1 (d) does not exist

16. $\lim\limits_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ **[Jan. 9, 2019 (I)]**

(a) exists and equals $\dfrac{1}{4\sqrt{2}}$

(b) exists and equals $\dfrac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(c) exists and equals $\dfrac{1}{2\sqrt{2}}$

(d) does not exist

17. For each $x\in\mathbf{R}$, let $[x]$ be greatest integer less than or equal to x. Then **[Jan. 09, 2019 (II)]**

$\lim\limits_{x\to 0}\dfrac{x\left([x]+|x|\right)\sin[x]}{x}$ is equal to:

(a) – sin 1 (b) 1 (c) sin 1 (d) 0

18. $\lim\limits_{x\to 0}\dfrac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ equals. **[Online April 15, 2018]**

(a) 1 (b) $-\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{2}$

19. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cot x-\cos x}{(\pi-2x)^3}$ equals : **[2017]**

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{24}$ (c) $\dfrac{1}{16}$ (d) $\dfrac{1}{8}$

20. $\lim\limits_{x\to 3}\dfrac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to : **[Online April 8, 2017]**

(a) $\sqrt{3}$ (b) $\dfrac{1}{\sqrt{2}}$ (c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{1}{2\sqrt{2}}$

21. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to : **[2015]**

(a) 2 (b) $\dfrac{1}{2}$ (c) 4 (d) 3

22. $\lim\limits_{x\to 0}\dfrac{e^{x^2}-\cos x}{\sin^2 x}$ is equal to : **[Online April 10, 2015]**

(a) 2 (b) 3 (c) $\dfrac{3}{2}$ (d) $\dfrac{5}{4}$

23. $\lim\limits_{x\to 0}\dfrac{\sin\left(\pi\cos^2 x\right)}{x^2}$ is equal to: **[2014]**

(a) $-\pi$ (b) π (c) $\dfrac{\pi}{2}$ (d) 1

24. If $\lim\limits_{x\to 2}\dfrac{\tan(x-2)\left\{x^2+(k-2)x-2k\right\}}{x^2-4x+4}=5,$

then k is equal to: **[Online April 11, 2014]**

(a) 0 (b) 1 (c) 2 (d) 3

25. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **[2013]**

(a) $-\dfrac{1}{4}$ (b) $\dfrac{1}{2}$ (c) 1 (d) 2

26. $\lim\limits_{x\to 0}\dfrac{\sin\left(\pi\cos^2 x\right)}{x^2}$ equals **[Online May 26, 2012]**

(a) $-\pi$ (b) 1 (c) – 1 (d) π

27. $\lim\limits_{x\to 0}\left(\dfrac{x-\sin x}{x}\right)\sin\left(\dfrac{1}{x}\right)$ **[Online May 7, 2012]**

(a) equals 1 (b) equals 0
(c) does not exist (d) equals – 1

28. Let $f : R \to [0,\infty)$ be such that $\lim_{x\to5} f(x)$ exists and

$$\lim_{x\to5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$$ **[2011RS]**

Then $\lim_{x\to5} f(x)$ equals :

(a) 0 (b) 1 (c) 2 (d) 3

29. $\lim_{x\to2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ **[2011]**

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$

(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

30. Let f : R → R be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$ then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$ **[2010]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 1

31. Let α and β be the distinct roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to **[2005]**

(a) $\frac{a^2}{2}(\alpha-\beta)^2$ (b) 0

(c) $\frac{-a^2}{2}(\alpha-\beta)^2$ (d) $\frac{1}{2}(\alpha-\beta)^2$

32. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is **[2003]**

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

33. $\lim_{x\to0}\frac{\log x^n-[x]}{[x]}, n\in N$, ([x] denotes greatest integer less than or equal to x) **[2002]**

(a) has value -1 (b) has value 0

(c) has value 1 (d) does not exist

34. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ **[2002]**

(a) e^4 (b) e^2 (c) e^3 (d) 1

35. $\lim_{x\to0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is **[2002]**

(a) 1 (b) –1

(c) zero (d) does not exist

TOPIC 2 Limits Using L-hospital's Rule, Evaluation of Limits of the form 1^∞, Limits by Expansion Method

36. $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}$ $(a\neq0)$ is equal to :

[Sep. 03, 2020 (II)]

(a) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$ (b) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

(c) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (d) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

37. If $\lim_{x\to1}\frac{x+x^2+x^3+...+x^n-n}{x-1}=820, (n\in\mathbf{N})$ then the value of n is equal to ________. **[NA Sep. 02, 2020 (I)]**

38. $\lim_{x\to0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to : **[Sep. 02, 2020 (II)]**

(a) e (b) 2 (c) 1 (d) e^2

39. $\lim_{x\to0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ (c) e^2 (d) e

40. $\lim_{x\to0}\frac{\int_0^x t\sin(10t)dt}{x}$ is equal to: **[Jan. 8, 2020 (II)]**

(a) 0 (b) $\frac{1}{10}$ (c) $-\frac{1}{5}$ (d) $-\frac{1}{10}$

41. If α and β are the roots of the equation $375x^2–25x–2=0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r+\lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to :

[April 12, 2019 (I)]

(a) $\frac{21}{346}$ (b) $\frac{29}{358}$ (c) $\frac{1}{12}$ (d) $\frac{7}{116}$

42. Let $f: R \to R$ be a differentiable function satisfying $f'(3)+f'(2)=0$. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to : **[April 08, 2019 (II)]**

(a) 1 (b) e^{-1} (c) e (d) e^2

43. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then **[2018]**

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+...+\left[\frac{15}{x}\right]\right)$$

(a) is equal to 15. (b) is equal to 120.
(c) does not exist (in R). (d) is equal to 0.

44. $\lim_{x\to 0}\frac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals. **[Online April 16, 2018]**

(a) $-\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $-\frac{1}{6}$ (d) $\frac{1}{3}$

45. Let $p=\lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to : **[2016]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 2 (d) 1

46. $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$ is : **[Online April 10, 2016]**

(a) 2 (b) $-\frac{1}{2}$ (c) –2 (d) $\frac{1}{2}$

47. If $\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, then 'a' is equal to : **[Online April 9, 2016]**

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

48. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are **[2004]**

(a) $a=1$ and $b=2$ (b) $a=1, b\in R$
(c) $a\in R, b=2$ (d) $a\in R, b\in R$

49. Let $f(x)=4$ and $f'(x)=4$. Then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ is given by **[2002]**

(a) 2 (b) –2 (c) -4 (d) 3

TOPIC 3 Derivatives of Polynomial & Trigonometric Functions, Derivative of Sum, Difference, Product & Quotient of two functions

50. Let $f(x)$ be a polynomial of degree 4 having extreme values at $x=1$ and $x=2$. If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right)=3$ then $f(-1)$ is equal to **[Online April 15, 2018]**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{9}{2}$

51. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then f(2) is equal to : **[2015]**

(a) 0 (b) 4
(c) -8 (d) -4

52. Let $f(1)=-2$ and $f'(x)\geq 4.2$ for $1\leq x\leq 6$. The possible value of $f(6)$ lies in the interval : **[April 25, 2013]**

(a) $[15, 19)$ (b) $(-\infty, 12)$
(c) $[12, 15)$ (d) $[19, \infty)$

53. If $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7$, then $\lim_{\alpha\to 0}\frac{f(1-\alpha)-f(1)}{\alpha^3+3\alpha}$ is **[Online May 19, 2012]**

(a) $-\frac{53}{3}$ (b) $\frac{53}{3}$ (c) $-\frac{55}{3}$ (d) $\frac{55}{3}$

Hints & Solutions

1. **(b)** $x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2, -1 \Rightarrow \alpha = 2$

$$\therefore \lim_{x\to 2^+} \frac{\sqrt{1-\cos(x^2-x-2)}}{x-2}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{2}\left|\sin\left(\frac{x^2-x-2}{2}\right)\right|}{x-2}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{2}\sin\frac{(x^2-x-2)}{2}}{\left(\frac{x^2-x-2}{2}\right)} \times \frac{(x^2-x-2)}{2(x-2)}$$

$$= \frac{1}{\sqrt{2}} \lim_{x\to 2^+} \left(\frac{\sin\left(\frac{x^2-x-2}{2}\right)}{\frac{x^2-x-2}{2}}\right) \times \lim_{x\to 2^+} \frac{(x-2)(x+1)}{(x-2)}$$

$$= \frac{1}{\sqrt{2}} \times 1 \times 3 = \frac{3}{\sqrt{2}}$$

2. **(b)** Let $L = \lim_{x\to 0} \frac{x\left(e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1\right)}{\sqrt{1+x^2+x^4}-1}$

$$= \lim_{x\to 0} \frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1}{\frac{\sqrt{1+x^2+x^4}-1}{x}}$$

Put $\frac{\sqrt{1+x^2+x^4}-1}{x} = t$ when $x \to 0 \Rightarrow t \to 0$

$$\therefore L = \lim_{t\to 0} \frac{e^t - 1}{t} = 1$$

3. **(b)** Given $\lim_{x\to 0} \left|\frac{1-x+|x|}{\lambda - x + [x]}\right| = L$

Here, L.H.L. $= \lim_{h\to 0} \left|\frac{1+h+h}{\lambda+h-1}\right| = \left|\frac{1}{\lambda-1}\right|$

R.H.L. $= \lim_{h\to 0} \left|\frac{1-h+h}{\lambda+h+0}\right| = \left|\frac{1}{\lambda}\right|$

Given that limit exists. Hence L.H.L. = R.H.L.

$\Rightarrow |\lambda - 1| = |\lambda|$

$\Rightarrow \lambda = \frac{1}{2}$ and $L = \left|\frac{1}{\lambda}\right| = 2$

4. **(8)**

$$\lim_{x\to 0} \frac{\left(1-\cos\frac{x^2}{2}\right)}{x^4} \frac{\left(1-\cos\frac{x^2}{4}\right)}{x^4} = 2^{-k}$$

$$\Rightarrow \lim_{x\to 0} \frac{2\sin^2\frac{x^2}{4}}{\frac{x^4}{16}\times 16} \times \frac{2\sin^2\frac{x^2}{8}}{\frac{x^4}{64}\times 64} = 2^{-k}$$

$$\Rightarrow \frac{4}{16\times 64} = 2^{-8} = 2^{-k} \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$\therefore k = 8$

5. **(36)** Let $3^x = t^2$

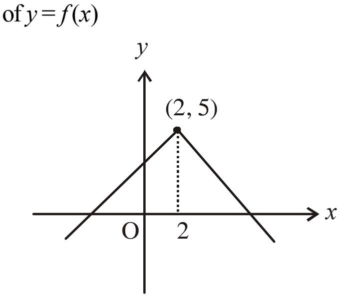
$$\lim_{t\to 3} \frac{t^2 + \frac{27}{t^2} - 12}{\frac{1}{t} - \frac{3}{t^2}} = \lim_{t\to 3} \frac{t^4 - 12t^2 + 27}{t-3}$$

$$= \lim_{t\to 3} \frac{(t^2-3)(t+3)(t-3)}{t-3}$$

$= (3^2 - 3)(3+3) = 36.$

6. **(a)** $f(x) = 5 - |x-2|$

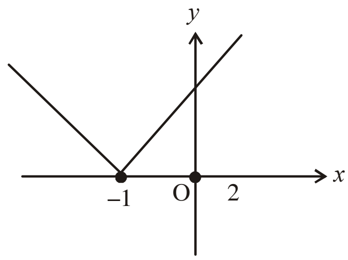
Graph of $y = f(x)$



By the graph $f(x)$ is maximum at $x = 2$

$\therefore \alpha = 2\ g(x) = |x+1|$

Graph of $y = g(x)$



By the graph $g(x)$ is minimum at $x = -1$

$\therefore \quad \beta = -1$

Now, $\lim\limits_{x \to 2} \dfrac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$

$= \lim\limits_{x \to 2} \dfrac{(x-1)(x-3)}{x-4} = \dfrac{1}{2}$

7. **(b)** Given limit is,

$$\lim_{x \to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$$

On rationalising,

$$= \lim_{x \to 0} \frac{(x + 2\sin x)\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{(x^2 - \sin^2 x) + (x + 2\sin x)}$$

$$= \lim_{x \to 0} \frac{\left[1 + 2\left(\frac{\sin x}{x}\right)\right]\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{\left(x - \frac{\sin^2 x}{x}\right) + \left(1 + 2\left(\frac{\sin x}{x}\right)\right)}$$

$$= \frac{3 \times 2}{3} = 2 \qquad \left[\because \lim_{x \to 0} \frac{\sin}{x} = 1\right]$$

8. **(a)** Given, $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1} = \lim\limits_{x \to K} \left(\dfrac{x^3 - k^3}{x^2 - k^2}\right)$

Taking L.H.S. $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1} \quad \left(\dfrac{0}{0}\text{form}\right)$

$\underset{x \to 1}{\text{Lt}} \dfrac{4x^3}{1} = 4 \qquad$ [Using L Hospital's Rule]

$\therefore \lim\limits_{x \to K} \dfrac{x^3 - k^3}{x^2 - k^2} = 4$

$\Rightarrow \lim\limits_{x \to K} \dfrac{3x^2}{2x} = 4 \qquad$ [Using L Hospital's Rule]

$\Rightarrow \dfrac{3}{2}k = 4 \Rightarrow k = \dfrac{8}{3}$

9. **(c)** $\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$

$\because$ limit is finite. $\therefore 1 - a + b = 0$

$\Rightarrow \lim\limits_{x \to 1} \dfrac{2x - a}{1} = 5 \quad \left(\dfrac{0}{0}\text{form}\right)$ (By L Hospital's rule)

$\Rightarrow 2 - a = 5 \Rightarrow a = -3$ and $b = -4$

Then $a + b = -3 - 4 = -7$

10. **(a)** $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$

$$= \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{2\cos^2 \frac{x}{2}}} \qquad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2}\left[1 - \cos \frac{x}{2}\right]} = \lim_{x \to 0} \frac{\sin^2 x}{2\sqrt{2} \sin^2 \frac{x}{4}}$$

$$= \lim_{x \to 0} \frac{\left(\frac{\sin x}{x}\right)^2 .16}{2\sqrt{2}\left(\frac{\sin \frac{x}{4}}{\frac{x}{4}}\right)^2} = \frac{16}{2\sqrt{2}} = 4\sqrt{2}$$

11. **(d)** $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)} = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cot^3 x\left(1 - \frac{\tan x}{\cos^3 x}\right)}{\cos(x + \pi/4)}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 - \tan^4 x)}{\tan^3 x \cos(x + \pi/4)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 + \tan^2 x)(1 - \tan x)(1 + \tan x)}{\tan^3 x\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 + \tan^2 x)(1 + \tan x)(\cos x - \sin x)}{\frac{\sin^3 x}{\cos^2 x}\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)}$$

$$= \frac{(2)(2)}{\frac{1}{(\sqrt{2})(\sqrt{2})}} = 8$$

12. **(b)** $\lim\limits_{x \to 1^-} \dfrac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1 - x}} = \lim\limits_{h \to 0} f(1 - h)$

$$= \lim_{h \to 0} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}(1 - h)}}{\sqrt{1 - (1 - h)}}$$

$$= \lim_{h\to 0}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{h}}$$

$$= \lim_{h\to 0}\frac{-\frac{1}{2\sqrt{2\sin^{-1}(1-h)}}\times 2\times\frac{1}{\sqrt{1-(1-h)^2}}(-1)}{\frac{1}{2\sqrt{h}}}$$

$$= \lim_{h\to 0}\frac{\frac{1}{\sqrt{2\sin^{-1}(1-h)}}\frac{1}{\sqrt{h(2-h)}}}{\frac{1}{2\sqrt{h}}}$$

$$=2\times\frac{1}{\sqrt{\pi}}\times\frac{1}{\sqrt{2}}=\sqrt{\frac{2}{\pi}}$$

13. (a) RHL is, $\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+(x-0)^2}{x^2}$

$$= \lim_{x\to 0^+}\left(\frac{\tan(\pi\sin^2 x}{x^2}+1\right)=1+\pi$$

And LHL is, $\lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

$$= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+x^2+\sin^2 x-2x\sin x}{x^2}$$

$=\pi+1+1-2=\pi$

Since, LHL $\neq$ RHL

Hence, limit does not exist.

14. (d) $\lim_{x\to 0}\frac{x\cot 4x}{\sin^2 x.\cot^2 2x}=\lim_{x\to 0}\frac{x.\tan^2 2x}{\sin^2 x.\tan 4x}$

$$= \lim_{x\to 0}\left(\frac{x}{\sin x}\right)^2.\left(\frac{\tan 2x}{2x}\right)^2.\left(\frac{4x}{\tan 4x}\right).\frac{4}{2^2}=1$$

15. (b) $\lim_{x\to 1^+}\frac{(1-|x|+\sin(|1-x|))\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$

$$=\lim_{h\to 0}\frac{(1-|1+h|+\sin(|1-1-h|))\sin\left(\frac{\pi}{2}[1-1-h]\right)}{|1-1-h|[1-1-h]}$$

$$=\lim_{h\to 0}\frac{(1-1-h+\sinh)\sin\left(\frac{\pi}{2}(-1)\right)}{h([0-h])}$$

$$= \lim_{h\to 0}\frac{(-h+\sin h)\sin\left(-\frac{\pi}{2}\right)}{h(-1)}=0$$

16. (a) $L=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$$=\lim_{y\to 0}\frac{\left(\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$=\lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$=\frac{1}{2\sqrt{2}\times 2}=\frac{1}{4\sqrt{2}}$$

17. (a) $\lim_{x\to 0^-}\frac{x([x]+|x|)\cdot\sin[x]}{|x|}$

$$=\lim_{h\to 0}\frac{(0-h)([0-h]+|0-h|)\cdot\sin[0-h]}{|0-h|}$$

$$=\lim_{h\to 0}\frac{(-h)(-1+h)\sin(-1)}{h}$$

$$=\lim_{h\to 0}(1-h)\sin(-1)=-\sin 1$$

18. (d) Let, $L=\lim_{x\to 0}\frac{(x\tan 2x-2x\tan x)}{(1-\cos 2x)^2}=\lim_{x\to 0}K$ (say)

$$\Rightarrow K=\frac{x\left[\frac{2\tan x}{1-(\tan x)^2}\right]-2x\tan x}{(1-(1-2\sin^2 x))^2}$$

$$=\frac{2x\tan x-[2x\tan x-2x\tan^3 x]}{4\sin^4 x\times(1-\tan^2 x)}$$

$$=\frac{2x\tan^3 x}{4\sin^4 x\times(1-\tan^2 x)}=\frac{2x\tan^3 x}{4\sin^4 x\times\left(\frac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)}$$

$$=\frac{2x\frac{\sin^3 x}{\cos^3 x}}{4\sin^4 x\times\left(\frac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)}$$

$$\Rightarrow K = \frac{x}{2\sin x \times (\cos^2 x - \sin^2 x)\cos x}$$

$$\therefore L = \lim_{x\to 0} \frac{x}{2\sin x} \times \lim_{x\to 0} \frac{1}{\cos x\,(\cos^2 x - \sin^2 x)}$$

$$= \lim_{x\to 0} \frac{x}{2\sin x} \times \lim_{x\to 0} \frac{1}{\cos 0\,(\cos^2 0 - \sin^2 0)} = \frac{1}{2}$$

19. **(c)** $\lim_{x\to\frac{\pi}{2}} \frac{\cot x(1-\sin x)}{-8\left(x-\frac{\pi}{2}\right)^3} = \lim_{x\to\frac{\pi}{2}} \frac{\cot x(1-\sin x)}{8\left(\frac{\pi}{2}-x\right)^3}$

Put $\frac{\pi}{2} - x = t \Rightarrow$ as $x \to \frac{\pi}{2} \Rightarrow t \to 0$

$$= \lim_{t\to 0} \frac{\cot\left(\frac{\pi}{2}-t\right)\left(1-\sin\left(\frac{\pi}{2}-t\right)\right)}{8t^3}$$

$$= \lim_{t\to 0} \frac{\tan t(1-\cos t)}{8t^3} = \lim_{t\to 0} \frac{\tan t}{8t} . \frac{1-\cos t}{t^2}$$

$$= \frac{1}{8}.1.\frac{1}{2} = \frac{1}{16}$$

20. **(b)** Let $A = \lim_{x\to 3} \frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

Rationalise

$$\Rightarrow A = \lim_{x\to 3} \frac{(3x-9)\times\left(2x-4+\sqrt{2}\right)}{\{(2x-4)-2\}\times\left(\sqrt{3x}+3\right)}$$

$$= \lim_{x\to 3} \frac{3(x-3)}{2(x-3)} \times \frac{\sqrt{2x-4}+\sqrt{2}}{\left(\sqrt{3x}+3\right)} = \frac{3}{2} \times \frac{2\sqrt{2}}{6} = \frac{1}{\sqrt{2}}$$

21. **(a)** Multiply and divide by x in the given expression, we get

$$\lim_{x\to 0} \frac{(1-\cos 2x)}{x^2} \frac{(3+\cos x)}{1} \cdot \frac{x}{\tan 4x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2} \cdot \frac{3+\cos x}{1} \cdot \frac{x}{\tan 4x}$$

$$= 2\lim_{x\to 0} \frac{\sin^2 x}{x^2} \cdot \lim_{x\to 0} 3+\cos x \cdot \lim_{x\to 0} \frac{x}{\tan 4x}$$

$$= 2.4\,\frac{1}{4} \lim_{x\to 0} \frac{4x}{\tan 4x} = 2.4.\frac{1}{4} = 2$$

22. **(c)** $\lim_{x\to 0} \frac{2xe^{x^2}+\sin x}{2\sin x\cos x}$

$$\lim_{x\to 0}\left(\frac{x}{\sin x}e^{x^2}+\frac{1}{2}\right)\frac{1}{\cos x} = 1 + \frac{1}{2} = \frac{3}{2}$$

23. **(b)** $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$

$$= \lim_{x\to 0} \frac{\sin\left[\pi(1-\sin^2 x)\right]}{x^2}$$

$$= \lim_{x\to 0} \sin\frac{(\pi - \pi\sin^2 x)}{x^2} \qquad [\because \sin(\pi-\theta) = \sin\theta]$$

$$= \lim_{x\to 0} \sin\frac{(\pi\sin^2 x)}{\pi\sin^2 x} \times \frac{\pi\sin^2 x}{x^2}$$

$$= \lim_{x\to 0} 1 \times \pi\left(\frac{\sin x}{x}\right)^2 = \pi$$

24. **(d)** $\lim_{x\to 2} \frac{\tan(x-2)\{x^2+(k-2)x-2k\}}{x^2-4x+4} = 5$

$$\Rightarrow \lim_{x\to 2} \frac{\tan(x-2)\{x^2+kx-2x-2k\}}{(x-2)^2} = 5$$

$$\Rightarrow \lim_{x\to 2} \frac{\tan(x-2)\{x(x-2)+k(x-2)\}}{(x-2)\times(x-2)} = 5$$

$$\Rightarrow \lim_{x\to 2}\left(\frac{\tan(x-2)}{(x-2)}\right) \times \lim_{x\to 2}\left(\frac{(k+x)\cancel{(x-2)}}{\cancel{(x-2)}}\right) = 5$$

$$\Rightarrow 1 \times \lim_{x\to 2}(k+x) = 5 \qquad \left\{\because \lim_{h\to 0}\frac{\tan h}{h} = 1\right\}$$

or $k + 2 = 5$

$\Rightarrow \boxed{k=3}$

25. **(d)** Multiply and divide by x in the given expression, we get

$$\lim_{x\to 0} \frac{(1-\cos 2x)}{x^2} \frac{(3+\cos x)}{1} \cdot \frac{x}{\tan 4x}$$

$$\left[\because 1-\cos 2x = 2\sin^2\frac{x}{2}\right]$$

$$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2} \cdot \frac{3+\cos x}{1} \cdot \frac{x}{\tan 4x}$$

$$= 2\lim_{x\to 0} \frac{\sin^2 x}{x^2} \cdot \lim_{x\to 0}(3+\cos x) \cdot \lim_{x\to 0} \frac{4x}{\tan 4x} \times \frac{1}{4}$$

$$= 2.4.\frac{1}{4} = 2$$

26. **(d)** Consider, $\lim_{x\to 0} \frac{\sin\left(\pi\cos^2 x\right)}{x^2}$

$$= \lim_{x\to 0} \frac{\sin\left(\pi - \pi\sin^2 x\right)}{x^2} \quad \left[\because \sin(\pi-\theta) = \sin\theta\right]$$

$$= \lim_{x\to 0} \frac{\sin\left(\pi\sin^2 x\right)}{\pi\sin^2 x} \times \frac{\left(\pi\sin^2 x\right)}{x^2} = \pi$$

27. **(b)** Consider $\lim_{x\to 0}\left(\frac{x-\sin x}{x}\right)\sin\left(\frac{1}{x}\right)$

$$=\lim_{x\to 0}\left[\frac{x\left(1-\frac{\sin x}{x}\right)}{x}\right]\times\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$

$$=\lim_{x\to 0}\left[1-\frac{\sin x}{x}\right]\times\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$

$$=\left[1-\lim_{x\to 0}\frac{\sin x}{x}\right]\times\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$

$$=0\times\lim_{x\to 0}\sin\left(\frac{1}{x}\right)=0$$

28. **(d)** Given that $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$

$$\Rightarrow \lim_{x\to 5}[(f(x))^2-9]=0$$

$$\Rightarrow \left[\lim_{x\to 5}f(x)\right]^2=9 \Rightarrow \lim_{x\to 5}f(x)=3$$

29. **(d)** $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ $\left[\because 1-\cos\theta=2\sin^2\frac{v}{2}\right]$

$$=\lim_{x\to 2}\frac{\sqrt{2}\,|\sin(x-2)|}{x-2}$$

$$\underset{(\text{at }x=2)}{\text{L.H.L}}=-\lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=-\sqrt{2}$$

$$\underset{(\text{at }x=2)}{\text{R.H.L}}=\lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=\sqrt{2}$$

Thus $\underset{(\text{at }x=2)}{\text{L.H.L}}\neq\underset{(\text{at }x=2)}{\text{R.H.L}}$

Hence, $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ does not exist.

30. **(d)** Given that f(x) is a positive increasing function.

$\therefore\ 0<f(x)<f(2x)<f(3x)$

Divided by $f(x)$

$$\Rightarrow 0<1<\frac{f(2x)}{f(x)}<\frac{f(3x)}{f(x)}$$

$$\Rightarrow \lim_{x\to\infty}1\le\lim_{x\to\infty}\frac{f(2x)}{f(x)}\le\lim_{x\to\infty}\frac{f(3x)}{f(x)}$$

By Sandwich Theorem.

$$\Rightarrow \lim_{x\to\infty}\frac{f(2x)}{f(x)}=1$$

31. **(a)** Given that

$ax^2+bx+c=a(x-\alpha)(x-\beta)$

$$\lim_{x\to\alpha}\frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2}{(x-\alpha)^2}\times\frac{\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}}\times\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}$$

$$=\frac{a^2(\alpha-\beta)^2}{2}.$$

32. **(d)** $\lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right).(1-\sin x)}{(\pi-2x)^3}$

Let $x=\frac{\pi}{2}+y;\ y\to 0$

$$=\lim_{y\to 0}\frac{\tan\left(-\frac{y}{2}\right).(1-\cos y)}{(-2y)^3}$$

$$=\lim_{y\to 0}\frac{-\tan\frac{y}{2}\,2\sin^2\frac{y}{2}}{(-8).\frac{y^3}{8}.8}\qquad\left[\because 1-\cos\theta=2\sin^2\frac{\theta}{2}\right]$$

$$=\lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left[\frac{\sin y/2}{y/2}\right]^2=\frac{1}{32}$$

$$\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=\lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right]$$

33. **(d)** Since, $\lim_{x\to 0^-}[x]=-1\neq\lim_{x\to 0^+}[x]=0$. So $\lim_{x\to 0}[x]$ does not exist, hence the required limit does not exist.

34. **(a)** $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x=\lim_{x\to\infty}\left(1+\frac{4x+1}{x^2+x+2}\right)^x$

$$=\lim_{x\to\infty}\left[\left(1+\frac{4x+1}{x^2+x+2}\right)^{\frac{x^2+x+2}{4x+1}}\right]^{\frac{(4x+1)x}{x^2+x+2}}$$

$$=e^{\lim_{x\to\infty}\frac{4x^2+x}{x^2+x+2}}\quad\left[\because \lim_{x\to\infty}(1+\lambda x)^{\frac{1}{x}}=e^{\lambda}\right]$$

$$=e^{\lim_{x\to\infty}\frac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{2}{x^2}}}=e^4\qquad\left[\because \frac{1}{\infty}=0\right]$$

35. **(d)** $\lim\limits_{x\to 0}\dfrac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$;

$$\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}\Rightarrow \lim_{x\to 0}\frac{|\sin x|}{x}\qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

The limit of above does not exist as

LHS $=-1\neq$ RHL $=1$

36. **(b)** $\lim\limits_{x\to a}\dfrac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}\qquad \left[\dfrac{0}{0}\text{ case}\right]$

Apply L'Hospital rule

$$\lim_{x\to a}\frac{\frac{1}{3}(a+2x)^{-2/3}\cdot 2-\frac{1}{3}\cdot(3x)^{-2/3}\cdot 3}{\frac{1}{3}(3a+x)^{-2/3}\cdot -\frac{1}{3}(4x)^{-2/3}\cdot 4}$$

$$=\frac{\frac{1}{3}(3a)^{-2/3}\cdot(2-3)}{\frac{1}{3}(4a)^{-2/3}\cdot(1-4)}=\frac{3^{-2/3}}{4^{-2/3}}\cdot\frac{1}{3}=\frac{2^{4/3}}{9^{1/3}}\cdot\frac{1}{3}=\frac{2}{3}\cdot\left(\frac{2}{9}\right)^{1/3}$$

37. **(40)**

$$\lim_{x\to 1}\frac{x+x^2+x^3+....+x^n-n}{x-1}=820\left(\frac{0}{0}\text{ case}\right)$$

$$\lim_{x\to 1}\frac{1+2x+3x^2+....+nx^{n-1}}{1}=820$$

(Using L' Hospital rule)

$\Rightarrow 1+2+3+....+n=820$

$\Rightarrow \dfrac{n(n+1)}{2}=820 \quad \Rightarrow n^2+n-1640=0$

$\Rightarrow n=40,\ n\in \mathrm{N}$

38. **(d)** $\lim\limits_{x\to 0}\left(\dfrac{1+\tan x}{1-\tan x}\right)^{1/x}$

$$\Rightarrow e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}\quad \Rightarrow e^{\lim\limits_{x\to 0}\frac{1}{x}\left(\frac{1+\tan x}{1-\tan x}-1\right)}$$

$$\Rightarrow e^{\lim\limits_{x\to 0}\left(\frac{2\tan x}{1-\tan x}\right)\frac{1}{x}}=e^{\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)\left(\frac{2}{1-\tan x}\right)}=e^2$$

39. **(b)** Let R $=\lim\limits_{x\to 0}\left(\dfrac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}=e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left\{\frac{3x^2+2}{7x^2+2}-1\right\}}$

$$=e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left\{\frac{-4x^2}{7x^2+2}\right\}}=e^{\frac{-4}{2}}=e^{-2}=\frac{1}{e^2}$$

40. **(a)** Using L' Hospital rule,

$$\lim_{x\to 0}\frac{x\sin(10x)}{1}=0$$

41. **(c)** Given equation is, $375x^2-25x-2=0$

Sum and product of the roots are,

$\alpha+\beta=\dfrac{25}{375}$ and $\alpha\beta=\dfrac{-2}{375}$

$$\lim_{n\to\infty}\sum_{r=1}^{n}(\alpha^r+\beta^r)$$

$=(\alpha+\alpha^2+\alpha^3+...\infty(\beta+\beta^2+\beta^3...+\infty)$

$$=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}=\frac{\alpha+\beta-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$$

$$=\frac{\frac{25}{375}+\frac{4}{375}}{1-\frac{25}{375}-\frac{2}{375}}=\frac{29}{375-25-2}=\frac{29}{348}=\frac{1}{12}$$

42. **(a)** $I=\lim\limits_{x\to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}\quad [1^\infty\text{ form}]$

$\Rightarrow I=e^{\ell}1,$ where

$$I_1=\lim_{x\to 0}\left(\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right)\right)\left(\frac{1}{x}\right)$$

$$=\lim_{x\to 0}\left(\frac{1}{x}\right)\left(\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)$$

$\left(\dfrac{0}{0}\text{ form}\right)$

By L. Hospital Rule,

$$I_1=\lim_{x\to 0}\left(\frac{f'(3+x)+f'(2-x)}{1}\right)\lim_{x\to 0}\left(\frac{1}{1+f(2-x)-f(2)}\right)$$

$=f'(3)+f'(2)=0$

$\Rightarrow I=e^{I_1}=e^0=1$

43. **(b)** Since, $\lim\limits_{x\to 0^+}x\left(\left[\dfrac{1}{x}\right]+\left[\dfrac{2}{x}\right]+...+\left[\dfrac{15}{x}\right]\right)$

$$=\lim_{x\to 0^+}x\left(\frac{1+2+3+...+15}{x}\right)-\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+...+\left\{\frac{15}{x}\right\}\right)$$

$\because\ 0\le\left\{\dfrac{r}{x}\right\}<1 \quad\Rightarrow\quad 0\le x\left\{\dfrac{r}{x}\right\}<x$

$$\therefore\ \lim_{x\to 0^+}x\left(\frac{1+2+3+...+15}{x}\right)=\frac{15\times 16}{2}=120$$

44. **(c)** Let $L = \lim_{x\to 0} \dfrac{(27+x)^{\frac{1}{3}} - 3}{9-(27+x)^{\frac{2}{3}}}$

Here 'L' is in the indeterminate form i.e., $\dfrac{0}{0}$

$\therefore$ using the L'Hospital rule we get:

$$L = \lim_{x\to 0} \frac{\frac{1}{3}(27+x)^{\frac{-2}{3}}}{-\frac{2}{3}(27+x)^{\frac{-1}{3}}} = \frac{\frac{1}{3}\times(27)^{\frac{-2}{3}}}{\frac{-2}{3}\times 27^{\frac{-1}{3}}} = -\frac{1}{6}$$

45. **(a)** $\ln p = \lim_{x\to 0^+} \dfrac{1}{2x}\ln(1+\tan^2\sqrt{x})$

$$\lim_{x\to 0^+} \frac{1}{x}\ln\left(\sec\sqrt{x}\right)$$

Applying L hospital's rule :

$$= \lim_{x\to 0^+} \frac{\sec\sqrt{x}\tan\sqrt{x}}{\sec\sqrt{x}\cdot 2\sqrt{x}} = \lim_{x\to 0^+} \frac{\tan\sqrt{x}}{2\sqrt{x}} = \frac{1}{2}$$

46. **(c)** $\lim_{x\to 0} \dfrac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$

$$= \lim_{x\to 0} \frac{(2\sin^2 x)^2}{2x\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+.....\right) - x\left(2x+\frac{2^3x^3}{3}+2\frac{2^5x^5}{15}+.....\right)}$$

$$= \lim_{x\to 0} \frac{4\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-.....\right)^4}{x^4\left(\frac{2}{3}-\frac{8}{5}\right)+x^6\left(\frac{4}{15}-\frac{64}{15}\right)}$$

$$= \lim_{x\to 0} \frac{4\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-.....\right)^4}{-2+x^2\left(-\frac{60}{15}\right)+.....}$$

(dividing numerator & denominator by x^4)

$= -2$

47. **(b)** $\lim_{x\to\infty}\left(1+\dfrac{a}{x}-\dfrac{4}{x^2}\right)^{2x}$ (1^∞ form)

$$= e\left[\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}-1\right)2x\right]$$

$$= e \lim_{x\to\infty}\left(2a-\frac{8}{x}\right) = e^{2a}$$

$\therefore\ 2a = 3 \Rightarrow a = 3/2$

48. **(b)** We know that $\lim_{x\to\infty}(1+x)^{\frac{1}{x}} = e$

Given that $\lim_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x} = e^2$

$$\Rightarrow \lim_{x\to\infty}\left[\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{\left(\frac{1}{\frac{a}{x}+\frac{b}{x^2}}\right)}\right]^{2x\left(\frac{a}{x}+\frac{b}{x^2}\right)} = e^2$$

$$\Rightarrow e^{\lim_{x\to\infty} 2\left[a+\frac{b}{x}\right]} = e^2 \Rightarrow e^{2a} = e^2$$

$\Rightarrow a = 1$ and $b \in R$

49. **(c)** Given that $f(2) = 4$ and $f(2) = 4$

We have, $\lim_{x\to 2} \dfrac{xf(2)-2f(x)}{x-2}$, $\left(\dfrac{0}{0}\right)$

Applying L-Hospital's rule, we get

$= \lim_{x\to 2} f(2) - 2f'(x) = f(2) - 2f'(2)$

$= 4 - 2\times 4 = -4.$

50. **(d)** $\because f(x)$ has extremum values at $x = 1$ and $x = 2$

$\because f'(1) = 0$ and $f'(2) = 0$

As, $f(x)$ is a polynomial of degree 4.

Suppose $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

$$\because \lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{Ax^4+Bx^3+Cx^2+Dx+E}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(Ax^2+Bx+C+\frac{D}{x}+\frac{E}{x^2}+1\right) = 3$$

As limit has finite value, so $D = 0$ and $E = 0$

Now $A(0)^2 + B(0) + C + 0 + 0 + 1 = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$

$f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$f'(1) = 0 \Rightarrow 4A(1) + 3B(1) + 2C(1) + D = 0$

$\Rightarrow 4A + 3B = -4$ ...(i)

$f'(2) = 0 \Rightarrow 4A(8) + 3B(4) + 2C(2) + D = 0$

$\Rightarrow 8A + 3B = -2$ ...(ii)

From equations (i) and (ii), we get

$A = \dfrac{1}{2}$ and $B = -2$

So, $f(x) = \dfrac{x^4}{2} - 2x^3 + 2x^2$

Therefore, $f(-1) = \dfrac{(-1)^4}{2} - 2(-1)^3 + 2(-1)^2$

$= \dfrac{1}{2} + 2 + 2 = \dfrac{9}{2}$. Hence $f(-1) = \dfrac{9}{2}$

51. **(a)** $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$

$\Rightarrow \lim_{x\to 0}\frac{f(x)}{x^2}=2$

So, f(x) contain terms in x^2, x^3 and x^4.

Let $f(x)=a_1x^2+a_2x^3+a_3x^4$

Since $\lim_{x\to 0}\frac{f(x)}{x^2}=2 \Rightarrow a_1=2$

Hence, $f(x)=2x^2+a_2x^3+a_3x^4$

$f'(x)=4x+3a_2x^2+4a_3x^3$

As given : $f'(1)=0$ adn $f'(2)=0$

Hence, $4+3a_2+4a_3=0$ ...(i)

and $8+12a_2+32a_3=0$ ...(ii)

By 4x(i) – (ii), we get

$16+12a_2+16a_3-(8+12a_2+32a_3)=0$

$\Rightarrow 8-16a_3=0 \Rightarrow a_3=1/2$

and by eqn. (i), $4+3a_2+4/2=0 \Rightarrow a_2=-2$

$\Rightarrow f(x)=2x^2-2x^3+\frac{1}{2}x^4$

$f(2)=2\times 4-2\times 8+\frac{1}{2}\times 16=0$

52. **(d)** Given $f(1)=-2$ and $f'(x)\ge 4.2$ for $1\le x\le 6$

Consider $f'(x)=\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f(x+h)-f(x)=f'(x).\,h\ge(4.2)h$

So, $f(x+h)\ge f(x)+(4.2)\,h$

put $x=1$ and $h=5$, we get

$f(6)\ge f(1)+5(4.2) \Rightarrow f(6)\ge 19$

Hence $f(6)$ lies in $[19, \infty)$

53. **(b)** Let $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7$

$f'(x)=30x^9-56x^7+30x^5-63x^2+6x$

$f'(1)=30-56+30-63+6$

$=66-63-56=-53$

Consider $\lim_{\alpha\to 0}\frac{f(1-\alpha)-f(1)}{\alpha^3+3\alpha}$

$=\lim_{\alpha\to 0}\frac{f'(1-\alpha)(-1)-0}{3\alpha^2+3}$ (By using L'hospital rule)

$=\frac{f'(1-0)(-1)}{3(0)^2+3}=\frac{-f'(1)}{3}=\frac{53}{3}$

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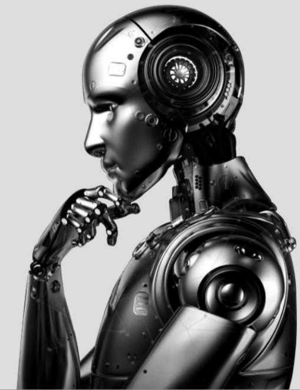


Mathematical Reasoning

TOPIC 1 Statement, Truth value of a statement, Logical Connectives, Truth Table, Logical Equivalance, Tautology & Contradiction, Duality

1. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to : **[Sep. 06, 2020 (I)]**
(a) $p \wedge \sim q$ (b) $\sim p \wedge \sim q$
(c) $\sim p \vee \sim q$ (d) $\sim p \vee q$

2. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to: **[Sep. 05, 2020 (I)]**
(a) $(x \wedge y) \vee (\sim x \wedge \sim y)$ (b) $(x \wedge y) \wedge (\sim x \vee \sim y)$
(c) $(x \wedge \sim y) \vee (\sim x \wedge y)$ (d) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

3. Given the following two statements :
$(S_1) : (q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.
$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then : **[Sep. 04, 2020 (I)]**
(a) both (S_1) and (S_2) are correct
(b) only (S_1) is correct
(c) only (S_2) is correct
(d) both (S_1) and (S_2) are not correct

4. The proposition $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to : **[Sep. 03, 2020 (I)]**
(a) q (b) $(\sim p) \vee q$
(c) $(\sim p) \wedge q$ (d) $(\sim p) \vee (\sim q)$

5. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively : **[Sep. 03, 2020 (II)]**
(a) T, F, T (b) T, T, T (c) F, T, F (d) T, T, F

6. If $p \rightarrow (p \wedge \sim q)$ is false, then the truth values of p and q are respectively: **[Jan. 9, 2020 (II)]**
(a) F, F (b) T, F (c) T, T (d) F, T

7. Which one of the following is a tautology? **[Jan. 8, 2020 (I)]**
(a) $(p \wedge (p \rightarrow q)) \rightarrow q$ (b) $q \rightarrow (p \wedge (p \rightarrow q))$
(c) $p \wedge (p \vee q)$ (d) $p \vee (p \wedge q)$

8. Which of the following statements is a tautology? **[Jan. 8, 2020 (II)]**
(a) $p \vee (\sim q) \rightarrow p \wedge q$ (b) $\sim (p \wedge \sim q) \rightarrow p \vee q$
(c) $\sim (p \vee \sim q) \rightarrow p \wedge q$ (d) $\sim (p \vee \sim q) \rightarrow p \vee q$

9. The logical statement
$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to: **[Jan. 7, 2020 (I)]**
(a) p (b) q (c) $\sim p$ (d) $\sim q$

10. If the truth value of the statement $p \rightarrow (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively. **[April 12, 2019 (I)]**
(a) T, T, F (b) T, F, F (c) T, F, T (d) F, T, T

11. The Boolean expression $\sim (p \Rightarrow (\sim q))$ is equivalent to : **[April 12, 2019 (II)]**
(a) $p \wedge q$ (b) $q \Rightarrow \sim p$ (c) $p \vee q$ (d) $(\sim p) \Rightarrow q$

12. Which one of the following Boolean expressions is a tautology ? **[April 10, 2019 (I)]**
(a) $(p \wedge q) \vee (p \wedge \sim q)$ (b) $(p \vee q) \vee (p \vee \sim q)$
(c) $(p \vee q) \wedge (p \vee \sim q)$ (d) $(p \vee q) \wedge (\sim p \vee \sim q)$

13. If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively: **[April 09, 2019 (II)]**
(a) F, T, T (b) T, F, F (c) T, T, F (d) F, F, F

14. Which one of the following statements is not a tautology? **[April 08, 2019 (II)]**
(a) $(p \vee q) \rightarrow (p \vee (\sim q))$ (b) $(p \wedge q) \rightarrow (\sim p) \vee q$
(c) $p \rightarrow (p \vee q)$ (d) $(p \wedge q) \rightarrow p$

15. The Boolean expression

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to : **[Jan. 12, 2019 (I)]**

(a) $p \wedge q$ (b) $p \wedge (\sim q)$

(c) $(\sim p) \wedge (\sim q)$ (d) $p \vee (\sim q)$

16. The expression $\sim(\sim p \to q)$ is logically equivalent to : **[Jan. 12, 2019 (II)]**

(a) $\sim p \wedge \sim q$ (b) $p \wedge \sim q$

(c) $\sim p \wedge q$ (d) $p \wedge q$

17. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology? **[Jan. 11, 2019 (I)]**

(a) $(p \vee r) \to (p \wedge r)$ (b) $(p \wedge r) \to (p \vee r)$

(c) $p \wedge r$ (d) $p \vee r$

18. Consider the following three statements:

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true? **[Jan. 10, 2019 (II)]**

(a) $(\sim P) \vee (Q \wedge R)$ (b) $(P \wedge Q) \vee (\sim R)$

(c) $(\sim P) \wedge (\sim Q \wedge R)$ (d) $P \vee (\sim Q \wedge R)$

19. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$ then the ordered pair $(\oplus, \odot)$ is: **[Jan. 09, 2019 (I)]**

(a) $(\vee, \wedge)$ (b) $(\vee, \vee)$ (c) $(\wedge, \vee)$ (d) $(\wedge, \wedge)$

20. The logical statement

$[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim p \wedge r)$

is equivalent to: **[Jan. 09, 2019 (II)]**

(a) $(\sim p \wedge \sim q) \wedge r$ (b) $\sim p \vee r$

(c) $(p \wedge r) \wedge \sim q$ (d) $(p \wedge \sim q) \vee r$

21. The Boolean expression

$\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to : **[2018]**

(a) p (b) q (c) ~q (d) ~p

22. If $p \to (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively. **[Online April 16, 2018]**

(a) T, F (b) F, F (c) F, T (d) T, T

23. If $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false, then the truth values of p, q and r are respectively **[Online April 15, 2018]**

(a) F, T, F (b) T, F, T (c) F, F, F (d) T, T, T

24. Which of the following is a tautology? **[2017]**

(a) $(\sim p) \wedge (p \vee q) \to q$ (b) $(q \to p) \vee \sim (p \to q)$

(c) $(\sim q) \vee (p \wedge q) \to q$ (d) $(p \to q) \wedge (q \to p)$

25. The following statement

$(p \to q) \to [(\sim p \to q) \to q]$ is : **[2017]**

(a) a fallacy (b) a tautology

(c) equivalent to $\sim p \to q$ (d) equivalent to $p \to \sim q$

26. The proposition $(\sim p) \vee (p \wedge \sim q)$ **[Online April 8, 2017]**

(a) $p \to \sim q$ (b) $p \wedge (\sim q)$

(c) $q \to p$ (d) $p \vee (\sim q)$

27. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to: **[2016]**

(a) $p \vee q$ (b) $p \vee \sim q$ (c) $\sim p \wedge q$ (d) $p \wedge q$

28. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[2015]**

(a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$

(c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$

29. The statement $\sim(p \leftrightarrow \sim q)$ is: **[2014]**

(a) a tautology

(b) a fallacy

(c) eqivalent to $p \leftrightarrow q$

(d) equivalent to $\sim p \leftrightarrow q$

30. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is: **[Online April 12, 2014]**

(a) $(p \vee q) \Rightarrow r$ (b) $(p \Rightarrow q) \vee (p \Rightarrow r)$

(c) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$ (d) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

31. The proposition $\sim(p \vee \sim q) \vee \sim(p \vee q)$ is logically equivalent to: **[Online April 11, 2014]**

(a) p (b) q (c) $\sim p$ (d) $\sim q$

32. Consider

Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

33. Let p and q be any two logical statements and $r : p \to (\sim p \vee q)$. If r has a truth value F, then the truth values of p and q are respectively : **[Online April 25, 2013]**

(a) F, F (b) T, T (c) T, F (d) F, T

34. For integers m and n, both greater than 1, consider the following three statements :

P : m divides n

Q : m divides n^2

R : m is prime,

then **[Online April 23, 2013]**

(a) $Q \wedge R \rightarrow P$ (b) $P \wedge Q \rightarrow R$

(c) $Q \rightarrow R$ (d) $Q \rightarrow P$

35. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to : **[Online April 22, 2013]**

(a) $p \rightarrow q$ (b) $p \rightarrow (p \vee q)$

(c) $p \rightarrow (p \rightarrow q)$ (d) $p \rightarrow (p \wedge q)$

36. **Statement-1:** The statement $A \rightarrow (B \rightarrow A)$ is equivalent to $A \rightarrow (A \vee B)$.

Statement-2: The statement $\sim [(A \wedge B) \rightarrow (\sim A \vee B)]$ is a Tautology. **[Online April 9, 2013]**

(a) Statement-1 is false; Statement-2 is true.

(b) Statement-1 is true; Statement-2 is true; Statement- 2 is **not** correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is true; Statement-2 is true; Statement- 2 is the correct explanation for Statement-1.

37. Let p and q be two Statements. Amongst the following, the Statement that is equivalent to $p \rightarrow q$ is **[Online May 19, 2012]**

(a) $p \wedge \sim q$ (b) $\sim p \vee q$ (c) $\sim p \wedge q$ (d) $p \vee \sim q$

38. The logically equivalent preposition of $p \Leftrightarrow q$ is **[Online May 12, 2012]**

(a) $(p \Rightarrow q) \wedge (q \Rightarrow p)$ (b) $p \wedge q$

(c) $(p \wedge q) \vee (q \Rightarrow p)$ (d) $(p \wedge q) \Rightarrow (q \vee p)$

39. The only statement among the following that is a tautology is **[2011RS]**

(a) $A \wedge (A \vee B)$ (b) $A \vee (A \wedge B)$

(c) $[A \wedge (A \rightarrow B)] \rightarrow B$ (d) $B \rightarrow [A \wedge (A \rightarrow B)]$

40. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tantology **[2009]**

(a) Statement-1 is true, Statement-2 is true;
Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true,
Statement-2 is a correct explanation for statement -1

41. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to **[2008]**

(a) $p \rightarrow (p \rightarrow q)$ (b) $p \rightarrow (p \vee q)$

(c) $p \rightarrow (p \wedge q)$ (d) $p \rightarrow (p \leftrightarrow q)$

42. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number". **[2008]**

Statement-1 : r is equivalent to either q or p

Statement-2 : r is equivalent to $\sim(p \leftrightarrow \sim q)$.

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

TOPIC 2 Converse, Inverse & Contrapositive of the Conditional Statement, Negative of a Compound Statement, Algebra of Statement

43. Consider the statement: "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is: **[Sep. 06, 2020 (II)]**

(a) For an integer n, if n is even, then $n^3 - 1$ is odd.

(b) For an intetger n, if $n^3 - 1$ is not even, then n is not odd.

(c) For an integer n, if n is even, then $n^3 - 1$ is even.

(d) For an integer n, if n is odd, then $n^3 - 1$ is even.

44. The statement $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is : **[Sep. 05, 2020 (II)]**

(a) equivalent to $(p \wedge q) \vee (\sim q)$

(b) a contradiction

(c) equivalent to $(p \vee q) \wedge (\sim p)$

(d) a tautology

45. Contrapositive of the statement :
'If a function f is differentiable at a, then it is also continuous at a', is : **[Sep. 04, 2020 (II)]**

(a) If a function f is continuous at a, then it is not differentiable at a.

(b) If a function f is not continuous at a, then it is not differentiable at a.

(c) If a function f is not continuous at a, then it is differentiable at a

(d) If a function f is continuous at a, then it is differentiable at a.

46. The contrapositive of the statement "If *I* reach the station in time, then *I* will catch the train" is : **[Sep. 02, 2020 (I)]**

(a) If *I* do not reach the station in time, then *I* will catch the train.

(b) If *I* do not reach the station in time, then *I* will not catch the train.

(c) If *I* will catch the train, then *I* reach the station in time.

(d) If *I* will not catch the train, then *I* do not reach the station in time.

47. Negation of the statement:

$\sqrt{5}$ is an integer of 5 is irrational is: **[Jan. 9, 2020 (I)]**

(a) $\sqrt{5}$ is not an integer or 5 is not irrational

(b) $\sqrt{5}$ is not an integer and 5 is not irrational

(c) $\sqrt{5}$ is irrational or 5 is an integer.

(d) $\sqrt{5}$ is an integer and 5 is irrational

48. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is: **[Jan. 7, 2020 (II)]**

(a) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(b) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(c) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

(d) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

49. The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[April 10, 2019 (II)]**

(a) $\sim s \wedge \sim r$ (b) r

(c) $s \vee r$ (d) $s \wedge r$

50. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is: **[April 9, 2019 (I)]**

(a) $\sim p \wedge \sim q$ (b) $p \wedge q$

(c) $p \leftrightarrow q$ (d) $\sim p \vee \sim q$

51. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is : **[April 8, 2019 (I)]**

(a) If you are not a citizen of India, then you are not born in India.

(b) If you are a citizen of India, then you are born in India.

(c) If you are born in India, then you are not a citizen of India.

(d) If you are not born in India, then you are not a citizen of India.

52. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal". is : **[Jan. 11, 2019 (II)]**

(a) If the squares of two numbers are not equal, then the numbers are equal.

(b) If the squares of two numbers are equal, then the numbers are not equal.

(c) If the squares of two numbers are equal, then the numbers are equal.

(d) If the squares of two numbers are not equal, then the numbers are not equal.

53. Consider the following two statements.

Statement p:

The value of sin 120° can be divided by taking $\theta = 240°$ in the equation $2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$.

Statement q:

The angles A, B, C and D of any quadrilateral $ABCD$ satisfy the equation $\cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$

Then the truth values of p and q are respectively. **[Online April 15, 2018]**

(a) F, T (b) T, T (c) F, F (d) T, F

54. Contrapositive of the statement

'If two numbers are not equal, then their squares are not equal', is : **[Online April 9, 2017]**

(a) If the squares of two numbers are equal, then the numbers are equal.

(b) If the squares of two numbers are equal, then the numbers are not equal.

(c) If the squares of two numbers are not equal, then the numbers are not equal.

(d) If the squares of two numbers are not equal, then the numbers are equal.

55. The contrapositive of the following statement,

"If the side of a square doubles, then its area increases four times", is : **[Online April 10, 2016]**

(a) If the area of a square increases four times, then its side is not doubled.

(b) If the area of a square increases four times, then its side is doubled.

(c) If the area of a square does not increases four times, then its side is not doubled.

(d) If the side of a square is not doubled, then its area does not increase four times.

56. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals: **[Online April 9, 2016]**

(a) (F, F) (b) (F, T) (c) (T, F) (d) (T, T)

57. Consider the following statements :

P : Suman is brilliant

Q : Suman is rich.

R : Suman is honest

the negation of the statement

"Suman is brilliant and dishonest if and only if suman is rich" can be equivalently expressed as : **[Online April 11, 2015]**

(a) $\sim Q \leftrightarrow \sim P \vee R$ (b) $\sim Q \leftrightarrow \sim P \wedge R$

(c) $\sim Q \leftrightarrow P \vee \sim R$ (d) $\sim Q \leftrightarrow P \wedge \sim R$

58. The contrapositive of the statement "If it is raining, then I will not come", is : **[Online April 10, 2015]**

(a) If I will not come, then it is raining.
(b) If I will not come, then it is not raining.
(c) If I will come, then it is raining.
(d) If I will come, then it is not raining.

59. The contrapositive of the statement "if I am not feeling well, then I will go to the doctor" is **[Online April 19, 2014]**

(a) If I am feeling well, then I will not go to the doctor
(b) If I will go to the doctor, then I am feeling well
(c) If I will not go to the doctor, then I am feeling well
(d) If I will go to the doctor, then I am not feeling well.

60. The contrapositive of the statement "I go to school if it does not rain" is **[Online April 9, 2014]**

(a) If it rains, I do not go to school.
(b) If I do not go to school, it rains.
(c) If it rains, I go to school.
(d) If I go to school, it rains.

61. The negation of the statement
"If I become a teacher, then I will open a school", is : **[2012]**

(a) I will become a teacher and I will not open a school.
(b) Either I will not become a teacher or I will not open a school.
(c) Neither I will become a teacher nor I will open a school.
(d) I will not become a teacher or I will open a school.

62. Let p and q denote the following statements
p : The sun is shining
q: I shall play tennis in the afternoon
The negation of the statement "If the sun is shining then I shall play tennis in the afternoon", is **[Online May 26, 2012]**

(a) $q \Rightarrow \sim p$ (b) $q \wedge \sim p$
(c) $p \wedge \sim q$ (d) $\sim q \Rightarrow \sim p$

63. The Statement that is TRUE among the following is **[Online May 7, 2012]**

(a) The contrapositive of $3x+2=8 \Rightarrow x=2$ is $x \neq 2 \Rightarrow 3x+2 \neq 8$.
(b) The converse of $\tan x = 0 \Rightarrow x = 0$ is $x \neq 0 \Rightarrow \tan x = 0$.
(c) $p \Rightarrow q$ is equivalent to $p \vee \sim q$.
(d) $p \vee q$ and $p \wedge q$ have the same truth table.

64. Let S be a non-empty subset of R. Consider the following statement :

P : There is a rational number $x \in$ S such that $x > 0$.

Which of the following statements is the negation of the statement P ? **[2010]**

(a) There is no rational number $x \in$ S such than $x \leq 0$.
(b) Every rational number $x \in$ S satisfies $x \leq 0$.
(c) $x \in$ S and $x \leq 0 \Rightarrow x$ is not rational.
(d) There is a rational number $x \in$ S such that $x \leq 0$.

Hints & Solutions

1. **(b)** Negation of given statement $= \sim (p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim (\sim p \wedge q) = \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$= F \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$

2. **(a)** $p : x \leftrightarrow \sim y = (x \to \sim y) \wedge (\sim y \to x)$

$= (\sim x \vee \sim y) \wedge (y \vee x)$

$= \sim (x \wedge y) \wedge (x \vee y)$ $\qquad (\because \sim (x \wedge y) = \sim x \vee \sim y)$

Negation of p is

$\sim p = (x \wedge y) \vee \sim (x \vee y) = (x \wedge y) \vee (\sim x \wedge \sim y)$

3. **(d)** The truth table of both the statements is

p	q	~p	~q	q ∨ p	p↔~q	(S1)	~p↔q	(S2)
T	T	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F
F	F	T	T	F	F	T	F	F

$\therefore$ S_1 is not tautology and

S_2 is not fallacy.

Hence, both the statements (S_1) and (S_2) are not correct.

4. **(b)**

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$p \to \sim(p \wedge \sim q)$	$\sim p \vee q$
T	T	F	F	F	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

$\therefore p \to \sim (p \wedge \sim q)$ is equivalent to $\sim p \vee q$

5. **(d)** $(p \wedge q) \to (\sim q \vee r)$

$= \sim (p \wedge q) \vee (\sim q \vee r)$

$= (\sim p \vee \sim q) \vee (\sim q \vee r)$

$= (\sim p \vee \sim q \vee r)$

$\because (\sim p \vee \sim q \vee r)$ is false, then $\sim p$, $\sim q$ and r all these must be false.

$\Rightarrow p$ is true, q is true and r is false.

6. **(c)**

p	q	$\sim q$	$p \wedge \sim q$	$p \to (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

7. **(a)**

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$	$q \to p \wedge (p \to q)$	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

8. **(d)** $(\sim p \wedge q) \to (p \vee q)$

$\Rightarrow \sim \{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\Rightarrow \sim \{\sim p \wedge f\}$

9. **(c)**

p	q	$p \Rightarrow q$	$\sim p$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \wedge (p \Rightarrow \sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to $\sim p$

10. **(a)** Given statement $p \to (\sim q \vee r)$ is False.

$\Rightarrow$ p is True and $\sim q \vee r$ is False

$\Rightarrow$ p is True and $\sim q$ is False and r is False

$\therefore$ truth values of $p, q\ r$ are T, T, F respectively.

11. **(a)** Given Boolean expression is,

$\sim (p \Rightarrow (\sim q))$ $\qquad \{\because p \Rightarrow q$ is same as $\sim p \vee q\}$

$\equiv \sim ((\sim p) \vee (\sim q)) \equiv p \wedge q$

12. **(b)** $(p \vee q) \vee (p \vee \sim q) = p \vee (q \vee p) \vee \sim q$

$= (p \vee p) \vee (q \vee \sim q) = p \vee T = T$

Hence first statement is tautology.

13. **(b)** For $p \Rightarrow q \vee r$ to be F.

r should be F and $p \Rightarrow q$ should be F

for $p \Rightarrow q$ to be F, $p \Rightarrow$ T and $q \Rightarrow$ F

$p, q, r \equiv$ T, F, F

14. **(a)** By truth table :

p	q	$\sim q$	$pv \sim q$	$\sim p$	$p \wedge \sim q$	pvq	$p \rightarrow pvq$	$p \wedge q$	$(p \wedge q) \rightarrow p$	$\sim pvq$
T	T	F	T	F	F	T	T	T	T	T
T	F	T	T	F	T	T	T	F	T	F
F	T	F	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	T	T

$(p \wedge q) \rightarrow (\sim p) \vee q$	$(p \vee q) \rightarrow (p \vee (\sim q))$
T	T
T	T
T	F
T	T

15. **(c)** Consider the Boolean expression

$((p \wedge q) \vee (pv \sim q)) \wedge (\sim p \wedge \sim q)$

$= (p \vee \sim q) \wedge (\sim p \wedge \sim q)$

$= ((p \vee \sim q) \wedge \sim p) \wedge ((p \vee \sim q) \wedge \sim q)$

$= ((p \wedge \sim p) \vee (\sim q \wedge \sim p)) \wedge \sim q$

$= (\sim p \wedge \sim q) \wedge \sim q = (\sim p \wedge \sim q)$

16. **(a)** $\sim (\sim p \rightarrow Q) \equiv \sim (p \vee q) \equiv \sim p \wedge \sim q$

17. **(b)** q is false and $[(p \wedge q) \leftrightarrow r]$ is true

As $(p \wedge q)$ is false

[False $\leftrightarrow r$] is true

Hence r is false

Option (a): says $p \vee r$,

Since r is false

Hence $(p \vee r)$ can either be true or false

Option (b): says $(p \wedge r) \rightarrow (p \vee r)$

$(p \wedge r)$ is false

Since, $F \rightarrow T$ is true and

$F \rightarrow F$ is also true

Hence, it is a tautology

Option (c): $(p \vee r) \rightarrow (p \wedge r)$

i.e. $(p \vee r) \rightarrow F$

It can either be true or false

Option (d): $(p \wedge r)$,

Since, r is false

Hence, $(p \wedge r)$ is false.

18. **(d)** P is True, Q is False and R is True

(a) $(\sim P) \vee (Q \wedge R) \equiv F \vee (F \wedge T) \equiv F \vee F = F$

(b) $(P \wedge Q) \vee (\sim R) \equiv (T \wedge F) \vee (F) \equiv F \vee F = F$

(c) $(\sim P) \wedge (\sim Q \wedge R) \equiv F \wedge (T \wedge T) \equiv F \wedge T = F$

(d) $P \vee (\sim Q \wedge R) \equiv T \vee (T \wedge T) \equiv T \vee T = T$

19. **(c)** Check each option

(a) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$

(b) $(p \vee q) \wedge (\sim p \vee q) = q$

(c) $(p \wedge q) \wedge (\sim p \vee q) = p \wedge q$

(d) $(p \wedge q) \wedge (\sim p \wedge q) = F$

20. **(c)** Logical statement,

$= [\sim (\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \wedge (\sim q \wedge r)] \vee [(p \wedge r) \wedge (\sim q \wedge r)]$

$= [p \wedge \sim q \wedge r] \vee [p \wedge r \wedge \sim q]$

$= (p \wedge \sim q) \wedge r$

$= (p \wedge r) \wedge \sim q$

21. **(d)** $\sim (p \vee q) \vee (\sim p \wedge q)$

$\Rightarrow (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow \sim p \wedge (\sim q \vee q)$

$\Rightarrow \sim p \wedge t \equiv \sim p$

22. **(d)**

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \rightarrow (\sim p \vee \sim q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

From the truth table,

$p \rightarrow (\sim p \vee \sim q)$ is false only when p and q both are true.

23. **(b)** As the truth table for the $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then only possible values of (p, q, r) is (T, F, T)

p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge r$	$\sim p$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (p \wedge r)$	$(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$
T	T	T	F	F	T	F	T	F	T
T	F	T	T	T	T	F	F	T	F
T	T	F	F	F	F	F	T	F	T
F	T	T	F	F	F	T	T	F	T
F	F	T	T	F	F	T	T	F	T
F	T	F	F	F	F	T	T	F	T
T	F	F	T	T	F	F	F	F	T
F	F	F	T	F	F	T	T	F	T

24. **(a)** Truth table

p	q	$\sim p$	$p \vee q$	$(\sim p)\wedge(p \vee q)$	$(\sim p)\wedge(p \vee q)\to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

$\therefore$ (a) $\sim p \wedge (p \vee q) \to q$ be a tautology

Other options are not tautology.

25. **(b)** We have

p	q	~ p	p → q	~ p → q	(~ p → q) → q	(p → q) → ((~p→q)→ q)
T	F	F	F	T	F	T
T	T	F	T	T	T	T
F	F	T	T	F	T	T
F	T	T	T	T	T	T

$\therefore$ It is tautology.

26. **(b)** $(\sim p) \vee (p \wedge \sim q)$

p	q	~ p	~ q	p∧ ~ q	(~ p)∨(p ∧ ~ q)
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F

27. **(a)** $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \wedge (\sim q \vee q)\} \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \wedge T\} \vee (\sim p \wedge q)$

$\Rightarrow (p \vee q) \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \vee \sim p\} \wedge (p \vee q \vee q)$

$\Rightarrow T \wedge (p \vee q)$

$\Rightarrow p \vee q$

28. **(b)** $\sim[\sim s \vee (\sim r \wedge s)]$

$= s \wedge \sim(\sim r \wedge s)$

$= s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s)$

$= (s \wedge r) \vee f$

$= s \wedge r$

29. **(c)**

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$ (i)	$p \leftrightarrow q$ (ii)
F	F	T	F	T	T
F	T	F	T	F	F
T	F	T	T	F	F
T	T	F	F	T	T

From column (i) and (ii) are equivalent.

Clearly equivalent to $p \leftrightarrow q$

30. **(b)** Given statement is

$p \Rightarrow (q \vee r)$ which is equivalent to

$(p \Rightarrow q) \vee (p \Rightarrow r)$

31. **(c)** Given $\sim(p \vee \sim q) \vee \sim(p \vee q)$

$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$\equiv \sim p \vee (q \vee \sim q)$

$\equiv \sim p$

32. **(b)** **Statement-2 :** $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\equiv (p \to q) \leftrightarrow (p \to q)$

which is always true.

So, statement 2 is true

Statement-1: $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$= p \wedge \sim q \wedge \sim p \wedge q$

$= p \wedge \sim p \wedge \sim q \wedge q$

$= f \wedge f = f$

So statement-1 is true

33. **(c)** $p \to (\sim p \vee q)$ has truth value F.

It means $p \to (\sim p \vee q)$ is false.

It means p is true and $\sim p \vee q$ is false.

$\Rightarrow$ p is true and both $\sim p$ and q are false.

$\Rightarrow$ p is true and q is false.

34. **(a)**

(b) $\frac{8}{4} = 2, \frac{64}{4} = 16$; but 4 is not prime.

Hence P $\wedge$ Q $\to$ R, false

(c) $\frac{(6)^2}{12} = \frac{36}{12} = 3$; but 12 is not prime

Hence Q $\to$ R, false

(d) $\frac{(4)^2}{8} = \frac{16}{8} = 2$; $\frac{4}{8}$ is not an integer

Hence Q $\to$ P, false

35. **(b)**

q	p	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Since truth value of $p \to (q \to p)$ and $p \to (p \vee q)$ are same, hence $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

36. **(c)**

A	B	~ A	A ∧ B	~ A ∨ B	(A ∧ B) → (~ A ∨ B)	~[(A ∧ B) → (~ A ∨ B)]
T	T	F	T	T	T	F
T	F	F	F	F	T	F
F	T	T	F	T	T	F
F	F	T	F	T	T	F

37. **(b)** Let p and q be two statements.
$p \to q$ is equivalent to $\sim p \vee q$.

38. **(a)** $(p \Rightarrow q) \wedge (q \Rightarrow p)$ means $p \Leftrightarrow q$

39. **(c)** Truth table of all options is as follows.

A	B	A ∨ B	A ∧ B	A ∧ (A ∨ B)	A ∨ (A ∧ B)	A → B	A ∧ (A → B)	[A ∧ (A → B) → B]	[B → [A ∧ (A → B)]
T	F	T	F	T	T	F	F	T	T
F	T	T	F	F	F	T	F	T	F
T	T	T	T	T	T	T	T	T	T
F	F	F	F	F	F	T	F	T	T

∴ It is tautology.

40. **(b)** The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	(i) $\sim(p \leftrightarrow \sim q)$	(ii) $p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns (i) and (ii) are identical, therefore $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

But $\sim (p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.

∴ Statement-1 is true but statement-2 is false.

41. **(b)** The truth table for the given statements, as follows :

p	q	p∨ q	q→ p	p → (q→ p)	p → (p∨ q)
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

From table we observe that

$p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

42. **(None)**

Given that

p : x is an irrational number

q : y is a transcendental number

r : x is a rational number iff y is a transcendental number.

clearly $r : \sim p \leftrightarrow q$

Truth table to check the equivalence of 'r' and 'q or p'; 'r' and $\sim (p \leftrightarrow \sim q)$

p	q	~p	~q	(i) ~p ↔ q	(ii) q or p	p↔ ~q	(iii) ~(p↔ ~q)
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

From columns (i), (ii) and (iii), we observe, that none of the these statements are equivalent to each other.

∴ Statement 1as well as statement 2 both are false.

∴ None of the options is correct.

43. **(a)** Contrapositive statement will be

"For an integer n, if n is not odd then $n^3 - 1$ is not even".

or

"For an integer n, if n is even then $n^3 - 1$ is odd".

44. **(d)** The truth table of $(p \to (q \to p)) \to (p \to (p \vee q))$ is

p	q	p∨ q	p→ (p∨ q)	q→ p	p→ (q→ p)	(p→ (q→ p)) → (p→ (p∨ q))
T	T	O	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	T

Hence, the statement is tautology.

45. **(b)** Contrapositive statement will be "If a function is not continuous at 'a', then it is not differentiable at 'a'.

46. **(d)** Contrapositive of $p \to q$ is $\sim q \to \sim p$

i.e. contrapositive of 'if p then q' is 'if not q then not p'.

47. **(b)** Let p and q the statements such that $p = \sqrt{5}$ is an integer $q = 5$ is an irrational number.

Then, negation of the given statement

$\sqrt{5}$ is not an integer and 5 is not an irrational Number

$\sim (p \vee q) = \sim p \wedge \sim q$

48. **(d)** Let $P = A \subseteq B, Q = B \subseteq D, R = A \subseteq C$

Contrapositive of $(P \wedge Q) \to R$ is $\sim R \to \sim (P \wedge Q)$

$\sim R \to \sim P \vee \sim Q$

49. **(d)** $\sim s \vee (\sim r \wedge s) \equiv (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$\equiv (\sim s \vee \sim r)$ $\quad$ ($\because \sim s \vee s$) is tautology)

$\equiv \sim (s \wedge r)$

Hence, its negation is $s \wedge r$.

50. **(d)** $\sim(p \vee (\sim p \wedge q)) = \sim(\sim p \wedge q) \wedge \sim p$
$= (\sim q \vee p) \wedge \sim p$
$= \sim p \wedge (p \vee \sim q)$
$= (\sim q \wedge \sim p) \vee (p \wedge \sim p)$
$= (\sim p \wedge \sim q)$

51. **(a)** S: ''If you are born in India, then you are a citizen of India.''
Contrapositive of $p \to q$ is $\sim q \to \sim p$
So contrapositive of statement S will be :
''If you are not a citizen of India, then you are not born in India.''

52. **(c)** Contrapositive of "If A then B" is "If ~B then ~A". Hence contrapositive of "If two numbers are not equal, then their squares are not equal" is "If squares of two numbers are equal, then the two numbers are equal".

53. **(a)** **Statement p:**

$$\sin 120° = \cos 30° = \frac{\sqrt{3}}{2} \Rightarrow 2\sin 120° = \sqrt{3}$$

So, $\sqrt{1+\sin 240°} - \sqrt{1-\sin 240°}$

$$= \sqrt{\frac{1-\sqrt{3}}{2}} - \sqrt{\frac{1+\sqrt{3}}{2}} \neq \sqrt{3}$$

Statement q:

$$\text{So, } A+B+C+D = 2\pi \Rightarrow \frac{A+C}{2} + \frac{B+D}{2} = \pi$$

$$\Rightarrow \cos\left(\frac{A+C}{2}\right) + \cos\left(\frac{B+D}{2}\right)$$

$$= \cos\left(\frac{A+C}{2}\right) - \cos\left(\frac{A+C}{2}\right) = 0$$

Therefore, statement p is false and statement q is true.

54. **(a)** $p \to q$
then $\sim q \to \sim p$
$\therefore$ If the square of two numbers are equal, then the numbers are equal.

55. **(c)** Contrapositive of $p \to q$ is given by $\sim q \to \sim p$
So (c) is the right option.

56. **(a)** Contrapositive of P :
T is not divisible by 2 $\Rightarrow$ T is not odd number
$T \Rightarrow F : F(V_1)$
Contra positive Q :
T is not odd number $\Rightarrow$ T is not a prime number
$F \Rightarrow F : T(V_2)$

57. **(d)** Suman is brilliant and dishonest can be expressed as $P \wedge \sim R$
therefore given statement is equal to $(P \wedge \sim R) \leftrightarrow Q$
Negation of the above statement is $\sim Q \leftrightarrow P \wedge \sim R$

58. **(d)** The centre positive of the statement is "If i will come, then it is not raining".

59. **(c)** Given statement can be written in implication form as
I am not feeling well $\Rightarrow$ I will go to the doctor.
Contrapositive form :
I will not go to the doctor $\Rightarrow$ I am feeling well.
i.e. If I will not go to the doctor, then I am feeling well.

60. **(b)** let p = If it does not rain
q = I go to school
According to law of contrapositive
$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$
i.e. $\sim q$ = I do not go to school
$\sim p$ = It rains
$\sim q \Rightarrow \sim p$ is If I do not go to school, it rains.

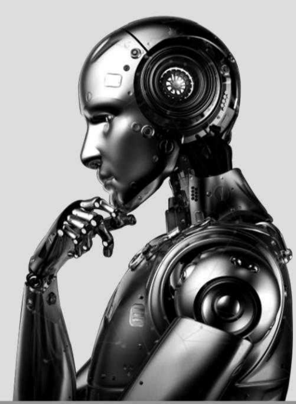
61. **(a)** Let p : I become a teacher.
q : I will open a school
Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$
i.e. I will become a teacher and I will not open a school.

62. **(c)** Let p : The sun is shining.
q : I shall play tennis in the afternoon.
Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$

63. **(a)** Only statement given in option
(a) is true.
(b) The converse of $\tan x = 0 \Rightarrow x = 0$ is
$x = 0 \Rightarrow \tan x = 0$
$\therefore$ Statement (b) is false
(c) $\sim(p \Rightarrow q)$ is equivalent to $p \wedge \sim q$
$\therefore$ Statement given in option (c) is false.
(d) No, $p \vee q$ and $p \wedge q$ does not have the same truth value.

64. **(b)** Given that P : there is a rational number $x \in S$ such that $x > 0$.
$\sim$P : Every rational number $x \in S$ satisfies $x \leq 0$.

14



Statistics

TOPIC 1 Arithmetic Mean, Geometric Mean, Harmonic Mean, Median & Mode

1. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ______. **[NA Sep. 06, 2020 (II)]**

2. The minimum value of $2^{\sin x} + 2^{\cos x}$ is : **[Sep. 04, 2020 (II)]**
(a) $2^{-1+\frac{1}{\sqrt{2}}}$ (b) $2^{-1+\sqrt{2}}$ (c) $2^{1-\sqrt{2}}$ (d) $2^{1-\frac{1}{\sqrt{2}}}$

3. If for some $x \in \mathbf{R}$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is : **[April 10, 2019 (I)]**
(a) 3.2 (b) 3.0 (c) 2.5 (d) 2.8

4. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to: **[April. 09, 2019 (II)]**
(a) 9/4 (b) 7/2 (c) 8/3 (d) 7/3

5. The mean of a set of 30 observations is 75. If each other observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. The λ is equal to **[Online April 15, 2018]**
(a) $\frac{10}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

6. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is : **[Online April 8, 2017]**
(a) 25 (b) 30 (c) 35 (d) 40

7. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is: **[2015]**
(a) 15.8 (b) 14.0 (c) 16.8 (d) 16.0

8. Let the sum of the first three terms of an A. P, be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is : **[Online April 10, 2015]**
(a) 28 (b) 26.5 (c) 29.5 (d) 31

9. A factory is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is ₹ 54 and per day mean wage of all the workers is ₹ 60, then per day mean wage of the night shift workers (in ₹) is : **[Online April 10, 2015]**
(a) 69 (b) 66 (c) 74 (d) 75

10. In a set of 2n distinct observations, each of the observations below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations: **[Online April 9, 2014]**
(a) increases by 1 (b) decreases by 1
(c) decreases by 2 (d) increases by 2

11. If the median and the range of four numbers $\{x, y, 2x + y, x - y\}$, where $0 < y < x < 2y$, are 10 and 28 respectively, then the mean of the numbers is : **[Online April 23, 2013]**
(a) 18 (b) 10 (c) 5 (d) 14

12. The mean of a data set consisting of 20 observations is 40. If one observation 53 was wrongly recorded as 33, then the correct mean will be : **[Online April 9, 2013]**
(a) 41 (b) 49 (c) 40.5 (d) 42.5

13. The median of 100 observations grouped in classes of equal width is 25. If the median class interval is 20 - 30 and the number of observations less than 20 is 45, then the frequency of median class is **[Online May 19, 2012]**
(a) 10 (b) 20 (c) 15 (d) 12

14. The frequency distribution of daily working expenditure of families in a locality is as follows:

Expenditure in ₹. (x):	0-50	50-100	100-150	150-200	200-250
No. of families (f):	24	33	37	b	25

If the mode of the distribution is ₹ 140, then the value of b is **[Online May 7, 2012]**

(a) 34 (b) 31 (c) 26 (d) 36

15. The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is **[2007]**

(a) 80 (b) 60 (c) 40 (d) 20

16. Let x_1, x_2, ... x_n be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then the possible value of n among the following is **[2005]**

(a) 15 (b) 18 (c) 9 (d) 12

17. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately **[2005]**

(a) 22.0 (b) 20.5 (c) 25.5 (d) 24.0

18. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set **[2003]**

(a) remains the same as that of the original set
(b) is increased by 2
(c) is decreased by 2
(d) is two times the original median.

19. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls? **[2002]**

(a) 73 (b) 65 (c) 68 (d) 74

TOPIC 2 Quartile, Measures of Dispersion, Quartile Deviation, Mean Deviation, Variance & Standard Deviation, Coefficient of Variation

20. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$, then the standard deviation of n observations $x_1, x_2, ..., x_n$ is : **[Sep. 06, 2020 (I)]**

(a) $a - 1$ (b) $n\sqrt{a-1}$

(c) $\sqrt{n(a-1)}$ (d) $\sqrt{a-1}$

21. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is : **[Sep. 05, 2020 (I)]**

(a) 1 (b) 4 (c) 2 (d) 3

22. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation : **[Sep. 05, 2020 (II)]**

(a) $x^2 - 10x + 18 = 0$ (b) $2x^2 - 20x + 19 = 0$

(c) $x^2 - 10x + 19 = 0$ (d) $x^2 - 20x + 18 = 0$

23. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is : **[Sep. 04, 2020 (I)]**

(a) 9 (b) 5 (c) 3 (d) 7

24. If a variance of the following frequency distribution :

Class	10-20	20-30	30-40
Frequency	2	x	2

is 50, then x is equal to _____________. **[NA Sep. 04, 2020 (II)]**

25. For the frequency distribution :

Variate (x) :	x_1	x_2	$x_1 ... x_{15}$
Frequency (f) :	f_1	f_2	$f_3 ... f_{15}$

where $0 < x_1 < x_2 < x_3 < ... < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation **cannot** be : **[Sep. 03, 2020 (I)]**

(a) 4 (b) 1 (c) 6 (d) 2

26. Let $x_i (1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in \mathbf{R}$, then the standard deviation of these observations is : **[Sep. 03, 2020 (II)]**

(a) $\sqrt{\frac{3}{5}}$ (b) $\frac{4}{5}$ (c) $\frac{9}{10}$ (d) $\frac{7}{10}$

27. Let $X = \{x \in \mathbf{N} : 1 \le x \le 17\}$ and $Y = \{ax + b : x \in X$ and $a, b \in \mathbf{R}, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then $a + b$ is equal to : **[Sep. 02, 2020 (I)]**

(a) 7 (b) –7 (c) –27 (d) 9

28. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, ..., b_{11}$ is 90, then the common difference of this A.P. is ___________. **[NA Sep. 02, 2020 (II)]**

29. Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, ..., x_{10} - 3$, then the ordered pair (μ, λ) is equal to: **[Jan. 9, 2020 (I)]**

(a) (3, 3) (b) (6, 3) (c) (6, 6) (d) (3, 6)

30. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to: **[Jan. 8, 2020 (I)]**

(a) –5 (b) 10 (c) –20 (d) –10

31. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is: **[Jan. 8, 2020 (II)]**

(a) 3.99 (b) 4.01 (c) 4.02 (d) 3.98

32. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to———. **[NA Jan. 7, 2020 (I)]**

33. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to _________. **[NA Jan. 7, 2020 (II)]**

34. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000 ; then the standard deviation of this data is : **[April 12, 2019 (I)]**

(a) $2\sqrt{2}$ (b) 2 (c) 4 (d) $\sqrt{2}$

35. If both the mean and the standard deviation of 50 observations $x_1, x_2, ..., x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, ..., (x_{50} - 4)^2$ is : **[April 10, 2019 (II)]**

(a) 400 (b) 380 (c) 525 (d) 480

36. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to: **[April 09, 2019 (I)]**

(a) $2\sqrt{6}$ (b) $2\sqrt{\frac{10}{3}}$ (c) $4\sqrt{\frac{5}{3}}$ (d) $\sqrt{6}$

37. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is : **[April 08, 2019 (I)]**

(a) 45 (b) 49 (c) 48 (d) 40

38. A student scores the following marks in five tests: 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is : **[April. 08, 2019 (II)]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{100}{3}$ (c) $\frac{10}{3}$ (d) $\frac{100}{\sqrt{3}}$

39. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is : **[Jan. 12, 2019 (I)]**

(a) 30 (b) 51 (c) 50 (d) 31

40. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations, is : **[Jan. 12, 2019 (II)]**

(a) 7 (b) 5 (c) 1 (d) 3

41. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals : **[Jan. 11, 2019 (I)]**

(a) $\frac{2}{3}$ (b) 2 (c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{2}$

42. A data consists of n observations: $x_1, x_2, ..., x_n$. If $\sum_{i=1}^{n}(x_i + 1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i - 1)^2 = 5n$ then the standard deviation of this data is: **[Jan. 09, 2019 (II)]**

(a) 2 (b) $\sqrt{5}$ (c) 5 (d) $\sqrt{7}$

43. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is: **[Jan. 10, 2019 (I)]**

(a) 10 : 3 (b) 4 : 9 (c) 5 : 8 (d) 6 : 7

44. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, ..., x_5$ and – 50 is equal to: **[Jan. 10, 2019 (II)]**

(a) 509.5 (b) 586.5 (c) 582.5 (d) 507.5

45. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is: **[Jan. 9, 2019 (I)]**

(a) 16 (b) 22 (c) 20 (d) 18

46. The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively.

If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is? **[Online April 16, 2018]**

(a) 0 (b) 4 (c) 2 (d) 1

47. If the mean of the data : 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is **[Online April 15, 2018]**

(a) $\frac{9}{8}$ (b) 2 (c) $\frac{7}{8}$ (d) 1

48. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is : **[2018]**

(a) 4 (b) 2 (c) 3 (d) 9

49. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is : **[Online April 9, 2017]**

(a) 8.25 (b) 8.50 (c) 8.00 (d) 9.00

50. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true? **[2016]**

(a) $3a^2 - 34a + 91 = 0$ (b) $3a^2 - 23a + 44 = 0$

(c) $3a^2 - 26a + 55 = 0$ (d) $3a^2 - 32a + 84 = 0$

51. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6; then the mean deviation from the mean of the data is : **[Online April 10, 2016]**

(a) 2.5 (b) 2.6 (c) 2.8 (d) 2.4

52. If the mean deviation of the numbers 1, 1 + d, ..., 1 + 100d from their mean is 255, then a value of d is : **[Online April 9, 2016]**

(a) 10.1 (b) 5.05 (c) 20.2 (d) 10

53. The variance of first 50 even natural numbers is **[2014]**

(a) 437 (b) $\frac{437}{4}$ (c) $\frac{833}{4}$ (d) 833

54. Let $\bar{x}$, M and σ^2 be respectively the mean, mode and variance of n observations $x_1, x_2, ..., x_n$ and $d_i = -x_i - a$, $i = 1, 2, ..., n$, where a is any number.

Statement I: Variance of $d_1, d_2, ... d_n$ is σ^2.

Statement II: Mean and mode of $d_1, d_2, ... d_n$ are $-\bar{x} - a$ and $-M - a$, respectively. **[Online April 19, 2014]**

(a) Statement I and Statement II are both false

(b) Statement I and Statement II are both true

(c) Statement I is true and Statement II is false

(d) Statement I is false and Statement II is true

55. Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observations x_i, i = 1, 2, ..., n. If each of the observations is increased by 5, then the new mean and the mean deviation about the new mean, respectively, are: **[Online April 12, 2014]**

(a) $\overline{X}$, M.D. (b) $\overline{X} + 5$, M.D.

(c) $\overline{X}$, M.D. + 5 (d) $\overline{X} + 5$, M.D. + 5

56. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ? **[2013]**

(a) mean (b) median (c) mode (d) variance

57. In a set of 2n observations, half of them are equal to 'a' and the remaining half are equal to '–a'. If the standard deviation of all the observations is 2 ; then the value of $|a|$ is : **[Online April 25, 2013]**

(a) 2 (b) $\sqrt{2}$ (c) 4 (d) $2\sqrt{2}$

58. Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing then the variance of all the five observations is : **[Online April 22, 2013]**

(a) 4 (b) 6 (c) 8 (d) 2

59. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.

Statement-1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.

Statement-2 : Arithmetic mean $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$. **[2012]**

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

60. **Statement 1:** The variance of first n odd natural numbers is $\frac{n^2 - 1}{3}$

Statement 2: The sum of first n odd natural number is n^2 and the sum of square of first n odd natural numbers is $\frac{n(4n^2 + 1)}{3}$. **[Online May 26, 2012]**

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(c) Statement 1 is false, Statement 2 is true.

(d) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

61. If the mean of 4, 7, 2, 8, 6 and a is 7, then the mean deviation from the median of these observations is **[Online May 12, 2012]**

(a) 8 (b) 5 (c) 1 (d) 3

62. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standarion deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : **[2011RS]**

(a) 32, 2 (b) 32, 4 (c) 28, 2 (d) 28, 4

63. If the mean deviation about the median of the numbers a, $2a$,.......,$50a$ is 50, then $|a|$ equals **[2011]**

(a) 3 (b) 4 (c) 5 (d) 2

64. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is **[2010]**

(a) $\frac{11}{2}$ (b) 6 (c) $\frac{13}{2}$ (d) $\frac{5}{2}$

65. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ... 1 + 100d from their mean is 255, then d is equal to: **[2009]**

(a) 20.0 (b) 10.1 (c) 20.2 (d) 10.0

66. **Statement-1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. **[2009]**

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

67. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b? **[2008]**

(a) $a = 0, b = 7$ (b) $a = 5, b = 2$

(c) $a = 1, b = 6$ (d) $a = 3, b = 4$

68. Suppose a population A has 100 observations 101, 102, ..., 200 and another population B has 100 obsevrations 151, 152, ... 250. If V_A and V_B represent the variances of the two populations, respectively then $\frac{V_A}{V_B}$ is **[2006]**

(a) 1 (b) $\frac{9}{4}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

69. In a series of 2 n observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals. **[2004]**

(a) $\frac{\sqrt{2}}{n}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{n}$

70. Consider the following statements :

(A) Mode can be computed from histogram

(B) Median is not independent of change of scale

(C) Variance is independent of change of origin and scale.

Which of these is / are correct ? **[2004]**

(a) (A), (B) and (C) (b) Only (B)

(c) Only (A) and (B) (d) Only (A)

71. In an experiment with 15 observations on x, the following results were available:

$\Sigma x^2 = 2830, \Sigma x = 170$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. The corrected variance is **[2003]**

(a) 8.33 (b) 78.00 (c) 188.66 (d) 177.33

Hints & Solutions

1. **(6.00)**

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{0\cdot {}^nC_0 + 2\cdot {}^nC_1 + 2^2\cdot {}^nC_2 + ... + 2^n \cdot {}^nC_n}{{}^nC_0 + {}^nC_1 + ... + {}^nC_n}$$

To find sum of numerator consider

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_n x^n \quad ...(i)$$

Put $x = 2 \Rightarrow 3^n - 1 = 2\cdot {}^nC_1 + 2^2\cdot {}^nC_2 + ... + 2^n \cdot {}^nC_n$

To find sum of denominator, put $x = 1$ in (i), we get

$$2^n = {}^nC_0 + {}^nC_1 + ... + {}^nC_n$$

$$\therefore \frac{3^n - 1}{2^n} = \frac{728}{2^n} \Rightarrow 3^n = 729 \Rightarrow n = 6$$

2. **(d)** $\dfrac{2^{\sin x} + 2^{\cos x}}{2} \ge (2^{\sin x + \cos x})^{\frac{1}{2}}$ $\quad (\because \text{AM} \ge \text{GM})$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \ge 2\cdot 2^{\frac{\sin x + \cos x}{2}}$$

Since, $-2 \le \sin x + \cos x \le \sqrt{2}$

$\therefore$ Minimum value of $2^{\frac{\sin x+\cos x}{2}} = 2^{-\frac{1}{\sqrt{2}}}$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \ge 2^{1-\frac{1}{\sqrt{2}}}.$$

3. **(d)** Number of students are,

$(x+1)^2 + (2x-5) + (x^2 - 3x) + x = 20$

$\Rightarrow 2x^2 + 2x - 4 = 20 \Rightarrow x^2 + x - 12 = 0$

$\Rightarrow (x+4)(x-3) = 0 \Rightarrow x = 3$

$\therefore$

Marks	2	3	5	7
No. of students	16	1	0	3

$$\text{Average marks} = \frac{32+3+21}{20} = \frac{56}{20} = 2.8$$

4. **(d)** Ten numbers in increasing order are

$10, 22, 26, 29, 34, x, 42, 67, 70, y$

$$\text{Mean} = \frac{\sum x_i}{n} = \frac{x+y+300}{10} = 42 \Rightarrow x + y = 120$$

$$\text{Median} = \frac{T_5 + T_6}{2} = 35 = \frac{34+x}{2} \Rightarrow x = 36 \text{ and } y = 84$$

$$\text{Hence, } \frac{y}{x} = \frac{84}{36} = \frac{7}{3}$$

5. **(b)** As mean is a linear operation, so if each observation is multiplied by λ and decreased by 25 then the mean becomes $75\lambda - 25$.

According to the question,

$$75\lambda - 25 = 75 \Rightarrow \lambda = \frac{4}{3}.$$

6. **(c)** Let; $\dfrac{x_1 + x_2 + ... + x_{25}}{25} = \bar{x} = 40$

$\Rightarrow x_1 + x_2 + ... + x_{25} = 1000$

$\therefore x_2 + x_2 + ... + x_{25} - 60 + A = 39 \times 25$

Let A be the age of new teacher.

$\Rightarrow 1000 - 60 + A = 975$

$\Rightarrow A = 975 - 940 = 35$

7. **(b)** Sum of 16 observations $= 16 \times 16 = 256$

Sum of resultant 18 observations

$= 256 - 16 + (3 + 4 + 5) = 252$

$$\text{Mean of observations} = \frac{252}{18} = 14$$

8. **(c)** $a_1 + a_2 + a_3 = 39$

$\Rightarrow a_1 + (a_1 + d) + (a_1 + 2d) = 39$

$\Rightarrow 3a_1 + 3d = 39 \quad [\because a_1 = 10]$

$\Rightarrow d = 3$

Sum of last four term $= 178$

$$\text{Their mean} = \frac{178}{4} = 44.5$$

$a_n = 44.5 + 1.5 + 3 = 49$

$$\text{Median} = \frac{10+49}{2} = \frac{59}{2} = 29.5$$

9. **(c)** Let average wage of Night shift worker is x

$70 \times 54 + 30 \times x = 60 \times 100$

$x = 74$

10. **(a)** There are $2n$ observations $x_1, x_2, ..., x_{2n}$

$$\text{So, mean} = \sum_{i=1}^{2n} \frac{x_i}{2n}$$

Let these observations be divided into two parts $x_1, x_2, ..., x_n$ and $x_{n+1}, ..., x_{2n}$

Each in 1st part 5 is added, so total of first part is $\sum_{i=1}^{n} x_i + 5n$.

In second part 3 is subtracted from each

So, total of second part is $\sum_{i=n+1}^{2n} x_i - 3n$

Total of $2n$ terms are

$$\sum_{i=1}^{n} x_i + 5n + \sum_{i=n+1}^{2n} x_i - 3n = \sum_{i=1}^{2n} x_i + 2n$$

$$\text{Mean} = \sum_{i=1}^{2n} \frac{x_i + 2n}{2n} = \sum_{i=1}^{2n} \frac{x_i}{2n} + 1$$

So, it increase by 1.

11. **(d)** Since $0 < y < x < 2y$

$\therefore\ y > \frac{x}{2} \Rightarrow x - y < \frac{x}{2}$

$\therefore\ x - y < y < x < 2x + y$

Hence median $= \frac{y + x}{2} = 10$

$\Rightarrow x + y = 20$ ...(i)

And range $= (2x + y) - (x - y) = x + 2y$

But range $= 28$

$\therefore x + 2y = 28$ ...(ii)

From equations (i) and (ii),

$x = 12, y = 8$

$$\therefore \text{Mean} = \frac{(x-y) + y + x + (2x + y)}{4} = \frac{4x + y}{4}$$

$$= x + \frac{y}{4} = 12 + \frac{8}{4} = 14$$

12. **(a)** Correct mean $= \frac{20 \times 40 - 33 + 55}{20} = 41.1$

Nearest option : (a) 41

13. **(a)** Median is given as

$$M = l + \frac{\frac{N}{2} - F}{f} \times C$$

where

l = lower limit of the median - class

f = frequency of the median class

N = total frequency

F = cumulative frequency of the class just before the median class

C = length of median class

Now, given, $M = 25, N = 100, F = 45,$

$C = 20 - 30 = 10,\ l = 20.$

$\therefore$ By using formula, we have

$$25 = 20 + \frac{50 - 45}{f} \times 10$$

$$25 - 20 = \frac{50}{f} \Rightarrow 5 = \frac{50}{f} \Rightarrow f = 10$$

14. **(d)** Frequency distribution is given as

Expenditure	No. of families (f)
0-50	24
50-100	33
100-150	37
150-200	b
200-250	25

Clearly, modal class is 100-150, as the maximum frequency occurs in this class.

Given, Mode = 140

We have

$$\text{Mode} = \ell + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times i$$

where

$\ell = 100, f_0 = 37, f_{-1} = 33, f_1 = \text{b}$

$i = 50$

Thus, we get

$$140 = 100 + \left[\frac{37 - 33}{2(37) - 33 - b}\right] \times 50$$

$$= 100 + \left[\frac{4}{74 - 33 - b}\right] \times 50 = 100 + \frac{200}{41 - b}$$

$\Rightarrow\ 5740 = 4300 + 40b \Rightarrow b = 36$

15. **(a)** Let the number of boys be x and girls be y.

$\Rightarrow 52x + 42y = 50(x + y)$

$\Rightarrow 52x - 50x = 50y - 42y$

$\Rightarrow\ 2x = 8y \Rightarrow \frac{x}{y} = \frac{4}{1} \Rightarrow \frac{x}{x + y} = \frac{4}{5}$

$\therefore$ Required % of boys $= \frac{x}{x + y} \times 100$

$= \frac{4}{5} \times 100 = 80\,\%$

16. **(b)** We know that for positive real numbers $x_1, x_2, ..., x_n$, A.M. of k^{th} powers of $x_i \geq k^{\text{th}}$ the power of A.M. of x_i

$$\Rightarrow \frac{\sum x_1^2}{n} \geq \left(\frac{\sum x_1}{n}\right)^2 \Rightarrow \frac{400}{n} \geq \left(\frac{80}{n}\right)^2$$

$\Rightarrow n \geq 16$. So only possible value for $n = 18$

17. **(d)** We know that Mode = 3 Median – 2Mean

$3 \times 22 - 2 \times 21 = 66 - 42 = 24$

18. **(a)** $n = 9$ then median term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ term. That means four observation followed by it. If last four observations are increased by 2. The median is 5th observation which is remaining unchanged.

$\therefore$ There will be no change in median.

19. **(b)** Total student = 100

Total marks of 70 boys = 75 × 70 = 5250

$\Rightarrow$ Total marks of girls = 7200 – 5250 = 1950

Number of girls = 100 – 70 = 30

Average of girls = $\frac{1950}{30} = 65$

20. **(d)** Standard deviation

$$= \sqrt{\frac{\sum_{i=1}^{n}(x_i - a)^2}{n} - \left(\frac{\sum_{i=1}^{n}(x_i - a)}{n}\right)^2} \qquad [\because n, a > 1]$$

$$= \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2} = \sqrt{a-1}$$

21. **(c)** Let two remaining observations are x_1, x_2.

So, $\bar{x} = \frac{2+4+10+12+14+x_1+x_2}{7} = 8$ (given)

$\Rightarrow x_1 + y_1 = 14$ ...(i)

Now, $\sigma^2 = \frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2 = 16$ (given)

$= \frac{4+16+100+144+196+x_1^2+x_2^2}{7} - 64 = 16$

$\Rightarrow 460 + x_1^2 + x_2^2 = (16+64) \times 7$

$\Rightarrow x_1^2 + x_2^2 = 100$ ...(ii)

$\because (x+y)^2 = x^2 + y^2 + 2xy \Rightarrow xy = 48$ ...(iii)

$\because (x-y)^2 = (x+y)^2 - 4xy = 196 - 192 = 4$

$\Rightarrow x - y = 2 \Rightarrow |x - y| = 2$

22. **(c)** Mean $= \frac{3+5+7+a+b}{5} = 5 \Rightarrow a + b = 10$

Variance $= \frac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2 = 4$

$\Rightarrow a^2 + b^2 = 62$

$\Rightarrow (a+b)^2 - 2ab = 62$

$\Rightarrow ab = 19$

Hence, a and b are the roots of the equation,

$x^2 - 10x + 19 = 0$.

23. **(d)** Let the two remaining observations be x and y.

$\because \bar{x} = \frac{5+7+10+12+14+15+x+y}{8}$

$\Rightarrow 10 = \frac{63+x+y}{8}$

$\Rightarrow x + y = 80 - 63$

$\Rightarrow x + y = 17$ ...(i)

$\because \text{var}(x) = 13.5$

$= \frac{25+49+100+144+196+225+x^2+y^2}{8} - (10)^2$

$\Rightarrow x^2 + y^2 = 169$ ...(ii)

From (i) and (ii) we get

$(x, y) = (12, 5)$ or $(5, 12)$

So, $|x - y| = 7$.

24. **(4)**

x_i	15	25	35
f_i	2	x	2

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{30+70+25x}{4+x} = 25$

$\sigma^2 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2$

$\Rightarrow 50 = \frac{450+625x+2450}{4+x} - 625$

$\Rightarrow 675 = \frac{2900+625x}{4+x} \Rightarrow 50x = 200$

$\therefore x = 4$

25. **(c)** If variate varries from a to b then variance

$\text{var}(x) \le \left(\frac{b-a}{2}\right)^2$

$\Rightarrow \text{var}(x) < \left(\frac{10-0}{2}\right)^2$

$\Rightarrow \text{var}(x) < 25$

$\Rightarrow$ standard deviation < 5

It is clear that standard deviation cann't be 6.

26. **(c)** S.D. $= \sqrt{\frac{\sum_{i=1}^{10}(x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10}(x_i - p)}{10}\right)^2}$

$$= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2} = \frac{9}{10}.$$

27. **(b)** $\because \bar{x} = \frac{1+2+3+....+17}{17} = \frac{17 \times 18}{17 \times 2} = 9$

$\bar{y} = a\bar{x} + b = \frac{a(1+2+3+.....+17)}{17} + b = 17$

$$\Rightarrow \frac{a\cdot(17\cdot18)}{17\cdot2}+b=17 \Rightarrow 9a+b=17 \quad ...(i)$$

$$\text{Var}(x)=\sigma A^2=\frac{\Sigma x^2}{n}-(\overline{x})^2$$

$$=\frac{1^2+2^2+....+17^2}{17}-(9)^2$$

$$=\frac{17\cdot18\cdot35}{6\cdot17}-(9)^2=105-81=24$$

$\text{Var}(y)=a^2\,\text{Var}(x)=a^2\cdot24=216$

$a^2=\frac{216}{24}=9\Rightarrow a=3$

$\therefore$ From (i), $b=17-9a=17-27=-10$

$\therefore a+b=3+(-10)=-7$

28. **(3)**

$$\text{Variance}=\frac{\sum_{i=1}^{11}b_i^2}{11}-\left(\frac{\sum_{i=1}^{11}b_i}{11}\right)^2$$

Let common difference of A.P. be d

$$=\frac{\sum_{r=0}^{10}(b_1+rd)^2}{11}-\left(\frac{\sum_{r=0}^{10}(b_1+rd)}{11}\right)^2$$

$$=\frac{11b_1^2+2b_1d\left(\frac{10\times11}{2}\right)+d^2\left(\frac{10\times11\times21}{6}\right)}{11}-\left(\frac{11b_1+\frac{10\times11}{2}d}{11}\right)^2$$

$=(b_1^2+10b_1d+35d^2)-(b_1+5d)^2=10d^2$

$\because$ Variance = 90 (Given)

$\Rightarrow 10d^2=90\Rightarrow d=3.$

29. **(a)** Mean of the observation $(x_i-5)=\frac{\Sigma(x_i-5)}{10}=1$

$\therefore\ \lambda=\{\text{Mean}\,(x_i-5)\}+2=3$

Variance of the observation

$$\mu=\text{var}\,(x_i-5)=\frac{\Sigma(x_i-5)^2}{10}-\frac{\Sigma(x_i-5)}{10}=3$$

30. **(c)** Let $\overline{x}$ and σ be the mean and standard deviations of given observations.

If each observation is multiplied with p and then q is subtracted.

New mean $(\overline{x}_1)=p\overline{x}-q$

$\Rightarrow\ 10=p(20)-q \quad ...(i)$

and new standard deviations $\sigma_1=|p|\,\sigma$

$\Rightarrow\ 1=|p|\,(2)\ \Rightarrow\ |p|=\frac{1}{2}\ \Rightarrow\ p=\pm\frac{1}{2}$

If $p=\frac{1}{2}$, then $q=0$ (from equation (i))

If $p=-\frac{1}{2}$, then $q=-20$

31. **(a)** Let $x_1, x_2, ..., x_{20}$ be 20 observations, then

$$\text{Mean}=\frac{x_1+x_2+.....+x_{20}}{20}=10$$

$$\Rightarrow \frac{\sum_{i=1}^{20}x_i}{20}=10 \quad ...(i)$$

$$\text{Variance}=\frac{\Sigma x_i^2}{n}-(\overline{x})^2$$

$$\Rightarrow \frac{\Sigma x_i^2}{20}-100=4 \quad ...(ii)$$

$\Sigma x_i^2=104\times20=2080$

$$\text{Actual mean}=\frac{200-9+11}{20}=\frac{202}{20}$$

$$\text{Variance}=\frac{2080-81+121}{20}-\left(\frac{202}{20}\right)^2$$

$$=\frac{2120}{20}-(10.1)^2=106-102.01=3.99$$

32. **(18)** Var $(1, 2, ..., n)=10$

$$\Rightarrow \frac{1^2+2^2+.....+n^2}{n}-\left(\frac{1+2+.....+n}{n}\right)^2=10$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2=10$$

$\Rightarrow\ n^2-1=120 \qquad \Rightarrow\ n=11$

Var $(2, 4, 6, ..., 2m)=16\ \Rightarrow$ Var $(1, 2, ..., m)=4$

$\Rightarrow\ m^2-1=48\ \Rightarrow\ m=7$

$\Rightarrow\ m+n=18$

33. **(52)** Mean $=\overline{x}=\frac{3+7+9+12+13+20+x+y}{8}=10$

$\Rightarrow\ x+y=16 \quad ...(i)$

Variance $= \sigma^2 = \frac{\Sigma(x_i)^2}{8} - (\bar{x})^2 = 25$

$$\sigma^2 = \frac{9+49+81+144+169+400+x^2+y^2}{8} - 100 = 25$$

$\Rightarrow x^2 + y^2 = 148$ ...(ii)

From eqn. (i), $(x+y)^2 = (16)^2$

$\Rightarrow x^2 + y^2 + 2xy = 256$

Using eqn. (ii), $148 + 2xy = 256$

$\Rightarrow xy = 52$

34. **(b)** According to the question,

$\frac{x_1+x_2+x_3+x_4}{4} = 11 \Rightarrow x_1+x_2+x_3+x_4 = 44$

$\frac{x_5+x_6+...+x_{10}}{6} = 16 \Rightarrow x_5+x_6+...+x_{10} = 96$

and $x_1^2 + x_2^2 + ...x_{10}^2 = 2000$

$\because$ standard deviation, $\sigma^2 = \frac{\sum x_i^2}{N} - (\bar{x})^2$

$= \frac{2000}{10} - \left(\frac{44+96}{10}\right)^2 = 4 \quad \Rightarrow \sigma = 2$

35. **(a)** Given, mean and standard deviation are equal to 16.

$\therefore \frac{x_1+x_2+....x_{50}}{50} = 16$

and $16^2 = \frac{x_1^2+x_2^2...x_{50}^2}{50} - 16^2$

$\Rightarrow 2(16)^2 50 = x_1^2 + x_2^2 + ...x_{50}^2$

Required mean $= \frac{(x_1-4)^2+(x_2-4)^2+...(x_{50}-4)^2}{50}$

$= \frac{x_1^2+x_2^2+...+x_{50}^2+50\times16-8(x_1+x_2+...+x_{50})}{50}$

$= \frac{16^2(100)+(50)-8(16\times50)}{50} = 400$

36. **(a)** Mean of given observation $= \frac{k}{4}$

$\because$ Standard deviation = 5

$\therefore \sigma^2 = 5$

$\Rightarrow \sigma^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2$

$$= \frac{\left(\frac{k}{4}+1\right)^2+\left(\frac{k}{4}\right)^2+\left(\frac{k}{4}-1\right)^2+\left(\frac{3k}{4}\right)^2}{4} = 5$$

$$\Rightarrow \frac{\frac{12k^2}{16}+2}{4} = 5 \Rightarrow k = 2\sqrt{6}$$

37. **(c)** Let the remaining numbers are a and b.

Mean $(\bar{x}) = \frac{\sum x_i}{N} = \frac{2+4+10+12+14+a+b}{7} = 8$

$\Rightarrow a + b = 14$ ...(i)

Variance $(\sigma^2) = \frac{\sum x_i^2}{N} - (\bar{x})^2 = 16$

$$\Rightarrow \frac{2^2+4^2+10^2+12^2+14^2+a^2+b^2}{7} - (8)^2 = 16$$

$\Rightarrow a^2 + b^2 = 100$ ...(ii)

From (i) and (ii), $(14-b)^2 + b^2 = 100$

$\Rightarrow 196 + b^2 - 28b + b^2 = 100$

$\Rightarrow b^2 - 14b + 48 = 0$

$\Rightarrow b = 6, 8$

$\therefore a = 8, 6.$

$\therefore (a, b) = (6, 8)$ or $(8, 6)$

Hence, the product of the remaining two observations $= ab = 48$

38. **(a)** $\because$ Mean score = 48

Let unknown score be x,

$\therefore \bar{x} = \frac{41+45+54+57+43+x}{6} = 48$

$\Rightarrow x + 240 = 288 \Rightarrow x = 48$

Now, $\sigma^2 = \frac{1}{6}[(48-41)^2 + (48-45)^2 + (48-54)^2 + (48-57)^2 + (48-43)^2 + (48-48)^2$

$= \frac{1}{6}(49+9+36+81+25) = \frac{200}{6} = \frac{100}{3}$

$\sigma = \frac{10}{\sqrt{3}}$

39. **(d)** Given, $\sum_{i=1}^{50}(x_i - 30) = 50$

$\sum_{i=1}^{50} x_i - 50(30) = 50$

$\Rightarrow \sum_{i=1}^{50} x_i = 1550$

Mean, $\bar{x} = \dfrac{\sum x_i}{50}$

$= \dfrac{1550}{50} = 31$

40. **(a)** Let two observations be x_1 and x_2, then

$\dfrac{x_1 + x_2 + 3 + 4 + 4}{5} = 4$

$\Rightarrow\ x_1 + x_2 = 9$ ...(i)

Variance $= \dfrac{\sum x_i^2}{N} - (\bar{x})^2$

$(5 \cdot 20) = \dfrac{9 + 16 + 16 + x_1^2 + x_2^2}{5} - 16$

$26 = 41 + x_1^2 + x_2^2 - 80$

$x_1^2 + x_2^2 = 65$ ...(ii)

From (i) and (ii);

$x_1 = 8, x_2 = 1$

Hence, the required value of the difference of other two observations $= |x_1 - x_2| = 7$

41. **(d)** Outcomes are $\left(\frac{1}{2} - d\right), \left(\frac{1}{2} - d\right), 0 \ldots$, 10 times, $\frac{1}{2}, \frac{1}{2}$, ..., 10 times, $\frac{1}{2} + d, \frac{1}{2} + d$, ..., 10 times

Mean $= \dfrac{1}{30}\left(\dfrac{1}{2} \times 30\right) = \dfrac{1}{2}$

Variance of the outcomes is,

$\sigma^2 = \dfrac{1}{30}\Sigma x_i^2 - (\bar{x})^2$

$= \dfrac{1}{30}\left[\left(\dfrac{1}{2} - d\right)^2 \times 10 + \left(\dfrac{1}{2}\right)^2 \times 10 + \left(\dfrac{1}{2} + d\right)^2 \times 10\right] - \dfrac{1}{4}$

$\Rightarrow\ \dfrac{4}{3} = \dfrac{1}{30}\left[30 \times \dfrac{1}{4} + 20d^2\right] - \dfrac{1}{4}$

$\Rightarrow\ \dfrac{4}{3} = \dfrac{1}{4} + \dfrac{2}{3}d^2 - \dfrac{1}{4}$

$\Rightarrow\ d^2 = 2 \Rightarrow |d| = \sqrt{2}$

42. **(b)** Variance is given by,

$\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\dfrac{1}{n}\sum_{i=1}^{n} x_i\right)^2$

$\sigma^2 = \dfrac{1}{n}A - \dfrac{1}{n^2}B^2$ ...(i)

Here, $A = \sum_{i=1}^{n} x_i^2$ and $B = \sum_{i=1}^{n} x_i$

$\because\ \sum_{i=1}^{n}(x_i + 1)^2 = 9n$

$\Rightarrow\ A + n + 2B = 9n \Rightarrow A + 2B = 8n$ ...(ii)

$\because\ \sum_{i=1}^{n}(x_i - 1)^2 = 5n$

$\Rightarrow\ A + n - 2B = 5n \Rightarrow A - 2B = 4n$ ...(iii)

From (ii) and (iii),

$A = 6n, B = n$

$\Rightarrow \sigma^2 = \dfrac{1}{n} \times 6n - \dfrac{1}{n^2} \times n^2 = 6 - 1 = 5$

$\Rightarrow \sigma = \sqrt{5}$

43. **(b)** Since mean of x_1, x_2, x_3, x_4 and x_5 is 5

$\Rightarrow\ x_1 + x_2 + x_3 + x_4 + x_5 = 25$

$\Rightarrow\ 1 + 3 + 8 + x_4 + x_5 = 25$

$\Rightarrow\ x_4 + x_5 = 13$ ...(i)

$\because\ \dfrac{\sum_{i=1}^{5} x_i^2}{5} - (5)^2 = 9.2 \Rightarrow \sum_{i=1}^{5} x_i^2 = 5(25 + 9.2)$

$= 125 + 46 = 171$

$\Rightarrow\ (1)^2 + (3)^2 + (8)^2 + x_4^2 + x_5^2 = 171$

$\Rightarrow\ x_4^2 + x_5^2 = 97$ ...(ii)

$\Rightarrow\ (x_4 + x_5)^2 - 2x_4 x_5 = 97$

$\Rightarrow\ 2x_4 x_5 = 13^2 - 97 = 72 \Rightarrow x_4 x_5 = 36$ ...(iii)

(i) and (iii) $\Rightarrow x_4 : x_5 = \dfrac{4}{9}$ or $\dfrac{9}{4}$

44. **(d)** $\because\ \bar{x} = \dfrac{\sum_{i=1}^{5} x_i}{5} \Rightarrow \sum_{i=1}^{5} x_i = 10 \times 5 = 50 \Rightarrow \sum_{i=1}^{6} xi = 50 - 50 = 0$

$\dfrac{\sum_{i=1}^{5} x_i^2}{5} - (10)^2 = 3^2 = 9$

$\Rightarrow\ \sum_{i=1}^{5} x_i^2 = 545$

Then,

$\Rightarrow\ \sum_{i=1}^{6} x_i^2 = \sum_{i=1}^{5} x_i^2 + (-50)^2$

$= 545 + (-50)^2 = 3045$

$$\text{Variance} = \frac{\sum_{i=1}^{6} x_i^2}{6} - \left(\frac{\sum_{i=1}^{6} x_i}{6}\right)^2 = \frac{3045}{6} - 0 = 507.5$$

45. **(c)** $\because$ Variance $= \sigma^2 = \frac{\Sigma x_i^2}{N} - (\bar{x})^2$

$\Rightarrow 18 = \frac{\Sigma x_i^2}{5} - (150)^2$

$\Rightarrow \Sigma x_i^2 = 90 + 112590 = 112590$

Then, variance of the height of six students

$$V' = \frac{112590 + (156)^2}{6} - \left(\frac{750 + 156}{6}\right)^2$$

$= 22821 - 22801 = 20$

46. **(c)** Here mean $= \bar{x} = 9$

$\Rightarrow \bar{x} = \frac{\Sigma x_i}{n} = 9$

$\Rightarrow \Sigma x_i = 9 \times 5 = 45$

Now, standard deviation = 0

$\therefore$ all the five terms are same i.e.; 9.

Now for changed observation

$\bar{x}_{\text{new}} = \frac{36 + x_5}{5} = 10$

$\Rightarrow x_5 = 14$

$$\therefore \ \sigma_{\text{new}} = \sqrt{\frac{\Sigma(x_i - \bar{x}_{\text{new}})^2}{n}}$$

$$= \sqrt{\frac{4(9-10)^2 + (14-10)^2}{5}} = 2$$

47. **(d)** $\bar{x} = \frac{7+8+9+7+8+7+\lambda+8}{8} = 8$

$\Rightarrow \frac{54+\lambda}{8} = 8 \Rightarrow \lambda = 10$

Now variance $= \sigma^2$

$$= \frac{(7-8)^2 + (8-8)^2 + (9-8)^2 + (7-8)^2 + (8-8)^2 + (7-8)^2 + (10-8)^2 + (8-8)^2}{8}$$

$\Rightarrow \sigma^2 = \frac{1+0+1+1+0+1+4+0}{8} = \frac{8}{8} = 1$

Hence, the variance is 1.

48. **(b)** Given $\sum_{i=1}^{9}(x_i - 5) = 9 \Rightarrow \sum_{i=1}^{9} x_i = 54$ ...(i)

Also, $\sum_{i=1}^{9}(x_i - 5)^2 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 10\sum_{i=1}^{9} x_i + 9(25) = 45$ ...(ii)

From (i) and (ii) we get,

$\sum_{i=1}^{9} x_i^2 = 360$

Since, variance $= \frac{\sum x_i^2}{9} - \left(\frac{\sum x_i}{9}\right)^2$

$= \frac{360}{9} - \left(\frac{54}{9}\right)^2 = 40 - 36 = 4$

$\therefore$ Standard deviation $= \sqrt{\text{Variance}} = 2$

49. **(d)** $\sum_{i=1}^{100} x_i = 400$ $\qquad \sum_{i=1}^{100} x_i^2 = 2475$

Variance $= \sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$

$= \frac{2475}{97} - \left(\frac{388}{97}\right)^2$

$= \frac{2425 - 1552}{97} = \frac{873}{97} = 9$

50. **(d)** $\bar{x} = \frac{2+3+a+11}{4} = \frac{a}{4} + 4$

$\sigma = \sqrt{\sum \frac{x_i^2}{n} - (\bar{x})^2}$

$\Rightarrow 3.5 = \sqrt{\frac{4+9+a^2+121}{4} - \left(\frac{a}{4}+4\right)^2}$

$\Rightarrow \frac{49}{4} = \frac{4(134+a^2) - (a^2+256+32a)}{16}$

$\Rightarrow 3a^2 - 32a + 84 = 0$

51. **(c)** $n = 5$

$\bar{x} = 5$

variance = 124

$x_1 = 1,\ x_2 = 2, x_3 = 6$

$\bar{x} = 5$

$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 5$

$\Rightarrow x_4 + x_5 + 9 = 25$
$\Rightarrow x_4 + x_5 = 16$
$\Rightarrow x_4 + x_5 + 10 - 10 = 16$
$\Rightarrow (x_4 - 5) + (x_5 - 5) = 16 - 10$
$\Rightarrow (x_4 - 5) + (x_5 - 5) = 6$

Mean deviation $= \dfrac{\Sigma|x_i - \bar{x}|}{N}$

$= |x_1 - 5| + |x_2 - 5| + |x_3 - 5| + \dfrac{|x_4 - 5| + |x_5 - 5|}{5}$

$= \dfrac{4+3+1+6}{5} = \dfrac{14}{5} = 2.8$

52. **(a)** $\bar{x} = \dfrac{1}{101}[1 + (1+d) + (1+2d)] \ldots\ldots (1+100d)]$

$= \dfrac{1}{101} \times \dfrac{101}{2}[1 + (1+100d)] = 1 + 50d$

mean deviation from mean

$= \dfrac{1}{101}[|1 - (1+50d)| + |(1+d) - (1+50d)| \ldots\ldots|$

$[1+100d] - (1+50d)|]$

$= \dfrac{2|d|}{101}(1+2+3\ldots\ldots+50)$

$= \dfrac{2|d|}{101} \times \dfrac{50 \times 51}{2} = \dfrac{2550}{101}|d|$

$= \dfrac{2550}{101}|d| = 225 \Rightarrow |d| = 10.1$

53. **(d)** First 50 even natural numbers are 2, 4, 6 ....., 100

Variance $= \dfrac{\sum x_i^2}{N} - (\bar{x})^2$

$\Rightarrow \sigma^2 = \dfrac{2^2 + 4^2 + \ldots + 100^2}{50} - \left(\dfrac{2+4+\ldots+100}{50}\right)^2$

$= \dfrac{4(1^2 + 2^2 + 3^2 + \ldots. + 50^2)}{50} - (51)^2$

$= 4\left(\dfrac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2$

$= 3434 - 2601 \Rightarrow \sigma^2 = 833$

54. **(b)** $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

$\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Mean of $d_1, d_2, d_3, \ldots., d_n$

$= \dfrac{d_1 + d_2 + d_3 + \ldots. + d_n}{n}$

$= \dfrac{(-x_1 - a) + (-x_2 - a) + (-x_3 - a) + \ldots. + (-x_n - a)}{n}$

$= -\left[\dfrac{x_1 + x_2 + x_3 + \ldots. + x_n}{n}\right] - \dfrac{na}{n} = -\bar{x} - a$

Since, $d_i = -x_i - a$ and we multiply or subtract each observation by any number the mode remains the same. Hence mode of $-x_i - a$ i.e. d_i and x_i are same.

Now variance of $d_1, d_2, \ldots., d_n$

$= \dfrac{1}{n}\sum_{i=1}^{n}[d_i - (-\bar{x} - a)]^2$

$= \dfrac{1}{n}\sum_{i=1}^{n}[-x_i - a + \bar{x} + a]^2$

$= \dfrac{1}{n}\sum_{i=1}^{n}(-x_i + \bar{x})^2 = \dfrac{1}{n}\sum_{i=1}^{n}(\bar{x} - x_i)^2 = \sigma^2$

55. **(b)** Let x_i be n observations, $i = 1, 2, \ldots n$

Let $\bar{X}$ be the mean and M.D be the mean deviation about $\bar{X}$.

If each observation is increased by 5 then new mean will be $\bar{X} + 5$ and new M.D. about new mean will be M.D.

$\left(\because \text{Mean} = \sum_{i=1}^{n}\dfrac{x_i}{n}\right)$

56. **(d)** If initially all marks were x_i then

$\sigma_1^2 = \dfrac{\sum_i (x_i - \bar{x})^2}{N}$

Now each is increased by 10

$\sigma_1^2 = \dfrac{\sum_i [(x_i + 10) - (\bar{x} + 10)]^2}{N} = \dfrac{\sum_i (x_i - \bar{x})^2}{N} = \sigma_1^2$

Hence, variance will not change even after the grace marks were given.

57. **(a)** Clearly mean A = 0

Now, standard deviation $\sigma = \sqrt{\dfrac{\Sigma(x - A)^2}{2n}}$

$2 = \sqrt{\dfrac{(a-0)^2 + (a-0)^2 + \ldots\ldots + (0-a)^2 + \ldots\ldots}{2n}}$

$= \sqrt{\dfrac{a^2.2n}{2n}} = |a|$

Hence, $|a| = 2$

58. **(d)** Let 5th observation be x.

Given mean = 7

$\therefore\ 7 = \dfrac{6+7+8+10+x}{5}$

$\Rightarrow x = 4$

Now, Variance

$$=\sqrt{\frac{(6-7)^2+(7-7)^2+(8-7)^2+(10-7)^2+(4-7)^2}{5}}$$

$$=\sqrt{\frac{1^2+0^2+1^2+3^2+3^2}{5}}=\sqrt{\frac{20}{5}}=\sqrt{4}=2$$

59. **(d)** A.M. of $2x_1, 2x_2, ..., 2x_n$ is

$$\frac{2x_1+2x_2+...+2x_n}{n}=2\left(\frac{x_1+x_2+....+x_n}{n}\right)=2\bar{x}$$

$$\left(\because \text{ Mean}=\frac{\text{Sum of observations}}{\text{Number of observations}}\right)$$

So statement-2 is false.

If each observations is multiply by 2 then mean multiply by 2 and variance multiply by 2^2.

variance $(2x_i)=2^2$ variance $(x_i)=4\sigma^2$ where $i=1, 2, ...n$

So statement-1 is true.

60. **(a)** Statement 2 : Sum of first n odd natural numbers is not equal to n^2.

So, statement - 2 is false.

61. **(d)** Given observations are 4, 7, 2, 8, 6, a and mean is 7.

We know

$$\text{Mean}=\frac{4+7+2+8+6+a}{6}$$

$$\Rightarrow\quad 7=\frac{4+7+2+8+6+a}{6}\Rightarrow a=15$$

Now, given observations can be written in ascending order which is 2, 4, 6, 7, 8, 15

Since, No. of observation is even

$\therefore$ Median

$$=\frac{\left(\frac{6}{2}\right)\text{th observation}+\left(\frac{6}{2}+1\right)\text{th observation}}{2}$$

$$=\frac{\text{3rd observation}+\text{4th observation}}{2}=\frac{6+7}{2}=\frac{13}{2}$$

$$\text{Now, Mean deviation}=\frac{\sum_{i=1}^{6}\left|x_i-\frac{13}{2}\right|}{6}$$

$$=\frac{\left|4-\frac{13}{2}\right|+\left|7-\frac{13}{2}\right|+\left|2-\frac{13}{2}\right|+\left|8-\frac{13}{2}\right|+\left|6-\frac{13}{2}\right|+\left|15-\frac{13}{2}\right|}{6}$$

$$=\frac{\frac{5}{2}+\frac{1}{2}+\frac{9}{2}+\frac{3}{2}+\frac{1}{2}+\frac{17}{2}}{6}=\frac{18}{6}=3$$

62. **(a)** We know that if each observation is increase by 2 then mean is increase by 2 but S.D. remains same.

Correct mean = observed mean + 2 $=30+2=32$

Correct S. D. = observed S.D. = 2

63. **(b)** $\because\ n=50$ (even)

$$\text{Median}=\frac{25^{\text{th}}\text{ obs.}+26^{\text{th}}\text{ obs.}}{2}$$

$$\therefore\ M=\frac{25a+26a}{2}=25.5a$$

$$M.D\,(M)=\frac{\sum|x_i-M|}{N}$$

$$\Rightarrow\ 50=\frac{1}{50}[2\times|a|\times(0.5+1.5+2.5+....24.5)]$$

$$\Rightarrow\ 2500=2|a|\times\frac{25}{2}(25)$$

$$\Rightarrow\ |a|=4$$

64. **(a)** $\sigma_x^2=4, \sigma_y^2=5, \bar{x}=2, \bar{y}=4$

$$\sigma_x^2=\frac{1}{5}\sum x_i^2-(2)^2=4\Rightarrow\sum x_i^2=40;$$

$$\sigma_y^2=\frac{1}{5}\sum y_i^2-(4)^2=5\Rightarrow\sum y_i^2=105$$

$$\Rightarrow\ \sum x_i^2+\sum y_i^2=\sum\left(x_i^2+y_i^2\right)=145$$

$$\Rightarrow\ \sum x_i+\sum y_i=\sum(x_i+y_i)=5(2)+5(4)=30$$

Variance of combined data

$$=\frac{1}{10}\sum\left(x_i^2+y_i^2\right)-\left(\frac{1}{10}\sum(x_i+y_i)\right)^2$$

$$=\frac{145}{10}-9=\frac{11}{2}$$

65. **(b)** $$\text{Mean}=\frac{101+d(1+2+3+......+100)}{101}$$

$$=1+\frac{d\times 100\times 101}{101\times 2}=1+50d$$

Given that mean deviation from the mean = 255

$$\Rightarrow\ \frac{1}{101}[|1-(1+50d)|+|(1+d)-(1+50d)|$$
$$+|(1+2d)-(1+50d)|+....+$$
$$|(1+100d)-(1+50d)|]=255$$

$$\Rightarrow\ 2d[1+2+3+...+50]=101\times 255$$

$$\Rightarrow\ 2d\times\frac{50\times 51}{2}=101\times 255$$

$$\Rightarrow\ d=\frac{101\times 255}{50\times 51}=10.1$$

66. **(c)** First n even natural numbers be $2, 4, 6, 8, ..., 2n$

$$\therefore\ \bar{x}=\frac{2(1+2+3+...+n)}{n}=\frac{2[n(n+1)]}{2n}=(n+1)$$

And $Var = \frac{\Sigma(x-\bar{x})^2}{2n} = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$= \frac{4\Sigma n^2}{n} - (n+1)^2 = \frac{4n(n+1)(2n+1)}{6n} - (n+1)^2$

$= \frac{2(2n+1)(n+1)}{3} - (n+1)^2 = (n+1)\left[\frac{4n+2-3n-3}{3}\right]$

$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$

$\therefore$ Statement-1 is false. Clearly, statement - 2 is true.

67. **(d)** Mean of $a, b, 8, 5, 10$ is 6

$\Rightarrow \frac{a+b+8+5+10}{5} = 6$

$\Rightarrow a+b=6$ ...(i)

Variance of $a, b, 8, 5, 10$ is 6.80

$\Rightarrow \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$

$= 6.80$

$\Rightarrow a^2-12a+36+(1-a)^2+21=34$

[using eq. (i)]

$\Rightarrow 2a^2-14a+24=0 \Rightarrow a^2-7a+12=0$

$\Rightarrow a=3$ or $4 \Rightarrow b=4$ or 3

$\therefore$ The possible values of a and b are $a=3$ and $b=4$ or, $a=4$ and $b=3$

68. **(a)** $\sigma_x^2 = \frac{\sum d_i^2}{n}$ (Here d_i = deviations are taken from the mean). Since population A and population B both have 100 consecutive integers, therefore both have same standard deviation and hence the variance is also same. $\therefore \frac{V_A}{V_B} = 1$

69. **(c)** Clearly sum of observations = 0,

$\therefore$ mean $A = 0$

Standard deviation $\sigma = \sqrt{\frac{\sum(x-A)^2}{2n}}$

$2 = \sqrt{\frac{(a-0)^2+(a-0)^2+...(0-a)^2+...}{2n}}$ $[\because \sigma = 2]$

$= \sqrt{\frac{a^2.2n}{2n}} = |a|$

Hence, $|a| = 2$

70. **(c)** Only first statement (A) and second statements (B) are correct.

71. **(b)** $\Sigma x = 170, \Sigma x^2 = 2830$

New, $\Sigma x' = 170 + (30-20) = 180$

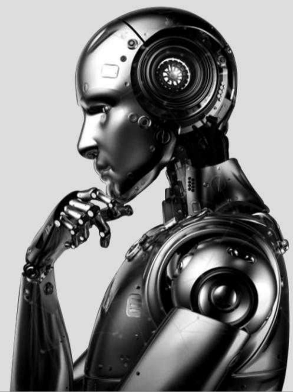
New, $\Sigma x'^2 = 2830 + (900-400)$

$= 2830 + 500 = 3330$

Now, Variance $= \frac{1}{n}\Sigma x'^2 - \left(\frac{1}{n}\Sigma x'\right)^2$

$= \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2 = 222 - 144 = 78.$

15



Probability

TOPIC 1 Random Experiment, Sample Space, Events, Probability of an Event, Mutually Exclusive & Exhaustive Events, Equally Likely

1. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is: **[Sep. 06, 2020 (I)]**

(a) $\frac{15}{101}$ (b) $\frac{5}{101}$

(c) $\frac{5}{33}$ (d) $\frac{10}{99}$

2. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is : **[Jan. 9, 2020 (II)]**

(a) $\frac{965}{2^{11}}$ (b) $\frac{965}{2^{10}}$

(c) $\frac{945}{2^{10}}$ (d) $\frac{945}{2^{11}}$

3. If three of the six vertices of a regular hexazon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is : **[April 12, 2019 (I)]**

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$

(c) $\frac{3}{10}$ (d) $\frac{3}{20}$

4. Let S = $\{1, 2, ..., 20\}$. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Than the probability that a randomly chosen subset of S is "nice" is : **[Jan. 11, 2019 (II)]**

(a) $\frac{7}{2^{20}}$ (b) $\frac{5}{2^{20}}$

(c) $\frac{4}{2^{20}}$ (d) $\frac{6}{2^{20}}$

5. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket.
If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is? **[Online April 16, 2018]**

(a) 4 (b) 6

(c) 3 (d) 5

6. A box $'A'$ contanis 2 white, 3 red and 2 black balls. Another box $'B'$ contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box $'B'$ is **[Online April 15, 2018]**

(a) $\frac{7}{16}$ (b) $\frac{9}{32}$

(c) $\frac{7}{8}$ (d) $\frac{9}{16}$

7. If the lengths of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given that it is an isosceles triangle, is : **[Online April 11, 2015]**

(a) $\frac{1}{21}$ (b) $\frac{1}{27}$

(c) $\frac{1}{15}$ (d) $\frac{1}{26}$

8. A number x is chosen at random from the set {1, 2, 3, 4, ..., 100}. Define the event: A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$

Then P (A) is: **[Online April 12, 2014]**

(a) 0.71 (b) 0.70
(c) 0.51 (d) 0.20

9. A set S contains 7 elements. A non-empty subset A of S and an element x of S are chosen at random. Then the probability that $x \in A$ is: **[Online April 11, 2014]**

(a) $\frac{1}{2}$ (b) $\frac{64}{127}$
(c) $\frac{63}{128}$ (d) $\frac{31}{128}$

10. There are two balls in an urn. Each ball can be either white or black. If a white ball is put into the urn and there after a ball is drawn at random from the urn, then the probability that it is white is **[Online May 26, 2012]**

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$
(c) $\frac{1}{5}$ (d) $\frac{1}{3}$

11. If six students, including two particular students *A* and *B*, stand in a row, then the probability that *A* and *B* are separated with one student in between them is **[Online May 19, 2012]**

(a) $\frac{8}{15}$ (b) $\frac{4}{15}$
(c) $\frac{2}{15}$ (d) $\frac{1}{15}$

12. A number *n* is randomly selected from the set {1, 2, 3, ..., 1000}. The probability that $\frac{\sum_{i=1}^{n} i^2}{\sum_{i=1}^{n} i}$ is an integer is **[Online May 12, 2012]**

(a) 0.331 (b) 0.333
(c) 0.334 (d) 0.332

13. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ...20}. **[2010]**

Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is $(\pm1, \pm2, \pm3, \pm4, \pm5)$.

(a) Statement -1 is true, Statement -2 is true; Statement -2 is **not** a correct explanation for Statement -1
(b) Statement -1 is true, Statment -2 is false
(c) Statement -1 is false, Statment -2 is true.
(d) Statement -1 is true, Statement -2 is true ; Statement -2 is a correct explanation for Statement -1.

14. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is **[2010]**

(a) $\frac{2}{7}$ (b) $\frac{1}{21}$
(c) $\frac{2}{23}$ (d) $\frac{1}{3}$

15. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is **[2003]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

TOPIC 2 Odds Against & Odds in Favour of an Event, Addition Theorem, Boole's Inequality, Demorgan's Law

16. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval: **[Sep. 06, 2020 (II)]**

(a) [0.35, 0.36] (b) [0.25, 0.35]
(c) [0.20, 0.25] (d) [0.36, 0.40]

17. Let *A* and *B* be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that *A* or *B* occurs is $\frac{1}{2}$, then the probability of both of them occur together is: **[Jan. 8, 2020 (II)]**

(a) 0.02 (b) 0.20
(c) 0.01 (d) 0.10

18. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is : **[Jan. 12, 2019 (II)]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

19. For three events A, B and C,
P(Exactly one of A or B occurs)
= P(Exactly one of B or C occurs)
= P(Exactly one of C or A occurs) = $\frac{1}{4}$ and

P(All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is : **[2017]**

(a) $\frac{3}{16}$ (b) $\frac{7}{32}$

(c) $\frac{7}{16}$ (d) $\frac{7}{64}$

20. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is : **[Online April 9, 2017]**

(a) $\frac{21}{220}$ (b) $\frac{3}{11}$

(c) $\frac{1}{11}$ (d) $\frac{2}{23}$

21. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : **(2015)**

(a) $220\left(\frac{1}{3}\right)^{12}$ (b) $22\left(\frac{1}{3}\right)^{11}$

(c) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (d) $55\left(\frac{2}{3}\right)^{10}$

22. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the incorrect statement amongst the following statements is: **[Online April 9, 2014]**

(a) A and B are equally likely

(b) $P(A \cap B') = 0$

(c) $P(A' \cap B) = 0$

(d) $P(A) + P(B) = 1$

23. If the events A and B are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$ and $P(B) = \frac{1-x}{4}$, then the set of possible values of x lies in the interval : **[Online April 25, 2013]**

(a) $[0, 1]$ (b) $\left[\frac{1}{3}, \frac{2}{3}\right]$

(c) $\left[-\frac{1}{3}, \frac{5}{9}\right]$ (d) $\left[-\frac{7}{9}, \frac{4}{9}\right]$

24. Let X and Y are two events such that $P(X \cup Y) = P(X \cap Y)$.

Statement 1: $P(X \cap Y') = P(X' \cap Y) = 0$

Statement 2: $P(X) + P(Y) = 2P(X \cap Y)$

[Online May 7, 2012]

(a) Statement 1 is false, Statement 2 is true.

(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

(c) Statement 1 is true, Statement 2 is false.

(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

25. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **[2008]**

(a) $\frac{3}{5}$ (b) 0

(c) 1 (d) $\frac{2}{5}$

26. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$ The set of possible values of x are in the interval. **[2003]**

(a) $[0, 1]$ (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$

(c) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

27. A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\overline{A}) = 2/3$ then $P(\overline{A} \cap B)$ is **[2002]**

(a) 5/12 (b) 3/8

(c) 5/8 (d) 1/4

Hints & Solutions

1. **(c)** For an A.P. $2b = a + c$ (even), so both a and c even numbers or odd numbers from given numbers and b number will be fixed automatically.

Required probability $= \dfrac{{}^6C_2 + {}^5C_2}{{}^{11}C_3} = \dfrac{25}{165} = \dfrac{5}{33}$

2. **(Bonus)** Total number of ways placing 10 different balls in 4 distinct boxes $= 4^{10}$
Since, two of the 4 distinct boxes contains exactly 2 and 3 balls.
Then, there are three cases to place exactly 2 and 3 balls in 2 of the 4 boxes.
Case-1: When boxes contains balls in order 2, 3, 0, 5
Then, number of ways of placing the balls

$$= \frac{10!}{2! \times 3! \times 0! \times 5!} \times 4!$$

Case-2: When boxes contains ball in order 2, 3, 1, 4.
Then, number of ways of placing the balls

$$= \frac{10!}{2! \times 3! \times 1! \times 4!} \times 4!$$

Case-3: When boxes contains ball in order 2, 3, 2, 3
Then, number of ways of placing the balls

$$= \frac{10!}{(2!)^2 \times (3!)^2 \times 2! \times 3!} \times 4!$$

Therefore, number of ways of placing the balls that contains exactly 2 and 3 balls.

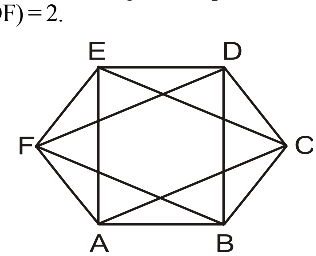
$$= \frac{10!}{2! \times 3! \times 0! \times 5!} \times 4! + \frac{10!}{2! \times 3! \times 1! \times 4!} \times 4!$$

$$+ \frac{10!}{(2!)^2 \times 2! \times (3!)^2 \times 2!} \times 4!$$

$= 2^5 \times 17 \times 945$
Hence, the required probability

$$= \frac{2^5 \times 17 \times 945}{4^{10}} = \frac{17 \times 945}{2^{15}}$$

3. **(1)** Total no. of triangles $= {}^6C_3$
Favorable no. of triangle i.e, equilateral triangles (ΔAEC and ΔBDF) = 2.



Hence, required probability $= \dfrac{2}{{}^6C_3} = \dfrac{1}{10}$

4. **(2)** Since total number of subsets of the set $S = 2^{20}$

Now, the sum of all number from 1 to 20 $= \dfrac{20 \times 21}{2} = 210$

Then, find the sets which has sum 7.
(1) $\{7\}$
(2) $\{1, 6\}$
(3) $\{2, 5\}$
(4) $\{3, 4\}$
(5) $\{1, 2, 4\}$
Then, there is only 5 sets which has sum 203

Hence required probability $= \dfrac{5}{2^{20}}$

5. **(d)** Let the number of children in each family be x.
Thus the total number of children in both the families are $2x$
Now, it is given that 3 tickets are distributed amongst the children of these two families.
Thus, the probability that all the three tickets go to the children in family B

$$= \frac{{}^xC_3}{{}^{2x}C_3} = \frac{1}{12}$$

$$\Rightarrow \frac{x(x-1)(x-2)}{2x(2x-1)(2x-2)} = \frac{1}{12}$$

$$\Rightarrow \frac{(x-2)}{(2x-1)} = \frac{1}{6}$$

$\Rightarrow x = 5$
Thus, the number of children in each family is 5.

6. **(a)** Probability of drawing a white ball and then a red ball from bag B is given by $\dfrac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \dfrac{2}{9}$

Probability of drawing a white ball and then a red ball from bag A is given by $\dfrac{{}^2C_1 \times {}^3C_1}{{}^7C_2} = \dfrac{2}{7}$

Hence, the probability of drawing a white ball and then a red ball from bag B $= \dfrac{\frac{2}{9}}{\frac{2}{7} + \frac{2}{9}} = \dfrac{2 \times 7}{18 + 14} = \dfrac{7}{16}$

7. **(b)** Favourable case = (6, 6, 6)
Total case = {(1, 1, 1) (2, 2, 1), (2, 2, 2), (2, 2, 3), (3, 3, 1) ..... (3, 3, 5) (4, 4, 1).... (4, 4, 6) (5, 5, 1).... (5, 5, 6) (6, 6, 1).... (6, 6, 6)}

which satisfies condition a + b > c
Number of total case = 27

Probability = $\frac{1}{27}$

8. **(a)** Given $\frac{(x-10)(x-50)}{(x-30)} \geq 0$

Let $x \geq 10,\ x \geq 50$ equation will be true $\forall x \geq 50$

as $\left(\frac{x-50}{x-30}\right) \geq 0,\ \forall\ x \in [10, 30)$

$\frac{(x-10)(x-50)}{x-30} \geq 0\ \ \forall\ x \in [10, 30)$

Total value of x between 10 to 30 is 20.
Total values of x between 50 to 100 including 50 and 100 is 51.
Total values of x = 51 + 20 = 71

$P(\text{A}) = \frac{71}{100} = 0.71$

9. **(b)** Let S = $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
Let the chosen element be x_i.
Total number of subsets of S = 2^7 = 128
No. of non-empty subsets of S = 128 – 1 = 127
We need to find number of those subsets that contains x_i.

2	2	2	2	1	2	2

$x_1\ x_2$ ------- x_i ---- x_7

For those subsets containing x_i, each element has 2 choices.
i.e., (included or not included) in subset,
However as the subset must contain x_i, x_i has only one choice. (included one)
So, total no. of subsets containing
x_i = 2 × 2 × 2 × 2 × 1 × 2 × 2 = 64

Required prob $= \frac{\text{No. of subsets containing } x_i}{\text{Total no. of non-empty subsets}}$

$= \frac{64}{127}$

10. **(b)** Total possible event when one ball is taken out $= {}^3C_1$
Let E : The event of 1 white ball coming out
No. of ways to 1 white ball coming out $= {}^2C_1$

$\therefore P(E) = \frac{{}^2C_1}{{}^3C_1} = \frac{2}{3}$

11. **(b)** Consider a group of three students A, B and an other student in between A and B. Choice for a student between A and B is 4. A and B can interchange their places in the group in 2 ways.
Now the group of three students (student A, student B and a student in between A and B) and the remaining 3 students can be stand in a row in 4! ways.
Hence total number of ways to stand in a row such that A and B are separated with one student in between them = 4 × 2 × 4!
Now total number of ways to stand 6 student stand in a row without any restriction = 6!
Hence required probability

$= \frac{4 \times 2 \times 4!}{6!} = \frac{4 \times 2}{6 \times 5} = \frac{4}{15}$

12. **(c)** $\frac{\sum_{i=1}^{n} i^2}{\sum_{i=1}^{n} i} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3}$

For n = 1, 2, 3, ..., 1000

Value of $\frac{2n+1}{3} = \frac{3}{3}, \frac{5}{3}, \frac{7}{3}, ..., \frac{2001}{3}$ respectively. Out of $\frac{3}{3}, \frac{5}{3}, \frac{7}{3}, ..., \frac{2001}{3}$ only first term $\left(\frac{3}{3} = 1\right)$, fourth term $\left(\frac{9}{3} = 3\right)$, 667th term $\left(\frac{2001}{3} = 667\right)$ are integers.

Hence, out of 1000 values of $\frac{2n+1}{3}$,

total number of integral values of $\frac{2n+1}{3}$

= 333 + 1 = 334

$\therefore$ Required probability $= \frac{334}{1000} = 0.334$

13. **(b)** Four numbers are chosen from {1, 2, 3...20}
n(S) = ${}^{20}C_4$
Statement-1:
Common difference is 1; total number of ways = 17
common difference is 2; total number of ways = 14
common difference is 3; total number of ways = 11
common difference is 4; total number of ways = 8
common difference is 5; total number of ways = 5
common difference is 6; total number of ways = 2

Prob. $= \frac{17+14+11+8+5+2}{{}^{20}C_4} = \frac{1}{85}$

Statement -2 is false, because common difference can be 6 also.

14. **(a)** $n(S) = {}^9C_3$

$n(E) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

Probability $= \frac{3 \times 4 \times 2}{{}^9C_3} = \frac{24 \times 3!}{9!} \times 6! = \frac{24 \times 6}{9 \times 8 \times 7} = \frac{2}{7}$

15. **(a)** Let 5 horses are H_1, H_2, H_3, H_4 and H_5.
Total ways of selecting pair of horses be

$= {}^5C_2 = 10$[i.e. $H_1H_2, H_1H_3, H_1H_4, H_1H_5,$

$H_2H_3, H_2H_4, H_2H_5, H_3H_4, H_3H_5, H_4H_5$]

Any horse can win the race in 4 ways
(e.g. for $H_1 : H_1H_2, H_1H_3, H_1H_4, H_1H_5$)

Hence required probability $= \frac{4}{10} = \frac{2}{5}$

16. **(b)** $\because P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 1 - 0.8 = 0.2$

Now,

$\because P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$\Rightarrow \alpha = 0.6 + 0.4 + 0.5 - 0.2 - \beta - 0.3 + 0.2$

$\Rightarrow \beta = 1.2 - \alpha$

$\because \alpha \in [0.85, 0.95]$ then $\beta \in [0.25, 0.35]$

17. **(4)** $P(\text{exactly one}) = \frac{2}{5}$

$\Rightarrow \quad P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$P(A \text{ or } B) = P(A \cup B) = \frac{1}{2}$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = \frac{1}{2}$

$\therefore \quad P(A \cap B) = \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10} = 0.10$

18. **(1)** P = Set of students who opted for NCC
Q = Set of Students who opted for NSS
$n(P) = 40, n(Q) = 30, n(P \cap Q) = 20$
$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$
$= 40 + 30 - 20$
$= 50$

$\therefore$ Hence, required probability $= 1 - \frac{50}{60}$

$= \frac{1}{6}$

19. **(c)** P (exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ ...(1)

P (Exactly one of B or C occurs)

$= P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ ...(2)

P (Exactly one of C or A occurs)

$= P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ ...(3)

Adding (1), (2) and (3), we get

$2\Sigma P(A) - 2\Sigma P(A \cap B) = \frac{3}{4}$

$\therefore \Sigma P(A) - \Sigma P(A \cap B) = \frac{3}{8}$

Now, $P(A \cap B \cap C) = \frac{1}{16}$

$\therefore P(A \cup B \cup C)$
$= \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

20. **(c)** Probability of 4 member committee which contain atleast one woman.

$\Rightarrow$ P(3M, 1W) + P(2M, 2W) + P(1M, 3W) + P(0M, 4W)

$\Rightarrow \frac{{}^{10}C_3\ {}^5C_1}{{}^{15}C_4} + \frac{{}^{10}C_2\ {}^5C_2}{{}^{15}C_4} + \frac{{}^{10}C_1\ {}^5C_3}{{}^{15}C_4} + \frac{{}^{10}C_0\ {}^5C_4}{{}^{15}C_4}$

$\Rightarrow \frac{600}{1365} + \frac{450}{1365} + \frac{100}{1365} + \frac{5}{1365}$

$\Rightarrow \frac{1155}{1365}$

$\therefore$ Probability of committees to have more women than men.

$= \frac{P(1M, 3W) + P(0M, 4W)}{P(3M, 1W) + P(2M, 2W) + P(1M, 3W) + P(0M, 4W)}$

$= \frac{\frac{105}{1365}}{\frac{1155}{1365}} = \frac{1}{11}$

21. **(c)** **Note:-** The question should state '3 different' boxes instead of '3 identical boxes' and one particular box has 3 balls. Then the solution would be:

Required probability $= \frac{{}^{12}C_3 \times 2^9}{3^{12}}$

$= \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

22. **(d)** Let A and B be two events such that
$P(A \cup B) = P(A \cap B)$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

option (a) : since $P(A \cup B) = P(A \cap B)$ (given)
therefore A and B are equally likely
Suppose option (b) and option (c) are correct.

$\therefore P(A \cap B') = 0$ and $P(A' \cap B) = 0$

$\Rightarrow P(A) - P(A \cap B) = 0$ and $P(B) - P(A \cap B) = 0$

$\Rightarrow P(A) = P(A \cap B)$ and $P(B) = P(A \cap B)$

Thus $P(A) = P(B) = P(A \cap B) = P(A \cup B)$

[$\because$ Given $P(A \cap B) = P(A \cup B)$]

Also, we know

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A \cap B) + P(A \cap B) - P(A \cap B)$

$= P(A \cap B)$

which is true from given condition
Hence, option (a), (b) and (c) are correct.

23. **(c)** Since events A and B are mutually exclusive

$\therefore P(A) + P(B) = 1$

$\Rightarrow \frac{3x+1}{3} + \frac{1-x}{4} = 1$

$\Rightarrow 12x + 4 + 3 - 3x = 12$

$\Rightarrow 9x = 5 \Rightarrow x = \frac{5}{9}$

$\therefore x \in \left[-\frac{1}{3}, \frac{5}{9}\right]$

24. **(b)** Let X and Y be two events such that

$P(X \cup Y) = P(X \cap Y)$ ...(1)

We know

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$P(X \cap Y) = P(X) + P(Y) - P(X \cap Y)$ (from (1)

$\Rightarrow P(X) + P(Y) = 2P(X \cap Y)$

Hence, Statement - 2 is true.

Now, $P(X \cap Y') = P(X) - P(X \cap Y)$

and $P(X' \cap Y) = P(Y) - P(X \cap Y)$

This implies statement-1 is also true.

25. **(c)** A (number is greater than 3) = {4, 5, 6}

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B (number is less than 5) = {1, 2, 3, 4} $\Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

$\therefore A \cap B = \{4\}$

$\Rightarrow P(A \cap B) = \frac{1}{6}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{3+4-1}{6} = 1$

26. **(b)** Given that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and

$P(C) = \frac{1-2x}{2}$

We know that $0 \le P(E) \le 1$

$\Rightarrow 0 \le \frac{3x+1}{3} \le 1, \ge -1 \le 3x \le 2$

$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3}$ ...(i)

$0 \le \frac{1-x}{4} \le 1 \Rightarrow -3 \le x \le 1$ ...(ii)

and $0 \le \frac{1-2x}{2} \le 1 \Rightarrow -1 \le 2x \le 1$

$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$ ...(iii)

Also for mutually exclusive events A, B, C,

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$\Rightarrow P(A \cup B \cup C) = \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2}$

$\therefore 0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13$

$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$ ...(iv)

From (i), (ii), (iii) and (iv), we get

$\frac{1}{3} \le x \le \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$

27. **(a)** We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{4} = 1 - P(\bar{A}) + P(B) - \frac{1}{4}$ $[\because P(A) = 1 - P(\bar{A})]$

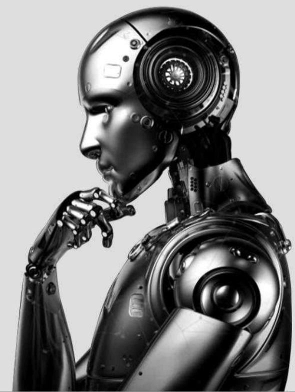
$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3}$;

Now, $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$.

16

Relations and Functions



TOPIC 1 Types of Relations, Inverse of a Relation, Mappings, Mapping of Functions, Kinds of Mapping of Functions

1. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$ is ___________. **[NA Sep. 05, 2020 (II)]**

2. Let a function $f:(0,\infty)\to(0,\infty)$ be defined by $f(x)=\left|1-\frac{1}{x}\right|$. Then f is : **[Jan. 11, 2019 (II)]**

(a) not injective but it is surjective
(b) injective only
(c) neither injective nor surjective
(d) both injective as well as surjective

3. The number of functions f from $\{1, 2, 3, ..., 20\}$ onto $\{1, 2, 3, ..., 20\}$ such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4 is : **[Jan. 11, 2019 (II)]**

(a) $6^5 \times (15)!$
(b) $5! \times 6!$
(c) $(15)! \times 6!$
(d) $5^6 \times 15$

4. Let $\mathbf{N}$ be the set of natural numbers and two functions f and g be defined as $f, g : \mathbf{N} \to \mathbf{N}$ such that

$$f(n) = \begin{cases} \dfrac{n+1}{2} & \text{if n is odd} \\ \dfrac{n}{2} & \text{if n is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is: **[Jan. 10, 2019 (II)]**

(a) onto but not one-one.
(b) one-one but not onto.
(c) both one-one and onto.
(d) neither one-one nor onto.

5. Let $A = \{x \in \mathbf{R} : x$ is not a positive integer$\}$. Define a function $f: A \to \mathbf{R}$ as $f(x) = \dfrac{2x}{x-1}$, then f is: **[Jan. 09, 2019 (II)]**

(a) not injective
(b) neither injective nor surjective
(c) surjective but not injective
(d) injective but not surjective

6. The function $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is : **[2017]**

(a) neither injective nor surjective
(b) invertible
(c) injective but not surjective
(d) surjective but not injective

7. The function $f : N \to N$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$, where N is set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x, is : **[Online April 9, 2017]**

(a) one-one and onto.
(b) one-one but not onto.
(c) onto but not one-one.
(d) neither one-one nor onto.

8. Let $A = \{x_1, x_2, ..., x_7\}$ and $B = \{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f : A \to B$ that are onto, if there exist exactly three elements x in A such that $f(x) = y_2$, is equal to : **(Online April 11, 2015)**

(a) $14.{}^7C_3$ (b) $16.{}^7C_3$ (c) $14.{}^7C_2$ (d) $12.{}^7C_2$

9. Let $f : R \to R$ be defined by $f(x) = \dfrac{|x|-1}{|x|+1}$ then f is: **[Online April 19, 2014]**

(a) both one-one and onto
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto.

10. Let P be the relation defined on the set of all real numbers such that
$P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$. Then P is: **[Online April 9, 2014]**
(a) reflexive and symmetric but not transitive.
(b) reflexive and transitive but not symmetric.
(c) symmetric and transitive but not reflexive.
(d) an equivalence relation.

11. Let $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$, where N is the set of all natural numbers. Then the relation R is : **[Online April 23, 2013]**
(a) reflexive but neither symmetric nor transitive.
(b) symmetric and transitive.
(c) reflexive and symmetric,
(d) reflexive and transitive.

12. Let $R = \{(3,3)(5,5),(9,9),(12,12),(5,12),(3,9),(3,12),(3,5)\}$ be a relation on the set $A = \{3, 5, 9, 12\}$. Then, R is : **[Online April 22, 2013]**
(a) reflexive, symmetric but not transitive.
(b) symmetric, transitive but not reflexive.
(c) an equivalence relation.
(d) reflexive, transitive but not symmetric.

13. Let $A = \{1, 2, 3, 4\}$ and $R : A \to A$ be the relation defined by $R = \{(1, 1), (2, 3), (3, 4), (4, 2)\}$. The correct statement is : **[Online April 9, 2013]**
(a) R does not have an inverse.
(b) R is not a one to one function.
(c) R is an onto function.
(d) R is not a function.

14. If $P(S)$ denotes the set of all subsets of a given set S, then the number of one-to-one functions from the set $S = \{1, 2, 3\}$ to the set $P(S)$ is **[Online May 19, 2012]**
(a) 24 (b) 8 (c) 336 (d) 320

15. If $A = \{x \in z^+ : x < 10$ and x is a multiple of 3 or 4$\}$, where z^+ is the set of positive integers, then the total number of symmetric relations on A is **[Online May 12, 2012]**
(a) 2^5 (b) 2^{15} (c) 2^{10} (d) 2^{20}

16. Let R be the set of real numbers. **[2011]**
Statement-1: $A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R.
Statement-2: $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.
(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

17. Consider the following relations:
R = $\{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$; $S = \{\left(\frac{m}{n}, \frac{p}{q}\right) \mid m, n, p$ and q are integers such that $n, q \neq 0$ and $qm = pn\}$.
Then **[2010]**
(a) Neither R nor S is an equivalence relation
(b) S is an equivalence relation but R is not an equivalence relation
(c) R and S both are equivalence relations
(d) R is an equivalence relation but S is not an equivalence relation

18. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y) : x - y \text{ is an integer}\}$,
Which one of the following is true? **[2008]**
(a) Neither S nor T is an equivalence relation on R
(b) Both S and T are equivalence relation on R
(c) S is an equivalence relation on R but T is not
(d) T is an equivalence relation on R but S is not

19. Let W denote the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W \mid$ the words x and y have at least one letter in common.$\}$ Then R is **[2006]**
(a) not reflexive, symmetric and transitive
(b) relexive, symmetric and not transitive
(c) reflexive, symmetric and transitive
(d) reflexive, not symmetric and transitive

20. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is **[2005]**
(a) reflexive and transitive only
(b) reflexive only
(c) an equivalence relation
(d) reflexive and symmetric only

21. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1 - x^2}$, then f is both one - one and onto when B is the interval **[2005]**
(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$
(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

22. Let $R = \{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A = \{1,2,3,4\}$.. The relation R is **[2004]**
(a) reflexive (b) transitive
(c) not symmetric (d) a function

23. If $f: R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is **[2004]**

(a) $[-1, 3]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[0, 3]$

24. A function f from the set of natural numbers to integers defined by **[2003]**

$$f(n) = \begin{cases} \dfrac{n-1}{2}, \text{when n is odd} \\ -\dfrac{n}{2}, \text{when n is even} \end{cases} \quad \text{is}$$

(a) neither one -one nor onto

(b) one-one but not onto

(c) onto but not one-one

(d) one-one and onto both.

TOPIC 2 Composite Functions & Relations, Inverse of a Function, Binary Operations

25. The inverse function of $f(x) = \dfrac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1, 1)$, is ________. **[Jan. 8, 2020 (I)]**

(a) $\dfrac{1}{4}\log_e\left(\dfrac{1+x}{1-x}\right)$ (b) $\dfrac{1}{4}(\log_8 e)\log_e\left(\dfrac{1-x}{1+x}\right)$

(c) $\dfrac{1}{4}\log_e\left(\dfrac{1-x}{1+x}\right)$ (d) $\dfrac{1}{4}(\log_8 e)\log_e\left(\dfrac{1+x}{1-x}\right)$

26. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\dfrac{5}{4}\right)$ is equal to: **[Jan. 7, 2020 (I)]**

(a) $\dfrac{3}{2}$ (b) $-\dfrac{1}{2}$ (c) $\dfrac{1}{2}$ (d) $-\dfrac{3}{2}$

27. For a suitably chosen real constant a, let a function, $f: \mathrm{R} - \{-\mathrm{a}\} \to \mathrm{R}$ be defined by $f(x) = \dfrac{\mathrm{a} - x}{\mathrm{a} + x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, $(fof)(x) = x$. Then $f\left(-\dfrac{1}{2}\right)$ is equal to: **[Sep. 06, 2020 (II)]**

(a) $\dfrac{1}{3}$ (b) $-\dfrac{1}{3}$ (c) -3 (d) 3

28. For $x \in \left(0, \dfrac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \dfrac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\dfrac{\pi}{3}\right)$ is equal to : **[April 12, 2019 (I)]**

(a) $\tan\dfrac{\pi}{12}$ (b) $\tan\dfrac{11\pi}{12}$ (c) $\tan\dfrac{7\pi}{12}$ (d) $\tan\dfrac{5\pi}{12}$

29. Let $f(x) = x^2, x \in \mathrm{R}$. For any $\mathrm{A} \subseteq \mathrm{R}$, define $\mathrm{g(A)} = \{x \in \mathrm{R} : f(x) \in \mathrm{A}\}$. If S = [0, 4], then which one of the following statements is not true ? **[April 10, 2019 (I)]**

(a) $\mathrm{g(f(S))} \neq \mathrm{S}$ (b) $\mathrm{f(g(S))} = \mathrm{S}$

(c) $\mathrm{g(f(S))} = \mathrm{g(S)}$ (d) $\mathrm{f(g(S))} \neq \mathrm{f(S)}$

30. For $x \in \mathbf{R} - \{0, 1\}$, let $f_1(x) = \dfrac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \dfrac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then $J(x)$ is equal to: **[Jan. 09, 2019 (I)]**

(a) $f_3(x)$ (b) $\dfrac{1}{x}f_3(x)$ (c) $f_2(x)$ (d) $f_1(x)$

31. Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then **[Online April 16, 2018]**

(a) Both R_1 and R_2 are transitive relations

(b) Both R_1 and R_2 are symmetric relations

(c) Range of R_2 is $\{1, 2, 3, 4\}$

(d) Range of R_1 is $\{2, 4, 8\}$

32. Consider the following two binary relations on the set $A = \{a, b, c\}$: $R_1 = \{(c, a)\,(b, b), (a, c), (c, c), (b, c), (a, a)\}$ and $R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)$. Then **[Online April 15, 2018]**

(a) R_2 is symmetric but it is not transitive

(b) Both R_1 and R_2 are transitive

(c) Both R_1 and R_2 are not symmetric

(d) R_1 is not symmetric but it is transitive

33. Let $f: \mathrm{A} \to \mathrm{B}$ be a function defined as $f(x) = \dfrac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$. Then f is **[Online April 15, 2018]**

(a) invertible and $f^{-1}(y) = \dfrac{2y+1}{y-1}$

(b) invertible and $f^{-1}(y) = \dfrac{3y-1}{y-1}$

(c) no invertible

(d) invertible and $f^{-1}(y) = \dfrac{2y-1}{y-1}$

34. Let $f(x) = 2^{10} \cdot x + 1$ and $g(x) = 3^{10} \cdot x - 1$. If $(fog)(x) = x$, then x is equal to : **[Online April 8, 2017]**

(a) $\dfrac{3^{10}-1}{3^{10}-2^{-10}}$ (b) $\dfrac{2^{10}-1}{2^{10}-3^{-10}}$

(c) $\dfrac{1-3^{-10}}{2^{10}-3^{-10}}$ (d) $\dfrac{1-2^{-10}}{3^{10}-2^{-10}}$

35. For $x \in R, x \neq 0$, let $f_0(x) = \dfrac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2, \ldots$. Then the value of $f_{100}(3) + f_1\left(\dfrac{2}{3}\right) + f_2\left(\dfrac{3}{2}\right)$ is equal to : **[Online April 9, 2016]**

(a) $\dfrac{8}{3}$ (b) $\dfrac{4}{3}$ (c) $\dfrac{5}{3}$ (d) $\dfrac{1}{3}$

36. If g is the inverse of a function f and $f'(x) = \dfrac{1}{1+x^5}$, then $g'(x)$ is equal to: **[2014]**

(a) $\dfrac{1}{1+\{g(x)\}^5}$ (b) $1+\{g(x)\}^5$

(c) $1+x^5$ (d) $5x^4$

37. Let A and B be non empty sets in R and $f : A \to B$ is a bijective function. **[Online May 26, 2012]**

Statement 1: f is an onto function.

Statement 2: There exists a function $g : B \to A$ such that $fog = I_B$.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1.

38. Let f be a function defined by

$f(x) = (x-1)^2 + 1, (x \geq 1)$. **[2011RS]**

Statement - 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$.

Statement - 2 : f is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

39. Let $f(x) = (x+1)^2 - 1, x \geq -1$

Statement -1 : The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}$

Statement-2 : f is a bijection. **[2009]**

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

40. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$.

Show that f is invertible and its inverse is **[2008]**

(a) $g(y) = \dfrac{3y+4}{3}$ (b) $g(y) = 4 + \dfrac{y+3}{4}$

(c) $g(y) = \dfrac{y+3}{4}$ (d) $g(y) = \dfrac{y-3}{4}$

Hints & Solutions

1. **(19.00)**
The desired functions will contain either one element or two elements in its codomain of which '2' always belongs to $f(A)$.
$\therefore$ The set B can be $\{2\}$, $\{1, 2\}$, $\{2, 3\}$, $\{2, 4\}$
Total number of functions $= 1 + (2^3 - 2)3 = 19$.

2. **(Bonus)** $f : (0, \infty) \to (0, \infty)$

$f(x) = \left|1 - \dfrac{1}{x}\right|$ is not a function

$\because$ $f(1) = 0$ and $1 \in$ domain but $0 \notin$ co-domain

Hence, $f(x)$ is not a function.

3. **(c)** Domain and codomain $= \{1, 2, 3, \ldots, 20\}$.
There are five multiple of 4 as 4, 8, 12, 16 and 20.
and there are 6 multiple of 3 as 3, 6, 9, 12, 15, 18.
Since, when ever k is multiple of 4 then $f(k)$ is multiple of 3 then total number of arrangement
$= {}^6c_5 \times 5! = 6!$
Remaining 15 elements can be arranged in 15! ways.
Since, for every input, there is an output
$\Rightarrow$ function $f(k)$ in onto
$\therefore$ Total number of arrangement = 15! . 6!

4. **(a)** $f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

$$g(n) = \begin{cases} 2, & n = 1 \\ 1, & n = 2 \\ 4, & n = 3 \\ 3, & n = 4 \\ 6, & n = 5 \\ 5, & n = 6 \end{cases}$$

Then,

$$f(g(n)) = \begin{cases} \dfrac{g(n)+1}{2}, & \text{if } g(n) \text{ is odd} \\ \dfrac{g(n)}{2}, & \text{if } g(n) \text{ is even} \end{cases}$$

$$f(g(n)) = \begin{cases} 1, & n = 1 \\ 1, & n = 2 \\ 2, & n = 3 \\ 2, & n = 4 \\ 3, & n = 5 \\ 3, & n = 6 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \end{cases}$$

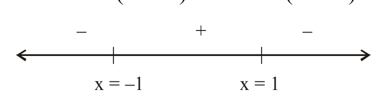
$\Rightarrow fog$ is onto but not one - one

5. **(d)** As $A = \{x \in R : x$ is not a positive integer$\}$
A function $f : A \to R$ given by $f(x) = \dfrac{2x}{x-1}$
$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$
So, f is one-one.
As $f(x) \neq 2$ for any $x \in A \Rightarrow f$ is not onto.
Hence f is injective but not surjective.

6. **(d)** We have $\mathrm{f} : \mathrm{R} \to \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$,

$\mathrm{f}(x) = \dfrac{x}{1+x^2}\ \forall x \in R$

$\Rightarrow \mathrm{f}'(\mathrm{x}) = \dfrac{(1+\mathrm{x}^2).1 - \mathrm{x}.2\mathrm{x}}{(1+\mathrm{x}^2)^2} = \dfrac{-(\mathrm{x}+1)(\mathrm{x}-1)}{(1+\mathrm{x}^2)^2}$



sign of $f'(x)$
$\Rightarrow f'(x)$ changes sign in different intervals.
$\therefore$ Not injective

Now $\mathrm{y} = \dfrac{\mathrm{x}}{1+\mathrm{x}^2}$
$\Rightarrow$ $\mathrm{y} + \mathrm{yx}^2 = \mathrm{x}$
$\Rightarrow$ $\mathrm{yx}^2 - \mathrm{x} + \mathrm{y} = 0$
For $\mathrm{y} \neq 0$, $\mathrm{D} = 1 - 4\mathrm{y}^2 \geq 0$

$\Rightarrow$ $\mathrm{y} \in \left[\dfrac{-1}{2}, \dfrac{1}{2}\right] - \{0\}$

For $\mathrm{y} = 0 \Rightarrow \mathrm{x} = 0$

$\therefore$ Range is $\left[\dfrac{-1}{2}, \dfrac{1}{2}\right]$

$\Rightarrow$ Surjective but not injective

7. **(d)** $\left.\begin{array}{l} f(1)=1-5[1/5]=1 \\ f(6)=6-5[6/5]=1 \end{array}\right\} \rightarrow$ Many one

$f(10)=10-5(2)=0$ which is not in co–domain.
Neither one–one nor onto.

8. **(a)** Number of onto function such that exactly three elements in x ∈ A such that f(x) = $\frac{1}{2}$ is equal to
$= {}^7C_3, \{2^4-2\} = 14.\,{}^7C_3$

9. **(c)** $f(x)=\frac{|x|-1}{|x|+1}$

for one-one function if $f(x_1)=f(x_2)$ then x_1 must be equal to x_2
Let $f(x_1)=f(x_2)$

$$\frac{|x_1|-1}{|x_1|+1}=\frac{|x_2|-1}{|x_2|+1}$$

$$|x_1||x_2|+|x_1|-|x_2|-1=|x_1||x_2|-|x_1|+|x_2|-1$$

$$\Rightarrow |x_1|-|x_2|=|x_2|-|x_1|$$

$$2|x_1|=2|x_2|$$

$$|x_1|=|x_2|$$

$x_1 = x_2,\ x_1 = -x_2$
here x_1 has two values therefore function is many one function.

For onto : $f(x)=\frac{|x|-1}{|x|+1}$

for every value of $f(x)$ there is a value of x in domain set.
If $f(x)$ is negative then $x = 0$
for all positive value of $f(x)$, domain contain atleast one element. Hence $f(x)$ is onto function.

10. **(d)** P = $\{(a,b): \sec^2 a - \tan^2 b = 1\}$

For reflexive :
$\sec^2 a - \tan^2 a = 1$ (true $\forall\, a$)
For symmetric :
$\sec^2 b - \tan^2 a = 1$
L.H.S
$1+\tan^2 b-(\sec^2 a-1) = 1+\tan^2 b-\sec^2 a+1$
$= -(\sec^2 a - \tan^2 b) + 2$
$= -1 + 2 = 1$
So, Relation is symmetric
For transitive :
if $\sec^2 a - \tan^2 b = 1$ and $\sec^2 b - \tan^2 c = 1$
$\sec^2 a - \tan^2 c = (1 + \tan^2 b) - (\sec^2 b - 1)$
$= -\sec^2 b + \tan^2 b + 2$
$= -1 + 2 = 1$
So, Relation is transitive.
Hence, Relation P is an equivalence relation

11. **(d)** $R = \{(x, y): x, y \in N \text{ and } x^2-4xy+3y^2=0\}$
Now, $x^2-4xy+3y^2=0$
$\Rightarrow (x-y)(x-3y)=0$
$\therefore$ $x = y$ or $x = 3y$
$\therefore$ R = {(1, 1), (3, 1), (2, 2), (6, 2), (3, 3), (9, 3),......}
Since (1, 1), (2, 2), (3, 3),...... are present in the relation, therefore R is reflexive.
Since (3, 1) is an element of R but (1, 3) is not the element of R, therefore R is not symmetric
Here $(3, 1) \in R$ and $(1, 1) \in R \Rightarrow (3, 1) \in R$
$(6, 2) \in R$ and $(2, 2) \in R \Rightarrow (6, 2) \in R$
For all such $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
Hence R is transitive.

12. **(d)** Let R = {(3, 3), (5, 5), (9, 9), (12, 12), (5, 12), (3, 9), (3, 12), (3, 5)} be a relation on set
A = {3, 5, 9, 12}
Clearly, every element of A is related to itself.
Therefore, it is a reflexive.
Now, R is not symmetry because 3 is related to 5 but 5 is not related to 3.
Also R is transitive relation because it satisfies the property that if a R b and b R c then a R c.

13. **(c)** Domain = {1, 2, 3, 4}
Range = {1, 2, 3, 4}
$\therefore$ Domain = Range
Hence the relation R is onto function.

14. **(c)** Let $S = \{1, 2, 3\} \Rightarrow n(S) = 3$
Now, $P(S)$ = set of all subsets of S
total no. of subsets = $2^3 = 8$
$\therefore n[P(S)] = 8$
Now, number of one-to-one functions from $S \rightarrow P(S)$ is

$${}^8P_3 = \frac{8!}{5!} = 8\times 7\times 6 = 336.$$

15. **(b)** A relation on a set A is said to be symmetric iff $(a,b)\in A \Rightarrow (b,a)\in A,\ \forall\, a,b \in A$

Here $A = \{3,4,6,8,9\}$

Number of order pairs of $A\times A = 5\times 5 = 25$
Divide 25 order pairs of $A \times A$ in 3 parts as follows :
Part – A : (3, 3), (4, 4), (6, 6), (8, 8), (9, 9)
Part – B : (3, 4), (3, 6), (3, 8), (3, 9), (4, 6), (4, 8),(4, 9), (6, 8), (6, 9), (8, 9)
Part – C : (4, 3), (6, 3), (8, 3),(9, 3), (6, 4), (8, 4), (9, 4), (8, 6), (9, 6), (9, 8)
In part – A, both components of each order pair are same.
In part – B, both components are different but not two such order pairs are present in which first component of one order pair is the second component of another order pair and vice-versa.

In part–C, only reverse of the order pairs of part –B are present i.e., if (a, b) is present in part – B, then (b, a) will be present in part –C

For example (3, 4) is present in part – B and (4, 3) present in part –C.

Number of order pair in A, B and C are 5, 10 and 10 respectively.

In any symmetric relation on set A, if any order pair of part –B is present then its reverse order pair of part –C will must be also present.

Hence number of symmetric relation on set A is equal to the number of all relations on a set D, which contains all the order pairs of part –A and part– B.

Now $n(D) = n(A) + n(B) = 5 + 10 = 15$

Hence number of all relations on set $D = (2)^{15}$

$\Rightarrow$ Number of symmetric relations on set $D = (2)^{15}$

16. **(a)** $\because x - x = 0 \in I$ ($\therefore R$ is reflexive)

Let $(x, y) \in R$ as $x - y$ and $y - x \in I$ ($\because R$ is symmetric)

Now $x - y \in I$ and $y - z \in I \Rightarrow x - z \in I$

So, R is transative.

Hence R is equivalence.

Similarly as $x = \alpha y$ for $\alpha = 1$. B is reflexive symmetric and transative. Hence B is equivalence.

Both relations are equivalence but not the correct explanation.

17. **(b)** Let $x\,R\,y$.

$\Rightarrow x = wy \Rightarrow y = \dfrac{x}{w}$

$\Rightarrow (y, x) \notin R$

R is not symmetric

Let $S : \dfrac{m}{n} S \dfrac{p}{q}$

$\Rightarrow qm = pn \Rightarrow \dfrac{p}{q} = \dfrac{m}{n}$

$\because \dfrac{m}{n} = \dfrac{m}{n}$ $\therefore$ reflexive.

$\dfrac{m}{n} = \dfrac{p}{q} \Rightarrow \dfrac{p}{q} = \dfrac{m}{n}$ $\therefore$ symmetric

Let $\dfrac{m}{n} S \dfrac{p}{q}, \dfrac{p}{q} S \dfrac{r}{s}$

$\Rightarrow qm = pn, ps = rq$

$\Rightarrow \dfrac{p}{q} = \dfrac{m}{n} = \dfrac{r}{s}$

$\Rightarrow ms = rn$ transitive.

S is an equivalence relation.

18. **(d)** Given that

$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$

$\because x \neq x + 1$ for any $x \in (0, 2)$

$\Rightarrow (x, x) \notin$ S

So, S is not reflexive.

Hence, S in not an equivalence relation.

Given $T = \{x, y) : x - y$ is an integer$\}$

$\because x - x = 0$ is an integer, $\forall\, x \in R$

So, T is reflexive.

Let $(x, y) \in$ T $\Rightarrow$ $x - y$ is an integer then $y - x$ is also an integer $\Rightarrow (y, x) \in R$

$\therefore T$ is symmetric

If $x - y$ is an integer and $y - z$ is an integer then $(x - y) + (y - z) = x - z$ is also an integer.

$\therefore$ T is transitive

Hence T is an equivalence relation.

19. **(b)** Clearly $(x, x) \in R, \forall x \in W$

$\because$ All letter are common in some word. So R is reflexive.

Let $(x, y) \in R$, then $(y, x) \in R$ as x and y have at least one letter in common. So, R is symmetric.

But R is not transitive for example

Let x = BOY, y = TOY and z = THREE

then $(x, y) \in R$ (O, Y are common) and $(y, z) \in R$ (T is common) but $(x, z) \notin R$. (as no letter is common)

20. **(a)** R is reflexive and transitive only.

Here (3, 3), (6, 6), (9, 9), (12, 12) $\in$ R [So, reflexive]

(3, 6), (6, 12), (3, 12) $\in$ R [So, transitive].

$\because$ (3, 6) $\in R$ but (6, 3) $\notin R$ [So, non-symmetric]

21. **(d)** Given $f(x) = \tan^{-1}\left(\dfrac{2x}{1 - x^2}\right) = 2\tan^{-1}$x

for $x \in (-1, 1)$

If $x \in (-1, 1) \Rightarrow \tan^{-1} x \in \left(\dfrac{-\pi}{4}, \dfrac{\pi}{4}\right)$

$\Rightarrow 2\tan^{-1} x \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

Clearly, range of $f(x) = \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

For f to be onto, codomain = range

$\therefore$ Co-domain of function $= B = \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

22. **(c)** $\because$ (1, 1) $\notin R \Rightarrow R$ is not reflexive

$\because$ (2, 3) $\in R$ but (3, 2) $\notin R$

$\therefore$ R is not symmetric

$\because$ (4, 2) and (2, 4) $\in R$ but (4, 4) $\notin R$

$\Rightarrow R$ is not transitive

23. **(a)** Given that $f(x)$ is onto

$\therefore$ range of $f(x)$ = codomain = S

Now, $f(x)=\sin x-\sqrt{3}\cos x+1$

$= 2\sin\left(x-\frac{\pi}{3}\right)+1$

we know that $-1\le \sin\left(x-\frac{\pi}{3}\right)\le 1$

$-1\le 2\sin\left(x-\frac{\pi}{3}\right)+1\le 3 \qquad \therefore f(x)\in[-1,\ 3]=S$

24. **(d)** We have $f:N\to I$

Let x and y are two even natural numbers,

and $f(x)=f(y)\Rightarrow \frac{-x}{2}=\frac{-y}{2} \Rightarrow x=y$

$\therefore f(n)$ is one-one for even natural number.

Let x and y are two odd natural numbers and

$f(x)=f(y)\Rightarrow \frac{x-1}{2}=\frac{y-1}{2} \Rightarrow x=y$

$\therefore f(n)$ is one-one for odd natural number.

Hence f is one-one.

Let $y=\frac{n-1}{2}\Rightarrow 2y+1=n$

This shows that n is always odd number for $y\in$ I. ....(i)

and $y=\frac{-n}{2}\Rightarrow -2y=n$

This shows that n is always even number for $y\in$ I. ....(ii)

From (i) and (ii)

Range of f = I = codomain

$\therefore f$ is onto.

Hence f is one one and onto both.

25. **(a)** $y=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$

$\frac{1+y}{1-y}=\frac{8^{2x}}{8^{-2x}} \Rightarrow 8^{4x}=\frac{1+y}{1-y}$

$\Rightarrow 4x=\log_8\left(\frac{1+y}{1-y}\right)$

$\Rightarrow x=\frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$

$\therefore f^{-1}(x)=\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

26. **(b)** $(gof)(x)=g(f(x))=f^2(x)+f(x)-1$

$g\left(f\left(\frac{5}{4}\right)\right)=4\left(\frac{5}{4}\right)^2-10.\frac{5}{4}+5=-\frac{5}{4}$

$[\because g(f(x))=4x^2-10x+5]$

$g\left(f\left(\frac{5}{4}\right)\right)=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$

$-\frac{5}{4}=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$

$f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)+\frac{1}{4}=0$

$\left(f\left(\frac{5}{4}\right)+\frac{1}{2}\right)^2=0$

$t\left(\frac{5}{4}\right)=-\frac{1}{2}$

27. **(d)** $f(f(x))=\dfrac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}=x$

$\Rightarrow \frac{a-ax}{1+x}=f(x)\Rightarrow \frac{a(1-x)}{1+x}=\frac{a-x}{a+x}\Rightarrow a=1$

$\therefore f(x)=\frac{1-x}{1+x}\Rightarrow f\left(-\frac{1}{2}\right)=3$

28. **(b)** $\because \phi(x)=((hof)og)(x)$

$\because \phi\left(\frac{\pi}{3}\right)=h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right)=h(f(\sqrt{3}))=h(3^{1/4})$

$=\frac{1-\sqrt{3}}{1+\sqrt{3}}=-\frac{1}{2}(1+3-2\sqrt{3})=\sqrt{3}-2=-(-\sqrt{3}+2)$

$=-\tan 15°=\tan(180°-15°)=\tan\left(\pi-\frac{\pi}{12}\right)=\tan\frac{11\pi}{12}$

29. **(c)** $f(x)=x^2; x\in$ R

$g(A)=\{x\in R: f(x)\in A\}$ $S=[0,4]$

$g(S)=\{x\in R: f(x)\in S\}$

$=\{x\in R: 0\le x^2\le 4\}=\{x\in R: -2\le x\le 2\}$

$\therefore g(S)\ne S \therefore f(g(S))\ne f(S)$

$g(f(S))=\{x\in R: f(x)\in f(S)\}$

$=\{x\in R: x^2\in S^2\}=\{x\in R: 0\le x^2\le 16\}$

$=\{x\in R: -4\le x\le 4\}$

$\therefore g(f(S))\ne g(S)$

$\therefore g(f(S))=g(S)$ is incorrect.

30. **(a)** The given relation is

$$(f_2 o J o f_1)(x) = f_3(x) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 o J)(f_1(x)) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 o J)\left(\frac{1}{x}\right) = \frac{1}{1-\frac{1}{\frac{1}{x}}} = \frac{\frac{1}{x}}{\frac{1}{x}-1} \quad \left[\because f_1(x) = \frac{1}{x}\right]$$

$$\Rightarrow (f_2 o J)(x) = \frac{x}{x-1} \quad \left[\frac{1}{x} \text{ is replaced by } x\right]$$

$$\Rightarrow f_2(J(x)) = \frac{x}{x-1}$$

$$\Rightarrow 1 - J(x) = \frac{x}{x-1} \quad [\because f_2(x) = 1-x]$$

$$\therefore J(x) = 1 - \frac{x}{x-1} = \frac{1}{1-x} = f_3(x)$$

31. **(c)** Here,
$R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and
$R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$
For R_1; $2x + y = 10$ and $x, y \in N$
So, possible values for x and y are:
$x = 1, y = 8$ i.e. $(1, 8)$;
$x = 2, y = 6$ i.e. $(2, 6)$;
$x = 3, y = 4$ i.e. $(3, 4)$ and
$x = 4, y = 2$ i.e. $(4, 2)$.
$R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$
Therefore, Range of R_1 is $\{2, 4, 6, 8\}$
R_1 is not symmetric
Also, R_1 is not transitive because $(3, 4), (4, 2) \in R_1$ but $(3, 2) \notin R_1$
Thus, options A, B and D are incorrect.
For R_2; $x + 2y = 10$ and $x, y \in N$
So, possible values for x and y are:
$x = 8, y = 1$ i.e. $(8, 1)$;
$x = 6, y = 2$ i.e. $(6, 2)$;
$x = 4, y = 3$ i.e. $(4, 3)$ and
$x = 2, y = 4$ i.e. $(2, 4)$
$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$
Therefore, Range of R_2 is $\{1, 2, 3, 4\}$
R_2 is not symmetric and transitive.

32. **(a)** Both R_1 and R_2 are symmetric as
For any $(x, y) \in R_1$, we have
$(y, x) \in R_1$ and similarly for R_2

Now, for R_2, $(b, a) \in R_2, (a, c) \in R_2$ but $(b, c) \notin R_2$.
Similarly, for R_1, $(b, c) \in R_1, (c, a) \in R_1$ but $(b, a) \notin R_1$.
Therefore, neither R_1 nor R_2 is transitive.

33. **(d)** Suppose $y = f(x)$

$$\Rightarrow y = \frac{x-1}{x-2}$$

$$\Rightarrow yx - 2y = x - 1$$

$$\Rightarrow (y-1)x = 2y - 1$$

$$\Rightarrow x = f^{-1}(y) = \frac{2y-1}{y-1}$$

As the function is invertible on the given domain and its inverse can be obtained as above.

34. **(d)** $f(g(x)) = x$

$$\Rightarrow f(3^{10}x - 1) = 2^{10}(3^{10} . x - 1) + 1 = x$$

$$\Rightarrow 2^{10}(3^{10}x - 1) + 1 = x$$

$$\Rightarrow x(6^{10} - 1) = 2^{10} - 1$$

$$\Rightarrow x = \frac{2^{10}-1}{6^{10}-1} = \frac{1-2^{-10}}{3^{10}-2^{-10}}$$

35. **(c)** $f_1(x) = f_{0+1}(x) = f_0(f_0(x)) = \dfrac{1}{1-\frac{1}{1-x}} = \dfrac{x-1}{x}$

$$f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = \frac{1}{1-\frac{x-1}{x}} = x$$

$$f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = f_0(x) = \frac{1}{1-x}$$

$$f_4(x) = f_{3+1}(x) = f_0(f_3(x)) = \frac{x-1}{x}$$

$$\therefore \quad f_0 = f_3 = f_6 = ... = \frac{1}{1-x}$$

$$f_1 = f_4 = f_7 = ... = \frac{x-1}{x}$$

$$f_2 = f_5 = f_8 = ... = x$$

$$f_{100}(3) = \frac{3-1}{3} = \frac{2}{3} f_1\left(\frac{2}{3}\right) = \frac{\frac{2}{3}-1}{\frac{2}{3}} = -\frac{1}{2}$$

$$f_2\left(\frac{3}{2}\right) = \frac{3}{2}$$

$$\therefore \quad f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = \frac{5}{3}$$

36. **(b)** Since $f(x)$ and $g(x)$ are inverse of each other

$\therefore \quad g'(f(x)) = \dfrac{1}{f'(x)}$

$\Rightarrow \quad g'(f(x)) = 1 + x^5 \qquad \left(\because \; f'(x) = \dfrac{1}{1+x^5}\right)$

Here $x = g(y)$

$\therefore \quad g'(y) = 1 + [g(y)]^5$

$\Rightarrow g'(x) = 1 + (g(x))^5$

37. **(d)** Let A and B be non-empty sets in R.

Let $f : A \to B$ is bijective function.

Clearly statement - 1 is true which says that f is an onto function.

Statement - 2 is also true statement but it is not the correct explanation for statement-1

38. **(a)** Given f is a bijective function

$\therefore \quad f : [1, \infty) \to [1, \infty)$

$f(x) = (x-1)^2 + 1, \; x \ge 1$

Let $\quad y = (x-1)^2 + 1 \Rightarrow (x-1)^2 = y - 1$

$\Rightarrow \; x = 1 \pm \sqrt{y-1} \Rightarrow f^{-1}(y) = 1 \pm \sqrt{y-1}$

$\Rightarrow \quad f^{-1}(x) = 1 + \sqrt{x-1} \; \{\therefore x \ge 1\}$

Hence statement-2 is correct

Now $f(x) = f^{-1}(x)$

$\Rightarrow f(x) = x \Rightarrow (x-1)^2 + 1 = x$

$\Rightarrow \quad x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$

Hence statement-1 is correct

39. **(d)** Given that $f(x) = (x+1)^2 - 1, \; x \ge -1$

Clearly $D_f = [-1, \infty)$ but co-domain is not given. Therefore $f(x)$ is onto.

Let $f(x_1) = f(x_2)$

$\Rightarrow \quad (x_1 + 1)^2 - 1 = (x_2 + 1)^2 - 1$

$\Rightarrow \quad x_1 = x_2$

$\therefore \quad f(x)$ is one-one, hence $f(x)$ is bijection

$\because \quad (x+1)$ being something +ve, $\forall \, x > -1$

$\therefore \quad f^{-1}(x)$ will exist.

Let $(x+1)^2 - 1 = y$

$\Rightarrow x + 1 = \sqrt{y+1} \qquad$ (+ve square root as $x + 1 \ge 0$)

$\Rightarrow x = -1 + \sqrt{y+1}$

$\Rightarrow f^{-1}(x) = \sqrt{x+1} - 1$

Then $\quad f(x) = f^{-1}(x)$

$\Rightarrow (x+1)^2 - 1 = \sqrt{x+1} - 1$

$\Rightarrow (x+1)^2 = \sqrt{x+1} \;\; \Rightarrow \;\; (x+1)^4 = (x+1)$

$\Rightarrow (x+1)\,[\,(x+1)^3 - 1] = 0 \;\; \Rightarrow \;\; x = -1, 0$

$\therefore$ The statement-1 and statement-2 both are true.

40. **(d)** Clearly $f(x) = 4x + 3$ is one one and onto, so it is invertible.

Let $\quad f(\mathrm{x}) = 4x + 3 = y$

$\Rightarrow x = \dfrac{y-3}{4} \qquad \therefore \; g(y) = \dfrac{y-3}{4}$

17

Inverse Trigonometric Functions

TOPIC 1 Trigometric Functions & Their Inverses, Domain & Range of Inverse Trigonometric Functions, Principal Value of Inverse Trigonometric Functions, Intervals for Inverse Trigonometric Functions

1. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to : **[April 8, 2019 (I)]**

   (a) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (b) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

   (c) $\tan^{-1}\left(\frac{9}{14}\right)$ (d) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

2. A value of x satisfying the equation $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1}x]$, is : **[Online April 9, 2017]**

   (a) $-\frac{1}{2}$ (b) -1 (c) 0 (d) $\frac{1}{2}$

3. The principal value of $\tan^{-1}\left(\cot\frac{43\pi}{4}\right)$ is: **[Online April 19, 2014]**

   (a) $-\frac{3\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $-\frac{\pi}{4}$ (d) $\frac{\pi}{4}$

4. The number of solutions of the equation, $\sin^{-1} x = 2\tan^{-1} x$ (in principal values) is : **[Online April 22, 2013]**

   (a) 1 (b) 4 (c) 2 (d) 3

5. A value of $\tan^{-1}\left(\sin\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)\right)\right)$ is **[Online May 19, 2012]**

   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

6. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function, $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$, is defined, is **[2007]**

   (a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$

   (c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

7. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

   (a) $[1, 2]$ (b) $[2, 3)$ **[2004]**

   (c) $[1, 2]$ (d) $[2, 3]$

8. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for **[2003]**

   (a) $|a| \le \frac{1}{\sqrt{2}}$ (b) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

   (c) all real values of a (d) $|a| < \frac{1}{2}$

9. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x =$

   (a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$ **[2002]**

   (c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

10. The domain of $\sin^{-1}[\log_3(x/3)]$ is **[2002]**

    (a) $[1, 9]$ (b) $[-1, 9]$

    (c) $[-9, 1]$ (d) $[-9, -1]$

TOPIC 2 Properties of Inverse Trigonometric Functions, Infinite Series of Inverse Trigonometric Functions

11. $2\pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right)$ is equal to : **[Sep. 03, 2020 (I)]**

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{4}$ (c) $\frac{3\pi}{2}$ (d) $\frac{7\pi}{4}$

12. If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \ldots,$ then tan (S) is equal to: **[Sep. 05, 2020 (I)]**

(a) $\frac{5}{6}$ (b) $\frac{5}{11}$

(c) $-\frac{6}{5}$ (d) $\frac{10}{11}$

13. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to : **[April 12, 2019 (I)]**

(a) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$ (b) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

(c) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$ (d) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

14. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1, -2 \le y \le 2,$ $x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to: **[April 10, 2019 (II)]**

(a) $4\sin^2\alpha$ (b) $2\sin^2\alpha$

(c) $4\sin^2\alpha - 2x^2y^2$ (d) $4\cos^2\alpha + 2x^2y^2$

15. Considering only the principal values of inverse functions, the set $A = \left\{x \ge 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$ **[Jan. 12, 2019 (I)]**

(a) contains two elements
(b) contains more than two elements
(c) is a singleton
(d) is an empty set

16. All x satisfying the inequality $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, lie in the interval : **[Jan. 11, 2019 (II)]**

(a) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(b) $(\cot 2, \infty)$

(c) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

(d) $(\cot 5, \cot 4)$

17. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1 + \sum_{p=1}^{n} 2p\right)\right)$ is: **[Jan. 10, 2019 (II)]**

(a) $\frac{21}{19}$ (b) $\frac{19}{21}$

(c) $\frac{22}{23}$ (d) $\frac{23}{22}$

18. If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to: **[Jan. 09, 2019 (II)]**

(a) 0 (b) 10 (c) 7π (d) π

19. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$, then x is equal to: **[Jan. 09, 2019 (I)]**

(a) $\frac{\sqrt{145}}{12}$ (b) $\frac{\sqrt{145}}{10}$ (c) $\frac{\sqrt{146}}{12}$ (d) $\frac{\sqrt{145}}{11}$

20. The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $|x| < \frac{1}{2}, x \ne 0,$ is equal to **[Online April 8, 2017]**

(a) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$ (b) $\frac{\pi}{4} + \cos^{-1}x^2$

(c) $\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$ (d) $\frac{\pi}{4} - \cos^{-1}x^2$

21. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where or $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is : **[2015]**

(a) $\frac{3x - x^3}{1+3x^2}$ (b) $\frac{3x + x^3}{1+3x^2}$

(c) $\frac{3x - x^3}{1-3x^2}$ (d) $\frac{3x + x^3}{1-3x^2}$

22. If $f(x) = 2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right), x > 1$ then $f(5)$ is equal to : **[Online April 10, 2015]**

(a) $\tan^{-1}\left(\frac{65}{156}\right)$ (b) $\frac{\pi}{2}$

(c) π (d) $4\tan^{-1}(5)$

23. **Statement I:** The equation $(\sin^{-1}x)^3 + (\cos^{-1} x)^3 - a\pi^3 = 0$ has a solution for all $a \geq \frac{1}{32}$.

Statement II: For any $x \in R$, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and $0 \leq \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$ **[Online April 12, 2014]**

(a) Both statements I and II are true.
(b) Both statements I and II are false.
(c) Statement I is true and statement II is false.
(d) Statement I is false and statement II is true.

24. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then **[2013]**

(a) $x = y = z$
(b) $2x = 3y = 6z$
(c) $6x = 3y = 2z$
(d) $6x = 4y = 3z$

25. Let $x \in (0, 1)$. The set of all x such that $\sin^{-1}x > \cos^{-1}x$, is the interval: **[Online April 25, 2013]**

(a) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
(b) $\left(\frac{1}{\sqrt{2}}, 1\right)$
(c) $(0, 1)$
(d) $\left(0, \frac{\sqrt{3}}{2}\right)$

26. $S = \tan^{-1}\left(\frac{1}{n^2+n+1}\right) + \tan^{-1}\left(\frac{1}{n^2+3n+3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+(n+19)(n+20)}\right)$, then $\tan S$ is equal to : **[Online April 23, 2013]**

(a) $\frac{20}{401+20n}$
(b) $\frac{n}{n^2+20n+1}$
(c) $\frac{20}{n^2+20n+1}$
(d) $\frac{n}{401+20n}$

27. A value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$, is : **[Online April 9, 2013]**

(a) $-\frac{1}{2}$
(b) 1
(c) 0
(d) $\frac{1}{2}$

28. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the values of x is **[2007]**

(a) 4
(b) 5
(c) 1
(d) 3

29. If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to **[2005]**

(a) $2\sin 2\alpha$
(b) 4
(c) $4\sin^2\alpha$
(d) $-4\sin^2\alpha$

Hints & Solutions

1. **(d)** $\because \cos\alpha = \frac{3}{5}$, then $\sin\alpha = \frac{4}{5}$

$\Rightarrow \tan\alpha = \frac{4}{3}$

and $\tan\beta = \frac{1}{3}$

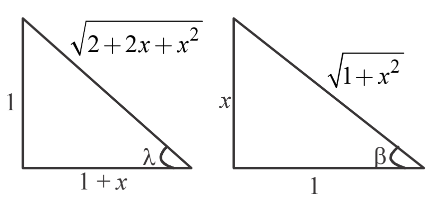
$\because \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha.\tan\beta}$

$= \frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{9}} = \frac{1}{\frac{13}{9}} = \frac{9}{13}$

$\therefore \alpha-\beta = \tan^{-1}\left(\frac{9}{13}\right) = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

$= \cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$

2. **(a)** $\sin\left[\cot^{-1}(1+x)\right] = \cos\left(\tan^{-1}x\right)$



Let; $\cot\lambda = 1+x$
$\tan\beta = x$
$\Rightarrow \sin\lambda = \cos\beta$

$\Rightarrow \frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{1\sqrt{1+x^2}}$

$\Rightarrow x^2+2x+2 = x^2+1$

$\Rightarrow x = -1/2$

3. **(c)** Consider

$\tan^{-1}\left[\cot\frac{43\pi}{4}\right] = \tan^{-1}\left[\cot\left(10\pi+\frac{3\pi}{4}\right)\right]$

$= \tan^{-1}\left[\cot\frac{3\pi}{4}\right]$ $\quad [\because \cot(2n\pi+\theta) = \cot\theta]$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{3\pi}{4}\right)\right]$

$= \frac{\pi}{2}-\frac{3\pi}{4} = \frac{2\pi-3\pi}{4} = \frac{-\pi}{4}$

4. **(a)** Given equation is
$\sin^{-1}x = 2\tan^{-1}x$
Now, this equation has only one solution.

$\therefore$ LHS $= \sin^{-1}1 = \frac{\pi}{2}$

and RHS $= 2\tan^{-1}1 = 2\times\frac{\pi}{4} = \frac{\pi}{2}$

Also, $x = 1$ gives angle value as $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

$\frac{5\pi}{4}$ is outside the principal value.

5. **(d)** Consider $\tan^{-1}\left[\sin\left(\cos^{-1}\sqrt{\frac{2}{3}}\right)\right]$

Let $\cos^{-1}\sqrt{\frac{2}{3}} = \theta \Rightarrow \cos\theta = \sqrt{\frac{2}{3}}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{2}{3}} = \sqrt{\frac{1}{3}}$

$\therefore \tan^{-1}\left[\sin\left(\cos^{-1}\sqrt{\frac{2}{3}}\right)\right] = \tan^{-1}[\sin\theta]$

$= \tan^{-1}\left[\sqrt{\frac{1}{3}}\right] = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

6. **(b)** Given that

$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2}-1\right) + \log(\cos x)$

$f(x)$ is defined if $-1 \le \left(\frac{x}{2}-1\right) \le 1$ and $\cos x > 0$

$\Rightarrow 0 \le \frac{x}{2} \le 2$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\Rightarrow 0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore x \in \left[0, \frac{\pi}{2}\right)$

7. **(b)** $f(x)=\dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is defined

When $-1 \le x-3 \le 1 \Rightarrow 2 \le x \le 4$ ....(i)

and $9-x^2>0 \Rightarrow -3<x<3$ ....(ii)

from (i) and (ii),

we get $2 \le x < 3$ $\therefore$ Domain $=[2,3)$

8. **(a)** Given that $\sin^{-1}x = 2\sin^{-1}a$

We know that $-\dfrac{\pi}{2} \le \sin^{-1}x \le \dfrac{\pi}{2}$

$$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}a \le \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} \le \sin^{-1}a \le \frac{\pi}{4} \Rightarrow \frac{-1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$$

$$\therefore \ |a| \le \frac{1}{\sqrt{2}}$$

9. **(a)** Given that, $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$$

$$\Rightarrow \tan^{-1}\frac{\dfrac{1}{\sqrt{\cos\alpha}}-\sqrt{\cos\alpha}}{1+\dfrac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

$$\Rightarrow \cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha} = \frac{B}{P}$$

$P=(1-\cos\alpha)$ and $B = 2\sqrt{\cos\alpha}$

$$H=\sqrt{P^2+B^2} = 1+\cos\alpha$$

$$\Rightarrow \sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1-(1-2\sin^2\alpha/2)}{1+2\cos^2\alpha/2-1}$$

$$\text{or} \quad \sin x = \tan^2\frac{\alpha}{2}$$

10. **(a)** $f(x)=\sin^{-1}\left(\log_3\left(\dfrac{x}{3}\right)\right)$

We know that domain of $\sin^{-1}x$ is $x \in [-1, 1]$

$$\therefore \ -1 \le \log_3\left(\frac{x}{3}\right) \le 1 \Rightarrow 3^{-1} \le \frac{x}{3} \le 3^1$$

$$\Rightarrow 1 \le x \le 9 \quad \text{or} \quad x \in [1, 9]$$

11. **(c)** $2\pi - \left(\sin^{-1}\dfrac{4}{5}+\sin^{-1}\dfrac{5}{13}+\sin^{-1}\dfrac{16}{65}\right)$

$$= 2\pi - \left(\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{16}{63}\right)$$

$$\left[\because \sin^{-1}\frac{4}{5} = \tan^{-1}\frac{4}{3}\right]$$

$$= 2\pi - \left\{\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}\right)+\tan^{-1}\frac{16}{63}\right\}$$

$$= 2\pi - \left(\tan^{-1}\frac{63}{16}+\tan^{-1}\frac{16}{63}\right)$$

$$= 2\pi - \left(\tan^{-1}\frac{63}{16}+\cot^{-1}\frac{63}{16}\right)$$

$$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}.$$

12. **(a)** $S = \tan^{-1}\left(\dfrac{1}{3}\right)+\tan^{-1}\dfrac{1}{7}+\tan^{-1}\dfrac{1}{13}+....$ upto 10 terms

$$= \tan^{-1}\left(\frac{2-1}{1+2\cdot 1}\right)+\tan^{-1}\left(\frac{3-2}{1+3\cdot 2}\right)$$

$$+\tan^{-1}\left(\frac{4-3}{1+3\cdot 4}\right)+......+\tan^{-1}\left(\frac{11-10}{1+11\cdot 10}\right)$$

$$= (\tan^{-1}2-\tan^{-1}1)+(\tan^{-1}3-\tan^{-1}2)+$$

$$(\tan^{-1}4-\tan^{-1}3)+....+(\tan^{-1}11-\tan^{-1}10)$$

$$= \tan^{-1}11-\tan^{-1}1 = \tan^{-1}\left(\frac{11-1}{1+11\cdot 1}\right) = \tan^{-1}\left(\frac{5}{6}\right)$$

$$\therefore \tan(S) = \frac{5}{6}$$

13. **(b)** $-\sin^{-1}\left(\dfrac{3}{5}\right)+\sin^{-1}\left(\dfrac{12}{13}\right) = -\sin^{-1}\left(\dfrac{3}{5}\times\dfrac{5}{13}-\dfrac{12}{13}\times\dfrac{4}{5}\right)$

($\because$ xy e” 0 and x^2+y^2 d” 1)

$$\left[\because \sin^{-1}x-\sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}\right]$$

$$= \sin^{-1}\left(\frac{-33}{65}\right) = \sin^{-1}\left(\frac{33}{65}\right)$$

$$= \cos^{-1}\left(\frac{56}{65}\right) = \frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$$

14. (a) Given, $\cos^{-1}x-\cos^{-1}\dfrac{y}{2}=\alpha$

$\Rightarrow \cos^{-1}\left(\dfrac{xy}{2}+\sqrt{1-x^2}.\sqrt{1-\dfrac{y^2}{4}}\right)=\alpha$

$\Rightarrow \dfrac{xy}{2}+\dfrac{\sqrt{1-x^2}\sqrt{4-y^2}}{2}=\cos\theta$

$\Rightarrow xy+\sqrt{1-x^2}\sqrt{4-y^2}=2\cos\alpha$

$\Rightarrow (xy-2\cos\alpha)^2=(1-x^2)(4-y^2)$

$\Rightarrow x^2y^2+4\cos^2\alpha-4xy\cos\alpha=4-y^2-4x^2+x^2y^2$

$\Rightarrow 4x^2-4xy\cos\alpha+y^2=4\sin^2\alpha$

15. (c) Consider, $\tan^{-1}(2x)+\tan^{-1}(3x)=\dfrac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\dfrac{5x}{1-6x^2}\right)=\dfrac{\pi}{4}$

$\Rightarrow \dfrac{5x}{1-6x^2}=1\Rightarrow 5x=1-6x^2$

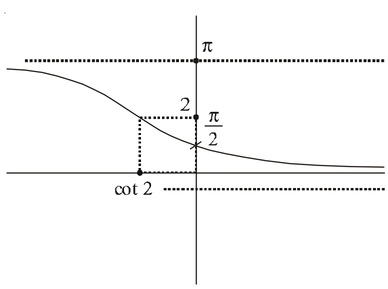
$\Rightarrow 6x^2+5x-1=0$

$\Rightarrow (6x-1)(x+1)=0$

$\Rightarrow x=\dfrac{1}{6}$ (as $x\geq 0$)

Therefore, A is a singleton set.

16. (b)



$(\cot^{-1}x)^2-7(\cot^{-1}x)+10>0$

$(\cot^{-1}x-5)(\cot^{-1}-2)>0$

$\cot^{-1}x\in(-\infty,2)\cup(5,\infty)$ ...(1)

But $\cot^{-1}x$ lies in $(0,\pi)$

Now, from equation (1)

$\cot^{-1}x\in(0,2)$

Now, it is clear from the graph

$x\in(\cot 2,\infty)$

17. (a) $\text{cat}\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$

$=\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1))\right)$

$=\cot\left(\sum_{n=1}^{19}\tan^{-1}\left(\dfrac{(n+1)-n}{1+(n+1)n}\right)\right)$

$\left[\cot^{-1}x=\tan^{-1}\left(\dfrac{1}{x}\right):\text{for } x>0\right]$

$=\cot\left(\sum_{n=1}^{19}(\tan^{-1}(n+1)-\tan^{-1}n)\right)$

$=\cot(\tan^{-1}20-\tan^{-1}1)$

$=\cot\left(\tan^{-1}\left(\dfrac{20-1}{1+20\times 1}\right)\right)$

$=\cot\left(\tan^{-1}\left(\dfrac{19}{21}\right)\right)=\cot\cot^{-1}\left(\dfrac{21}{19}\right)=\dfrac{21}{19}$

18. (d) $x=\sin^{-1}(\sin 10)$

$\Rightarrow x=3\pi-10$ $\quad\begin{cases}3\pi-\dfrac{\pi}{2}<10<3\pi+\dfrac{\pi}{2}\\ \Rightarrow 3\pi-x=10\end{cases}$

and $y=\cos^{-1}(\cos 10)$ $\quad\begin{cases}3\pi<10<4\pi\\ \Rightarrow 4\pi-x=10\end{cases}$

$\Rightarrow y=4\pi-10$

$\therefore\ y-x=(4\pi-10)-(3\pi-10)=\pi$

19. (a) $\cos^{-1}\left(\dfrac{2}{3x}\right)+\cos^{-1}\left(\dfrac{3}{4x}\right)=\dfrac{\pi}{2};\left(x>\dfrac{3}{4}\right)$

$\Rightarrow \cos^{-1}\left(\dfrac{2}{3x}\right)=\dfrac{\pi}{2}-\cos^{-1}\left(\dfrac{3}{4x}\right)$

$\Rightarrow \cos^{-1}\left(\dfrac{2}{3x}\right)=\sin^{-1}\left(\dfrac{3}{4x}\right)\left[\because \sin^{-1}x+\cos^{-1}x=\dfrac{\pi}{2}\right]$

Put $\sin^{-1}\left(\dfrac{3}{4x}\right)=\theta\Rightarrow \sin\theta=\dfrac{3}{4x}$

$\Rightarrow \cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\dfrac{9}{16x^2}}$

$\Rightarrow \theta=\cos^{-1}\left(\dfrac{\sqrt{16x^2-9}}{4x}\right)$

$\therefore\ \cos^{-1}\left(\dfrac{2}{3x}\right)=\cos^{-1}\left(\dfrac{\sqrt{16x^2-9}}{4x}\right)$

$\Rightarrow \dfrac{2}{3x}=\dfrac{\sqrt{16x^2-9}}{4x}\Rightarrow x^2=\dfrac{64+81}{9\times 16}\Rightarrow x=\pm\sqrt{\dfrac{145}{144}}$

$\Rightarrow x = \dfrac{\sqrt{145}}{12} \quad \left(\because x > \dfrac{3}{4}\right)$

20. **(a)** Let $x^2 = \cos 2\theta$; $\Rightarrow \theta = \dfrac{1}{2}\cos^{-1} x^2$

$$\Rightarrow \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$$

$$= \left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$$

$$= \frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2$$

21. **(c)** Given that, $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left[\dfrac{2x}{1-x^2}\right]$

$$= \tan^{-1} x + 2\tan^{-1} x = 3\tan^{-1} x$$

$$\tan^{-1} y = \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$$

$$\Rightarrow y = \frac{3x-x^3}{1-3x^2}$$

22. **(c)** $f(x) = 2\tan^{-1} x + \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$\Rightarrow f(x) = 2\tan^{-1} x + \pi - 2\tan^{-1} x$

$\Rightarrow f(x) = \pi$

$\Rightarrow f(5) = \pi$

23. **(a)** $\sin^{-1} x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$$\Rightarrow -\frac{3\pi}{4} \le \left(\sin^{-1} x - \frac{\pi}{4}\right) \le \frac{\pi}{4}$$

$$0 \le \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \le \frac{9}{16}\pi^2 \quad ..(1)$$

Statement II is true

$(\sin^{-1} x)^3 + (\cos^{-1} x)^3 = a\pi^3$

$\Rightarrow (\sin^{-1} x + \cos^{-1} x)\ [(\sin^{-1} x + \cos^{-1} x)^2 - 3\sin^{-1} x \cos^{-1} x] = a\pi^3$

$$\Rightarrow \frac{\pi^2}{4} - 3\sin^{-1} x \cos^{-1} x = 2a\pi^2$$

$$\Rightarrow \sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right) = \frac{\pi^2}{12}(1-8a)$$

$$\Rightarrow \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 = \frac{\pi^2}{12}(8a-1) + \frac{\pi^2}{16}$$

$$\Rightarrow \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 = \frac{\pi^2}{48}(32a-1)$$

Putting this value in equation (1)

$$0 \le \frac{\pi^2}{48}(32a-1) \le \frac{9}{16}\pi^2$$

$\Rightarrow 0 \le 32a - 1 \le 27$

$$\frac{1}{32} \le a \le \frac{7}{8}$$

Statement-I is also true

24. **(a)** Since, x, y, z are in A.P.

$\therefore \quad 2y = x + z$

Also, we have

$2\tan^{-1} y = \tan^{-1} x + \tan^{-1}(z)$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \quad (\because 2y = x+z)$$

$\Rightarrow y^2 = xz$ or $x + z = 0 \Rightarrow x = y = z = 0$

25. **(b)** Given $\sin^{-1} x > \cos^{-1} x$ where $x \in (0, 1)$

$$\Rightarrow \sin^{-1} x > \frac{\pi}{2} - \sin^{-1} x$$

$$\Rightarrow 2\sin^{-1} x > \frac{\pi}{2} \Rightarrow \sin^{-1} x > \frac{\pi}{4}$$

$$\Rightarrow x > \sin\frac{\pi}{4} \Rightarrow x > \frac{1}{\sqrt{2}}$$

Maximum value of $\sin^{-1} x$ is $\dfrac{\pi}{2}$

So, maximum value of x is 1. So, $x \in \left(\dfrac{1}{\sqrt{2}}, 1\right)$.

26. **(c)** We know that,

$$\tan^{-1}\frac{1}{1+2} + \tan^{-1}\frac{1}{1+2\times 3} + \tan^{-1}\frac{1}{1+3\times 4} + \ldots\ldots +$$

$$\tan^{-1}\frac{1}{1+(n-1)n} + \tan^{-1}\frac{1}{1+n(n+1)} + \ldots\ldots +$$

$$\tan^{-1}\frac{1}{1+(n+19)(n+20)} = \tan^{-1}\frac{n+19}{n+21}$$

$$\Rightarrow \tan^{-1}\frac{n-1}{n+1} + \tan^{-1}\frac{1}{1+n(n+1)}$$

$$+ \tan^{-1}\frac{1}{1+(n+1)(n+2)} + \ldots\ldots + \frac{1}{1+(n+19)(n+20)}$$

$$= \tan^{-1}\frac{n+19}{n+21}$$

$$\Rightarrow \tan^{-1}\frac{1}{1+n(n+1)}+\tan^{-1}\frac{1}{1+(n+1)(n+2)}+......+$$

$$\frac{1}{1+(n+19)\,(n+20)} = \tan^{-1}\frac{n+19}{n+21}-\tan^{-1}\frac{n-1}{n+1}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{n^2+n+1}\right)+\tan^{-1}\left(\frac{1}{n^2+3n+3}\right)+......+$$

$$\tan^{-1}\frac{1}{1+(n+19)\,(n+20)}$$

$$=\tan^{-1}\left(\frac{\frac{n+19}{n+21}-\frac{n-1}{n+1}}{1+\frac{n+19}{n+21}\times\frac{n-1}{n+1}}\right)=\tan^{-1}\frac{20}{n^2+20n+1}=\text{S}$$

$$\therefore\ \tan^{-1}\text{S}=\frac{20}{n^2+20n+1}$$

27. **(a)** $\sin(\cot^{-1}(1+x))=\cos(\tan^{-1}x)$

$\Rightarrow \text{cosec}^2(\cot^{-1}(1+x))=\sec^2(\tan^{-1}x)$

$\Rightarrow 1+[\cot(\cot^{-1}(1+x))]^2=1+[\tan(\tan^{-1}x)]^2$

$\Rightarrow (1+x)^2=x^2 \Rightarrow x=-\frac{1}{2}$

28. **(d)** $\sin^{-1}\left(\frac{x}{5}\right)+\text{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right)=\frac{\pi}{2}-\text{cosec}^{-1}\left(\frac{5}{4}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right)=\frac{\pi}{2}-\sin^{-1}\left(\frac{4}{5}\right)$$

$[\because \sin^{-1}x+\cos^{-1}x=\pi/2]$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right)=\cos^{-1}\left(\frac{4}{5}\right)$$

$$\sin^{-1}\frac{x}{5}=\sin^{-1}\sqrt{1-\left(\frac{4}{5}\right)^2}\quad\left[\because\ \cos^{-1}x=\sin^{-1}\sqrt{1-x^2}\right]$$

$$\Rightarrow \sin^{-1}\frac{x}{5}=\sin^{-1}\frac{3}{5} \Rightarrow \frac{x}{5}=\frac{3}{5}$$

$\Rightarrow x=3$

29. **(c)** $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$

$$\Rightarrow \cos^{-1}\left(\frac{xy}{2}+\sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right)=\alpha$$

$$\Rightarrow \cos^{-1}\left(\frac{xy+\sqrt{4-y^2-4x^2+x^2y^2}}{2}\right)=\alpha$$

$$\Rightarrow xy+\sqrt{4-y^2-4x^2+x^2y^2}=2\cos\alpha$$

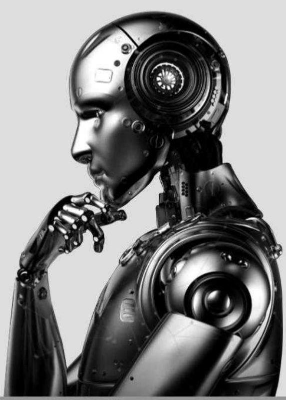
$$\Rightarrow \sqrt{4-y^2-4x^2+x^2y^2}=2\cos\alpha-xy$$

Squaring both sides, we get

$$\Rightarrow 4-y^2-4x^2+x^2y^2=4\cos^2\alpha+x^2y^2-4xy\cos\alpha$$

$$\Rightarrow 4x^2+y^2-4xy\cos\alpha=4\sin^2\alpha.$$

18



Matrices

TOPIC 1 Order of Matrices, Types of Matrices, Addition & Subtraction of Matrices, Scalar Multiplication of Matrices, Multiplication of Matrices

1. Let α be a root of the equation $x^2 + x + 1 = 0$

and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31}

is equal to: **[Jan. 7, 2020 (I)]**

(a) A (b) I_3 (c) A^2 (d) A^3

2. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

where $i = \sqrt{-1}$, then which one of the following is **not** true? **[Sep. 04, 2020 (I)]**

(a) $0 \le a^2 + b^2 \le 1$ (b) $a^2 - d^2 = 0$

(c) $a^2 - c^2 = 1$ (d) $a^2 - b^2 = \frac{1}{2}$

3. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbf{R}$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to __________. **[NA Sep. 03, 2020 (I)]**

4. Let $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, $(\alpha \in R)$ such that $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Then a value of α is : **[April 8, 2019 (I)]**

(a) $\frac{\pi}{32}$ (b) 0 (c) $\frac{\pi}{64}$ (d) $\frac{\pi}{16}$

5. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices

such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to : **[Jan. 12, 2019 (I)]**

(a) 10 (b) 135 (c) 15 (d) 9

6. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is? **[Online April 16, 2018]**

(a) 211 (b) 210 (c) 231 (d) 251

7. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is not correct? **[Online April 10, 2015]**

(a) $A^2 + I = A(A^2 - I)$ (b) $A^4 - I = A^2 + I$

(c) $A^3 + I = A(A^3 - I)$ (d) $A^3 - I = A(A - I)$

8. If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$,

then: **[Online April 12, 2014]**

(a) $y = 2x$ (b) $y = -2x$

(c) $y = x$ (d) $y = -x$

9. If p, q, r are 3 real numbers satisfying the matrix equation,

$[p\,q\,r]\begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = [3 \quad 0 \quad 1]$ then

$2p + q - r$ equals : **[Online April 22, 2013]**

(a) -3 (b) -1

(c) 4 (d) 2

10. The matrix $A^2 + 4A - 5I$, where I is identity matrix and $A=\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}$, equals **[Online April 9, 2013]**

(a) $4\begin{bmatrix}2 & 1\\ 2 & 0\end{bmatrix}$ (b) $4\begin{bmatrix}0 & -1\\ 2 & 2\end{bmatrix}$

(c) $32\begin{bmatrix}2 & 1\\ 2 & 0\end{bmatrix}$ (d) $32\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

11. If $A=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ -3 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ 7 & -2 & 1\end{bmatrix}$ then AB equals **[Online May 26, 2012]**

(a) I (b) A (c) B (d) 0

12. If $\omega \neq 1$is the complex cube root of unity and matrix $H=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}$, then H^{70} is equal to **[2011RS]**

(a) 0 (b) $-H$ (c) H^2 (d) H

13. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[2010]**

(a) 5 (b) 6

(c) at least 7 (d) less than 4

14. Let $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}$, $a, b \in N$. Then **[2006]**

(a) there cannot exist any B such that AB = BA
(b) there exist more than one but finite number of B′s such that AB = BA
(c) there exists exactly one B such that AB = BA
(d) there exist infinitely many B′s such that AB = BA

15. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A-B)(A+B)$, then which of the following will be always true? **[2006]**

(a) $A = B$
(b) $AB = BA$
(c) either of A or B is a zero matrix
(d) either of A or B is identity matrix

16. If $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction **[2005]**

(a) $A^n = nA-(n-1)I$

(b) $A^n = 2^{n-1}A-(n-1)I$

(c) $A^n = nA+(n-1)I$

(d) $A^n = 2^{n-1}A+(n-1)I$

17. If $A=\begin{bmatrix}a & b\\ b & a\end{bmatrix}$ and $A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}$, then **[2003]**

(a) $\alpha = 2ab, \beta = a^2+b^2$

(b) $\alpha = a^2+b^2,\ \beta = ab$

(c) $\alpha = a^2+b^2,\ \beta = 2ab$

(d) $\alpha = a^2+b^2,\ \beta = a^2-b^2$.

TOPIC 2 Transpose of Matrices, Symmetric & Skew Symmetric Matrices, Inverse of a Matrix by Elementary Row Operations

18. Let $a, b, c \in \mathbf{R}$ be all non-zero and satisfy $a^3+b^3+c^3=2$. If the matrix $A=\begin{pmatrix}a & b & c\\ b & c & a\\ c & a & b\end{pmatrix}$ satisfies $A^TA = I$, then a value of abc can be : **[Sep. 02, 2020 (II)]**

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{2}{3}$

19. The number of all 3×3 matrices A, with enteries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is ________. **[NA Jan. 8, 2020 (I)]**

20. If $A=\begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to: **[Jan. 8, 2020 (II)]**

(a) $A-4I$ (b) $6I-A$ (c) $A-6I$ (d) $4I-A$

21. If A is a symmetric matrix and B is a skew-symmetrix matrix such that $A+B=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}$, then AB is equal to : **[April 12, 2019 (I)]**

(a) $\begin{bmatrix}-4 & -1\\ -1 & 4\end{bmatrix}$ (b) $\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$

(c) $\begin{bmatrix}4 & -2\\ 1 & -4\end{bmatrix}$ (d) $\begin{bmatrix}-4 & 2\\ 1 & 4\end{bmatrix}$

22. The total number of matrices $A=\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix}$, $(x, y \in R, x \neq y)$ for which $A^TA = 3I_3$ is: **[April 09, 2019 (II)]**

(a) 2 (b) 3 (c) 6 (d) 4

23. Let $A=\begin{pmatrix}0 & 2q & r\\ p & q & -r\\ p & -q & r\end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is : **[Jan. 11, 2019 (I)]**

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{6}}$

24. For two 3 × 3 matrices A and B, let $A + B = 2B^T$ and $3A + 2B = I_3$, where B^T is the transpose of B and I_3 is 3 × 3 identity matrix. Then : **[Online April 9, 2017]**

(a) $5A + 10B = 2I_3$ (b) $10A + 5B = 3I_3$

(c) $B + 2A = I_3$ (d) $3A + 6B = 2I_3$

25. If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is ; **[Online April 9, 2016]**

(a) $\begin{bmatrix}0 & 2015\\ 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}2015 & 0\\ 1 & 2015\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2015\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}2015 & 1\\ 0 & 2015\end{bmatrix}$

26. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to: **[2015]**

(a) (2, 1) (b) (–2, – 1)

(c) (2, – 1) (d) (–2, 1)

27. Let A and B be any two 3 × 3 matrices. If A is symmetric and B is skewsymmetric, then the matrix AB – BA is: **[Online April 19, 2014]**

(a) skewsymmetric

(b) symmetric

(c) neither symmetric nor skewsymmetric

(d) I or – I, where I is an identity matrix.

28. If $A=\begin{pmatrix}\alpha-1\\ 0\\ 0\end{pmatrix}$, $B=\begin{pmatrix}\alpha+1\\ 0\\ 0\end{pmatrix}$ be two matrices, then AB^T is a non-zero matrix for $|\alpha|$ not equal to **[Online May 7, 2012]**

(a) 2 (b) 0 (c) 1 (d) 3

29. Let A and B be two symmetric matrices of order 3. **[2011]**

Statement-1: $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Hints & Solutions

1. **(d)** Solution of $x^2 + x + 1 = 0$ is ω, ω^2
So, $\alpha = \omega$ and
$\omega^4 = \omega^3.\omega = 1.\omega = \omega$

$$A^2 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow \quad A^4 = I$
$\Rightarrow \quad A^{30} = A^{28} \times A^3 = A^3$

2. **(d)** $\because A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$$\therefore A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}, n \in \mathbf{N}$$

$$\because A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\therefore A^5 = \begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore a = \cos 5\theta,\ b = i\sin 5\theta = c,\ d = \cos 5\theta$

$\therefore a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 1$

$a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos\frac{10\pi}{24}$

and $0 < \cos\frac{5\pi}{12} < 1 \Rightarrow 0 \le a^2 + b^2 \le 1$

$\therefore a^2 - b^2 = \frac{1}{2}$ is wrong.

3. **(10)**

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2 + x^2 & x(x^2+1) + x \\ x(x^2+1) + x & x^2+1 \end{bmatrix}$$

Given that $(x^2+1)^2 + x^2 = 109$

$x^4 + 3x^2 - 108 = 0$

$\Rightarrow (x^2+12)(x^2-9) = 0$

$\therefore x^2 = 9$

$a_{22} = x^2 + 1 = 9 + 1 = 10.$

4. **(c)** $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

Similarly, $A^4 = A^2.A^2 = \begin{bmatrix} \cos 4\alpha & -\sin 4\alpha \\ \sin 4\alpha & \cos 4\alpha \end{bmatrix}$

and so on $A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Then $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$\Rightarrow \quad 32\alpha = n\pi + (-1)^n\frac{\pi}{2}$ and $32\alpha = 2n\pi + \frac{\pi}{2}$

$\Rightarrow \quad \alpha = \frac{n\pi}{32} + (-1)^n\frac{\pi}{64}$ and $\alpha = \frac{n\pi}{16} + \frac{\pi}{64}$ where $n \in z$

Put $n = 0, \alpha = \frac{\pi}{64}$

Hence, required value of α is $\frac{\pi}{64}$.

5. **(a)** $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$

$$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}$$

$$\Rightarrow P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$\because \quad Q - P^5 = I_3$

$\therefore \quad Q = I_3 + P^5 = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$

$\frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$

6. **(c)** Here $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore \quad A^2 = A.\,A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

also $A^3 = A^2.\,A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$

and, $A^4 = A^3.\,A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$

On observing the pattern, we come to a conclusion that,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ \frac{n\,(n+1)}{2} & n & 1 \end{bmatrix}$$

$\therefore \quad A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 20 & 1 & 0 \\ 210 & 20 & 1 \end{bmatrix}$

Therefore, sum of first column of $A^{20} = [1+20+210] = 231$

7. **(a)** Given that

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow A^2 = -I$

$A^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A^2 + I = A^3 - A$

$-I + I = A^3 - A$

$A^3 \neq A$

8. **(a)** Let $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y+2x+x \\ 3y-x+2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix}$

$\Rightarrow y + 3x = 6$ and $3y - x = 6$

On solving, we get

$x = \frac{6}{5}$ and $y = \frac{12}{5}$

$\Rightarrow \quad y = 2x$

9. **(a)** Given

$[p \quad q \quad r] \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = [3 \quad 0 \quad 1]$

$\Rightarrow [3p+3q+2r \quad 4p+2q \quad p+3q+2r] = [3 \quad 0 \quad 1]$

$\Rightarrow 3p+3q+2r = 3$ ...(i)

$4p+2q = 0 \Rightarrow q = -2p$ ...(ii)

$p+3q+2r = 1$ ...(iii)

On solving (i), (ii) and (iii), we get

$p = 1, q = -2, r = 3$

$\therefore\ 2p+q-r = 2(1)+(-2)-(3) = -3.$

10. **(a)** $A^2 + 4A - 5I = A \times A + 4A - 5I$

$= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 9+4-5 & -4+8-0 \\ -8+16-0 & 17-12-5 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix} = 4\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

11. (a) $A=\begin{bmatrix}1&0&0\\2&1&0\\-3&2&1\end{bmatrix}$, $B=\begin{bmatrix}1&0&0\\-2&1&0\\7&-2&1\end{bmatrix}$

$AB=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$

12. (d) $H^2=\begin{bmatrix}\omega&0\\0&\omega\end{bmatrix}\begin{bmatrix}\omega&0\\0&\omega\end{bmatrix}=\begin{bmatrix}\omega^2&0\\0&\omega^2\end{bmatrix}$

We observed that $H^k=\begin{bmatrix}\omega^k&0\\0&\omega\end{bmatrix}$

$\therefore\ H^{70}=\begin{bmatrix}\omega^{70}&0\\0&\omega^{70}\end{bmatrix}=\begin{bmatrix}\omega^{69}\omega&0\\0&\omega^{69}\omega\end{bmatrix}=\begin{bmatrix}\omega&0\\0&\omega\end{bmatrix}=H$

$[\because \omega^{3n}=1]$

13. (c) $\begin{bmatrix}1&...&...\\...&1&...\\...&...&1\end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.

Similarly, $\begin{bmatrix}...&...&1\\...&1&...\\1&...&...\end{bmatrix}$ are 6 non-singular matrices.

Total $=6+6=12$

So, required cases are more than 7, non-singular 3×3 matrices.

14. (d) Given that $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ $B=\begin{bmatrix}a&0\\0&b\end{bmatrix}$

$AB=\begin{bmatrix}a&2b\\3a&4b\end{bmatrix}$

$BA=\begin{bmatrix}a&0\\0&b\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}a&2a\\3b&4b\end{bmatrix}$

Hence, $AB=BA$ only when $a=b$

$\therefore$ There can be infinitely many B's for which $AB=BA$

15. (b) Given that $A^2-B^2=(A-B)(A+B)$

$A^2-B^2=A^2+AB-BA-B^2$

$\Rightarrow\ AB=BA$

16. (a) Given that $A=\begin{bmatrix}1&0\\1&1\end{bmatrix}$

$A^2=\begin{bmatrix}1&0\\2&1\end{bmatrix}$, $A^3=\begin{bmatrix}1&0\\3&1\end{bmatrix}$

Therefore we observed that $A^n=\begin{bmatrix}1&0\\n&1\end{bmatrix}$

Now $nA-(n-1)I=\begin{bmatrix}n&0\\n&n\end{bmatrix}-\begin{bmatrix}n-1&0\\0&n-1\end{bmatrix}$

$=\begin{bmatrix}1&0\\n&1\end{bmatrix}=A^n$

$\therefore nA-(n-1)I=A^n$

17. (c) $A^2=\begin{bmatrix}\alpha&\beta\\\beta&\alpha\end{bmatrix}\Rightarrow A\cdot A=\begin{bmatrix}a&b\\b&a\end{bmatrix}\begin{bmatrix}a&b\\b&a\end{bmatrix}$

$=\begin{bmatrix}a^2+b^2&2ab\\2ab&a^2+b^2\end{bmatrix}$

$\alpha=a^2+b^2;\ \beta=2ab$

18. (b) Given : $A^TA=I$

$\Rightarrow\begin{bmatrix}a&b&c\\b&c&a\\c&a&b\end{bmatrix}\begin{bmatrix}a&b&c\\b&c&a\\c&a&b\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}\Sigma a^2&\Sigma ab&\Sigma ab\\\Sigma ab&\Sigma a^2&\Sigma ab\\\Sigma ab&\Sigma ab&\Sigma a^2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

So, $\Sigma a^2=1$ and $\Sigma ab=0$

Now, $a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=(a+b+c)(1-0)$

$=\sqrt{(a+b+c)^2}=\sqrt{\Sigma a^2+2\Sigma ab}=\pm 1$

$\Rightarrow 2-3abc=1\Rightarrow abc=\frac{1}{3}$

or $2-3abc=-1\ \Rightarrow abc=1.$

19. (672) Let $A=[a_{ij}]_{3\times 3}$

It is given that sum of diagonal elements of AA^T is 3 i.e., $tr(AA^T)=3$

$a^2_{11}+a^2_{12}+a^2_{13}+a^2_{21}+.....+a^2_{33}=3$

Possible cases are

$$\left.\begin{array}{ll}0,0,0,0,0,0,1,1,1 & \to 1\\ 0,0,0,0,0,0,-1,-1,-1 & \to 1\\ 0,0,0,0,0,0,1,1,-1 & \to 3\\ 0,0,0,0,0,0,-1,1,-1 & \to 3\end{array}\right\}\ {}^9C_6\times 8=84\times 8=672$$

20. (c) Characteristics equation of matrix 'A' is $|A-\lambda I|=0$

$\begin{vmatrix}2-\lambda&2\\9&4-\lambda\end{vmatrix}=0\Rightarrow \lambda^2-6\lambda-10=0$

$\therefore\quad A^2-6A-10I=0$

$\Rightarrow\quad A^{-1}(A^2)-6A^{-1}-10IA^{-1}=0$

$\Rightarrow\quad 10A^{-1}=A-6I$

21. (b) Let $A=\begin{bmatrix} a & c \\ c & b\end{bmatrix}$ and $B=\begin{bmatrix} 0 & d \\ -d & 0\end{bmatrix}$

Then, $A+B=\begin{bmatrix} a & c+d \\ c-d & b\end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 5 & -1\end{bmatrix}$

On comparing each term,

$a=2, b=-1, c-d=5, c+d=3$

$\Rightarrow\quad a=2, b=-1, c=4, d=-1$

Now, $AB=\begin{bmatrix} 2 & 4 \\ 4 & -1\end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0\end{bmatrix}=\begin{bmatrix} 4 & -2 \\ -1 & -4\end{bmatrix}$

22. (d) Given, $A^TA=3I$

$$\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1\end{bmatrix}=3I$$

$$\Rightarrow\begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3\end{bmatrix}=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3\end{bmatrix}$$

$\Rightarrow 8x^2=3$ and $6y^2=3 \Rightarrow x=\pm\sqrt{\frac{3}{8}}$ and $y=\pm\sqrt{\frac{1}{2}}$

Number of combinations of $(x, y)=2\times 2=4$

23. (c) $A=\begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r\end{bmatrix}$

$$\therefore\quad A\cdot A^T=\begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r\end{bmatrix}\times\begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r\end{bmatrix}$$

$$=\begin{bmatrix} 4q^2+r^2 & 2q^2-r^2 & -2q^2+r^2 \\ 2q^2-r^2 & p^2+q^2+r^2 & p^2-q^2-r^2 \\ -2q^2+r^2 & p^2-q^2-r^2 & p^2+q^2+r^2\end{bmatrix}$$

Given, $AA^T=I$

$\therefore\quad 4q^2+r^2=p^2+q^2+r^2=1$

$\Rightarrow\quad p^2-3q^2=0$ and $r^2=1-4q^2$

and $2q^2-r^2=0 \Rightarrow r^2=2q^2$

$\therefore\quad p^2=\frac{1}{2}, q^2=\frac{1}{6}$ and $r^2=\frac{1}{3}$

$\therefore\quad |p|=\frac{1}{\sqrt{2}}.$

24. (b) $A^T+B^T=2B$

$\because\quad [(A+B)^T=(2B^T)^T]$

$$\Rightarrow\quad B=\frac{A^T+B^T}{2}=A+\left(\frac{B^T+A^T}{2}\right)=2B^T$$

$$\Rightarrow\quad 2A+A^T=3B^T \Rightarrow A=\frac{3B^T-A^T}{2}$$

Also, $\quad 3A+2B=I_3$ ...(i)

$$\Rightarrow\quad 3\left(\frac{3B^T-A^T}{2}\right)+2\left(\frac{A^T+B^T}{2}\right)=I_3$$

$\Rightarrow\quad 11B^T-A^T=2I_3$ ....(ii)

Add (i) and (ii)

$35B=7I_3$

$$\Rightarrow\quad B=\frac{I_3}{5} \Rightarrow 11\frac{I_3}{5}-A=2I_3$$

$$\Rightarrow\quad 11\frac{I_3}{5}-2I_3=A \Rightarrow A=\frac{I_3}{5}$$

$\because\quad 5A=5B=I_3$

$\Rightarrow\quad 10A+5B=3I_3$

25. (c) $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$ $P^T=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

$PP^T=P^TP=I$

$Q^{2015}=(PAP^T)(PAP^T)$ ——— (2015 terms)

$\qquad=PA^{2015}P^T$

$P^TQ^{2015}P=A^{2015}$

$A^2=\begin{bmatrix} 1 & 1 \\ 0 & 1\end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1\end{bmatrix}=\begin{bmatrix} 1 & 2 \\ 0 & 1\end{bmatrix}$

$A^3=\begin{bmatrix} 1 & 2 \\ 0 & 1\end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1\end{bmatrix}=\begin{bmatrix} 1 & 3 \\ 0 & 1\end{bmatrix}$

$\therefore\quad A^{2015}=\begin{bmatrix} 1 & 2015 \\ 0 & 1\end{bmatrix}$

26. **(b)** Given that $AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2-4 & a+4+2b \\ 2+2-4 & 4+1+4 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow a+4+2b = 0 \Rightarrow a+2b = -4$ ...(i)

$2a+2-2b = 0 \Rightarrow 2a-2b = -2$

$\Rightarrow a-b = -1$ ...(ii)

Subtract (ii) from (i)

$$\begin{array}{l} a+2b=-4 \\ a\ -\ b=-1 \\ \underline{-\quad +\quad +} \\ 3b=-3 \end{array}$$

$b = -1$

and $a = -2$

$(a, b) = (-2, -1)$

27. **(b)** Let A be symmetric matrix and B be skew symmetric matrix.

$\therefore A^T = A$ and $B^T = -B$

Consider

$(AB - BA)^T = (AB)^T - (BA)^T$

$= B^TA^T - A^TB^T = (-B)(A) - (A)(-B)$

$= -BA + AB = AB - BA$

This shows AB – BA is symmetric matrix.

28. **(c)** Let $A = \begin{pmatrix} \alpha-1 \\ 0 \\ 0 \end{pmatrix}$, $B = \begin{pmatrix} \alpha+1 \\ 0 \\ 0 \end{pmatrix}$

be two matrices.

$$AB^T = \begin{pmatrix} \alpha-1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} \alpha+1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \alpha^2-1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus, AB^T is non-zero matrix for $|\alpha| \neq 1$

29. **(a)** Given that A and B are symmetric matrix

$A' = A$

$B' = B$

Now $(A(BA))' = (BA)'A' = (A'B')A' = (AB)A = A(BA)$

($\because$ product of matrices are associative)

Similarly, $((AB)A)' = A'(B'A') = A(BA) = (AB)A$

So, A(BA) and (AB)A are symmetric matrices.

Again $(AB)' = B'A' = BA$

Now if BA = AB, then AB is symmetric matrix.

19

Determinants

TOPIC 1 Minor & Co-factor of an Element of a Determinant, Value of a Determinant, Property of Determinant of Matrices, Singular & Non-Singular Matrices, Multiplication of two Determinants

1. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then det (B): **[Sep. 06, 2020 (II)]**
(a) is one (b) lies in (2, 3)
(c) is zero (d) lies in (1, 2)

2. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D,$
then $B + C$ is equal to : **[Sep. 03, 2020 (I)]**
(a) –1 (b) 1 (c) –3 (d) 9

3. Let $a - 2b + c = 1$.
If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then : **[Jan. 9, 2020 (II)]**
(a) $f(-50) = 501$ (b) $f(-50) = -1$
(c) $f(50) = -501$ (d) $f(50) = 1$

4. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and
$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x \neq 0$;
then for all $\theta \in \left(0, \frac{\pi}{2}\right)$: **[April 10, 2019 (I)]**
(a) $\Delta_1 - \Delta_2 = -2x^3$
(b) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$
(c) $\Delta_1 \times \Delta_2 = -2(x^3 + x - 1)$
(d) $\Delta_1 + \Delta_2 = -2x^3$

5. The sum of the real roots of the equation
$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to : **[April 10, 2019 (II)]**
(a) 6 (b) 0 (c) 1 (d) –4

6. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is: **[Jan. 10, 2019 (II)]**
(a) $2\sqrt{3}$ (n) $-2\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}$

7. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to : **[2018]**
(a) (–4, 3) (b) (–4, 5)
(c) (4, 5) (d) (–4, –5)

8. If $S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\}$, then $\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to **[Online April 8, 2017]**
(a) $4 + 2\sqrt{3}$ (b) $-2 + \sqrt{3}$
(c) $-2 - \sqrt{3}$ (d) $-4 - 2\sqrt{3}$

9. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is : **[Online April 10, 2016]**
(a) –175 (b) 2014 (c) 2016 (d) –25

10. if $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax-12$, then '$a$' is equal to : **[Online April 11, 2015]**

(a) 24 (b) –12 (c) –24 (d) 12

11. The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is non-negative, is : **[Online April 10, 2015]**

(a) $-2\sqrt{2}$ (b) –1
(c) $-16\sqrt{2}$ (d) –8

12. If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ and A and B are respectively the maximum and the minimum values of f(θ), then (A, B) is equal to: **[Online April 12, 2014]**

(a) $(3,-1)$ (b) $\left(4, 2-\sqrt{2}\right)$
(c) $\left(2+\sqrt{2}, 2-\sqrt{2}\right)$ (d) $\left(2+\sqrt{2}, -1\right)$

13. If B is a 3 × 3 matrix such that $B^2 = 0$, then det. $[(I+B)^{50} - 50B]$ is equal to: **[Online April 9, 2014]**

(a) 1 (b) 2 (c) 3 (d) 50

14. Let $S = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : a_{ij} \in \{0,1,2\}, a_{11} = a_{22} \right\}$

Then the number of non-singular matrices in the set S is : **[Online April 25, 2013]**

(a) 27 (b) 24
(c) 10 (d) 20

15. Let A, other than I or – I, be a 2 × 2 real matrix such that $A^2 = I$, I being the unit matrix. Let Tr (A) be the sum of diagonal elements of A. **[Online April 23, 2013]**

Statement-1: Tr (A) = 0

Statement-2: det (A) = – 1

(a) Statement-1 is true; Statement-2 is false.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is false; Statement-2 is true.

16. **Statement - 1:** **[2011RS]**
Determinant of a skew-symmetric matrix of order 3 is zero.

Statement - 2 :
For any matrix A, det $(A)^T$ = det (A) and det (– A) = – det (A).
Where det (B) denotes the determinant of matrix B. Then :

(a) Both statements are true
(b) Both statements are false
(c) Statement-1 is false and statement-2 is true
(d) Statement-1 is true and statement-2 is false

17. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define
$\mathrm{Tr}(A)$ = sum of diagonal elements of A and
$|A|$ = determinant of matrix A.

Statement - 1 : $\mathrm{Tr}(A) = 0$.

Statement -2 : $|A| = 1$. **[2010]**

(a) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

18. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of a. Assume that $A^2 = I$. **[2008]**

Statement-1 : If $A \neq I$ and $A \neq -I$, then det $(A) = -1$

Statement-2 : If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

19. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $\left|A^2\right| = 25$, then $|\alpha|$ equals **[2007]**

(a) 1/5 (b) 5
(c) 5^2 (d) 1

20. If $1, \omega, \omega^2$ are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$ is equal to **[2003]**

(a) ω^2 (b) 0
(c) 1 (d) ω

TOPIC 2 Properties of Determinants, Area of a Triangle

21. If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$ are m and M respectively, then the ordered pair (m, M) is equal to : **[Sep. 05, 2020 (I)]**

(a) $(0, 2\sqrt{2})$ (b) $(-4, 0)$
(c) $(-4, 4)$ (d) $(0, 4)$

22. If $a+x=b+y=c+z+1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix}x & a+y & x+a\\ y & b+y & y+b\\ z & c+y & z+c\end{vmatrix}$ is equal to : **[Sep. 05, 2020 (II)]**

(a) $y(b-a)$ (b) $y(a-b)$
(c) 0 (d) $y(a-c)$

23. Let two points be $A(1,-1)$ and $B(0,2)$. If a point $P(x', y')$ be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x+y-4\lambda=0$, then a value of λ is: **[Jan. 8, 2020 (I)]**

(a) 4 (b) 3 (c) 1 (d) –3

24. Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two 3×3 real matrices such that $b_{ij}=(3)^{(i+j-2)}a_{ij}$, where $i, j=1, 2, 3$. If the determinant of B is 81, then the determinant of A is: **[Jan. 7, 2020 (II)]**

(a) 1/3 (b) 3 (c) 1/81 (d) 1/9

25. A value of $\theta\in(0,\pi/3)$, for which

$$\begin{vmatrix}1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta\\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta\\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta\end{vmatrix}=0, \text{ is :}$$

[April 12, 2019 (II)]

(a) $\frac{\pi}{9}$ (b) $\frac{\pi}{18}$ (c) $\frac{7\pi}{24}$ (d) $\frac{7\pi}{36}$

26. Let α and β be the roots of the equation $x^2+x+1=0$. Then for y "' 0 in R, $\begin{vmatrix}y+1 & \alpha & \beta\\ \alpha & y+\beta & 1\\ \beta & 1 & y+\alpha\end{vmatrix}$ is equal to:

[April 09, 2019 (I)]

(a) $y(y^2-1)$ (b) $y(y^2-3)$
(c) y^3 (d) y^3-1

27. Let the numbers 2, b, c be in an A.P. and

$A=\begin{bmatrix}1 & 1 & 1\\ 2 & b & c\\ 4 & b^2 & c^2\end{bmatrix}$. If det(A)?[2, 16], then c lies in the interval : **[April 08, 2019 (II)]**

(a) $[2,3)$ (b) $(2+2^{3/4},4)$
(c) $[4,6]$ (d) $[3,2+2^{3/4}]$

28. If $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$; then for all $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, det (A) lies in the interval : **[Jan. 12, 2019 (II)]**

(a) $\left(1,\frac{5}{2}\right]$ (b) $\left[\frac{5}{2},4\right)$ (c) $\left(0,\frac{3}{2}\right]$ (d) $\left(\frac{3}{2},3\right]$

29. If $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$ $=(a+b+c)(x+a+b+c)^2$, $x\neq 0$ and $a+b+c\neq 0$, then x is equal to : **[Jan. 11, 2019 (II)]**

(a) abc (b) $-(a+b+c)$
(c) $2(a+b+c)$ (d) $-2(a+b+c)$

30. Let $d\in\mathbf{R}$, and

$$A=\begin{bmatrix}-2 & 4+d & (\sin\theta)^{-2}\\ 1 & (\sin\theta)+2 & d\\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d\end{bmatrix},$$

$\theta\in[0, 2\pi]$. If the minimum value of det (A) is 8, then a value of d is: **[Jan 10, 2019 (I)]**

(a) -5 (b) -7
(c) $2(\sqrt{2}+1)$ (d) $2(\sqrt{2}+2)$

31. Let $a_1, a_2, a_3, ..., a_{10}$ be in G.P. with $a_i>0$ for $i=1, 2, ..., 10$ and S be the set of pairs (r, k), r, k$\in$N (the set of natural numbers) for which

$$\begin{vmatrix}\log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k\\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k\\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k\end{vmatrix}=0$$

Then the number of elements in S, is : **[Jan. 10, 2019 (II)]**

(a) 4 (b) infinitely many
(c) 2 (d) 10

32. If

$$A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t-e^{-t}\sin t & -e^{-t}\sin t+e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

then A is: **[Jan. 09, 2019 (II)]**

(a) invertible for all $t\in\mathbf{R}$.

(b) invertible only if $t=\pi$.

(c) not invertible for any $t\in\mathbf{R}$.

(d) invertible only if $t=\frac{\pi}{2}$.

33. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[2017]**

(a) $\left(2,\frac{1}{2}\right)$ (b) $\left(2,-\frac{1}{2}\right)$ (c) $\left(1,\frac{3}{4}\right)$ (d) $\left(1,-\frac{3}{4}\right)$

34. Let ω be a complex number such that $2\omega+1=z$ where $z=\sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}=3k$, then k is equal to : **[2017]**

(a) 1 (b) –z (c) z (d) –1

35. The number of distinct real roots of the equaiton,

$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix}=0$ in the interval $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ is : **[Online April 9, 2016]**

(a) 1 (b) 4 (c) 2 (d) 3

36. If $\alpha,\beta\neq 0$, and $f(n)=\alpha^n+\beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}=K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2,$$

then K is equal to: **[2014]**

(a) 1 (b) –1 (c) αβ (d) $\frac{1}{\alpha\beta}$

37. If $\Delta_r=\begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n-4) \end{vmatrix}$

then the value of $\sum_{r=1}^{n-1}\Delta_r$ **[Online April 19, 2014]**

(a) depends only on a

(b) depends only on n

(c) depends both on a and n

(d) is independent of both a and n

38. If

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}=k\lambda\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix},\lambda\neq 0$$

then k is equal to: **[Online April 12, 2014]**

(a) $4\lambda abc$ (b) $-4\lambda abc$ (c) $4\lambda^2$ (d) $-4\lambda^2$

39. If a, b, c are sides of a scalene triangle, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is : **[Online April 9, 2013]**

(a) non - negative (b) negative

(c) positive (d) non-positive

40. If a, b, c, are non zero complex numbers satisfying $a^2+b^2+c^2=0$ and

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}=ka^2b^2c^2,$$ then k is equal to

[Online May 19, 2012]

(a) 1 (b) 3 (c) 4 (d) 2

41. If $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}=\alpha(a+b)(b+c)(c+a)\neq 0$

then α is equal to **[Online May 12, 2012]**

(a) $a+b+c$ (b) abc

(c) 4 (d) 1

42. The area of the triangle whose vertices are complex numbers $z, iz, z+iz$ in the Argand diagram is **[Online May 12, 2012]**

(a) $2|z|^2$ (b) $1/2|z|^2$ (c) $4|z|^2$ (d) $|z|^2$

43. The area of triangle formed by the lines joining the vertex of the parabola, $x^2=8y$, to the extremities of its latus rectum is **[Online May 12, 2012]**

(a) 2 (b) 8 (c) 1 (d) 4

44. Let a, b, c be such that $b(a+c)\neq 0$ if **[2009]**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}+\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix}=0,$$

then the value of n is :

(a) any even integer (b) any odd integer

(c) any integer (d) zero

45. If $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is

(a) divisible by x but not y **[2007]**
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y

46. If a_1, a_2, a_3, ..., a_n, ... are in G. P., then the determinant

$$\Delta = \begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}$$

is equal to **[2005]**
(a) 1 (b) 0 (c) 4 (d) 2

47. If $a^2+b^2+c^2=-2$ and **[2005]**

$$f(x)=\begin{vmatrix}1+a^2x & (1+b^2)x & (1+c^2)x\\ (1+a^2)x & 1+b^2x & (1+c^2)x\\ (1+a^2)x & (1+b^2)x & 1+c^2x\end{vmatrix},$$

then f (x) is a polynomial of degree
(a) 1 (b) 0 (c) 3 (d) 2

48. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots.$ are in G.P., then the value of the determinant **[2004]**

$$\begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}, \text{ is}$$

(a) –2 (b) 1
(c) 2 (d) 0

49. If $a > 0$ and discriminant of $ax^2+2bx+c$ is –ve, then

$$\begin{vmatrix}a & b & ax+b\\ b & c & bx+c\\ ax+b & bx+c & 0\end{vmatrix} \text{ is equal to}$$ **[2002]**

(a) +ve (b) $(ac-b^2)(ax^2+2bx+c)$
(c) –ve (d) 0

50. l, m, n are the p^{th}, q^{th} and r^{th} term of a G. P. all positive, then $\begin{vmatrix}\log l & p & 1\\ \log m & q & 1\\ \log n & r & 1\end{vmatrix}$ equals **[2002]**

(a) –1 (b) 2
(c) 1 (d) 0

TOPIC 3 Adjoint of a Matrix, Inverse of a Matrix, Some Special Cases of Matrix, Rank of a Matrix

51. Let A be a 3×3 matrix such that adj $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 0 & 2\\ 1 & -2 & -1\end{bmatrix}$ and $B=\text{adj}(\text{adj } A)$. If $|A|=\lambda$ and $|(B^{-1})^T|=\mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to : **[Sep. 03, 2020 (II)]**

(a) $\left(3, \frac{1}{81}\right)$ (b) $\left(9, \frac{1}{9}\right)$
(c) $(3, 81)$ (d) $\left(9, \frac{1}{81}\right)$

52. If the matrices $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, B = adj A and C = 3A, then $\frac{|\text{adj } B|}{|C|}$ is equal to : **[Jan. 9, 2020 (I)]**

(a) 8 (b) 16 (c) 72 (d) 2

53. If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all values of α for which det (A) + 1 = 0, is : **[April 12, 2019 (I)]**

(a) 0 (b) –1 (c) 1 (d) 2

54. If $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}\ldots\ldots\ldots\begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ is: **[April 09, 2019 (II)]**

(a) $\begin{bmatrix}1 & 0\\ 12 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & -13\\ 0 & 1\end{bmatrix}$
(c) $\begin{bmatrix}1 & -12\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 13 & 1\end{bmatrix}$

55. Let A and B be two invertible matrices of order 3 × 3. If det $(ABA^T)=8$ and det $(AB^{-1})=8$, then det $(BA^{-1}B^T)$ is equal to : **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{4}$ (b) 1
(c) $\frac{1}{16}$ (d) 16

56. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then the matrix A^{-50} when $\theta=\frac{\pi}{12}$, is equal to: **[Jan 09, 2019 (I)]**

(a) $\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

(c) $\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$ (d) $\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$

57. Let A be a matrix such that $A.\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and $|3A|=108$. Then A^2 equals **[Online April 15, 2018]**

(a) $\begin{bmatrix}4 & -32\\ 0 & 36\end{bmatrix}$ (b) $\begin{bmatrix}4 & 0\\ -32 & 36\end{bmatrix}$

(c) $\begin{bmatrix}36 & 0\\ -32 & 4\end{bmatrix}$ (d) $\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$

58. Suppose A is any 3×3 non-singular matrix and $(A-3I)(A-5I)=O$, where $I=I_3$ and $O=O_3$. If $\alpha A+\beta A^{-1}=4I$, then $\alpha+\beta$ is equal to **[Online April 15, 2018]**

(a) 8 (b) 12 (c) 13 (d) 7

59. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$, then adj $(3A^2+12A)$ is equal to : **[2017]**

(a) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$ (b) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$

(c) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$ (d) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$

60. Let A be any 3×3 invertible matrix. Then which one of the following is not always true ? **[Online April 8, 2017]**

(a) adj (A)= |A| . A^{-1}
(b) adj (adj(A)) = |A|.A
(c) adj (adj(A)) = $|A|^2$.$(\text{adj}(A))^{-1}$
(d) adj (adj(A)) = |A|.$(\text{adj}(A))^{-1}$

61. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to: **[2016]**

(a) 4 (b) 13 (c) –1 (d) 5

62. Let A be a 3×3 matrix such that $A^2-5A+7I=0$.

Statement–I : $A^{-1}=\frac{1}{7}(5I-A)$.

Statement II : the polynomial $A^3-2A^2-3A+\alpha$ can be reduced to $5(A-4I)$. **[Online April 10, 2016]**

Then :

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement–I is true, but Statement-II is false.
(d) Statement I is false, but Statement-II is true.

63. If A is a 3×3 matrix such that |5.adjA| = 5, then |A| is equal to : **[Online April 11, 2015]**

(a) $\pm\frac{1}{5}$ (b) $\pm\frac{1}{25}$ (c) ±1 (d) ±5

64. If A is an 3×3 non-singular matrix such that $AA'=A'A$ and $B=A^{-1}A'$, then BB' equals: **[2014]**

(a) B^{-1} (b) $(B^{-1})'$ (c) $I+B$ (d) I

65. Let A be a 3×3 matrix such that

$$A\begin{bmatrix}1 & 2 & 3\\ 0 & 2 & 3\\ 0 & 1 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

Then A^{-1} is: **[Online April 11, 2014]**

(a) $\begin{bmatrix}3 & 1 & 2\\ 3 & 0 & 2\\ 1 & 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}3 & 2 & 1\\ 3 & 2 & 0\\ 1 & 1 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1 & 3\\ 0 & 2 & 3\\ 1 & 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 1\\ 0 & 2 & 3\end{bmatrix}$

66. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and |A| = 4, then α is equal to : **[2013]**

(a) 4 (b) 11 (c) 5 (d) 0

67. Let P and Q be 3×3 matrices $P\neq Q$. If $P^3=Q^3$ and $P^2Q=Q^2P$ then determinant of (P^2+Q^2) is equal to : **[2012]**

(a) – 2 (b) 1
(c) 0 (d) – 1

68. Let $A=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2 & 1\end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1=\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\ 1\\ 0\end{pmatrix}$, then u_1+u_2 is equal to : **[2012]**

(a) $\begin{pmatrix}-1\\ 1\\ 0\end{pmatrix}$ (b) $\begin{pmatrix}-1\\ 1\\ -1\end{pmatrix}$ (c) $\begin{pmatrix}-1\\ -1\\ 0\end{pmatrix}$ (d) $\begin{pmatrix}1\\ -1\\ -1\end{pmatrix}$

69. If A^T denotes the transpose of the matrix $A=\begin{bmatrix}0 & 0 & a\\ 0 & b & c\\ d & e & f\end{bmatrix}$, where a, b, c, d, e and f are integers such that $abd \neq 0$, then the number of such matrices for which $A^{-1}=A^T$ is **[Online May 19, 2012]**
(a) 2(3!) (b) 3(2!) (c) 2^3 (d) 3^2

70. Let A and B be real matrices of the form $\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix}$ and $\begin{bmatrix}0 & \gamma\\ \delta & 0\end{bmatrix}$, respectively. **[Online May 12, 2012]**
Statement 1: $AB-BA$ is always an invertible matrix.
Statement 2: $AB-BA$ is never an identity matrix.
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

71. Consider the following relation R on the set of real square matrices of order 3. **[2011RS]**
$R=\{(A,B) \mid A=P^{-1}BP \text{ for some invertible matrix } P\}$
Statement-1 : R is equivalence relation.
Statement-2 : For any two invertible 3×3 matrices M and N, $(MN)^{-1}=N^{-1}M^{-1}$.
(a) Statement-1 is true, statement-2 is true and statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(c) Statement-1 is true, stement-2 is false.
(d) Statement-1 is false, statement-2 is true.

72. Let A be a 2×2 matrix
Statement -1 : adj (adj A) = A
Statement -2 : |adj A| = |A| **[2009]**
(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement -1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

73. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? **[2008]**
(a) If det $A=\pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(b) If det $A\neq\pm 1$, then A^{-1} exists and all its entries are non integers
(c) If det $A=\pm 1$, then A^{-1} exists but all its entries are integers
(d) If det $A=\pm 1$, then A^{-1} need not exists

74. If $A^2-A+I=0$, then the inverse of A is **[2005]**
(a) $A+I$ (b) A (c) $A-I$ (d) $I-A$

75. Let $A=\begin{pmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{pmatrix}$. and $B=\begin{pmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{pmatrix}$. If B is the inverse of matrix A, then α is **[2004]**
(a) 5 (b) –1 (c) 2 (d) –2

76. Let $A=\begin{pmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{pmatrix}$. The only correct statement about the matrix A is **[2004]**
(a) $A^2=I$
(b) $A=(-1)I$, where I is a unit matrix
(c) A^{-1} does not exist
(d) A is a zero matrix

TOPIC 4 Solution of System of Linear Equations

77. The values of λ and μ for which the system of linear equations **[Sep. 06, 2020 (I)]**
$x+y+z=2$
$x+2y+3z=5$
$x+3y+\lambda z=\mu$
has infinitely many solutions are, respectively :
(a) 6 and 8 (b) 5 and 7
(c) 5 and 8 (d) 4 and 9

78. The sum of distinct values of λ for whcih the system of equations
$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$
$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$
$2x+(3\lambda+1)y+3(\lambda-1)z=0,$
has non-zero solutions, is ______. **[NA Sep. 06, 2020 (II)]**

79. Let $\lambda\in\mathbb{R}$. The system of linear equations
$2x_1-4x_2+\lambda x_3=1$ **[Sep. 05, 2020 (I)]**
$x_1-6x_2+x_3=2$
$\lambda x_1-10x_2+4x_3=3$
(a) exactly one negative value of λ
(b) exactly one positive value of λ
(c) every value of λ
(d) exactly two value of λ

80. If the system of linear equations

$x+y+3z=0$

$x+3y+k^2z=0$

$3x+y+3z=0$

has a non-zero solution (x, y, z) for some $k \in \mathbf{R}$, then $x+\left(\frac{y}{z}\right)$ is equal to : **[Sep. 05, 2020 (II)]**

(a) -3 (b) 9 (c) 3 (d) -9

81. If the system of equations $x-2y+3z=9$, $2x+y+z=b$, $x-7y+az=24$, has infinitely many solutions, then $a-b$ is equal to __________. **[NA Sep. 04, 2020 (I)]**

82. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1=\begin{bmatrix}1\\1\\1\end{bmatrix}$, $x_2=\begin{bmatrix}0\\2\\1\end{bmatrix}$, $x_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $b_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $b_2=\begin{bmatrix}0\\2\\0\end{bmatrix}$ and $b_3=\begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to : **[Sep. 04, 2020 (II)]**

(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{3}{2}$

83. If the system of equations

$x+y+z=2$

$2x+4y-z=6$

$3x+2y+\lambda z=\mu$

has infinitely many solutions, then : **[Sep. 04, 2020 (II)]**

(a) $\lambda+2\mu=14$ (b) $2\lambda-\mu=5$

(c) $\lambda-2\mu=-5$ (d) $2\lambda+\mu=14$

84. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x-2y+5z=0$

$-2x+4y+z=0$

$-7x+14y+9z=0$

such that $15 \le x^2+y^2+z^2 \le 150$. Then, the number of elements in the set S is equal to ____________. **[NA Sep. 03, 2020 (II)]**

85. Let S be the set of all $\lambda \in \mathbf{R}$ for which the system of linear equations **[Sep. 02, 2020 (I)]**

$2x-y+2z=2$

$x-2y+\lambda z=-4$

$x+\lambda y+z=4$

has no solution. Then the set S

(a) contains more than two elements.

(b) is an empty set.

(c) is a singleton.

(d) contains exactly two elements.

86. Let $A = \{X=(x,\ y,\ z)^T : PX=0 \text{ and } x^2+y^2+z^2=1\}$,

where $P=\begin{bmatrix}1 & 2 & 1\\-2 & 3 & -4\\1 & 9 & -1\end{bmatrix}$, then the set A : **[Sep. 02, 2020 (II)]**

(a) is a singleton

(b) is an empty set

(c) contains more than two elements

(d) contains exactly two elements

87. The following system of linear equations

$7x+6y-2z=0$

$3x+4y+2z=0$

$x-2y-6z=0$, has **[Jan. 9, 2020 (II)]**

(a) infinitely many solutions, (x, y, z) satisfying $y=2z$.

(b) no solution.

(c) infinitely many solutions, (x, y, z) satisfying $x=2z$.

(d) only the trivial solution.

88. For which of the following ordered pairs (μ, δ), the system of linear equations

$x+2y+3z=1$

$3x+4y+5z=\mu$

$4x+4y+4z=\delta$

is inconsistent? **[Jan. 8, 2020 (I)]**

(a) $(4, 3)$ (b) $(4, 6)$

(c) $(1, 0)$ (d) $(3, 4)$

89. The system of linear equations

$\lambda x+2y+2z=5$

$2\lambda x+3y+5z=8$

$4x+\lambda y+6z=10$ has: **[Jan. 8, 2020 (II)]**

(a) no solution when $\lambda=8$

(b) a unique solution when $\lambda=-8$

(c) no solution when $\lambda=2$

(d) infinitely many solutions when $\lambda=2$

90. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0,$

where $a, b, c \in \boldsymbol{R}$ are non-zero and distinct; has a non-zero solution, then: **[Jan. 7, 2020 (I)]**

(a) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(b) a, b, c are in G.P.

(c) $a + b + c = 0$

(d) a, b, c are in A.P.

91. If the system of linear equations,

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more than two solutions, then $\mu - \lambda^2$ is equal to _________. **[NA Jan. 7, 2020 (II)]**

92. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in \mathbf{R})$, has infinitely many solutions, then the value of $\lambda + \mu$ is : **[April 10, 2019 (I)]**

(a) 12 (b) 9 (c) 7 (d) 10

93. Let λ be a real number for which the system of linear equations:

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation : **[April 10, 2019 (II)]**

(a) $\lambda^2 + 3\lambda - 4 = 0$ (b) $\lambda^2 - 3\lambda - 4 = 0$

(c) $\lambda^2 + \lambda - 6 = 0$ (d) $\lambda^2 - \lambda - 6 = 0$

94. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to: **[April 09, 2019 (II)]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) -4

95. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$; $cx - y + cz = 0$; $cx + cy - z = 0$

has a non-trivial solution, is : **[April 08, 2019 (I)]**

(a) -1 (b) $\frac{1}{2}$ (c) 2 (d) 0

96. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is : **[April 08, 2019 (II)]**

(a) $3x - 4y - 1 = 0$ (b) $4x - 3y - 4 = 0$

(c) $4x - 3y - 1 = 0$ (d) $3x - 4y - 4 = 0$

97. An ordered pair (α, β) for which the system of linear equations

$(1 + \alpha)x + \beta y + z = 2$

$ax + (1 + \beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has a unique solution, is : **[Jan. 12, 2019 (I)]**

(a) $(2, 4)$ (b) $(-3, 1)$

(c) $(-4, 2)$ (d) $(1, -3)$

98. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda 2$

has a non-trivial solution : **[Jan. 12, 2019 (II)]**

(a) is a singleton

(b) contains exactly two elements

(c) is an empty set

(d) contains more than two elements

99. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where, a, b, c are non-zero real numbers, has more than one solution, then : **[Jan. 11, 2019 (I)]**

(a) $b - c + a = 0$ (b) $b - c - a = 0$

(c) $a + b + c = 0$ (d) $b + c - a = 0$

100. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$

has a non-trivial solution, is: **[Jan. 10, 2019 (II)]**

(a) three (b) two

(c) four (d) one

101. If the system of equations **[Jan 10, 2019 (I)]**

$$x+y+z=5$$
$$x+2y+3z=9$$
$$x+3y+\alpha z=\beta$$

has infinitely many solutions, then $\beta-\alpha$ equals:

(a) 21 (b) 8 (c) 18 (d) 5

102. If the system of linear equations

$$x-4y+7z=g$$
$$3y-5z=h$$
$$-2x+5y-9z=k$$

is consistent, then : **[Jan. 09, 2019 (II)]**

(a) $g+2h+k=0$
(b) $g+h+2k=0$
(c) $2g+h+k=0$
(d) $g+h+k=0$

103. If the system of linear equations

$$x+ky+3z=0$$
$$3x+ky-2z=0$$
$$2x+4y-3z=0$$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to : **[2018]**

(a) 10 (b) –30 (c) 30 (d) –10

104. The number of values of k for which the system of linear equations, $(k+2)x+10y=k$, $kx+(k+3)y=k-1$ has no solution, is **[Online April 16, 2018]**

(a) Infinitely many (b) 3
(c) 1 (d) 2

105. Let S be the set of all real values of k for which the system of linear equations

$$x+y+z=2$$
$$2x+y-z=3$$
$$3x+2y+kz=4$$

has a unique solution. Then S is **[Online April 15, 2018]**

(a) an empty set (b) equal to $R-\{0\}$
(c) equal to $\{0\}$ (d) equal to R

106. If the system of linear equations

$$x+ay+z=3$$
$$x+2y+2z=6$$
$$x+5y+3z=b$$

has no solution, then **[Online April 15, 2018]**

(a) $a=1, b\neq 9$ (b) $a\neq -1, b=9$
(c) $a=-1, b=9$ (d) $a=-1, b\neq 9$

107. If S is the set of distinct values of 'b' for which the following system of linear equations **[2017]**

$$x+y+z=1$$
$$x+ay+z=1$$
$$ax+by+z=0$$

has no solution, then S is :

(a) a singleton
(b) an empty set
(c) an infinite set
(d) a finite set containing two or more elements

108. The number of real values of λ for which the system of linear equations

$$2x+4y-\lambda z=0$$
$$4x+\lambda y+2z=0$$
$$\lambda x+2y+2z=0$$

has infinitely many solutions, is : **[Online April 8, 2017]**

(a) 0 (b) 1 (c) 2 (d) 3

109. The system of linear equations

$$x+\lambda y-z=0$$
$$\lambda x-y-z=0$$
$$x+y-\lambda z=0$$

has a non-trivial solution for: **[2016]**

(a) exactly two values of λ.
(b) exactly three values of λ.
(c) infinitely many values of λ.
(d) exactly one value of λ.

110. The set of all values of λ for which the system of linear equations : **[2015]**

$$2x_1-2x_2+x_3=\lambda x_1$$
$$2x_1-3x_2+2x_3=\lambda x_2$$
$$-x_1+2x_2=\lambda x_3$$

has a non-trivial solution,

(a) contains two elements.
(b) contains more than two elements
(c) is an empty set.
(d) is a singleton

111. If a, b, c are non-zero real numbers and if the system of equations **[Online April 9, 2014]**

$$(a-1)x=y+z,$$
$$(b-1)y=z+x,$$
$$(c-1)z=x+y,$$

has a non-trivial solution, then ab + bc + ca equals:

(a) $a+b+c$ (b) abc
(c) 1 (d) -1

112. The number of values of k, for which the system of equations:

$$(k+1)x+8y=4k$$
$$kx+(k+3)y=3k-1$$

has no solution, is **[2013]**

(a) infinite (b) 1
(c) 2 (d) 3

113. Consider the system of equations :

$x+ay=0$, $y+az=0$ and $z+ax=0$. Then the set of all real values of 'a' for which the system has a unique solution is: **[Online April 25, 2013]**

(a) $R-\{1\}$ (b) $R-\{-1\}$
(c) $\{1,-1\}$ (d) $\{1,0,-1\}$

114. **Statement-1:** The system of linear equations

$x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$x - (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution for only one value of α lying in the interval $\left(0, \frac{\pi}{2}\right)$.

Statement-2: The equation in α

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

has only one solution lying in the interval $\left(0, \frac{\pi}{2}\right)$.

[Online April 23, 2013]

(a) Statement-1 is true, Statement-2 is true, Statement-2 is **not** correct explantion for Statement-1.

(b) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explantion for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statememt-1 is false, Statement-2 is true.

115. If the system of linear equations :

$x_1 + 2x_2 + 3x_3 = 6$

$x_1 + 3x_2 + 5x_3 = 9$

$2x_1 + 5x_2 + ax_3 = b$

is consistent and has infinite number of solutions, then : **[Online April 22, 2013]**

(a) $a = 8$, b can be any real number

(b) $b = 15$, a can be any real number

(c) $a \in R - \{8\}$ and $b \in R - \{15\}$

(d) $a = 8, b = 15$

116. **Statement 1:** If the system of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ has a non-trivial solution, then the value of k is $\frac{31}{2}$.

Statement 2: A system of three homogeneous equations in three variables has a non trivial solution if the determinant of the coefficient matrix is zero. **[Online May 26, 2012]**

(a) Statement 1 is false, Statement 2 is true.

(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(c) Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1.

(d) Statement 1 is true, Statement 2 is false.

117. If the system of equations **[Online May 7, 2012]**

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = 0$

has a unique solution, then λ is not equal to

(a) 1 (b) 0 (c) 2 (d) 3

118. If the trivial solution is the only solution of the system of equations **[2011RS]**

$x - ky + z = 0$

$kx + 3y - kz = 0$

$3x + y - z = 0$

then the set of all values of k is :

(a) $R - \{2, -3\}$ (b) $R - \{2\}$

(c) $R - \{-3\}$ (d) $\{2, -3\}$

119. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is **[2011]**

(a) 2 (b) 1 (c) zero (d) 3

120. Consider the system of linear equations; **[2010]**

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(a) exactly 3 solutions

(b) a unique solution

(c) no solution

(d) infinite number of solutions

121. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) 2 (b) –1 **[2008]**

(c) 0 (d) 1

122. The system of equations

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has infinite solutions, if α is **[2005]**

(a) – 2 (b) either – 2 or 1

(c) not – 2 (d) 1

123. If the system of linear equations **[2003]**

$x + 2ay + az = 0$; $x + 3by + bz = 0$;

$x + 4cy + cz = 0$ has a non - zero solution, then a, b, c.

(a) satisfy $a + 2b + 3c = 0$

(b) are in A.P

(c) are in G.P

(d) are in H.P.

Hints & Solutions

1. **(d)** $\because A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\therefore A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}, n\in\mathbf{N}$

$\therefore B = A + A^4$

$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\begin{bmatrix}\cos 4\theta & \sin 4\theta\\ -\sin 4\theta & \cos 4\theta\end{bmatrix}$

$\therefore B=\begin{bmatrix}\cos\frac{\pi}{5}+\cos\frac{4\pi}{5} & \sin\frac{\pi}{5}+\sin\frac{4\pi}{5}\\ -\sin\frac{\pi}{5}-\sin\frac{4\pi}{5} & \cos\frac{\pi}{5}+\cos\frac{4\pi}{5}\end{bmatrix}$

Then, $\det(B)=2\sin\left(\frac{\pi}{5}\right)\cdot\begin{vmatrix}0 & 1\\ -1 & 0\end{vmatrix}$

$=\frac{\sqrt{10-2\sqrt{5}}}{2}\approx\frac{2.35}{2}\approx 1.175$

$\therefore \det B\in(1,2)$

2. **(c)** $\Delta=\begin{vmatrix}x-2 & 2x-3 & 3x-4\\ 2x-3 & 3x-4 & 4x-5\\ 3x-5 & 5x-8 & 10x-17\end{vmatrix}$

$\Rightarrow\Delta=\begin{vmatrix}x-2 & x-1 & x-1\\ 2x-3 & x-1 & x-1\\ 3x-5 & 2x-3 & 5x-9\end{vmatrix}\quad\begin{bmatrix}C_3\to C_3-C_2\\ C_2\to C_2-C_1\end{bmatrix}$

$\Rightarrow\Delta=\begin{vmatrix}x-2 & x-1 & x-1\\ x-1 & 0 & 0\\ 3x-5 & 2x-3 & 5x-9\end{vmatrix}\quad[R_2\to R_2-R_1]$

$\Rightarrow\Delta=-(x-1)[(x-1)(5x-9)-(x-1)(2x-3)]$

$\Rightarrow\Delta=-(x-1)[(5x^2-14x+9)-(2x^2-5x+3)]$

$=-3x^3+12x^2-15x+6$

So, $B + C = -3$

3. **(d)** If $f(x)=\begin{vmatrix}x+a & x+2 & x+1\\ x+b & x+3 & x+2\\ x+c & x+4 & x+3\end{vmatrix}$

$R_1=R_1+R_3-2R_2$

$\Rightarrow\quad f(x)=\begin{vmatrix}1 & 0 & 0\\ x+b & x+3 & x+2\\ x+c & x+4 & x+3\end{vmatrix}$

$\Rightarrow\quad f(x)=1\ \Rightarrow\ f(50)=1$

4. **(d)** $\Delta_1=\begin{vmatrix}x & \sin\theta & \cos\theta\\ -\sin\theta & -x & 1\\ \cos\theta & 1 & x\end{vmatrix}$

$=(x-x^2-1)-\sin\theta(-x\sin\theta-\cos\theta)$
$+\cos\theta(-\sin\theta+x\cos\theta)$

$=-x^3-x+x\sin^2\theta+\sin\theta\cos\theta-\cos\theta\sin\theta+x\cos^2\theta$

$=-x^3-x+x=-x^3$

Similarly, $\Delta_2=-x^3$ Then, $\Delta_1+\Delta_2=-2x^3$

5. **(b)** Given $\begin{vmatrix}x & -6 & -1\\ 2 & -3x & x-3\\ -3 & 2x & x+2\end{vmatrix}=0$

On expanding,

$x(-3x^2-6x-2x^2+6x)-6(-3x+9-2x-4)$
$-(4x-9x)=0$

$\Rightarrow x(-5x^2)-6(-5x+5)-4x+9x=0$

$\Rightarrow x^3-7x+6=0$

$\because$ all the roots are real.

$\therefore$ sum of real roots $=\frac{0}{1}=0$

6. **(a)** $|A|=\begin{vmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{vmatrix}$

$=2(2b^2+2-b^2)-b(2b-b)+1(b^2-b^2-1)$

$=2b^2+4-b^2-1=b^2+3$

$\frac{|A|}{b}=b+\frac{3}{b}$

$\because\quad \frac{b+\frac{3}{b}}{2}\ge\left(b\frac{3}{b}\right)^{\frac{1}{2}}\Rightarrow b+\frac{3}{b}\ge 2\sqrt{3}$

$\therefore\quad \frac{|A|}{b}\ge 2\sqrt{3}$

Minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$.

7. **(b)** Here, $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Put $x=0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A^3 = (-4)^3$

$\Rightarrow A = -4$

$\Rightarrow \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx-4)(x+4)^2$

Now take x common from both the sides

$\therefore \begin{vmatrix} 1-\frac{4}{x} & 2x & 2x \\ 2x & 1-\frac{4}{x} & 2x \\ 2x & 2x & 1-\frac{4}{x} \end{vmatrix} = (B-\frac{4}{x})(1+\frac{4}{x})^2$

Now take $x \to \infty$, then $\frac{1}{x} \to 0$

$\Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

$\therefore$ ordered pair (A, B) is (–4, 5)

8. **(c)** Since the given determinant is equal to zero.

$\Rightarrow$ $0(0-\cos x \sin x) - \cos x(0-\cos^2 x) - \sin x (\sin^2 x - 0) = 0$

$\Rightarrow$ $\cos^3 x - \sin^3 x = 0$

$\Rightarrow$ $\tan^3 = 1 \Rightarrow \tan x = 1$

$\therefore \sum_{x\in s} \tan\left(\frac{\pi}{3}+x\right) = \sum_{x\in s} \frac{\tan \pi/3 + \tan x}{1-\tan \pi/3 . \tan x}$

$\sum_{x\in s} \frac{\sqrt{3}+1}{1-\sqrt{3}}$

$\sum_{x\in s} \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \Rightarrow \sum_{x\in s} \frac{1+3+2\sqrt{3}}{-2}$

$= -2-\sqrt{3}$

9. **(d)** $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix}$ and $|A| = 1$.

Now, $A^{2016} - 2A^{2015} - A^{2014} = A^{2014}(A^2 - 2A - I)$

$\Rightarrow |A^{2016} - 2A^{2015} - A^{2014}| = |A^{2014}||A^2 - 2A - I|$

$= |A|^{2014} \begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = -25$

10. **(a)** Let $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax - 12$

Put $x = -1$, we get

$\begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & 0 \\ 2 & -3 & -3 \end{vmatrix} = -a - 12$

$\Rightarrow -3(6+6) = -a - 12 \Rightarrow -36 + 12 = a$

$\Rightarrow a = 24$

11. **(d)** $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} \geq 0$

$xyz - x - y - z + 2 \geq 0$

$xyz + 2 \geq x + y + z \geq 3(xyz)^{1/3}$

$xyz + 2 - 3(xyz)^{1/3} \geq 0$

$ut(xyz) = t^3$

$t^3 - 3t + 2 \geq 0$

$(t+2)(t-1)^2 \geq 0$

$[t = -2]\ t^3 = -8$

12. **(c)** Let $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$

$= (1+\sin\theta\cos\theta) - \cos\theta(-\sin\theta - \cos\theta) + 1(-\sin^2\theta + 1)$

$= 1 + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta - \sin^2\theta + 1$

$= 2 + 2\sin\theta\cos\theta + \cos 2\theta$

$= 2 + \sin 2\theta + \cos 2\theta$ ...(1)

Now, maximum value of (1)

is $2 + \sqrt{1^2 + 1^2} = 2 + \sqrt{2}$

and minimum value of (1) is

$2 - \sqrt{1^2 + 1^2} = 2 - \sqrt{2}$.

13. **(a)** det $[(I + B)^{50} - 50B]$

$= \det [{}^{50}C_0 I + {}^{50}C_1 B + {}^{50}C_2 B^2 + {}^{50}C_3 B^3 + \ldots + {}^{50}C_{50} B^{50} B^{50} - 50B]$

{All terms having B^n, $2 \leq n \leq 50$ will be zero because given that $B^2 = 0$}

$= \det [I + 50B - 50B] = \det [I] = 1$

14. **(d)** The matrices in the form

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, a_{ij} \in \{0, 1, 2\}, a_{11} = a_{12} \text{ are}$$

$$\begin{bmatrix} 0 & 0/1/2 \\ 0/1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0/1/2 \\ 0/1/2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0/1/2 \\ 0/1/2 & 2 \end{bmatrix}$$

At any place, 0/1/2 means 0, 1 or 2 will be the element at that place.

Hence there are total $27 = 3 \times 3 + 3 \times 3 + 3 \times 3)$ matrices of the above form. Out of which the matrices which are singular are

$$\begin{bmatrix} 0 & 0/1/2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Hence there are total $7(= 3 + 2 + 1 + 1)$ singular matrices.

Therefore number of all non-singular matrices in the given form $= 27 - 7 = 20$

15. **(b)** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$b(a + d) = 0, b = 0$ or $a = -d$ ...(1)

$c(a + d) = 0, c = 0$ or $a = -d$ ...(2)

$a^2 + bc = 1, bc + d^2 = 1$ ...(3)

'a' and 'd' are diagonal elements $a + d = 0$ statement-1 is correct.

Now, $\det(A) = ad - bc$

Now, from (3) $a^2 + bc = 1$ and $d^2 + bc = 1$

So, $a^2 - d^2 = 0$

Adding $a^2 + d^2 + 2bc = 2$

$\Rightarrow (a + d)^2 - 2ad + 2bc = 2$

or $0 - 2(ad - bc) = 2$

So, $ad - bc = 1 \Rightarrow \det(A) = -1$

So, statement – 2 is also true.

But statement – 2 is not the correct explanation of statement-1.

16. **(d)** We know that determinant of skew symmetric matrix of odd order is zero.

So, statement-1 is true.

We know that $\det\left(A^T\right) = \det(A)$.

$\det(-A) = -(-1)^n \det(A)$.

where A is a $n \times n$ order matrix.

So, statement-2 is false.

17. **(b)** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d $\neq 0$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$

$ab + bd = ac + cd = 0$

$c \neq 0$ and $b \neq 0 \Rightarrow a + d = 0$

$|A| = ad - bc = -a^2 - bc = -1$

Also if $A \neq I$, then $tr(A) = a + d = 0$.

$\therefore$ Statement-1 true and statement-2 false.

18. **(d)** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given that $A^2 = I$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $a^2 + bc = 1$ and $ab + bd = 0$

$ac + cd = 0$ and $bc + d^2 = 1$

From these four equations,

$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$

and $b(a + d) = 0 = c(a + d) \Rightarrow a = -d$

$|A| = ad - bc = -a^2 - bc = -1$

Also if $A \neq I$ then $tr(A) = a + d = 0$

$\therefore$ Statement 2 is false.

19. **(a)** Given that $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$

$$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$\therefore |A^2| = 25\ (25\alpha^2)$

$\therefore 25 = 25\ (25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$

20. **(b)** $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

Expand through R_1

$= 1\left(\omega^{3n}-1\right)-\omega^{n}\left(\omega^{2n}-\omega^{2n}\right)+\omega^{2n}\left(\omega^{n}-\omega^{4n}\right)$

$= \omega^{3n}-1-0+\omega^{3n}-\omega^{6n}$

$= 1-1+1-1=0\left[\because \omega^{3n}=1\right]$

21. **(b)** Applying $C_2 \to C_2 - C_1$

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1 & 1\\ -\cos^2\theta & -1 & 1\\ 12 & -2 & -2\end{vmatrix}$$

$= 4(\cos^2\theta-\sin^2\theta)$

$= 4\cos 2\theta,\ \theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

Max. $f(\theta)=M=0$

Min. $f(\theta)=m=-4$

So, $(m, M)=(-4, 0)$

22. **(b)** Use properties of determinant

$$\begin{vmatrix}x & a+y & x+a\\ y & b+y & y+b\\ z & c+y & z+c\end{vmatrix}=\begin{vmatrix}x & a & x+a\\ y & b & y+b\\ z & c & z+c\end{vmatrix}+y\begin{vmatrix}x & 1 & x+a\\ y & 1 & y+b\\ z & 1 & z+c\end{vmatrix}$$

$$=0+y\begin{vmatrix}x & 1 & x+a\\ y-x & 0 & 0\\ z-x & 0 & -1\end{vmatrix}\qquad\begin{bmatrix}R_2\to R_2-R_1,\\ R_3\to R_3-R_1\end{bmatrix}$$

$= -y(x-y)=-y(b-a)=y(a-b)$

23. **(b)** $D=\frac{1}{2}\begin{vmatrix}0 & 2 & 1\\ 1 & -1 & 1\\ x' & y' & 1\end{vmatrix}=5$

$\Rightarrow\ -2(1-x')+(y'+x')=\pm 10$

$\Rightarrow\ -2+2x'+y'+x'=\pm 10$

$\Rightarrow\ 3x'+y'=12$ or $3x'+y'=-8$

$\therefore\ \lambda=3,-2$

24. **(d)** It is given that $|B|=81$

$$\therefore\ |B|=\begin{vmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{vmatrix}=\begin{vmatrix}3^0a_{11} & 3^1a_{12} & 3^2a_{13}\\ 3^1a_{21} & 3^2a_{22} & 3^3a_{23}\\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33}\end{vmatrix}$$

$\Rightarrow\ 81=3^3\cdot 3^2\cdot 3^1|A|$

$\Rightarrow\ 3^4=3^6|A|\ \Rightarrow\ |A|=\frac{1}{9}$

25. **(a)** $C_1\to C_1+C_2$

$$\begin{vmatrix}2 & \sin^2\theta & 4\cos 6\theta\\ 2 & 1+\sin^2\theta & 4\cos 6\theta\\ 1 & \sin^2\theta & 1+4\cos 6\theta\end{vmatrix}=0$$

$R_1\to R_1-R_2,\ R_2\to R_2-R_3$

$$\begin{vmatrix}0 & -1 & 0\\ 1 & 1 & -1\\ 1 & \sin^2\theta & (1+4\cos 6\theta)\end{vmatrix}=0$$

On expanding, we get $2+4\cos 6\theta=0$

$\cos 6\theta=-\frac{1}{2}\ \because\ \theta\in\left(0,\frac{\pi}{3}\right)\Rightarrow\ 6\theta\ (0, 2\pi)$

Therefore, $6\theta=\frac{2\pi}{3}$ or $\frac{4\pi}{3}\ \Rightarrow\ \theta=\frac{\pi}{9}$ or $\frac{2\pi}{9}$

26. **(c)** Let $\alpha=\omega$ and $\beta=\omega^2$ are roots of $x^2+x+1=0$

$$\&\ Let\ \Delta=\begin{vmatrix}y+1 & \omega & \omega^2\\ \omega & y+\omega^2 & 1\\ \omega^2 & 1 & y+\omega\end{vmatrix}=\Delta$$

Applying $C_1\to C_1+C_2+C_3$

$$\Delta=\begin{vmatrix}y+1+\omega+\omega^2 & \omega & \omega^2\\ y+1+\omega+\omega^2 & y+\omega^2 & 1\\ 1+\omega+\omega^2+y & 1 & y+\omega\end{vmatrix}$$

$$\Delta=\begin{vmatrix}y & \omega & \omega^2\\ y & y+\omega^2 & 1\\ y & 1 & y+\omega\end{vmatrix}(\because 1+\omega+\omega^2=0)$$

$$\Delta=y\begin{vmatrix}1 & \omega & \omega^2\\ 1 & y+\omega^2 & 1\\ 1 & 1 & y+\omega\end{vmatrix}$$

Applying $R_2\to R_2-R_1\ \&\ R_3\to R_3-R_1$

$$\Delta=y\begin{vmatrix}y+\omega^2-\omega & 1-\omega^2\\ 1-\omega & y+\omega-\omega^2\end{vmatrix}$$

$\Rightarrow \Delta=y\left[y-(\omega-\omega^2)(y+(\omega-\omega^2)-(1-\omega)(1-\omega^2)\right]$

$\Rightarrow \Delta=y\left[y^2-(\omega-\omega^2)^2-1+\omega^2+\omega-\omega^3\right]$

$$\Rightarrow \Delta = y\left[y^2 - \omega^2 - \omega^4 + 2\omega^3 - 1 + \omega^2 + \omega^4 - \omega^3\right]$$

$(\because \omega^4 = \omega)$

$\Rightarrow \Delta = y\,(y^2) = y^3$

27. **(c)** Consider, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$$

$= (b-2)(c-2)(c-b)$

$\therefore$ 2, b, c are in A.P.

$\therefore$ $(b-2) = (c-b) = d$ and $c - 2 = 2d$

$\Rightarrow |A| = d.2d.d = 2d^3$

$\because |A| \in [2, 16] \Rightarrow 1 \le d^3 \le 8 \Rightarrow 1 \le d \le 2$

$4 \le 2d + 2 \le 6 \Rightarrow 4 \le c \le 6$

28. **(d)** $|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$$= \begin{vmatrix} 0 & 0 & 2 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} \quad R_1 \to R_1 + R_3$$

$= 2(\sin^2\theta + 1)$

Since, $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \sin^2\theta \in \left(0, \frac{1}{2}\right)$

$\therefore$ $\det(A) \in [2, 3)$

$[2, 3) \subset \left(\frac{3}{2}, 3\right]$

29. **(d)** $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$

$$\Delta = (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$$

$= (a+b+c)\,(a+b+c)^2$

Hence, $x = -2(a+b+c)$

30. **(a)** $\det(A) = \begin{vmatrix} -2 & 4+d & \sin\theta - 2 \\ 1 & \sin\theta + 2 & d \\ 5 & 2\sin\theta - d & -\sin\theta + 2 + 2d \end{vmatrix}$

Applying $R_3 \to R_3 - 2R_2 + R_1$ we get

$$\det(A) = \begin{vmatrix} -2 & 4+d & \sin\theta - 2 \\ 1 & \sin\theta + 2 & d \\ 1 & 0 & 0 \end{vmatrix}$$

$= \quad d(4+d) - (\sin^2\theta - 4)$

$\Rightarrow$ $\det(A) = d^2 + 4d + 4 - \sin^2\theta = (d+2)^2 - \sin^2\theta$

Minimum value of $\det(A)$ is attained when $\sin^2\theta = 1$

$\therefore$ $(d+2)^2 - 1 = 8 \Rightarrow (d+2)^2 = 9 \Rightarrow d + 2 = \pm 3$

$\Rightarrow$ $d = -5$ or 1

31. **(b)** Let common ratio of G.P. be R

$\Rightarrow$ $a_2 = a_1R, a_3 = a_1R^2, \dots a^{10} = a_1R^9$

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} \ln\left(\frac{a_1^r a_2^k}{a_2^r a_3^k}\right) & \ln\left(\frac{a_2^r a_3^k}{a_3^r a_4^k}\right) & \ln a_3^r a_4^k \\ \ln\left(\frac{a_4^r a_5^k}{a_5^r a_6^k}\right) & \ln\left(\frac{a_5^r a_6^k}{a_6^r a_7^k}\right) & \ln a_6^r a_7^k \\ \ln\frac{a_7^r a_8^k}{a_8^r a_9^k} & \ln\left(\frac{a_8^r a_9^k}{a_9^r a_{10}^k}\right) & \ln a_9^r a_{10}^k \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_3^r a_4^k \\ \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_6^r a_7^k \\ \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_9^r a_{10}^k \end{vmatrix} = 0$$

$\forall r, K \in N$

Hence, number of elements in S is infinitely many.

32. **(a)** $\det(A) = |A|$

$$= \begin{vmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{vmatrix}$$

$$= e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$= e^{-t} \begin{vmatrix} 0 & 2\cos t + \sin t & 2\sin t - \cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 + R_3 \end{matrix}$$

$$= e^{-t} \begin{vmatrix} 0 & -5\sin t & 5\cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} R_1 \to R_1 + 2R_2$$

$= e^{-t}[(-5 \sin t)(-\sin t + 3 \cos t) - 5 \cos t (-\cos t - 3 \sin t)$

$= 5e^{-t} \neq 0, \forall t \in R$

$\therefore A$ is invertible.

33. **(a)** Let A (k, –3k), B(5, k) and C(–k + 2), we have

$$\frac{1}{2}\left\|\begin{matrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{matrix}\right\| = 28$$

$\Rightarrow 5k^2 + 13k - 46 = 0$

or $5k^2 + 13k + 66 = 0$

Now, $5k^2 + 13k - 46 = 0$

$\Rightarrow k = \dfrac{-13 \pm \sqrt{1089}}{10} \therefore k = \dfrac{-23}{5}; k = 2$

since k is an integer, $\therefore k = 2$

Also $5k^2 + 13k + 66 = 0$

$\Rightarrow k = \dfrac{-13 \pm \sqrt{-1151}}{10}$

So no real solution exist

A(2, –6), B(5, 2) and C(–2, 2)

For orthocentre H (α, β)

$BH \perp AC$

$\therefore \left(\dfrac{\beta - 2}{\alpha - 5}\right)\left(\dfrac{8}{-4}\right) = -1$

$\Rightarrow \alpha - 2\beta = 1$ ...(1)

Also $CH \perp AB$

$\therefore \left(\dfrac{\beta - 2}{\alpha + 2}\right)\left(\dfrac{8}{3}\right) = -1$

$\Rightarrow 3\alpha + 8\beta = 1$ ... (2)

Solving (1) and (2), we get

$\alpha = 2, \beta = \dfrac{1}{2}$

orthocentre is $\left(2, \dfrac{1}{2}\right)$

34. **(b)** Given $2\omega + 1 = z$;

and $z = \sqrt{3}i \Rightarrow \omega = \dfrac{\sqrt{3}i - 1}{2}$

$\Rightarrow \omega$ is complex cube root of unity

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

$= 3(-1 - \omega - \omega) = -3(1 + 2\omega) = -3z$

$\Rightarrow k = -z$

35. **(c)** $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & \cos x - \sin x & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$C_2 \to C_2 + C_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Expanding using second row

$2 \sin x (\sin x - \cos x)^2 = 0$

$\sin x = 0$ or $\sin x = \cos x$

$x = 0$ or $x = \dfrac{\pi}{4}$

36. **(a)** Consider

$$\begin{vmatrix} 3 & 1 + f(1) & 1 + f(2) \\ 1 + f(1) & 1 + f(2) & 1 + f(3) \\ 1 + f(2) & 1 + f(3) & 1 + f(4) \end{vmatrix}$$

$$= \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \quad [\because |A| = |A^1|]$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2 = [(1-\alpha)(1-\beta)(\alpha-\beta)]^2$$

So, $K = 1$

37. (d) $\sum_{r=1}^{n-1} r = 1 + 2 + 3 + ... + (n-1) = \dfrac{n(n-1)}{2}$

$$\sum_{r=1}^{n-1} (2r-1) = 1 + 3 + 5 + ... + [2(n-1) - 2]$$
$$= (n-1)^2$$

$$\sum_{r=1}^{n-1} (3r-2) = 1 + 4 + 7 + .. + (3n - 3 - 2)$$

$$= \frac{(n-1)(3n-4)}{2}$$

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \Sigma r & \Sigma(2r-1) & \Sigma(3r-2) \\ \dfrac{n}{2} & n-1 & a \\ \dfrac{n(n-1)}{2} & (n-1)^2 & \dfrac{(n-1)(3n-4)}{2} \end{vmatrix}$$

$\sum_{r=1}^{n-1} \Delta_r$ consists of $(n-1)$ determinants in L.H.S. and in R.H.S every constituent of first row consists of (n – 1) elements and hence it can be splitted into sum of $(n-1)$ determinants.

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \dfrac{n(n-1)}{2} & (n-1)^2 & \dfrac{(n-1)(3n-4)}{2} \\ \dfrac{n}{2} & n-1 & a \\ \dfrac{n(n-1)}{2} & (n-1)^2 & \dfrac{(n-1)(3n-4)}{2} \end{vmatrix}$$

$= 0$

($\because$ R_1 and R_3 are identical)

Hence, value of $\sum_{r=1}^{n-1} \Delta_r$ is independent of both 'a' and 'n'.

38. (c) Let $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$

Apply $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2-(a-\lambda)^2 & (b+\lambda)^2-(b-\lambda)^2 & (c+\lambda)^2-(c-\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$(\because (x+y)^2 - (x-y)^2 = 4xy)$

Taking out 4 common from R_2

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ a^2+\lambda^2-2a\lambda & b^2+\lambda^2-2b\lambda & c^2+\lambda^2-2c\lambda \end{vmatrix}$$

Apply $R_3 \to [R_3 - (R_1 - 2R_2)]$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ \lambda^2 & \lambda^2 & \lambda^2 \end{vmatrix}$$

Taking out λ common from R_2 and λ^2 from R_3.

$$= 4\lambda(\lambda^2)\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = k\lambda\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow k = 4\lambda^2$

39. (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$$

$= (a + b + c)[ab + bc + ca - a^2 - b^2 - c^2]$

$= -(a + b + c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$

Since a, b, c are sides of a scalene triangle, therefore at least two of the a, b, c will be unequal.

$\therefore \quad (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$

Also $a + b + c > 0$

$\therefore \quad -(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] < 0$

40. **(c)** Let $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$

Multiply C_1 by a, C_2 by b and C_3 by c and hence divide by abc.

$$= \frac{1}{abc}\begin{vmatrix} a(b^2+c^2) & ab^2 & ac^2 \\ a^2b & b(c^2+a^2) & bc^2 \\ a^2c & b^2c & c(a^2+b^2) \end{vmatrix}$$

Take out a, b, c common from R_1, R_2 and R_3 respectively.

$$\therefore \quad \Delta = \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & b^2 & c^2 \\ a^2 & c^2+a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$$

Apply $C_1 \to C_1 - C_2 - C_3$

$$\Delta = \begin{vmatrix} 0 & b^2 & c^2 \\ -2c^2 & c^2+a^2 & c^2 \\ -2b^2 & b^2 & a^2+b^2 \end{vmatrix}$$

$$= -2\begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & c^2+a^2 & c^2 \\ b^2 & b^2 & a^2+b^2 \end{vmatrix}$$

Apply $C_2 - C_1$ and $C_3 - C_1$

$$= -2\begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & a^2 & 0 \\ b^2 & 0 & a^2 \end{vmatrix} = -2\,[-b^2(c^2a^2) + c^2(-a^2b^2)]$$

$= 2a^2b^2c^2 + 2a^2b^2c^2 = 4a^2b^2c^2$

But $\Delta = ka^2b^2c^2 \therefore k = 4$

41. **(c)** Let $\Delta = \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}$

Applying $C_1 + C_3$ and $C_2 + C_3$

$$\Delta = \begin{vmatrix} -a+c & 2a+b+c & a+c \\ 2b+a+c & -b+c & b+c \\ a-c & b-c & -2c \end{vmatrix}$$

Now, applying $R_1 + R_3$ and $R_2 + R_3$

$$\Delta = \begin{vmatrix} 0 & 2(a+b) & a-c \\ 2(a+b) & 0 & b-c \\ a-c & b-c & -2c \end{vmatrix}$$

On expanding, we get

$\Delta = -2(a+b)\{-2c[2(a+b)] - (a-c)(b-c)\}$
$\qquad + (a-c)[2(a+b)(b-c)]$

$\Delta = 8c(a+b)(a+b) + 4(a+b)(a-c)(b-c)$

$= 4(a+b)[2ac + 2bc + ab - bc - ac + c^2]$

$= 4(a+b)[ac + bc + ab + c^2]$

$= 4(a+b)[c(a+c) + b(a+c)]$

$= 4(a+b)(b+c)(c+a)$

$= \alpha(a+b)(b+c)(c+a)$

Hence, $\alpha = 4$

42. **(b)** Vertices of triangle in complex form is

$z, iz, z + iz$

In cartesian form vertices are

$(x, y), (-y, x)$ and $(x-y, x+y)$

$$\therefore \text{ Area of triangle} = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x-y & x+y & 1 \end{vmatrix}$$

$= \frac{1}{2}[x(x-x-y) - y(-y-x+y) + 1(-yx-y^2-x^2+xy)]$

$= \frac{1}{2}[-xy+xy-y^2-x^2] = \frac{1}{2}(x^2+y^2)$

(∵ Area can not be negative)

$= \frac{1}{2}|z|^2 \qquad \left(\because z = x+iy, |z|^2 = x^2+y^2\right)$

43. **(b)** Given parabola is $x^2 = 8y$

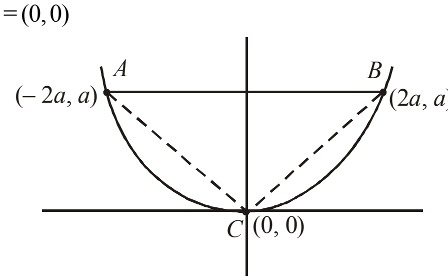
$\Rightarrow 4a = 8 \Rightarrow a = 2$

To find: Area of ΔABC

$A = (-2a, a) = (-4, 2)$

$B = (2a, a) = (4, 2)$

$C = (0, 0)$



$$\therefore \text{ Area} = \frac{1}{2}\begin{vmatrix} -4 & 2 & 1 \\ 4 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}[-4(2) - 2(4) + 1(0)]$$

$= \frac{-16}{2} = -8 \approx 8$ sq. unit (∵ area cannot be negative)

44. **(b)**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^n c \end{vmatrix} = 0$$

(Taking transpose of second determinant)

$$C_1 \Leftrightarrow C_3$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2}c & c+1 & c-1 \end{vmatrix} = 0$$

$$C_2 \Leftrightarrow C_3$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$C_2 - C_1,\ C_3 - C_1$$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0 \quad R_1 + R_3$$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $[1+(-1)^{n+2}](a+c)(2b+1+2b-1)=0$

$\Rightarrow$ $4b(a+c)[1+(-1)^{n+2}]=0$

$\Rightarrow$ $1+(-1)^{n+2}=0$ as $b(a+c) \neq 0$

$\Rightarrow$ n should be an odd integer.

45. **(d)** Given that, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\therefore\ D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$$

Hence, D is divisible by both x and y

46. **(b)** Let r be the common ratio of an G.P., then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$$

$= 0$

47. **(d)** Applying, $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$[\because\ a^2+b^2+c^2=-2]$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\therefore\ f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$f(x) = (x-1)^2$

Hence degree $= 2$.

48. **(d)** Let r be the common ratio of an G.P., then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$$

$= 0$

49. (c) Given that $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

Applying $R_3 \to R_3 - (xR_1 + R_2)$;

$$= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+c) \end{vmatrix}$$

$= (ax^2 + 2bx + c)(b^2 - ac) = (+)(-) = -$ve.

[Given that discriminant of $ax^2 + 2bx + c$ is $-$ve

$\therefore 4b^2 - 4ac < 0 \Rightarrow b^2 - ac < 0$]

50. (d) $l = AR^{p-1} \Rightarrow \log l = \log A + (p-1)\log R$

$m = AR^{q-1} \Rightarrow \log m = \log A + (q-1)\log R$

$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1)\log R$

Now, $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$

Operating

$C_1 - (\log R)C_2 + (\log R - \log A)C_3 = \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$

51. (a) $|\text{adj} A| = |A|^2 = 9$

$[\because |\text{adj} A| = |A|^{n-1}]$

$\Rightarrow |A| = \pm 3 = \lambda \Rightarrow |\lambda| = 3$

$\Rightarrow |B| = |\text{adj} A|^2 = 81$

$\mu = |(B^{-1})^T| = |B^{-1}| = |B|^{-1} = \dfrac{1}{|B|} = \dfrac{1}{81}$

52. (a) $|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = ((9+4) - 1(3-4) + 2(-1-3))$

$= 13 + 1 - 8 = 6$

$|adjB| = |adj(adjA)| = |A|^{(n-1)^2} = |A|^4 = (36)^2$

$|C| = |3A| = 3^3 \times 6$

Hence, $\dfrac{|adjB|}{|C|} = \dfrac{36 \times 36}{3^3 \times 6} = 8$

53. (c) $\because B = A^{-1} = \Rightarrow |B| = \dfrac{1}{|A|}$

Now, $|B| = \begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = 2\alpha^2 - 2\alpha - 25$

Given, det. $(A) + 1 = 0$

$\Rightarrow \dfrac{1}{2\alpha^2 - 2\alpha - 25} + 1 = 0$

$\Rightarrow \dfrac{2\alpha^2 - 2\alpha - 24}{2\alpha^2 - 2\alpha - 25} = 0$

$\Rightarrow \alpha = 4, -3 \Rightarrow$ Sum of values $= 1$

54. (b)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\ldots+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \dfrac{(n-1)n}{2} = 78 \Rightarrow n^2 - n - 15 = 0$

$\Rightarrow n = 13$

Now, the matrix $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$

Then, the required inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

55. (c) Let $|A| = a, |B| = b$

$\Rightarrow |A^T| = a\ |A^{-1}| = \dfrac{1}{a}, |B^T| = b, |B^{-1}| = \dfrac{1}{b}$

$\because |ABA^T| = 8 \Rightarrow |A|\,|B|\,|A^T| = 8 \ldots(1)$

$\Rightarrow a.b.a = 8 \Rightarrow a^2b = 8$

$\because |AB^{-1}| = 8 \Rightarrow |A|\,|B^{-1}| = 8 \Rightarrow a.\dfrac{1}{b} = 8$ ...(2)

From (1) & (2)

$a = 4, b = \dfrac{1}{2}$

Then, $|BA^{-1}B^T| = |B|\,|A^{-1}|\,|B^T| = b.\dfrac{1}{a}.b = \dfrac{b^2}{a} = \dfrac{1}{16}$

56. (c) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow |A| = 1$

$\text{adj}(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$

$$B^2=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$$

$$\Rightarrow \quad B^3=\begin{bmatrix}\cos 3\theta & \sin 3\theta\\ -\sin 3\theta & \cos 3\theta\end{bmatrix}$$

$$\Rightarrow \quad A^{-50}=B^{50}=\begin{bmatrix}\cos(50\theta) & \sin(50\theta)\\ -\sin(50\theta) & \cos(50\theta)\end{bmatrix}$$

$$(A^{-50})_{\theta=\frac{\pi}{12}}=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$\left[\because \cos\left(\frac{50\pi}{12}\right)=\cos\left(4\pi+\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}\right]$$

57. **(d)** Since $A.\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and $|3A|=108$

suppose the scalar matrix is $\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$

$$\therefore A.\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}^{-1}$$

$[\therefore AB=C\Rightarrow ABB^{-1}=CB^{-1}\Rightarrow A=CB^{-1}]$

$$\Rightarrow A=\frac{1}{3}\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}\begin{bmatrix}3 & -2\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}\begin{bmatrix}1 & -\frac{2}{3}\\ 0 & \frac{1}{3}\end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix}k & -\frac{2}{3}k\\ 0 & \frac{k}{3}\end{bmatrix} \text{ ... (1)}$$

$\because |3A|=108$

$$\Rightarrow 108=\begin{vmatrix}3k & -2k\\ 0 & k\end{vmatrix}$$

$\Rightarrow 3k^2=108 \Rightarrow k^2=36 \quad \Rightarrow k=\pm 6$

For $k=6$

$$A=\begin{bmatrix}6 & -4\\ 0 & 2\end{bmatrix} \text{ ... From (1)}$$

$$\Rightarrow A^2=\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$$

For $k=-6$

$$\Rightarrow A=\begin{bmatrix}-6 & 4\\ 0 & -2\end{bmatrix} \text{.... From (1)}$$

$$\Rightarrow A^2=\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$$

58. **(a)** We have

$(A-3I)(A-5I)=O$

$\Rightarrow \quad A^2-8A+15I=O$

Multiplying both sides by A^{-1}, we get;

$A^{-1}A.A-8A^{-1}A+15A^{-1}I=A^{-1}O$

$\Rightarrow \quad A-8I+15A^{-1}=O$

$A+15A^{-1}=8I$

$$\frac{A}{2}+\frac{15A^{-1}}{2}=4I$$

$$\therefore \quad \alpha+\beta=\frac{1}{2}+\frac{15}{2}=\frac{16}{2}=8$$

59. **(c)** We have $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$

$$\Rightarrow A^2=\begin{bmatrix}16 & -9\\ -12 & 13\end{bmatrix} \Rightarrow 3A^2=\begin{bmatrix}48 & -27\\ -36 & 39\end{bmatrix}$$

Also $12A=\begin{bmatrix}24 & -36\\ -48 & 12\end{bmatrix}$

$$\therefore 3A^2+12A=\begin{bmatrix}48 & -27\\ -36 & 39\end{bmatrix}+\begin{bmatrix}24 & -36\\ -48 & 12\end{bmatrix}$$

$$=\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$$

$$\text{adj}(3A^2+12A)=\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$$

60. **(b)**

61. **(d)** Given that $A(\text{adj } A)=AA^T$

Pre-multiply by A^{-1} both side, we get

$\Rightarrow A^{-1}A(\text{adj } A)=A^{-1}AA^T$

$\text{adj } A=A^T$

$$\Rightarrow \begin{bmatrix}2 & b\\ -3 & 5a\end{bmatrix}=\begin{bmatrix}5a & 3\\ -b & 2\end{bmatrix}$$

$$\Rightarrow a=\frac{2}{5} \text{ and } b=3$$

$\Rightarrow 5a+b=5$

62. **(a)** $A^2 - 5A = -7I$

$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$

$AI - 5I = -7A^{-1}$

$A - 5I = -7A^{-1}$

$A^{-1} = \frac{1}{7}(5I - A)$

$A^3 - 2A^2 - 3A + I = A(5A - 7I) - 2A^2 - 3A + I$

$= 5A^2 - 7A - 2A^2 - 3A + I = 3A^2 - 10A + I$

$= 3(5A - 7I) - 10A + I = 5A - 20I = 5(A - 4I)$

63. **(a)** $|5.\text{ adj A}| = 5 \Rightarrow 5^3.\ |A|^{3-1} = 5$

$\Rightarrow 125\,|A|^2 = 5 \Rightarrow |A| = \pm\frac{1}{5}$

64. **(d)** $BB' = B(A^{-1}A')' = B(A')'(A^{-1})'$

$= BA(A^{-1})' = (A^{-1}A')(A(A^{-1})')$

$= A^{-1}A.A'.(A^{-1})'$ {as $AA' = A'A$}

$= I(A^{-1}A)' = I \cdot I = I^2 = I$

65. **(a)** Given $A\begin{bmatrix}1&2&3\\0&2&3\\0&1&1\end{bmatrix} = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

Applying $C_1 \leftrightarrow C_3$

$A\begin{bmatrix}3&2&1\\3&2&0\\1&1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$

Again Applying $C_2 \leftrightarrow C_3$

$A\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

pre-multiplying both sides by A^{-1}

$A^{-1}A\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix} = A^{-1}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$I\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix} = A^{-1}\ I = A^{-1}$

($\because A^{-1}A = I$ and I = Identity matrix)

Hence, $A^{-1} = \begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}$

66. **(b)** $|P| = 1(12-12) - \alpha(4-6) + 3(4-6) = 2\alpha - 6$

Now, adj A = P $\Rightarrow$ |adj A| = |P|

$\Rightarrow |A|^2 = |P|$

$\Rightarrow |P| = 16$

$\Rightarrow 2\alpha - 6 = 16$

$\Rightarrow \alpha = 11$

67. **(c)** Given that $P^3 = Q^3$ ...(1)

and $P^2Q = Q^2P$ ...(2)

Subtracting (1) and (2), we get

$P^3 - P^2Q = Q^3 - Q^2P$

$\Rightarrow P^2(P-Q) + Q^2(P-Q) = 0$

$\Rightarrow (P^2 + Q^2)(P-Q) = 0$

$\because P \neq Q,\ \therefore P^2 + Q^2 = 0$

Hence $|P^2 + Q^2| = 0$

68. **(d)** Let $Au_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$ and $Au_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$

Then, $Au_1 + Au_2 = \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix}$

$\Rightarrow A(u_1 + u_2) = \begin{pmatrix}1\\1\\0\end{pmatrix}$ ...(1)

Given that $A = \begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix}$

$\Rightarrow |A| = 1(1) - 0(2) + 0(4-3) = 1$

$C_{11} = 1 \quad C_{21} = 0 \quad C_{31} = 0$

$C_{12} = -2 \quad C_{22} = 1 \quad C_{32} = 0$

$C_{13} = 1 \quad C_{23} = -2 \quad C_{33} = 1$

$\therefore adjA = \begin{bmatrix}1&0&0\\-2&0&0\\1&-2&1\end{bmatrix}$

We know,

$A^{-1} = \frac{1}{|A|}adjA$

$\Rightarrow A^{-1} = adj(A) \quad (\because |A| = 1)$

Now, from equation (1), we have

$u_1 + u_2 = A^{-1}\begin{pmatrix}1\\1\\0\end{pmatrix}$

$= \begin{bmatrix}1&0&0\\-2&1&0\\1&-2&1\end{bmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{bmatrix}1\\-1\\-1\end{bmatrix}$

69. **(c)** $A = \begin{bmatrix}0&0&a\\0&b&c\\d&e&f\end{bmatrix}$, $|A| = -abd \neq 0$

$c_{11} = +(bf - ce)$, $c_{12} = -(-cd) = cd$, $c_{13} = +(-bd) = -bd$

$c_{21} = -(-ea) = ae$, $c_{22} = +(-ad) = -ad$, $c_{23} = -(0) = 0$

$c_{31} = +(-ab) = -ab$, $c_{32} = -(0) = 0$, $c_{33} = 0$

$$\text{Adj } A = \begin{bmatrix} (bf-ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{abd}\begin{bmatrix} bf-ce & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

Now $A^{-1} = A^T$

$$\Rightarrow \frac{1}{-abd}\begin{bmatrix} bf-ce & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} bf-ce & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -abd^2 \\ 0 & -ab^2d & -abde \\ -a^2bd & -abcd & -abdf \end{bmatrix}$$

$\therefore\ bf - ce = ae = cd = 0$ ...(i)

$abd^2 = ab,\ ab^2d = ad,\ a^2bd = bd$ ...(ii)

$abde = abcd = abdf = 0$ ...(iii)

From (ii),

$(abd^2).\ (ab^2d).\ (a^2bd) = ab.\ ad.\ bd$

$\Rightarrow (abd)^4 - (abd)^2 = 0$

$\Rightarrow (abd)^2\ [(abd)^2 - 1] = 0$

$\because abd \neq 0$, $\therefore\ abd = \pm 1$ ...(iv)

From (iii) and (iv),

$e = c = f = 0$ ...(v)

From (i) and (v),

$bf = ae = cd = 0$ ...(vi)

From (iv), (v) and (vi), it is clear that a, b, d can be any non-zero integer such that $abd = \pm 1$

But it is only possible, if $a = b = d = \pm 1$

Hence, there are 2 choices for each a, b and d. there fore, there are $2\times2\times2$ choices for a, b and d. Hence number of required matrices $= 2\times2\times2 = (2)^3$

70. **(a)** Let A and B be real matrices such that $A = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$

and $B = \begin{bmatrix} 0 & \gamma \\ \delta & 0 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 0 & \alpha\gamma \\ \beta\delta & 0 \end{bmatrix}$

and $BA = \begin{bmatrix} 0 & \gamma\beta \\ \delta\alpha & 0 \end{bmatrix}$

Statement - 1 :

$$AB - BA = \begin{bmatrix} 0 & \gamma(\alpha-\beta) \\ \delta(\beta-\alpha) & 0 \end{bmatrix}$$

$|AB - BA| = (\alpha-\beta)^2\,\gamma\delta \neq 0$

$\therefore\ AB - BA$ is always an invertible matrix.

Hence, statement - 1 is true.

But $AB - BA$ can be identity matrix if $\gamma = -\delta$ or $\delta = -\gamma$

So, statement - 2 is false.

71. **(b)** **For reflexive**

$A = P^{-1}AP$ is true,

For $P = I$, which is an invertible matrix.

$(A, A) \in R$

$\therefore$ R is reflexive.

For symmetry

As $(A, B) \in R$ for matrix P

$A = P^{-1}BP$

$\Rightarrow\ PAP^{-1} = B$

$\Rightarrow\ B = PAP^{-1}$

$\Rightarrow\ B = \left(P^{-1}\right)^{-1} A\,(P^{-1})$

$\therefore$ $(B, A) \in R$ for matrix P^{-1}

$\therefore$ R is symmetric.

For transitivity

$A = P^{-1}BP$

and $B = P^{-1}CP$

$\Rightarrow\ A = P^{-1}\left(P^{-1}CP\right)P$

$\Rightarrow\ A = \left(P^{-1}\right)^2 CP^2$

$\Rightarrow\ A = \left(P^2\right)^{-1} C\left(P^2\right)$

$\therefore$ $(A, C) \in R$ for matrix P^2

$\therefore$ R is transitive.

So R is equivalence.

So, statement-1 is true.

We know that if A and B are two invertible matrices of order n, then

$(AB)^{-1} = B^{-1}A^{-1}$

So, statement-2 is true.

72. **(a)** We know that if A is square matrix of order n then

$adj\,(adj\,A) = |A|^{n-2}A$.

$= |A|^0 A = A$

Also $|adj\, A| = |A|^{n-1} = |A|^{2-1} = |A|$

$\therefore$ Both the statements are true but statement-2 is not a correct explanation for statement-1 .

73. **(c)** Given that all entries of square matrix A are integers, therefore all cofactors should also be integers.
If det $A = \pm 1$ then A^{-1} exists. Also all entries of A^{-1} are integers.

74. **(d)** Given that $A^2 - A + I = 0$
Pre-multiply by A^{-1} both side, we get
$A^{-1}A^2 - A^{-1}A + A^{-1}.I = A^{-1}.0$
$\Rightarrow A - I + A^{-1} = 0$ or $A^{-1} = I - A$.

75. **(a)** Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$$\Rightarrow B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

Given that $B = A^{-1} \Rightarrow AB = I$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$$

76. **(a)** Given that $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$\therefore A^{-1}$ exists, further $(-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

Also $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

77. **(c)** $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 5$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0 \Rightarrow \mu = 8$$

78. **(3.00)**
For non-zero solution, $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$$

$\Rightarrow 6\lambda^3 - 36\lambda^2 + 54\lambda = 0$

$\Rightarrow 6\lambda[\lambda^2 - 6\lambda + 9] = 0$

$\Rightarrow \lambda = 0, \lambda = 3$ [Distinct values]

Then, the sum of distinct values of $\lambda = 0 + 3 = 3$.

79. **(a)** $\because \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0 \Rightarrow 3\lambda^2 - 7\lambda - 12 = 0$

$\Rightarrow \lambda = 3$ or $-\frac{2}{3}$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3-\lambda)$$

$\therefore$ When $\lambda = -\frac{2}{3}$, $D_1 \neq 0$.

Hence, equations will be inconsistent when $\lambda = -\frac{2}{3}$.

80. **(a)** Since, system of linear equations has non-zero solution

$\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 1(9-k^2) - 1(3-3k^2) + 3(1-9) = 0$

$\Rightarrow 9 - k^2 - 3 + 3k^2 - 24 = 0$

$\Rightarrow 2k^2 = 18 \Rightarrow k^2 = 9, k = \pm 3$

So, equations are

$x + y + 3z = 0$ ...(i)

$x + 3y + 9z = 0$ ...(ii)

$3x + y + 3z = 0$ ...(iii)

Now, from equation (i) – (ii),

$-2y - 6z = 0 \Rightarrow y = -3z \Rightarrow \frac{y}{z} = -3$ ...(iv)

Now, from equation (i) – (iii),

$-2x = 0 \Rightarrow x = 0$

So, $x + \frac{y}{z} = 0 - 3 = -3$

81. (5.00)

For infinitely many solutions,

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$\Rightarrow (a+7) - 2(1-2a) + 3(-15) = 0$

$\Rightarrow a = 8$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 9 \\ 2 & 1 & b \\ 1 & -7 & 24 \end{vmatrix} = 0$$

$\Rightarrow (24 + 7b) - 2(b - 48) + 9(-15) = 0$

$\Rightarrow b = 3$

$\therefore a - b = 5.$

82. (b) Given that $Ax = b$ has solutions x_1, x_2, x_3 and b is equal to b_1, b_2 and b_3

$\therefore x_1 + y_1 + z_1 = 1$

$\Rightarrow 2y_1 + z_1 = 2 \Rightarrow z_1 = 2$

Determinant of coefficient matrix

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2$$

83. (d) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$ [$\because$ Equation has many solutions]

$\Rightarrow -15 + 6 + 2\lambda = 0 \Rightarrow \lambda = \frac{9}{2}$

$$\therefore D_Z = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 2\mu \end{vmatrix} = 0 \Rightarrow \mu = 5$$

$\therefore 2\lambda + \mu = 14.$

84. (8)

The given system of equations

$x - 2y + 5z = 0$ ...(i)

$-2x + 4y + z = 0$ ...(ii)

$-7x + 14y + 9z = 0$ ...(iii)

From equation, $2 \times$ (i) + (ii) $\Rightarrow z = 0$

Put $z = 0$ in equation (i), we get $x = 2y$

$\because 15 \le x^2 + y^2 + z^2 \le 150$

$\Rightarrow 15 \le 4y^2 + y^2 \le 150$

[$\because x = 2y,\ z = 0$]

$\Rightarrow 3 \le y^2 \le 30$

$\Rightarrow y = \pm 2, \pm 3, \pm 4, \pm 5$

$\Rightarrow$ 8 solutions.

85. (d) $\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = -(\lambda - 1)(2\lambda + 1)$

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = -2(\lambda^2 + 6\lambda - 4)$$

For no solution $\Delta = 0$ and at least one of Δ_1, Δ_2 and Δ_3 is non-zero.

$\therefore \Delta = 0 \Rightarrow \lambda = 1, -\frac{1}{2}$ and $\Delta_1 \ne 0$

Hence, $S = \left\{1, -\frac{1}{2}\right\}$

86. (d) $\because |P| = 1(-3+36) - 2(2+4) + 1(-18-3) = 0$

Given that $PX = 0$

$\therefore$ System of equations

$x + 2y + z = 0$; $2x - 3y + 4z = 0$

and $x + 9y - z = 0$ has infinitely many solution.

Let $z = k \in \mathbf{R}$ and solve above equations, we get

$x = -\frac{11k}{7},\ y = \frac{2k}{7},\ z = k$

But given that $x^2 + y^2 + z^2 = 1$

$\therefore k = \pm \frac{7}{\sqrt{174}}$

$\therefore$ Two solutions only.

87. (c) The given system of linear equations

$7x + 6y - 2z = 0$ ...(i)

$3x + 4y + 2z = 0$ ...(ii)

$x - 2y - 6z = 0$ ...(iii)

Now, determinant of coefficient matrix

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$

$= 7(-20) - 6(-20) - 2(-10)$

$= -140 + 120 + 20 = 0$

So, there are infinite non-trivial solutions.

From eqn. (i) + 3 × (iii); we get

$10x - 20z = 0 \Rightarrow x = 2z$

Hence, there are infinitely many solutions (x, y, z) satisfying $x = 2z$.

88. (a) From the given linear equation, we get

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix} (R_3 \to R_3 - 2R_2 + 3R_3)$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Now, let $P_3 = 4x + 4y + 4z - \delta = 0$. If the system has solutions it will have infinite solution.

So, $P_3 \equiv \alpha P_1 + \beta P_2$

Hence, $3\alpha + \beta = 4$ and $4\alpha + 2\beta = 4$

$\Rightarrow \alpha = 2$ and $\beta = -2$

So, for infinite solution $2\mu - 2 = \delta$

$\Rightarrow$ For $2\mu \neq \delta + 2$ system is inconsistent

89. (c) $D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$

$D = \lambda^2 + 6\lambda - 16$

$D = (\lambda + 8)(2 - \lambda)$

For no solutions, D = 0

$\Rightarrow \lambda = -8, 2$

when $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$$

$= 5[18 - 10] - 2[48 - 50] + 2(16 - 30)$

$= 40 + 4 - 28 \neq 0$

There exist no solutions for $\lambda = 2$

90. (a) For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$

$\Rightarrow -bc + 2ac - ab = 0$

$\Rightarrow ab + bc + 2ac$

$\Rightarrow \dfrac{2}{b} = \dfrac{1}{a} + \dfrac{1}{c}$

$\Rightarrow \dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ in A.P.

91. (13) $x + y + z = 6$ ...(i)

$x + 2y + 3z = 10$ ...(ii)

$3x + 2y + \lambda z = \mu$ ...(iii)

From (i) and (ii),

If $z = 0 \Rightarrow x + y = 6$ and $x + 2y = 10$

$\Rightarrow y = 4, x = 2$

(2, 4, 0)

If $y = 0 \Rightarrow x + z = 6$ and $x + 3z = 10$

$\Rightarrow z = 2$ and $x = 4$

(4, 0, 2)

So, $3x + 2y + \lambda z = \mu$, must pass through (2, 4, 0) and (4, 0, 2)

So, $6 + 8 = \mu \Rightarrow \mu = 14$

and $12 + 2\lambda = \mu$

$12 + 2\lambda = 14 \Rightarrow \lambda = 1$

So, $\mu - \lambda^2 = 14 - 1 = 13$

92. (d) Given system of linear equations: $x + y + z = 5$; $x + 2y + 2z = 6$ and $x + 3y + \lambda z = \mu$ have infinite solution.

$\therefore \Delta = 0, \Delta x = \Delta y = \Delta z = 0$

Now, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 2) + 1(3 - 2) = 0$

$\Rightarrow 2\lambda - 6 - \lambda + 2 + 1 = 0 \Rightarrow \lambda = 3$

$$\Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu - 5 & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(2 - \mu + 5) = 0 \Rightarrow \mu = 7$

$\therefore \lambda + \mu = 10$

93. (d) $\because$ system of equations has infinitely many solutions.

$\therefore \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

Here, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4 - \lambda & 2\lambda & -\lambda \\ 1 & 6 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

Now, for $\lambda = 3$, $\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ \lambda-2 & \lambda & -\lambda \\ -5 & 2 & -4 \end{vmatrix} = 0$

For $\lambda = 3$, $\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 4 & \lambda-2 & -\lambda \\ 3 & -5 & -4 \end{vmatrix} = 0$

For $\lambda = 3$, $\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 4 & \lambda & \lambda-2 \\ 3 & 2 & -5 \end{vmatrix} = 0$

$\therefore$ for $\lambda = 3$, system of equations has infinitely many solutions.

94. **(b)** Given system of equations has a non-trivial solution.

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow k = \frac{9}{2}$$

$\therefore$ equations are $2x + 3y - z = 0$ ...(i)

$2x - y + z = 0$ ...(ii)

$2x + 9y - 4z = 0$ ...(iii)

By (i) – (ii), $2y = z$

$\therefore z = -4x$ and $2x + y = 0$

$$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2} = \frac{1}{2}$$

95. **(b)** If the system of equations has non-trivial solutions, then the determinant of coefficient matrix is zero

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$\Rightarrow (1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0$

$\Rightarrow (1 + c)(1 - c) - 2c^2(1 + c) = 0$

$\Rightarrow (1 + c)(1 - c - 2c^2) = 0$

$\Rightarrow (1 + c)^2(1 - 2c) = 0$

$\Rightarrow c = -1$ or $\frac{1}{2}$

Hence, the greatest value of c is $\frac{1}{2}$ for which the system of linear equations has non-trivial solution.

96. **(b)** Given system of linear equations,

$x - 2y + kz = 1$, $2x + y + z = 2$, $3x - y - kz = 3$

$$\Delta = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix}$$

$= 1(-k + 1) + 2(-2k - 3) + k(-2 - 3)$

$= -k + 1 - 4k - 6 - 5k = -10k - 5 = -5(2k + 1)$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = -5(2k + 1)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & k \\ 2 & 2 & 1 \\ 3 & 3 & -k \end{vmatrix} = 0, \ \Delta_3 = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$\because z \neq 0 \Rightarrow \Delta = 0$

$\Rightarrow -5(2k + 1) = 0 \Rightarrow k = -\frac{1}{2}$

$\therefore$ System of equation has infinite many solutions.

Let $z = \lambda \neq 0$ then $x = \dfrac{10 - 3\lambda}{10}$ and $y = -\dfrac{2\lambda}{5}$

$\therefore$ (x, y) must lie on line $4x - 3y - 4 = 0$

97. **(a)** $\because$ The system of linear equations has a unique solution.

$\therefore \quad \Delta \neq 0$

$$\Delta = \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow \begin{vmatrix} 1+\alpha+\beta+1 & \beta & 1 \\ \alpha+1+\beta+1 & \beta+1 & 1 \\ \alpha+\beta+2 & \beta & 2 \end{vmatrix} \neq 0 \quad [C_1 \to C_1 + C_2 + C_3]$$

$$\Rightarrow (\alpha+\beta+2)\begin{vmatrix} 1 & \beta & 1 \\ 1 & \beta+1 & 1 \\ 1 & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow (\alpha+\beta+2)\begin{vmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0 \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\Rightarrow (\alpha + \beta + 2)\, 1(1) \neq 0$

$\Rightarrow \alpha + \beta + 2 \neq 0$

$\because$ Ordered pair (2, 4) satisfies this condition

$\therefore \alpha = 2$ and $\beta = 4$.

98. **(a)** Consider the given system of linear equations

$x(1 - \lambda) - 2y - 2z = 0$

$x + (2 - \lambda)y + z = 0$

$-x - y - \lambda z = 0$

Now, for a non-trivial solution, the determinant of coefficient matrix is zero.

$$\begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)^3 = 0$

$\lambda = 1$

99. **(b)** $\because$ System of equations has more than one solution

$\therefore \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ for infinite solution

$$\Delta_1 = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = a(13) + 2(5c - 2b) + 3(-3b + c)$$

$= 13a - 13b + 13c = 0$

i.e, $a - b + c = 0$

or $b - c - a = 0$

100. **(b)** Since, the system of linear equations has, non-trivial solution then determinant of coefficient matrix = 0

i.e., $\begin{vmatrix} \sin 3\theta & \cos 2\theta & 2 \\ 1 & 3 & 7 \\ -1 & 4 & 7 \end{vmatrix} = 0$

$\sin 3\theta(21-28) - \cos 2\theta(7+7) + 2(4+3) = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0$

$3\sin\theta - 4\sin^3\theta + 2 - 4\sin^2\theta - 2 = 0$

$4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$

$\sin\theta(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\sin\theta(4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3) = 0$

$\sin\theta[2\sin\theta(2\sin\theta - 1) + 3(2\sin\theta - 1)] = 0$

$\sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\sin\theta = 0, \sin\theta = \frac{1}{2} \quad \left(\because \sin\theta \neq -\frac{3}{2}\right)$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, for two values of θ, system of equations has non-trivial solution

101. **(b)** $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 2\alpha - 9 - \alpha + 3 + 1 = \alpha - 5$

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & \alpha \end{vmatrix} = 5(2\alpha - 9) - 1(9\alpha - 3\beta) + (27 - 2\beta)$$

$= 10\alpha - 45 - 9\alpha + 3\beta + 27 - 2\beta$

$= \alpha + \beta - 18$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 9 & 3 \\ 1 & \beta & \alpha \end{vmatrix} = 9\alpha - 3\beta - 5(\alpha - 3) + 1(\beta - 9)$$

$= 9\alpha - 3\beta - 5\alpha + 15 + \beta - 9 = 4\alpha - 2\beta + 6$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 2\beta - 27 - \beta + 9 + 5 = \beta - 13$$

Since, the system of equations has infinite many solutions. Hence,

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$

$\Rightarrow \quad \alpha = 5, \beta = 13 \Rightarrow \beta - \alpha = 8$

102. **(c)** Consider the system of linear equations

$x - 4y + 7z = g$ ...(i)

$3y - 5z = h$ ...(ii)

$-2x + 5y - 9z = k$ ...(iii)

Multiply equation (i) by 2 and add equation (i), equation (ii) and equation (iii)

$\Rightarrow \quad 0 = 2g + h + k. \therefore 2g + h + k = 0$

then system of equation is consistent.

103. **(a)** For non zero solution of the system of linear equations;

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow k = 11$

Now equations become

$x + 11y + 3z = 0$ ...(1)

$3x + 11y - 2z = 0$ ...(2)

$2x + 4y - 3z = 0$ ...(3)

Adding equations (1) & (3) we get

$3x + 15y = 0$

$\Rightarrow x = -5y$

Now put $x = -5y$ in equation (1), we get

$-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

$\therefore \frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$

104. **(c)** Here, the equations are;

$(k+2)x + 10y = k$

& $kx + (k+3)y = k - 1$.

These equations can be written in the form of $Ax = B$ as

$$\begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ k-1 \end{bmatrix}$$

For the system to have no solution

$$|A| = 0$$

$\Rightarrow \begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} = 0 \Rightarrow (k+2)(k+3) - k \times 10 = 0$

$\Rightarrow \quad k^2 - 5k + 6 = (k-2)(k-3) = 0$

$\therefore \quad k = 2, 3$

For $k = 2$, equations become:

$4x + 10y = 2$

& $2x + 5y = 1$

& hence infinite number of solutions.

For $k = 3$, equations becomes;

$5x + 10y = 3$

$3x + 6y = 2$

& hence no solution.

$\therefore$ required number of values of k is 1

105. **(b)** The system of linear equations is:

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

As, system has unique solution.

So, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

$\Rightarrow k + 2 - (2k+3) + 1 \neq 0$

$\Rightarrow k \neq 0$

Hence, $k \in R - \{0\} \equiv S$

106. **(d)** As the system of equations has no solution then Δ should be zero and at least one of Δ_1, Δ_2 and Δ_3 should not be zero.

$\therefore \quad \Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$

$\Rightarrow \quad -a - 1 = 0 \Rightarrow a = -1$

$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 2 \\ 1 & b & 3 \end{vmatrix} \neq 0$

$\Rightarrow \quad b \neq 9$

107. **(a)** $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$

$\Rightarrow 1[a - b] - 1[1 - a] + 1[b - a^2] = 0$

$\Rightarrow (a-1)^2 = 0$

$\Rightarrow a = 1$

For $a = 1$, First two equations are identical

i.e., $x + y + z = 1$

To have no solution with $x + by + z = 0$

$b = 1$

So $b = \{1\} \Rightarrow$ It is singleton set.

108. **(b)** Since the given system of linear equations has infinitely many solutions.

$\therefore \quad \begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$

$\Rightarrow \quad \lambda^3 + 4\lambda - 40 = 0$

λ has only 1 real root.

109. **(b)** For non-trivial solution,

$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow -\lambda(\lambda+1)(\lambda-1) = 0 \Rightarrow \lambda = 0, +1, -1$

110. **(a)** $\left.\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}\right\}$

$\Rightarrow \quad (2-\lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

For non-trivial solution,

$\Delta = 0$

i.e. $\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)[\lambda(3+\lambda) - 4] + 2[-2\lambda + 2] + 1[4 - (3+\lambda)] = 0$

$\Rightarrow \lambda^3 + \lambda^2 - 5\lambda + 3 = 0$

$\Rightarrow \lambda = 1, 1, 3$

Hence λ has 2 values.

111. **(b)** Given system of equations can be written as

$(a-1)x - y - z = 0$

$-x + (b-1)y - z = 0$

$-x - y + (c-1)z = 0$

For non-trivial solution, we have

$\begin{vmatrix} a-1 & -1 & -1 \\ -1 & b-1 & -1 \\ -1 & -1 & c-1 \end{vmatrix} = 0$

$R_2 \to R_2 - R_3$

$\begin{vmatrix} a-1 & -1 & -1 \\ 0 & b & -c \\ -1 & -1 & c-1 \end{vmatrix} = 0$

$C_2 \to C_2 - C_3$

$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -1 & -c & c-1 \end{vmatrix} = 0$

Apply $R_3 \to R_3 - R_1$

$$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -a & -c & c \end{vmatrix} = 0$$

$\Rightarrow (a-1)[bc+c^2-c^2]-1[a(b+c)] = 0$

$\Rightarrow (a-1)[bc]-ab-ac = 0$

$\Rightarrow abc-bc-ab-ac = 0$

$\Rightarrow ab+bc+ca = abc$

112. (b) Since, system of equations have no solution

$\therefore \frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$ ($\because$ System has no solution)

$\Rightarrow k^2+4k+3 = 8k \Rightarrow k^2-4k+3=0$

$\Rightarrow k = 1, 3$

If $k=1$ then $\frac{8}{1+3} \neq \frac{4.1}{2}$ which is false

and if $k=3$ then $\frac{8}{6} \neq \frac{4.3}{9-1}$ which is true, therefore $k=3$

Hence for only one value of k. System has no solution.

113. (b) Given system of equations is homogeneous which is

$x + ay = 0$

$y + az = 0$

$z + ax = 0$

It can be written in matrix form as

$$A = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix}$$

Now, $|A| = [1-a(-a^2)] = 1+a^3 \neq 0$

So, system has only trivial solution.

Now, $|A| = 0$ only when $a = -1$

So, system of equations has infinitely many solutions which is not possible because it is given that system has a unique solution.

Hence set of all real values of 'a' is $R - \{-1\}$.

114. (c) $\Delta_1 = \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix}$

$$= \begin{vmatrix} 0 & \sin\alpha-\cos\alpha & \cos\alpha-\sin\alpha \\ 0 & \cos\alpha+\sin\alpha & \sin\alpha-\cos\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix}$$

$= (\sin\alpha-\cos\alpha)^2 - (\cos^2\alpha - \sin^2\alpha)$

$= \sin^2\alpha + \cos^2\alpha - 2\sin\alpha \,.\, \cos\alpha - \cos^2\alpha + \sin^2\alpha$

$= 2\sin^2\alpha - 2\sin\alpha \,.\, \cos\alpha$

$= 2\sin\alpha(\sin\alpha - \cos\alpha)$

Now, $\sin\alpha - \cos\alpha = 0$ for only

$\alpha = \frac{\pi}{4}$ in $\left(0, \frac{\pi}{2}\right)$

$\therefore \Delta_1 = 2(\sin\alpha) \times 0 = 0,$

since value of $\sin\alpha$ is finite for $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence non-trivivial solution for only one value of α in $\left(0, \frac{\pi}{2}\right)$

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ 2\cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$\Rightarrow 2\cos\alpha(\sin^2\alpha - \cos^2\alpha) = 0$

$\therefore \cos\alpha = 0$ or $\sin^2\alpha - \cos^2\alpha = 0$

But $\cos\alpha = 0$ not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

$\therefore \sin^2\alpha - \cos^2\alpha = 0 \Rightarrow \sin\alpha = -\cos\alpha$, which is also not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence, there is no solution.

115. (d) Given system of equations can be written in matrix form as $AX = B$ where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{pmatrix} \text{ and } B = \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix}$$

Since, system is consistent and has infinitely many solutions

$\therefore$ (adj. A) B = 0

$$\Rightarrow \begin{pmatrix} 3a-25 & 15-2a & 1 \\ 10-a & a-6 & -2 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow -6-9+b=0 \Rightarrow b=15$

and $6(10-a)+9(a-6)-2(b)=0$

$\Rightarrow 60-6a+9a-54-30=0$

$\Rightarrow 3a=24 \Rightarrow a=8$

Hence, $a=8, b=15$.

116. (a) Given system of equations is

$x+ky+3z=0$

$3x+ky-2z=0$

$2x+3y-4z=0$

Since, system has non-trivial solution

$$\therefore \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow \quad 1(-4k+6)-k(-12+4)+3(9-2k)=0$

$\Rightarrow \quad 4k+33-6k=0 \Rightarrow k=\dfrac{33}{2}$

Hence, statement - 1 is false.
Statement-2 is the property.
It is a true statement.

117. **(d)** Given system of equations is

$x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=0$

It has unique solution.

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} \neq 0$$

$\Rightarrow \quad 1(2\lambda-6)-1(\lambda-3)+1(2-2)\neq 0$

$\Rightarrow \quad 2\lambda-6-\lambda+3\neq 0 \Rightarrow \lambda-3\neq 0 \Rightarrow \lambda\neq 3$

118. **(a)** $x-ky+z=0$

$kx+3y-kz=0$

$3x+y-z=0$

The given that system of equations have trivial solution,

$$\therefore \begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$\Rightarrow \quad 1(-3+k)+k(-k+3k)+1(k-9)\neq 0$

$\Rightarrow \quad k-3+2k^2+k-9\neq 0$

$\Rightarrow \quad k^2+k-6\neq 0 \quad \Rightarrow \quad k=-3, k\neq 2$

So, the equation will have only trivial solution,
when $k \in R-\{2,-3\}$

119. (a) Given that system of equations have non-zero solution $\Delta=0$

$$\Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 4(4-2)-k(k-2)+2(2k-8)=0$

$\Rightarrow \quad 8-k^2+2k+4k-16=0$

$k^2-6k+8=0$

$\Rightarrow (k-4)(k-2)=0 \Rightarrow k=4,2$

120. **(c)** $D=\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix}=0$

$$D_x=\begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix}\neq 0$$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$ No solution

121. **(d)** The given equations are

$-x+cy+bz=0$

$cx-y+az=0$

$bx+ay-z=0$

Given that x, y, z are not all zero

$\therefore$ The above system have non-zero solution

$$\Rightarrow \Delta=0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix}=0$$

$\Rightarrow -1(1-a^2)-c(-c-ab)+b(ac+b)=0$

$\Rightarrow -1+a^2+b^2+c^2+2abc=0$

$\Rightarrow a^2+b^2+c^2+2abc=1$

122. **(a)** $\alpha x+y+z=\alpha-1;$

$x+\alpha y+z=\alpha-1;$

$x+y+z\alpha=\alpha-1$

$$\Delta=\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$$

$=\alpha(\alpha^2-1)-1(\alpha-1)+1(1-\alpha)$

$=\alpha(\alpha-1)(\alpha+1)-1(\alpha-1)-1(\alpha-1)$

$=(\alpha-1)[\alpha^2+\alpha-1-1]$

$=(\alpha-1)[\alpha^2+\alpha-2]$

$=(\alpha-1)[\alpha^2+2\alpha-\alpha-2]$

$=(\alpha-1)[\alpha(\alpha+2)-1(\alpha+2)]$

$=(\alpha-1)^2(\alpha+2)$

$\because$ Equations has infinite solutions

$\therefore$ $\Delta=0$

$\Rightarrow \quad (\alpha-1)=0, \ \alpha+2=0$

$\Rightarrow \quad \alpha=-2, 1;$

But $\alpha\neq 1$.

$\therefore \alpha=-2$

123. **(d)** For homogeneous system of equations to have non zero solution, $\Delta=0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix}=0$$

Applying $C_2 \to C_2-2C_3$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix}=0 \ R_3\to R_3-R_1, R_2\to R_2-R_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c & c-a \end{vmatrix}=0$$

$\Rightarrow bc-ab=2bc-2ac$

$\Rightarrow \dfrac{2}{b}=\dfrac{1}{a}+\dfrac{1}{c}$

$\therefore a, b, c$ are in Harmonic Progression.

20

Continuity and Differentiability

TOPIC 1 Continuity

1. Let $f(x) = x.\left[\frac{x}{2}\right]$, for $-10 < x < 10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ______.

[NA Sep. 05, 2020 (I)]

2. If a function $f(x)$ defined by

$$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$$ be continuous for some

$a, b, c \in \mathbf{R}$ and $f'(0) + f'(2) = e$, then the value of a is :

[Sep. 02, 2020 (I)]

(a) $\frac{1}{e^2 - 3e + 13}$ (b) $\frac{e}{e^2 - 3e - 13}$

(c) $\frac{e}{e^2 + 3e + 13}$ (d) $\frac{e}{e^2 - 3e + 13}$

3. Let $[t]$ denote the greatest integer $\le t$ and $\lim\limits_{x \to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to : **[Jan. 9, 2020 (II)]**

(a) $\sqrt{A+1}$ (b) $\sqrt{A+5}$

(c) $\sqrt{A+21}$ (d) $\sqrt{A}$

4. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by

$$f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \ne 0 \\ k, & \text{when } x = 0 \end{cases}$$ is continuous, then k

is equal to __________. **[NA Jan. 7, 2020 (II)]**

5. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \ne \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$$

is continuous, then k is equal to: **[April 09, 2019 (I)]**

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

6. If $f(x) = [x] - \left[\frac{x}{4}\right], x \in R$, where $[x]$ denotes the greatest integer function, then: **[April 09, 2019 (II)]**

(a) f is continuous at $x = 4$.

(b) $\lim\limits_{x \to 4+} f(x)$ exists but $\lim\limits_{x \to 4-} f(x)$ does not exist.

(c) Both $\lim\limits_{x \to 4-} f(x)$ and $\lim\limits_{x \to 4+} f(x)$ exist but are not equal.

(d) $\lim\limits_{x \to 4-} f(x)$ exists but $\lim\limits_{x \to 4+} f(x)$ does not exist.

7. If the function

$$f(x) = \begin{cases} a|\pi - x| + 1, x \le 5 \\ b|x - \pi| + 3, x > 5 \end{cases}$$

is continuous at $x = 5$, then the value of $a - b$ is:

[April 09, 2019 (II)]

(a) $\frac{2}{\pi+5}$ (b) $\frac{-2}{\pi+5}$ (c) $\frac{2}{\pi-5}$ (d) $\frac{2}{5-\pi}$

8. Let $f: [-1, 3] \to R$ be defined as

$$f(x) = \begin{cases} |x| + [x], & -1 \le x < 1 \\ x + |x|, & 1 \le x < 2 \\ x + [x], & 2 \le x \le 3, \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t. Then, f is discontinuous at : **[April 08, 2019 (II)]**

(a) only one point (b) only two points

(c) only three points (d) four or more points

9. Let f: $\mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x)=\begin{cases} 5, & \text{if} \quad x \le 1 \\ a+bx, & \text{if} \quad 1<x<3 \\ b+5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

Then, f is : **[Jan 09, 2019 (I)]**

(a) continuous if a = 5 and b = 5
(b) continuous if a = – 5 and b = 10
(c) continous if a = 0 and b = 5
(d) not continuous for any values of a and b

10. If the function f defined as

$$f(x)=\frac{1}{x}-\frac{k-1}{e^{2x}-1}$$

$x \ne 0$, is continuous at $x = 0$,
then the ordered pair $(k, f(0))$ is equal to? **[Online April 16, 2018]**

(a) (3, 1) (b) (3, 2) (c) $\left(\frac{1}{3}, 2\right)$ (d) (2, 1)

11. Let $f(x)=\begin{cases} (x-1)^{\frac{1}{2-x}}, & x>1, x \ne 2 \\ k, & x=2 \end{cases}$

The value of k for which f is continuous at $x = 2$ is **[Online April 15, 2018]**

(a) e^{-2} (b) e (c) e^{-1} (d) 1

12. The value of k for which the function

$$f(x)=\begin{cases} \left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0<x<\frac{\pi}{2} \\ k+\frac{2}{5}, & x=\frac{\pi}{2} \end{cases}$$ is continuous at $x=\frac{\pi}{2}$, is :

[Online April 9, 2017]

(a) $\frac{17}{20}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $-\frac{2}{5}$

13. Let a, b $\in$ R, $(a \ne 0)$. if the function f defined as

$$f(x)=\begin{cases} \frac{2x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^3}, & \sqrt{2} \le x < \infty \end{cases}$$

is continuous in the interval $[0, \infty)$, then an ordered pair (a, b) is : **[Online April 10, 2016]**

(a) $(-\sqrt{2}, 1-\sqrt{3})$ (b) $(\sqrt{2}, -1+\sqrt{3})$
(c) $(\sqrt{2}, 1-\sqrt{3})$ (d) $(-\sqrt{2}, 1+\sqrt{3})$

14. Let k be a non–zero real number. **[Online April 11, 2015]**

If $f(x)=\begin{cases} \dfrac{(e^x-1)}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}, & x \ne 0 \\ 12, & x=0 \end{cases}$

is a continuous function then the value of k is:

(a) 4 (b) 1 (c) 3 (d) 2

15. If the function

$$f(x)=\begin{cases} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}, & x \ne \pi \\ k, & x=\pi \end{cases}$$

is continuous at $x = \pi$, then k equals: **[Online April 19, 2014]**

(a) 0 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$

16. If f(x) is continuous and $f\left(\frac{9}{2}\right)=\frac{2}{9}$, then $\lim\limits_{x \to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is equal to: **[Online April 9, 2014]**

(a) $\frac{9}{2}$ (b) $\frac{2}{9}$ (c) 0 (d) $\frac{8}{9}$

17. Consider the function :
$f(x) = [x] + |1-x|$, $-1 \le x \le 3$ where $[x]$ is the greatest integer function.
Statement 1 : f is not continuous at $x = 0$, 1, 2 and 3.

Statement 2 : $f(x)=\begin{cases} -x, & -1 \le x < 0 \\ 1-x, & 0 \le x < 1 \\ 1+x, & 1 \le x < 2 \\ 2+x, & 2 \le x \le 3 \end{cases}$

[Online April 25, 2013]

(a) Statement 1 is true ; Statement 2 is false,
(b) Statement 1 is true; Statement 2 is true; Statement 2 is not correct explanation for Statement 1.
(c) Statement 1 is true; Statement 2 is true; Statement It is a correct explanation for Statement 1.
(d) Statement 1 is false; Statement 2 is true.

18. Let f be a composite function of x defined by

$$f(u)=\frac{1}{u^2+u-2}, u(x)=\frac{1}{x-1}.$$

Then the number of points x where f is discontinuous is : **[Online April 23, 2013]**

(a) 4 (b) 3 (c) 2 (d) 1

19. Let $f(x) = -1 + |x-2|$, and $g(x) = 1 - |x|$; then the set of all points where fog is discontinuous is : **[Online April 22, 2013]**

(a) $\{0, 2\}$ (b) $\{0, 1, 2\}$
(c) $\{0\}$ (d) an empty set

20. If $f : R \to R$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is . **[2012]**

(a) continuous for every real x.
(b) discontinuous only at $x = 0$
(c) discontinuous only at non-zero integral values of x.
(d) continuous only at $x = 0$.

21. Let $f : [1, 3] \to R$ be a function satisfying $\frac{x}{[x]} \le f(x) \le \sqrt{6-x}$, for all $x \ne 2$ and $f(2) = 1$, where R is the set of all real numbers and $[x]$ denotes the largest integer less than or equal to x.

Statement 1: $\lim\limits_{x \to 2^-} f(x)$ exists. **[Online May 19, 2012]**

Statement 2: f is continuous at $x = 2$.

(a) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(d) Statement 1 is true, Statement 2 is false.

22. **Statement 1:** A function $f : R \to R$ is continuous at x_0 if and only if $\lim\limits_{x \to x_0} f(x)$ exists and $\lim\limits_{x \to x_0} f(x) = f(x_0)$

Statement 2: A function $f : R \to R$ is discontinuous at x_0 if and only if, $\lim\limits_{x \to x_0} f(x)$ exists and $\lim\limits_{x \to x_0} f(x) \ne f(x_0)$.

[Online May 12, 2012]

(a) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2 is false.

23. Define $f(x)$ as the product of two real functions **[2011RS]**

$$f_1(x) = x, x \in R, \text{and } f_2(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \ne 0 \\ 0, & \text{if } x = 0 \end{cases}$$

as follows :

$$f(x) = \begin{cases} f_1(x).f_2(x), & \text{if } x = 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Statement - 1 : $f(x)$ is continuous on R.

Statement - 2 : $f_1(x)$ and $f_2(x)$ are continuous on R.

(a) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

24. The values of p and q for which the function **[2011]**

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$$ is continuous for all x in R,

are

(a) $p = \frac{5}{2}, q = \frac{1}{2}$ (b) $p = -\frac{3}{2}, q = \frac{1}{2}$

(c) $p = \frac{1}{2}, q = \frac{3}{2}$ (d) $p = \frac{1}{2}, q = -\frac{3}{2}$

25. The function $f : R/\{0\} \to R$ given by **[2007]**

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as

(a) 0 (b) 1
(c) 2 (d) – 1

26. Let $f(x) = \frac{1-\tan x}{4x-\pi}, x \ne \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$.

If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is **[2004]**

(a) –1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) 1

27. f is defined in [-5, 5] as **[2002]**

$f(x) = x$ if x is rational
$= -x$ if x is irrational. Then

(a) $f(x)$ is continuous at every x, except $x = 0$
(b) $f(x)$ is discontinuous at every x, except $x = 0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere

TOPIC 2 Differentiability

28. Let $f: R \to R$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then: **[Sep. 06, 2020 (II)]**

(a) $\{0, 1\}$ (b) $\{0\}$

(c) ϕ (an empty set) (d) $\{1\}$

29. If the function $f(x)\begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to: **[Sep. 05, 2020 (I)]**

(a) $\left(\frac{1}{2}, 1\right)$ (b) $(1, 0)$

(c) $\left(\frac{1}{2}, -1\right)$ (d) $(1, 1)$

30. Let f be a twice differentiable function on $(1, 6)$. If $f(2) = 8$, $f'(2) = 5, f'(x) \ge 1$ and $f''(x) \ge 4$, for all $x \in (1, 6)$, then : **[Sep. 04, 2020 (I)]**

(a) $f(5) + f'(5) \le 26$ (b) $f(5) + f'(5) \ge 28$

(c) $f'(5) + f''(5) \le 20$ (d) $f(5) \le 10$

31. Suppose a differentiable function $f(x)$ satisfies the identity $f(x+y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim\limits_{x \to 0} \frac{f(x)}{x} = 1$, then $f'(3)$ is equal to ________. **[NA Sep. 04, 2020 (I)]**

32. The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \le 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$

is : **[Sep. 04, 2020 (II)]**

(a) continuous on $\mathbf{R} - \{1\}$ and differentiable on $\mathbf{R} - \{-1, 1\}$.

(b) both continuous and differentiable on $\mathbf{R} - \{1\}$.

(c) continuous on $\mathbf{R} - \{-1\}$ and differentiable on $\mathbf{R} - \{-1, 1\}$.

(d) both continuous and differentiable on $\mathbf{R} - \{-1\}$.

33. If $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x} & ; \quad x < 0 \\ b & ; \quad x = 0 \\ \frac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}} & ; \quad x > 0 \end{cases}$

is continuous at $x = 0$, then $a + 2b$ is equal to: **[Jan. 9, 2020 (I)]**

(a) 1 (b) –1 (c) 0 (d) –2

34. Let f and g be differentiable functions on **R** such that fog is the identity function. If for some $a, b \in \mathbf{R}$, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to: **[Jan. 9, 2020 (II)]**

(a) $\frac{1}{5}$ (b) 1 (c) 5 (d) $\frac{2}{5}$

35. Let S be the set of all functions $f: [0,1] \to R$, which are continuous on $[0, 1]$ and differentiable on $(0,1)$. Then for every f in S, there exists a $c \in (0,1)$, depending on f, such that: **[Jan. 8, 2020 (II)]**

(a) $|f(c) - f(1)| < (1-c)|f'(c)|$

(b) $\frac{f(1) - f(c)}{1-c} = f'(c)$

(c) $|f(c) + f(1)| < (1+c)|f'(c)|$

(d) $|f(c) - f(1)| < |f'(c)|$

36. Let the function, $f: [-7, 0] \to R$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x)$ d" 2, for all $x \in (-7, 0)$, then for all such functions f, $f'(-1) + f(0)$ lies in the interval: **[Jan. 7, 2020 (I)]**

(a) $(-\infty, 20]$ (b) $[-3, 11]$

(c) $(-\infty, 11]$ (d) $[-6, 20]$

37. Let S be the set of points where the function, $f(x) = |2 - |x-3||, x \in \boldsymbol{R}$, is not differentiable. Then $\sum\limits_{x \in S} f(f(x))$ is equal to———. **[NA Jan. 7, 2020 (I)]**

38. If $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$

is continuous at $x = 0$, then the ordered pair (p, q) is equal to: **[April 10, 2019 (I)]**

(a) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

(c) $\left(-\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{5}{2}, \frac{1}{2}\right)$

39. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \ge 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then: **[April 10, 2019 (II)]**

(a) $a\alpha^2 + b\alpha + a = 0$ (b) $a\alpha^2 - b\alpha - a = 1$

(c) $a\alpha^2 - b\alpha - a = 0$ (d) $a\alpha^2 + b\alpha - a = -2a^2$

40. Let f : $\mathbf{R} \to \mathbf{R}$ be differentiable at c $\in \mathbf{R}$ and f (c) = 0. If $g(x) = |f(x)|$, then at x = c, g is : **[April 10, 2019 (I)]**

(a) not differentiable if f'(c) = 0
(b) differentiable if f"(c) $\neq 0$
(c) differentiable if f' (c) = 0
(d) not differentiable

41. Let f(x) = 15 – |x – 10|; x $\in$ R. Then the set of all values of x, at which the function, g(x) = f (f(x)) is not differentiable, is: **[April 09, 2019 (I)]**

(a) {5, 10, 15} (b) {10, 15}
(c) {5, 10, 15, 20} (d) {10}

42. If $f(1) = 1, f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is : **[April 08, 2019 (II)]**

(a) 33 (b) 12 (c) 15 (d) 9

43. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to : **[Jan. 12, 2019 (II)]**

(a) $2e^2$ (b) 4e (c) 2e (d) $4e^2$

44. Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and

$g(x) = |f(x)| + f(|x|)$. Then, in the interval (–2, 2), g is : **[Jan. 11, 2019 (I)]**

(a) differentiable at all points
(b) not continuous
(c) not differentiable at two points
(d) not differentiable at one point

45. If $x\log_e(\log_e x) - x^2 + y^2 = 4(y > 0)$, then $\frac{dy}{dx}$ at x = e is equal to : **[Jan. 11, 2019 (I)]**

(a) $\frac{(1+2e)}{2\sqrt{4+e^2}}$ (b) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(c) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (d) $\frac{e}{\sqrt{4+e^2}}$

46. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x-\pi)\cos|x|$ is not differentiable. Then the set K is equal to : **[Jan. 11, 2019 (II)]**

(a) ϕ (an empty set) (b) {π}
(c) {0} (d) {0, π}

47. Let $f(x) = \begin{cases} \max\{|x|, x^2\} & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval (– 4, 4) at which f is not differentiable. Then S: **[Jan 10, 2019 (I)]**

(a) is an empty set
(b) equals {– 2, – 1, 0, 1, 2}
(c) equals {– 2, – 1, 1, 2}
(d) equals {– 2, 2}

48. Let f : (– 1, 1) $\to$ **R** be a function defined by $f(x)$ = max $\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly: **[Jan. 10, 2019 (II)]**

(a) five elements (b) one element
(c) three elements (d) two elements

49. Let S = $\{t \in R : f(x) = |x - \pi|(e^{|x|} - 1)\sin|x|$ is not differentiable at t$\}$. Then the set S is equal to : **[2018]**

(a) {0} (b) {π}
(c) {0, π} (d) ϕ (an empty set)

50. Let $S = \{(\lambda, \mu) \in R \times R : f(t) = (|\lambda|e^{|t|} - \mu).\sin(2|t|), t \in R$, is a differentiable function$\}$. Then S is a subest of? **[Online April 15, 2018]**

(a) $R \times [0, \infty)$ (b) $(-\infty, 0) \times R$
(c) $[0, \infty) \times R$ (d) $R \times (-\infty, 0)$

51. If the function

$f(x) = \begin{cases} -x, & x < 1 \\ a + \cos^{-1}(x+b), & 1 \le x \le 2 \end{cases}$ is differentiable at

x = 1, then $\frac{a}{b}$ is equal to : **[Online April 9, 2016]**

(a) $\frac{\pi+2}{2}$ (b) $\frac{\pi-2}{2}$

(c) $\frac{-\pi-2}{2}$ (d) $-1 - \cos^{-1}(2)$

52. If the function.

$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx + 2, & 3 < x \le 5 \end{cases}$ is differentiable, then the

value of k + m is : **[2015]**

(a) $\frac{10}{3}$ (b) 4

(c) 2 (d) $\frac{16}{5}$

53. Let f: R $\to$ R be a function such that $|f(x)| \le x^2$, for all $x \in R$. Then, at $x = 0$, f is: **[Online April 19, 2014]**

(a) continuous but not differentiable.
(b) continuous as well as differentiable.
(c) neither continuous nor differentiable.
(d) differentiable but not continuous.

54. Let f, g: R → R be two functions defined by

$$f(x)=\begin{cases} x\sin\left(\dfrac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}, \text{ and } g(x)=x\,f(x)$$

Statement I: f is a continuous function at $x = 0$.

Statement II: g is a differentiable function at $x = 0$. **[Online April 12, 2014]**

(a) Both statement I and II are false.
(b) Both statement I and II are true.
(c) Statement I is true, statement II is false.
(d) Statement I is false, statement II is true.

55. Consider the function, $f(x)=|x-2|+|x-5|, x\in R$.

Statement-1 : $f'(4)=0$

Statement-2 : f is continuous in [2,5], differentiable in (2,5) and $f(2)=f(5)$. **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

56. If $f(x) = a\,|\sin x| + be^{|x|} + c|x|^3$, where $a, b, c \in R$, is differentiable at $x = 0$, then **[Online May 26, 2012]**

(a) $a = 0$, b and c are any real numbers
(b) $c = 0$, $a = 0$, b is any real number
(c) $b = 0$, $c = 0$, a is any real number
(d) $a = 0$, $b = 0$, c is any real number

57. If $x + |y| = 2y$, then y as a function of x, at $x = 0$ is **[Online May 7, 2012]**

(a) differentiable but not continuous
(b) continuous but not differentiable
(c) continuous as well as differentiable
(d) neither continuous nor differentiable

58. If function $f(x)$ is differentiable at $x = a$,

then $\lim\limits_{x\to a}\dfrac{x^2 f(a)-a^2 f(x)}{x-a}$ is : **[2011RS]**

(a) $-a^2 f'(a)$ (b) $a\,f(a)-a^2 f'(a)$
(c) $2af(a)-a^2 f'(a)$ (d) $2af(a)+a^2 f'(a)$

59. Let $f(x)=\begin{cases}(x-1)\sin\dfrac{1}{x-1} & \text{if } x\neq 1\\ 0 & \text{if } x=1\end{cases}$ **[2008]**

Then which one of the following is true?

(a) f is neither differentiable at $x = 0$ nor at $x = 1$
(b) f is differentiable at $x = 0$ and at $x = 1$
(c) f is differentiable at $x = 0$ but not at $x = 1$
(d) f is differentiable at $x = 1$ but not at $x = 0$

60. Let $f: R\to R$ be a function defined by

$f(x)=\min\{x+1, |x|+1\}$, Then which of the following is true?

(a) $f(x)$ is differentiable everywhere **[2007]**
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x)\geq 1$ for all $x\in R$
(d) $f(x)$ is not differentiable at $x = 1$

61. The set of points where $f(x)=\dfrac{x}{1+|x|}$ is differentiable is **[2006]**

(a) $(-\infty,0)\cup(0,\infty)$ (b) $(-\infty,-1)\cup(-1,\infty)$
(c) $(-\infty,\infty)$ (d) $(0,\infty)$

62. If f is a real valued differentiable function satisfying $|f(x)-f(y)|\leq (x-y)^2, x, y\in R$ and $f(0) = 0$, then $f(1)$ equals **[2005]**

(a) -1 (b) 0
(c) 2 (d) 1

63. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim\limits_{h\to 0}\dfrac{1}{h}f(1+h)=5$, then $f'(1)$ equals **[2005]**

(a) 3 (b) 4 (c) 5 (d) 6

64. If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & , x\neq 0\\ 0 & , x=0\end{cases}$ then f(x) is

(a) discontinuous every where **[2003]**
(b) continuous as well as differentiable for all x
(c) continuous for all x but not differentiable at $x = 0$
(d) neither differentiable nor continuous at $x = 0$

TOPIC 3 Chain Rule of Differentiation, Differentiation of Explicit & Implicit Functions, Parametric & Composite Functions, Logarithmic & Exponential Functions, Inverse Functions, Differentiation by Trigonometric Substitution

65. The derivative of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=\dfrac{1}{2}$ is : **[Sep. 05, 2020 (II)]**

(a) $\dfrac{2\sqrt{3}}{5}$ (b) $\dfrac{\sqrt{3}}{12}$ (c) $\dfrac{2\sqrt{3}}{3}$ (d) $\dfrac{\sqrt{3}}{10}$

66. If $(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos y)=a^2-b^2$, where $a>b>0$, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is: **[Sep. 04, 2020 (I)]**

(a) $\frac{a-2b}{a+2b}$ (b) $\frac{a-b}{a+b}$ (c) $\frac{a+b}{a-b}$ (d) $\frac{2a+b}{2a-b}$

67. If $y=\sum_{k=1}^{6}k\cos^{-1}\left\{\frac{3}{5}\cos kx-\frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at $x=0$ is ________. **[NA Sep. 02, 2020 (II)]**

68. If $x=2\sin\theta-\sin 2\theta$ and $y=2\cos\theta-\cos 2\theta$, $\theta\in[0,2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta=\pi$ is : **[Jan. 9, 2020 (II)]**

(a) $\frac{3}{4}$ (b) $-\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{4}$

69. If $y(\alpha)=\sqrt{2\left(\frac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}\right)+\frac{1}{\sin^2\alpha}}$, $\alpha\in\left(\frac{3\pi}{4},\pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha=\frac{5\pi}{6}$ is: **[Jan. 7, 2020 (I)]**

(a) 4 (b) $\frac{4}{3}$ (c) –4 (d) $-\frac{1}{4}$

70. Let $y=y(x)$ be *a* function of x satisfying $y\sqrt{1-x^2}=k-x\sqrt{1-y^2}$ where k is *a* constant and $y\left(\frac{1}{2}\right)=-\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x=\frac{1}{2}$, is equal to: **[Jan. 7, 2020 (II)]**

(a) $-\frac{\sqrt{5}}{4}$ (b) $-\frac{\sqrt{5}}{2}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{2}$

71. If $e^y+xy=e$, the ordered pair $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)$ at $x=0$ is equal to : **[April 12, 2019 (I)]**

(a) $\left(\frac{1}{e},-\frac{1}{e^2}\right)$ (b) $\left(-\frac{1}{e},\frac{1}{e^2}\right)$

(c) $\left(\frac{1}{e},\frac{1}{e^2}\right)$ (d) $\left(-\frac{1}{e},-\frac{1}{e^2}\right)$

72. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ is : **[April 12, 2019 (II)]**

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) 2

73. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2$, $x\in\left(0,\frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to : **[April 08, 2019 (I)]**

(a) $\frac{\pi}{6}-x$ (b) $x-\frac{\pi}{6}$ (c) $\frac{\pi}{3}-x$ (d) $2x-\frac{\pi}{3}$

74. Let S be the set of all points in $(-\pi,\pi)$ at which the function $f(x)=\min\{\sin x,\cos x\}$ is not differentiable. Then S is a subset of which of the following? **[Jan. 12, 2019 (I)]**

(a) $\left\{-\frac{\pi}{4},0,\frac{\pi}{4}\right\}$ (b) $\left\{-\frac{3\pi}{4},-\frac{\pi}{4},\frac{3\pi}{4},\frac{\pi}{4}\right\}$

(c) $\left\{-\frac{\pi}{2},-\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right\}$ (d) $\left\{-\frac{3\pi}{4},-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{4}\right\}$

75. For $x>1$, if $(2x)^{2y}=4e^{2x-2y}$, then $(1+\log_e 2x)^2\frac{dy}{dx}$ is equal to : **[Jan. 12, 2019 (I)]**

(a) $\frac{x\log_e 2x-\log_e 2}{x}$ (b) $\log_e 2x$

(c) $\frac{x\log_e 2x+\log_e 2}{x}$ (d) $x\log_e 2x$

76. Let $f:\mathbf{R}\to\mathbf{R}$ be a function such that $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3), x\in\mathbf{R}$. Then $f(2)$ equals: **[Jan 10, 2019 (I)]**

(a) –4 (b) 30 (c) –2 (d) 8

77. If x = 3 tan t and y = 3 sec t, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$, is: **[Jan. 09, 2019 (II)]**

(a) $\frac{1}{3\sqrt{2}}$ (b) $\frac{1}{6\sqrt{2}}$ (c) $\frac{3}{2\sqrt{2}}$ (d) $\frac{1}{6}$

78. If $x=\sqrt{2^{\text{cosec}^{-1}t}}$ and $y=\sqrt{2^{\sec^{-1}t}}$ $(|t|\geq 1)$, then $\frac{dy}{dx}$ is equal to. **[Online April 16, 2018]**

(a) $\frac{y}{x}$ (b) $-\frac{y}{x}$ (c) $-\frac{x}{y}$ (d) $\frac{x}{y}$

79. If $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$, then $\lim_{x\to 0}\frac{f'(x)}{x}$ **[Online April 15, 2018]**

(a) Exists and is equal to – 2
(b) Does not exist
(c) Exist and is equal to 0
(d) Exists and is equal to 2

80. If $f(x)=\sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$, then $f'\left(-\frac{1}{2}\right)$ equals. **[Online April 15, 2018]**

(a) $\sqrt{3}\log_e\sqrt{3}$ (b) $-\sqrt{3}\log_e\sqrt{3}$

(c) $-\sqrt{3}\log_e 3$ (d) $\sqrt{3}\log_e 3$

81. If $x^2+y^2+\sin y=4$, then the value of $\frac{d^2y}{dx^2}$ at the point $(-2, 0)$ is **[Online April 15, 2018]**

(a) -34 (b) -32 (c) -2 (d) 4

82. If for $x\in\left(0,\frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$, then g(x) equals : **[2017]**

(a) $\frac{3}{1+9x^3}$ (b) $\frac{9}{1+9x^3}$

(c) $\frac{3x\sqrt{x}}{1-9x^3}$ (d) $\frac{3x}{1-9x^3}$

83. For $x\in R$, $f(x)=|\log 2-\sin x|$ and $g(x)=f(f(x))$, then : **[2016]**

(a) $g'(0)=-\cos(\log 2)$

(b) g is differentiable at x = 0 and $g'(0)=-\sin(\log 2)$

(c) g is not differentiable at x = 0

(d) $g'(0)=\cos(\log 2)$

84. If $f(x)=x^2-x+5$, $x>\frac{1}{2}$, and g(x) is its inverse function, then $g'(7)$ equals: **[Online April 12, 2014]**

(a) $-\frac{1}{3}$ (b) $\frac{1}{13}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{13}$

85. If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x=1$ is equal to : **[2013]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

86. If the curves $\frac{x^2}{\alpha}+\frac{y^2}{4}=1$ and $y^3=16x$ intersect at right angles, then a value of α is : **[Online April 23, 2013]**

(a) 2 (b) $\frac{4}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

87. For $a>0$, $t\in\left(0,\frac{\pi}{2}\right)$, let $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$,

Then, $1+\left(\frac{dy}{dx}\right)^2$ equals : **[Online April 22, 2013]**

(a) $\frac{x^2}{y^2}$ (b) $\frac{y^2}{x^2}$ (c) $\frac{x^2+y^2}{y^2}$ (d) $\frac{x^2+y^2}{x^2}$

88. Let $f(x)=\frac{x^2-x}{x^2+2x}.x\neq 0,-2$. Then $\frac{d}{dx}[f^{-1}(x)]$ (wherever it is defined) is equal to : **[Online April 9, 2013]**

(a) $\frac{-1}{(1-x)^2}$ (b) $\frac{3}{(1-x)^2}$

(c) $\frac{1}{(1-x)^2}$ (d) $\frac{-3}{(1-x)^2}$

89. If $f'(x)=\sin(\log x)$ and $y=f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ equals **[Online May 12, 2012]**

(a) $\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

(b) $\frac{12}{(3-2x)^2}$

(c) $\frac{12}{(3-2x)^2}\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

(d) $\frac{12}{(3-2x)^2}\cos\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

90. Let $f:(-1,1)\to R$ be a differentiable function with $f(0)=-1$ and $f'(0)=1$. Let $g(x)=[f(2f(x)+2)]^2$. Then $g'(0)=$ **[2010]**

(a) -4 (b) 0 (c) -2 (d) 4

91. Let y be an implicit function of x defined by $x^{2x}-2x^x\cot y-1=0$. Then $y'(1)$ equals **[2009]**

(a) 1 (b) log 2 (c) $-\log 2$ (d) -1

92. If $x^m.y^n=(x+y)^{m+n}$, then $\frac{dy}{dx}$ is **[2006]**

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$ (c) xy (d) $\frac{x}{y}$

93. If $x=e^{y+e^{y+\cdots\text{to}\infty}}$, $x>0$, then $\frac{dy}{dx}$ is **[2004]**

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

94. Let $f(x)$ be a polynomial function of second degree. If $f(1)=f(-1)$ and a,b,c are in A. P, then $f'(a), f'(b), f'(c)$ are in **[2003]**

(a) Arithmetic -Geometric Progression

(b) A.P

(c) G.P

(d) H.P.

95. If $f(x+y)=f(x).f(y)\forall x.y$ and $f(5)=2$, $f'(0)=3$, then $f'(5)$ is **[2002]**

(a) 0 (b) 1 (c) 6 (d) 2

TOPIC 4 Differentiation of Infinite Series, Successive Differentiation, nth Derivative of Some Standard Functions, Leibnitz's Theorem, Rolle's Theorem, Lagrange's Mean Value Theorem

96. For all twice differentiable functios f : R→R, with $f(0)=f(1)=f'(0)=0$ **[Sep. 06, 2020 (II)]**

(a) $f''(x)\neq 0$ at every point $x\in(0,1)$

(b) $f''(x)=0$, for some $x\in(0,1)$

(c) $f''(0)=0$

(d) $f''(x)=0$, at every point $x\in(0,1)$

97. If $y^2+\log_e(\cos^2 x)=y,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then : **[Sep. 03, 2020 (I)]**

(a) $y''(0)=0$ (b) $|y'(0)|+|y''(0)|=1$

(c) $|y''(0)|=2$ (d) $|y'(0)|+|y''(0)|=3$

98. If c is a point at which Rolle's theorem holds for the function, $f(x)=\log_e\left(\frac{x^2+a}{7x}\right)$ in the interval [3, 4], where $\alpha\in R$, then $f''(c)$ is equal to: **[Jan. 8, 2020 (I)]**

(a) $-\frac{1}{12}$ (b) $\frac{1}{12}$ (c) $-\frac{1}{24}$ (d) $\frac{\sqrt{3}}{7}$

99. Let $x^k+y^k=a^k$, $(a, k>0)$ and $\frac{dy}{dx}+\left(\frac{y}{x}\right)^{\frac{1}{3}}=0$, then k is: **[Jan. 7, 2020 (I)]**

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

100. The value of c in the Lagrange's mean value theorem for the function $f(x)=x^3-4x^2+8x+11$, when $x\in[0,1]$ is: **[Jan. 7, 2020 (II)]**

(a) $\frac{4-\sqrt{5}}{3}$ (b) $\frac{4-\sqrt{7}}{3}$

(c) $\frac{2}{3}$ (d) $\frac{\sqrt{7}-2}{3}$

101. If $2x=y^{\frac{1}{5}}+y^{-\frac{1}{5}}$ = and $(x^2-1)\frac{d^2y}{dx^2}+\lambda x\frac{dy}{dx}+ky=0$, then $\lambda+k$ is equal to : **[Online April 9, 2017]**

(a) −23 (b) −24 (c) 26 (d) −26

102. Let f be a polynomial function such that $f(3x)=f'(x)$, $f''(x)$, for all $x\in\mathbf{R}$. Then : **[Online April 9, 2017]**

(a) $f(b)+f'(b)=28$

(b) $f''(b)-f'(b)=0$

(c) $f''(b)-f'(b)=4$

(d) $f(b)-f'(b)+f''(b)=10$

103. If $y=\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$, then $(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is equal to **[Online April 8, 2017]**

(a) $12y$ (b) $224y^2$ (c) $225y^2$ (d) $225y$

104. If Rolle's theorem holds for the function $f(x)\ 2x^3+bx^2+cx$, $x\in[-1, 1]$, at the point $x=\frac{1}{2}$, then $2b+c$ equals : **[Online April 10, 2015]**

(a) –3 (b) –1 (c) 2 (d) 1

105. If f and g are differentiable functions in [0, 1] satisfying $f(0)=2=g(1)$, $g(0)=0$ and $f(1)=6$, then for some $c\in]0,1[$ **[2014]**

(a) $f'(c)=g'(c)$ (b) $f'(c)=2g'(c)$

(c) $2f'(c)=g'(c)$ (d) $2f'(c)=3g'(c)$

106. Let $f(x)=x|x|$, $g(x)=\sin x$ and $h(x)=(gof)(x)$. Then **[Online April 11, 2014]**

(a) h(x) is not differentiable at x = 0.

(b) h(x) is differentiable at x = 0, but h′(x) is not continuous at x = 0

(c) h′(x) is continuous at x = 0 but it is not differentiable at x = 0

(d) h′(x) is differentiable at x = 0

107. Let for i = 1, 2, 3, $p_i(x)$ be a polynomial of degree 2 in x, $p'_i(x)$ and $p''_i(x)$ be the first and second order derivatives of $p_i(x)$ respectively. Let,

$$A(x)=\begin{bmatrix} p_1(x) & p_1'(x) & p_1''(x)\\ p_2(x) & p_2'(x) & p_2''(x)\\ p_3(x) & p_3'(x) & p_3''(x)\end{bmatrix}$$

and $B(x)=[A(x)]^T A(x)$. Then determinant of B(x): **[Online April 11, 2014]**

(a) is a polynomial of degree 6 in x.

(b) is a polynomial of degree 3 in x.

(c) is a polynomial of degree 2 in x.

(d) does not depend on x.

108. If the Rolle's theorem holds for the function $f(x)=2x^3+ax^2+bx$ in the interval [– 1, 1] for the point $c=\frac{1}{2}$, then the value of 2a + b is: **[Online April 9, 2014]**

(a) 1 (b) – 1 (c) 2 (d) – 2

109. If $f(x) = \sin(\sin x)$ and $f''(x) + \tan x\, f'(x) + g(x) = 0$, then $g(x)$ is : **[Online April 23, 2013]**

(a) $\cos^2 x \cos(\sin x)$ (b) $\sin^2 x \cos(\cos x)$

(c) $\sin^2 x \sin(\cos x)$ (d) $\cos^2 x \sin(\sin x)$

110. Consider a quadratic equation $ax^2 + bx + c = 0$, where $2a + 3b + 6c = 0$ and let $g(x) = a\dfrac{x^3}{3} + b\dfrac{x^2}{2} + cx$. **[Online May 19, 2012]**

Statement 1: The quadratic equation has at least one root in the interval (0, 1).

Statement 2: The Rolle's theorem is applicable to function $g(x)$ on the interval [0, 1].

(a) Statement 1 is false, Statement 2 is true.

(b) Statement 1 is true, Statement 2 is false.

(c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(d) Statement 1 is true, Statement 2 is true, , Statement 2 is a correct explanation for Statement 1.

111. $\dfrac{d^2x}{dy^2}$ equals : **[2011]**

(a) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ (b) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

(c) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$ (d) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

112. Let $f(x) = x\,|\,x\,|$ and $g(x) = \sin x$.

Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : gof is twice differentiable at $x = 0$. **[2009]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

113. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is **[2007]**

(a) $\log_3 e$ (b) $\log_e 3$ (c) $2\log_3 e$ (d) $\dfrac{1}{2}\log_3 e$

114. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then **[2005]**

(a) $f(6) \geq 8$ (b) $f(6) < 8$ (c) $f(6) < 5$ (d) $f(6) = 5$

115. If the equation $a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots\ldots\ldots + a_1 x = 0$, $a_1 \neq 0, n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots\ldots\ldots + a_1 = 0$ has a positive root, which is **[2005]**

(a) greater than α

(b) smaller than α

(c) greater than or equal to α

(d) equal to α

116. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval **[2004]**

(a) (1, 3) (b) (1, 2) (c) (2, 3) (d) (0, 1)

117. If $f(x) = x^n$, then the value of **[2003]**

$f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + \ldots\ldots\ldots \dfrac{(-1)^n f^n(1)}{n!}$ is

(a) 1 (b) 2^n (c) $2^n - 1$ (d) 0.

118. Let $f(a) = g(a) = k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some n. Further if

$$\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$$

then the value of k is **[2003]**

(a) 0 (b) 4 (c) 2 (d) 1

119. If $y = (x + \sqrt{1 + x^2})^n$, then $(1 + x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is **[2002]**

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

120. If $2a + 3b + 6c = 0$, $(a, b, c \in R)$ then the quadratic equation $ax^2 + bx + c = 0$ has **[2002]**

(a) at least one root in [0, 1]

(b) at least one root in [2, 3]

(c) at least one root in [4, 5]

(d) None of these

Hints & Solutions

1. **(8)** We know $[x]$ discontinuous for $x \in Z$

$f(x) = x\left[\frac{x}{2}\right]$ may be discontinuous where $\frac{x}{2}$ is an integer.

So, points of discontinuity are,

$x = \pm 2, \pm 4, \pm 6, \pm 8$ and 0

but at $x = 0$

$\lim_{x\to 0^+} f(x) = 0 = f(0) = \lim_{x\to 0^-} f(x)$

So, $f(x)$ will be discontinuous at $x = \pm 2, \pm 4, \pm 6$ and ± 8.

2. **(d)** Since, function $f(x)$ is continuous at $x = 1, 3$

$\therefore f(1) = f(1^+)$

$\Rightarrow ae + be^{-1} = c$ ...(i)

$f(3) = f(3^+)$

$\Rightarrow 9c = 9a + 6c \Rightarrow c = 3a$ ...(ii)

From (i) and (ii),

$b = ae(3-e)$ ...(iii)

$$f'(x) = \begin{bmatrix} ae^x - be^{-x} & -1 < x < 1 \\ 2cx & 1 < x < 3 \\ 2ax + 2c & 3 < x < 4 \end{bmatrix}$$

$f'(0) = a - b,\ f'(2) = 4c$

Given, $f'(0) + f'(2) = e$

$a - b + 4c = e$ ...(iv)

From eqs. (i), (ii), (iii) and (iv),

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow 13a - 3ae + ae^2 = e$

$\Rightarrow a = \dfrac{e}{e^2 - 3e + 13}$

3. **(a)** $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A \Rightarrow \lim_{x\to 0} x\left[\frac{4}{x} - \left\{\frac{4}{x}\right\}\right] = A$

$\Rightarrow \lim_{x\to 0} 4 - x\left\{\frac{4}{x}\right\} = A \Rightarrow 4 - 0 = \text{A}$

As, $f(x) = [x^2]\sin(\pi x)$ will be discontinuous at non-integers

And, when $x = \sqrt{A+1} \Rightarrow x = \sqrt{5}$, which is not an integer.

Hence, $f(x)$ is discontinuous when x is equal to $\sqrt{A+1}$

4. **(5)** $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{1}{x}\ln\left(\frac{1+3x}{1-2x}\right)\right)$

$= \lim_{x\to 0}\left(\frac{\ln(1+3x)}{x} - \frac{\ln(1-2x)}{x}\right)$

$= \lim_{x\to 0}\left(\frac{3\ln(1+3x)}{3x} - \frac{2\ln(1-2x)}{-2x}\right)$

$= 3 + 2 = 5$

$\because$ $f(x)$ will be continuous

$\therefore$ $k = f(0) = \lim_{x\to 0} f(x) = 5$

5. **(b)** Since, f(x) is continuous, then

$\lim_{x\to \frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$

$\lim_{x\to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$

Now by L- hospital's rule

$\lim_{x\to \frac{\pi}{4}} \frac{\sqrt{2}\sin x}{\operatorname{cosec}^2 x} = k \Rightarrow \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{(\sqrt{2})^2} = k \quad \Rightarrow k = \frac{1}{2}$

6. **(a)** L.H.L. $\lim_{x\to 4^-}\left([x] - \left[\frac{x}{4}\right]\right) = 3 - 0 = 3$

R.H.L. $\lim_{x\to 4^+} [x] - \left[\frac{x}{4}\right] = 4 - 1 = 3$

$f(4) = [4] - \left[\frac{4}{4}\right] = 4 - 1 = 3$

$\because$ LHL $= f(4) =$ RHL

$\therefore f(x)$ is continuous at $x = 4$

7. **(d)** R.H.L. $\lim_{x\to 5^+} b|(x-\pi)| + 3 = (5-\pi)b + 3$

$f(5) =$ L.H.L. $\lim_{x\to 5^-} a|(\pi - x)| + 1 = a(5-\pi) + 1$

$\because$ function is continuous at $x = 5$

$\therefore$ LHL = RHL

$(5-\pi)b + 3 = (5-\pi)a + 1$

$\Rightarrow 2 = (a-b)(5-\pi) \Rightarrow a - b = \dfrac{2}{5-\pi}$

8. **(c)** Given function is,

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$$

$$=\begin{cases}-x-1, & -1\le x<0\\ x, & 0\le x<1\\ 2x, & 1\le x<2\\ x+2, & 2\le x<3\\ 6, & x=3\end{cases}$$

$\Rightarrow f(-1)=0, f(-1^+)=0;$

$f(0^-)=-1, f(0)=0, f(0^+)=0;$

$f(1^-)=1, f(1)=2, f(1^+)=2;$

$f(2^-)=4, f(2)=4, f(2^+)=4;$

$f(3^-)=5, f(3)=6$

$f(x)$ is discontinuous at $x=\{0, 1, 3\}$

Hence, $f(x)$ is discontinuous at only three points.

9. **(d)** Let $f(x)$ is continuous at $x=1$, then

$f(1^-)=f(1)=f(1^+)$

$\Rightarrow\ 5=a+b \quad ...(1)$

Let $f(x)$ is continuous at $x=3$, then

$f(3^-)=f(3)=f(3^+)$

$\Rightarrow\ a+3b=b+15 \quad ...(2)$

Let $f(x)$ is continuous at $x=5$, then

$f(5^-)=f(5)=f(5^+)$

$\Rightarrow\ b+25=30$

$\Rightarrow\ b=30-25=5$

From (1), $a=0$

But $a=0$, $b=5$ do not satisfy equation (2)

Hence, $f(x)$ is not continuous for any values of a and b

10. **(a)** If the function is continuous at $x=0$, then

$\lim_{x\to 0} f(x)$ will exist and $f(0)=\lim_{x\to 0} f(x)$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(\frac{1}{x}-\frac{k-1}{e^{2x}-1}\right)$

$$=\lim_{x\to 0}\left(\frac{e^{2x}-1-kx+x}{(x)(e^{2x}-1)}\right)$$

$$=\lim_{x\to 0}\left[\frac{\left(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+...\right)-1-kx+x}{(x)\left(\left(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+...\right)-1\right)}\right]$$

$$=\lim_{x\to 0}\left[\frac{(3-k)x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+...}{\left(2x^2+\frac{4x^3}{2!}+\frac{8x^3}{3!}+...\right)}\right]$$

For the limit to exist, power of x in the numerator should be greater than or equal to the power of x in the denominator. Therefore, coefficient of x in numerator is equal to zero

$\Rightarrow\ 3-k=0$

$\Rightarrow\ k=3$

So the limit reduces to

$$\lim_{x\to 0}\frac{(x^2)\left(\frac{4}{2!}+\frac{8x}{3!}+...\right)}{(x^2)\left(2+\frac{4x}{2!}+\frac{8x^2}{3!}+...\right)}$$

$$=\lim_{x\to 0}\frac{\frac{4}{2!}+\frac{8x}{3!}+...}{2+\frac{4x}{2!}+\frac{8x^2}{3!}+...}=1$$

Hence, $f(0)=1$

11. **(c)** Since $f(x)$ is continuous at $x=2$.

$\therefore\ \lim_{x\to 2} f(x)=f(2)$

$\Rightarrow\ \lim_{x\to 2}(x-1)^{\frac{1}{2-x}}=k \qquad (1^\infty \text{ form})$

$\therefore\ e^l=k$

where $l=\lim_{x\to 2}(x-1-1)\times\frac{1}{2-x}=\lim_{x\to 2}\frac{x-2}{2-x}$

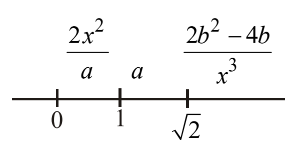
$$=\lim_{x\to 2}\left(\frac{x-2}{x-2}\right)$$

$\Rightarrow\ k=e^{-1}$

12. **(c)** $\lim_{x\to\pi/2} f(x)=f(\pi/2)$

$\Rightarrow\ k+2/5=1\ \Rightarrow\ k=1-\frac{2}{5}\ \Rightarrow\ k=\frac{3}{5}$

13. **(c)**



Continuity at $x=1$

$\frac{2}{a}=a\Rightarrow a=\pm\sqrt{2}$

Continuity at $x=\sqrt{2}$ $a=\sqrt{2}$

$a=\frac{2b^2-4b}{2\sqrt{2}}$

Put $a=\sqrt{2}$

$2=b^2-2b\ \Rightarrow b^2-2b-2=0$

$b=\frac{2\pm\sqrt{4+4.2}}{2}=1\pm\sqrt{3}$

So, $(a, b)=(\sqrt{2}, 1-\sqrt{3})$

14. **(c)** Since f(x) is a continuous function therefore limit of f(x) at $x \to 0$ = value of f(x) at 0.

$$\therefore \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(e^x-1)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}$$

$$= \lim_{x\to 0} \frac{x^2\left(\frac{e^x-1}{x}\right)^2}{\frac{x}{R}\left[\frac{\sin\left(\frac{x}{R}\right)}{\frac{x}{R}}\right].\frac{\log\left(1+\frac{x}{4}\right)}{\left(\frac{x}{4}\right)}} \times \left(\frac{x}{4}\right)$$

$$= \lim_{x\to 0} \frac{x^2\left(\frac{e^x-1}{x}\right)^2 4k}{\frac{\sin\frac{x}{k}}{\frac{x}{k}}.\frac{\log\left(1+\frac{x}{4}\right)}{\frac{x}{4}}}$$

on applying limit we get

$4k = 12 \Rightarrow k = 3$

15. **(d)** Since $f(x) = \frac{\sqrt{2+\cos x}-1}{(\pi - x)^2}$ is

Continuous at $x = \pi$

$\therefore$ L.H.L = R.H.L = f (π)

Let $(\pi - x) = \theta$, $\theta \to 0$ when $x \to \pi$

$$\therefore \lim_{\theta\to 0} \frac{\sqrt{2-\cos\theta}-1}{\theta^2}$$

$$= \lim_{\theta\to 0} \frac{(2-\cos\theta)-1}{\theta^2} \times \frac{1}{\sqrt{2-\cos\theta}+1}$$

$$= \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2}.\frac{1}{2} \quad (\because \cos 0 = 1)$$

$$= \frac{1}{2}\lim_{\theta\to 0} \frac{2\sin^2\theta/2}{\theta^2} = \frac{2}{2}\lim_{\theta\to 0} \frac{\sin^2\theta/2}{\frac{\theta^2}{4}.4}$$

$$= \frac{1}{4}\left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

16. **(b)** Given that $f\left(\frac{9}{2}\right) = \frac{2}{9}$

$$\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right) = \lim_{x\to 0}\left(\frac{x^2}{1-\cos 3x}\right)$$

$$= \lim_{x\to 0}\left(\frac{x^2}{2\sin^2\frac{3x}{2}}\right) = \frac{1}{2}\lim_{x\to 0}\left(\frac{\frac{9}{4}.x^2.\frac{4}{9}}{\sin^2\frac{3x}{2}}\right)$$

$$= \frac{4}{9\times 2}\lim_{x\to 0}\left(\frac{1}{\frac{\sin^2\frac{3x}{2}}{\left(\frac{3x}{2}\right)^2}}\right)$$

$$= \frac{2}{9}\left[\frac{\lim\limits_{x\to 0} 1}{\lim\limits_{x\to 0}\frac{\sin^2\frac{3x}{2}}{\left(\frac{3x}{2}\right)^2}}\right] \quad \left\{\lim_{x\to 0}\frac{\sin x}{x} = 1\right\}$$

$$= \frac{2}{9}.\left[\frac{1}{1}\right] = \frac{2}{9}$$

17. **(a)** Let $f(x) = [x] + |1-x|,\ -1 \le x \le 3$

where $[x]$ = greatest integer function.

f is not continuous at $x = 0, 1, 2, 3$

But in statement-2 $f(x)$ is continuous at $x = 3$.

Hence, statement-1 is true and 2 is false.

18. **(b)** $\mu(x) = \frac{1}{x-1}$, which is discontinous at $x = 1$

$f(u) = \frac{1}{u^2+u-2} = \frac{1}{(u+2)(u-1)}$,

which is discontinous at $u = -2, 1$

when $u = -2$, then $\frac{1}{x-1} = -2 \Rightarrow x = \frac{1}{2}$

when $u = 1$, then $\frac{1}{x-1} = 1 \Rightarrow x = 2$

Hence given composite function is discontinous at three points, $x = 1, \frac{1}{2}$ and 2.

19. **(d)** $fog = f(g(x)) = f(1-|x|)$

$= -1 + \big|1-|x|-2\big|$

$= -1 + \big|-|x|-1\big| = -1 + \big||x|+1\big|$

Let $fog = y$

$\therefore y = -1 + \big||x|+1\big|$

$$\Rightarrow y = \begin{cases} -1+x+1, & x \ge 0 \\ -1-x+1, & x < 0 \end{cases}$$

$$\Rightarrow y = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$

LHL at $(x=0) = \lim\limits_{x\to 0}(-x) = 0$

RHL at $(x=0) = \lim\limits_{x\to 0}(x) = 0$

When $x = 0$, then $y = 0$

Hence, LHL at $(x = 0)$ = RHL at $(x = 0)$

= value of y at $(x = 0)$

Hence y is continuous at $x = 0$.

Clearly at all other point y continuous. Therefore, the set of all points where fog is discontinuous is an empty set.

20. **(a)** Let $f(x)=[x]\cos\left(\frac{2x-1}{2}\right)$

We know that $[x]$ is discontinuous at all integral points and cos x is continuous at $x\in R$.

So, check at $x=n, n\in I$

L.H.L $=\lim\limits_{x\to n^-}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$=(n-1)\cos\left(\frac{2n-1}{2}\right)\pi=0$

($\because$ $[x]$ is the greatest integer function)

R.H.L $=\lim\limits_{x\to n^+}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$=n\cos\left(\frac{2n-1}{2}\right)\pi=0$

Now, value of the function at $x=n$ is

$f(n)=0$

Since, L.H.L = R.H.L. $=f(n)$

$\therefore f(x)=[x]\cos\left(\frac{2x-1}{2}\right)$ is continuous for every real x.

21. **(d)** Consider $\frac{x}{[x]}\le f(x)\le\sqrt{6-x}$

$\Rightarrow \lim\limits_{x\to 2^-}\frac{x}{[x]}=\frac{2}{1}=2$

$\Rightarrow \lim\limits_{x\to 2^-}\sqrt{6-x}=2$

$\therefore \lim\limits_{x\to 2^-}f(x)=2$ [By Sandwich theorem]

Now $\lim\limits_{x\to 2^+}\frac{x}{[x]}=1,\ \lim\limits_{x\to 2^+}\sqrt{6-x}=2$

Hence by Sandwich theorem $\lim\limits_{x\to 2^+}f(x)$ does not exists.

Therefore f is not continuous at $x=2$. Thus statement-1 is true but statement-2 is not true

22. **(d)** Statement - 1 is true.

It is the definition of continuity.

Statement - 2 is false.

23. **(c)** Given that $f(x)=\begin{cases}x\sin(1/x), & x\ne 0\\ 0 & , x=0\end{cases}$

At $x=0$

LHL $=\lim\limits_{h\to 0^-}\left\{-h\sin\left(-\frac{1}{h}\right)\right\}$

= 0 × a finite quantity between – 1 and 1= 0

RHL $=\lim\limits_{h\to 0^+}h\sin\frac{1}{h}=0$

Also, $f(0)=0$

Thus LHL = RHL = $f(0)$

$\therefore$ $f(x)$ is continuous on R.

but $f_2(x)$ is not continuous at $x=0$

24. **(b)** $\underset{(at\ x=0)}{L.H.L}=\lim\limits_{x\to 0^-}f(x)$

$=\lim\limits_{h\to 0}\frac{\sin\{(p+1)(-h)\}-\sinh}{-h}=p+1+1=p+2$

$\underset{(at\ x=0)}{R.H.L}=\lim\limits_{x\to 0^+}f(x)$

$=\lim\limits_{h\to 0}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}\times\frac{\sqrt{x+x^2}+\sqrt{x}}{\sqrt{x+x^2}+\sqrt{x}}=\frac{1}{1+1}=\frac{1}{2}$

$f(0)=2$

Given that f(x) is continuous at $x=0$

$\therefore p+2=q=\frac{1}{2}$

$\Rightarrow p=-\frac{3}{2}, q=\frac{1}{2}$

25. **(b)** Given, $f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$ is continuous at $x=0$

$\Rightarrow \quad f(0)=\lim\limits_{x\to 0}\frac{1}{x}-\frac{2}{e^{2x}-1}$

$=\lim\limits_{x\to 0}\frac{(e^{2x}-1)-2x}{x(e^{2x}-1)};\ \left[\frac{0}{0}\text{ form}\right]$

$\therefore$ Applying, L'Hospital rule

Differentiate two times, we get

$f(0)=\lim\limits_{x\to 0}\frac{4e^{2x}}{2(xe^{2x}2+e^{2x}.1)+e^{2x}.2}$

$=\lim\limits_{x\to 0}\frac{4e^{2x}}{4xe^{2x}+2e^{2x}+2e^{2x}}\quad\left[\frac{0}{0}\text{ form}\right]$

$=\lim\limits_{x\to 0}\frac{4e^{2x}}{4(xe^{2x}+e^{2x})}=\frac{4.e^0}{4(0+e^0)}=1$

26. **(c)** Given that $f(x)=\frac{1-\tan x}{4x-\pi}$ is continuous in $\left[0,\frac{\pi}{2}\right]$

$\therefore f\left(\frac{\pi}{4}\right)=\lim\limits_{x\to\frac{\pi}{4}^-}f(x)=\lim\limits_{x\to\frac{\pi}{4}^+}f(x)$

$\lim\limits_{x\to\frac{\pi}{4}^-}f(x)=\lim\limits_{h\to 0}f\left(\frac{\pi}{4}+h\right)$

$=\lim\limits_{h\to 0}\frac{1-\tan\left(\frac{\pi}{4}+h\right)}{4\left(\frac{\pi}{4}+h\right)-\pi}, h>0=\lim\limits_{h\to 0}\frac{1-\frac{1+\tan h}{1-\tan h}}{4h}$

$=\lim\limits_{h\to 0}\frac{-2}{1-\tan h}.\frac{\tan h}{4h}=\frac{-2}{4}=-\frac{1}{2}\quad\left[\because\lim\limits_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right]$

27. **(b)** Let a is a rational number other than 0, in [–5, 5], then $f(a)=a$ and $\lim\limits_{x\to a}f(x)=-a$

$\therefore x\to a^-$ and $x\to a^+$ is tends to irrational number

$\therefore f(x)$ is discontinuous at any rational number

If a is irrational number, then

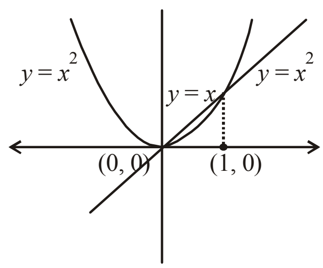
$f(a) = -a$ and $\lim_{x \to a} f(x) = a$

$\therefore f(x)$ is not continuous at any irrational number. For $x = 0$,

$\lim_{x \to 0} f(x) = f(0) = 0$

$\therefore f(x)$ is continuous at $x = 0$

28. **(a)**



$f(x) = \max.\{x, x^2\}$

$$\Rightarrow f(x) = \begin{cases} x^2, & x < 0 \\ x, & 0 \le x < 1 \\ x^2, & x \ge 1 \end{cases}$$

$\therefore f(x)$ is not differentiable at $x = 0, 1$

29. **(a)** $f(x)$ is differentiable then, $f(x)$ is also continuous.

$\therefore \lim_{x \to \pi^+} f(x) = \lim_{x \to \pi^-} f(x) = f(\pi)$

$\Rightarrow -1 = -K_2 \Rightarrow K_2 = 1$

$$\therefore f'(x) = \begin{cases} 2K_1(x - \pi) & : \ x \le \pi \\ -K_2 \sin x & \ \ x > \pi \end{cases}$$

Then, $\lim_{x \to \pi^+} f(x) = \lim_{x \to \pi^-} f(x) = 0$

$$f''(x) = \begin{cases} 2K_1 & ; \ x \le \pi \\ -K_2 \cos x & ; \ x > \pi \end{cases}$$

Then, $\lim_{x \to \pi^+} f(x) = \lim_{x \to \pi^-} f(x)$

$\Rightarrow 2K_1 = K_2 \Rightarrow K_1 = \frac{1}{2}$

So, $(K_1, K_2) = \left(\frac{1}{2}, 1\right)$

30. **(b)** Let f be twice differentiable function

$\because f'(x) \ge 1$

$\Rightarrow \frac{f(5) - f(2)}{3} \ge 1$

$\Rightarrow f(5) \ge 3 + f(2)$

$\Rightarrow f(5) \ge 3 + 8 \Rightarrow f(5) \ge 11$

and also $f''(x) \ge 4$

$\Rightarrow \frac{f'(5) - f'(2)}{5 - 2} \ge 4 \Rightarrow f'(5) \ge 12 + f'(2)$

$\Rightarrow f'(5) \ge 17$

Hence, $f(5) + f'(5) \ge 28$

31. **(10.00)**

$f(x + y) = f(x) + f(y) + xy^2 + x^2 y$

Differentiate w.r.t. x:

$f'(x + y) = f'(x) + 0 + y^2 + 2xy$

Put $y = -x$

$f'(0) = f'(x) + x^2 - 2x^2$ ...(i)

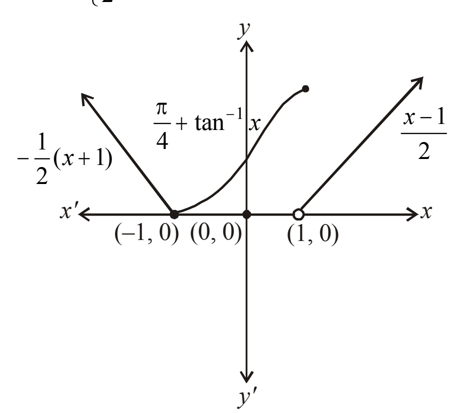
$\because \lim_{x \to 0} \frac{f(x)}{x} = 1 \Rightarrow f(0) = 0$

$\therefore f'(0) = 1$ ...(ii)

From equations (i) and (ii),

$f'(x) = (x^2 + 1) \Rightarrow f'(3) = 10.$

32. **(a)** $$f(x) = \begin{cases} \frac{-x-1}{2}, & x < -1 \\ \frac{\pi}{4} + \tan^{-1} x, & -1 \le x \le 1 \\ \frac{1}{2}(x - 1), & x > 1 \end{cases}$$



It is clear from above graph that,
$f(x)$ is discontinuous at $x = 1$.
i.e. continuous on $R - \{1\}$
$f(x)$ is non-differentiable at $x = -1, 1$
i.e. differentiable on $R - \{-1, 1\}$.

33. **(c)** LHL $= \lim_{x \to 0} \frac{\sin(a+2)x + \sin x}{x}$

$= \lim_{x \to 0} \left(\frac{\sin(a+2)x}{(a+2)x}\right)(a + 2) + \lim_{x \to 0} \frac{\sin x}{x} = a + 3$

$f(0)=b$

$$\text{RHL} = \lim_{h\to 0}\left(\frac{(1+3h)^{\frac{1}{3}}-1}{h}\right)=1$$

$\because$ Function $f(x)$ is continuous

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\therefore a+3=1 \Rightarrow a=-2$

and $b=1$

Hence, $a+2b=0$

34. **(a)** It is given that functions f and g are differentiable and fog is identity function.

$\therefore (fog)(x)=x \Rightarrow f(g(x))=x$

Differentiating both sides, we get

$f'(g(x))\cdot g'(x)=1$

Now, put $x=a$, then

$f'(g(a))\cdot g'(a)=1$

$f'(b)\cdot 5=1$

$f'(b)=\frac{1}{5}$

35. **(Bonus)** For a constant function $f(x)$, option (1), (3) and (4) doesn't hold and by LMVT theorem, option (2) is incorrect.

36. **(a)** From, LMVT for $x\in[-7,-1]$

$$\frac{f(-1)-f(-7)}{(-1+7)}\le 2 \Rightarrow \frac{f(-1)+3}{6}\le 2 \Rightarrow f(-1)\le 9$$

From, LMVT for $x\in[-7,0]$

$$\frac{f(0)-f(-7)}{(0+7)}\le 2$$

$$\frac{f(0)+3}{7}\le 2 \Rightarrow f(0)\le 11$$

$\therefore f(0)+f(-1)\le 20$

37. **(c)** $\because$ $f(x)$ is non differentiable at $x=1,3,5$

[$\because |x-3|$ is not differentiable at $x=3$]

$\Sigma f(f(x)) = f(f(1))+f(f(3))+f(f(5))$

$=1+1+1=3$

38. **(c)** $f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x} & x<0\\ q & x=0 \text{ is continuous at } x=0\\ \dfrac{\sqrt{x^2+x}-\sqrt{x}}{x^{\frac{3}{2}}} & x>0\end{cases}$

Therefore, $f(0^-)=f(0)=f(0^+)$ ...(1)

$$f(0^-)=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\frac{\sin(p+1)(-h)+\sin(-h)}{-h}$$

$$=\lim_{h\to 0}\left[\frac{-\sin(p+1)h}{-h}+\frac{\sin h}{h}\right]$$

$$=\lim_{h\to 0}\frac{\sin(p+1)h}{h(p+1)}\times(p+1)+\lim_{h\to 0}\frac{\sin h}{h}$$

$$=(p+1)+1=p+2 \qquad ...(2)$$

$$\text{And } f(0^+)=\lim_{h\to 0} f(0+h)=\frac{\sqrt{h^2+h}-\sqrt{h}}{h^{3/2}}$$

$$=\lim_{h\to 0}\frac{(h)^{\frac{1}{2}}\left[\sqrt{h+1}-1\right]}{h\left(h^{\frac{1}{2}}\right)}$$

$$=\lim_{h\to 0}\frac{\sqrt{h+1}-1}{h}\times\frac{\sqrt{h+1}+1}{\sqrt{h+1}+1}=\lim_{h\to 0}\frac{h+1-1}{h(\sqrt{h+1}+1)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{h+1}+1}=\frac{1}{1+1}=\frac{1}{2} \qquad ...(3)$$

Now, from equation (1),

$f(0^-)=f(0)=f(0^+)\Rightarrow p+2=q=\frac{1}{2}$

$\Rightarrow q=\frac{1}{2}$ and $p=\frac{-3}{2}$ $\quad\therefore (p,q)=\left(-\frac{3}{2},\frac{1}{2}\right)$

39. **(b)** $f(x)=\ln(\sin x)$, $g(x)=\sin^{-1}(e^{-x})$

$\Rightarrow f(g(x))=\ln(\sin(\sin^{-1}e^{-x}))=-x$

$\Rightarrow f(g(x))=-\alpha$

But given that $(fog)(\alpha)=b$

$\therefore -\alpha=b$ and $f'(g(\alpha))=a$, i.e., $a=-1$

$\therefore a\alpha^2-b\alpha-a=-\alpha^2+\alpha^2-(-1)$

$\Rightarrow a\alpha^2-b\alpha-a=1.$

40. **(c)** $g'(c)=\lim_{x\to c}\frac{g(x)-g(c)}{x-c}$

$\Rightarrow g'(c)=\lim_{x\to c}\frac{|f(x)|-|f(c)|}{x-c}$

Since, $f(c)=0$

Then, $g'(c)=\lim_{x\to c}\frac{|f(x)|}{x-c}$

$\Rightarrow g'(c)=\lim_{x\to c}\frac{f(x)}{x-c}$; if $f(x)>0$

and $g'(c)=\lim_{x\to c}\frac{-f(x)}{x-c}$; if $f(x)<0$

$\Rightarrow g'(c)=f'(c)=-f'(c)$

$\Rightarrow 2f'(c)=0\Rightarrow f'(c)=0$

Hence, $g(x)$ is differentiable if $f'(c)=0$

41. **(a)** Since, $f(x) = 15 - |(10 - x)|$

$\therefore g(x) = f(f(x)) = 15 - |10 - [15 - |10 - x|]|$

$= 15 - ||10 - x| - 5|$

$\therefore$ Then, the points where function g(x) is Non-differentiable are

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow x = 10$ and $x - 10 = \pm 5$

$\Rightarrow x = 10$ and $x = 15, 5$

42. **(a)** Let $g(x) = f(f(f(x))) + (f(x))^2$

Differentiating both sides w.r.t. x, we get

$g'(x) = f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x)$

$g'(1) = f'(f(f(1)))f'(f(1))f'(1) + 2f(1)f'(1)$

$= f'(f(1))f'(1)f'(1) + 2f(1)f'(1)$

$= 3 \times 3 \times 3 + 2 \times 1 \times 3 = 27 + 6 = 33$

43. **(b)** Since, $f'(x) = f(x)$

Then, $\dfrac{f'(x)}{f(x)} = 1$

$\Rightarrow \dfrac{f'(x)}{f(x)} = dx \Rightarrow \dfrac{f'(x)}{f(x)}\, dx = \int dx$

$\Rightarrow$ $\ln|f(x)| = x + c$

$f(x) = \pm e^{x+c}$ ...(1)

Since, the given condition

$f(1) = 2$

From eq^n (1) $f(x) = e^{x+c} = e^c e^x$

Then, $f(1) = e^c \cdot e^1$

$\Rightarrow$ $2 = e^c \cdot e$

$\Rightarrow$ $\dfrac{2}{e} = e^c$

Then, from eq^n (1)

$f(x) = \dfrac{2}{e}e^x$

$\Rightarrow$ $f'(x) = \dfrac{2}{e}e^x$

Now $h(x) = f(f(x))$

$\Rightarrow$ $h'(x) = f'(f(x)) \cdot f'(x)$

$h'(1) = f'(2) \cdot f'(1) = \dfrac{2}{e}e^2 \cdot \dfrac{2}{e} \cdot e = 4e$

44. **(d)** $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$

Then, $f(|x|) = \begin{cases} -1, & -2 \le |x| < 0 \\ |x|^2 - 1, & 0 \le |x| \le 2 \end{cases}$

$\Rightarrow$ $f(|x|) = x^2 - 1, -2 \le x \le 2$

$\Rightarrow$ $g(x) = \begin{cases} 1 + x^2 - 1, & -2 \le x < 0 \\ (x^2 - 1) + |x^2 - 1|, & 0 \le x \le 2 \end{cases}$

$= \begin{cases} x^2, & -2 \le x < 0 \\ 0, & 0 \le x < 1 \\ 2(x^2 - 1), & 1 \le x \le 2 \end{cases}$

$g'(0^-) = 0, g'(0^+) = 0, g'(1^-) = 0, g'(1^+) = 4$

$\Rightarrow$ $g(x)$ is non-differentiable at $x = 1$

$\Rightarrow$ $g(x)$ is not differentiable at one point.

45. **(b)** Consider the equation,

$x \log_e(\log_e x) - x^2 + y^2 = 4$

Differentiate both sides w.r.t. x,

$$\log_e(\log_e x) + x \cdot \frac{1}{x \cdot \log_e x} - 2x + 2y\frac{dy}{dx} = 0$$

$$\log_e(\log_e x) + \frac{1}{\log_e x} - 2x + 2y\frac{dy}{dx} = 0 \quad ...(1)$$

When $x = e, y = \sqrt{4 + e^2}$. Put these values in (1),

$$0 + 1 - 2e + 2\sqrt{4 + e^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}}.$$

46. **(a)** $f(x) = \sin|x| - |x| + 2(x - \pi)\cos|x|$

There are two cases,

Case (1), $x > 0$

$f(x) = \sin x - x + 2(x - \pi)\cos x$

$f'(x) = \cos x - 1 + 2(1 - 0)\cos x - 2\sin(x - \pi)$

$f'(x) = 3\cos x - 2(x - \pi)\sin x - 1$

Then, function $f(x)$ is differentiable for all $x > 0$

Case (2) $x < 0$

$f(x) = -\sin x + x + 2(x - \pi)\cos x$

$f'(x) = -\cos x + 1 - 2(x - \pi)\sin x + 2\cos x$

$f'(x) = \cos x + 1 - 2(x - \pi)\sin x$

Then, function $f(x)$ is differentiable for all $x < 0$

Now check for $x = 0$

$f'(0^+)$ R.H.D. $= 3 - 1 = 2$

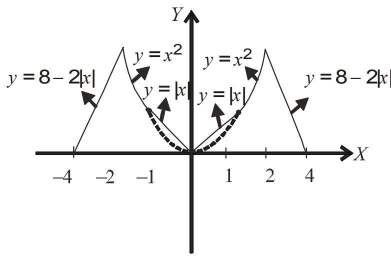
$f'(0^-)$ L.H.D. $= 1 + 1 = 2$

L.H.D. = R.H.D.

Then, function $f(x)$ is differentiable for $x = 0$. So it is differentiable everywhere

Hence, $k = \phi$

47. (b) Given $f(x)=\begin{cases}\max\{|x|,x^2\} & |x|\le 2\\ 8-2|x| & 2<|x|\le 4\end{cases}$



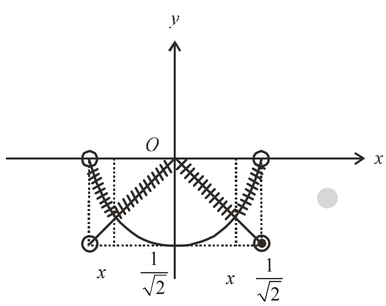
$\because f(x)$ is not differentiable at –2, –1, 0, 1 and 2.

$\therefore S=\{-2,-1,0,1,2\}$

48. (c) Consider the function

$f(x)=\max\left\{-|x|,-\sqrt{1-x^2}\right\}$

Now, the graph of the function



From the graph, it is clear that $f(x)$ is not differentiable at $x=0,\ -\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$

Then, $K=\left\{-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\}$

Hence, K has exactly three elements.

49. (d) $f(x)=|x-\pi|(e^{|x|}-1)\sin|x|$

Check differentiability of f(x) at $x=\pi$ and $x=0$

at $x=\pi$:

$$\text{R.H.D.}=\lim_{h\to 0}\frac{|\pi+h-\pi|(e^{|x+h|}-1)\sin|\pi+h|-0}{h}$$

$$\text{L.H.D}=\lim_{h\to 0}\frac{|\pi-h-\pi|(e^{|\pi-h|}-1)\sin|\pi-h|-0}{-h}=0$$

$\because$ RHD = LHD

Therefore, function is differentiable at $x=\pi$

at $x=0$:

$$\text{R.H.D.}=\lim_{h\to 0}\frac{|h-\pi|(e^{|h|}-1)\sin|h|-0}{h}=0$$

$$\text{L.H.D.}=\lim_{h\to 0}\frac{|-h-\pi|(e^{|-h|}-1)\sin|-h|-0}{-h}=0$$

$\therefore$ RHD = LHD

Therefore, function is differentiable.

at $x=0$.

Since, the function f(x) is differentiable at all the points including π and 0.

i.e., f(x) is every where differentiable .

Therefore, there is no element in the set S.

$\Rightarrow S=\phi$ (an empty set)

50. (a) $S=\{(\lambda,\mu)\in R\times R: f(t)=(|\lambda|e^{|t|}-\mu)\sin(2|t|),\ t\in R$

$f(t)=(|\lambda|e^{|t|}-\mu)\sin(2|t|)$

$$=\begin{cases}(|\lambda|e^{t}-\mu)\sin 2t, & t>0\\ (|\lambda|e^{-t}-\mu)(-\sin 2t), & t<0\end{cases}$$

$$f'(t)=\begin{cases}(|\lambda|e^{t})\sin 2t+(|\lambda|e^{t}-\mu)(2\cos 2t), & t>0\\ |\lambda|e^{-t}\sin 2t+(|\lambda|e^{-t}-\mu)(-2\cos 2t), & t<0\end{cases}$$

As, $f(t)$ is differentiable

$\therefore$ LHD = RHD at $t=0$

$\Rightarrow\ |\lambda|.\sin 2(0)+(|\lambda|e^{0}-\mu)2\cos(0)$

$=|\lambda|e^{-0}.\sin 2(0)-2\cos(0)(|\lambda|e^{-0}-\mu)$

$\Rightarrow\ 0+(|\lambda|-\mu)2=0-2(|\lambda|-\mu)$

$\Rightarrow\ 4(|\lambda|-\mu)=0$

$\Rightarrow\ |\lambda|=\mu$

So, $S\equiv(\lambda,\mu)=\{\lambda\in R\ \&\ \mu\in[0,\infty)\}$

Therefore set S is subset of $R\times[0,\infty)$

51. (a) $f(x)=\begin{cases}-x & x<1\\ a+\cos^{-1}(x+b) & 1\le x\le 2\end{cases}$

$f(x)$ is continuous

$\Rightarrow\ \lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^+}a+\cos^{-1}(x+b)=f(x)$

$\Rightarrow\ -1=a+\cos^{-1}(1+b)$

$\cos^{-1}(1+b)=-1-a$ .....(a)

$f(x)$ is differentiate

$\Rightarrow$ LHD = RHD

$$\Rightarrow\ -1=\frac{-1}{\sqrt{1-(1+b)^2}}$$

$\Rightarrow\ 1-(1+b)^2=1\ \Rightarrow\ b=-1$ .....(b)

From (a) $\Rightarrow \cos^{-1}(0)=-1-a$

$$\therefore\ -1-a=\frac{\pi}{2}$$

$$a=-1-\frac{\pi}{2}\ \Rightarrow\ a=\frac{-\pi-2}{2}\quad \text{.....(c)}$$

$$\therefore\ \frac{a}{b}=\frac{\pi+2}{2}$$

52. **(c)** Since g (x) is differentiable, it will be continuous at x = 3

$\therefore \lim_{x\to 3^-} g(x) = \lim_{x\to 3^+} g(x)$

$2k = 3m + 2$ ...(1)

Also g(x) is differentiable at x = 0

$\therefore \lim_{x\to 3^-} g'(x) = \lim_{x\to 3^+} g'(x)$

$\frac{k}{2\sqrt{3+1}} = m$

$k = 4m$ ...(2)

Solving (1) and (2), we get

$m = \frac{2}{5},\ k = \frac{8}{5}$

$k + m = 2$

53. **(b)** Let $|f(x)| \le x^2,\ \forall x \in R$

Now, at $x = 0$, $|f(0)| \le 0$

$\Rightarrow f(0) = 0$

$\therefore f'(0) = \lim_{h\to 0} \frac{f(h) - f(0)}{h - 0} = \lim_{h\to 0} \frac{f(h)}{h}$ ...(1)

Now, $\left|\frac{f(h)}{h}\right| \le |h| \quad (\because |f(x)| \le x^2)$

$\Rightarrow -|h| \le \frac{f(h)}{h} \le |h|$

$\Rightarrow \lim_{h\to 0} \frac{f(h)}{h} \to 0$ ...(2)

(using sandwich Theorem)

$\therefore$ from (1) and (2), we get $f'(0) = 0$,

i.e. $- f(x)$ is differentiable, at $x = 0$

Since, differentiability $\Rightarrow$ Continutity

$\therefore |f(x)| \le x^2$, for all $x \in R$ is continuous as well as differentiable at $x = 0$.

54. **(b)** $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \ne 0 \\ 0, & x = 0 \end{cases}$

and $g(x) = x\,f(x)$

For $f(x)$

$\text{LHL} = \lim_{h\to 0^-} \left\{-h\sin\left(-\frac{1}{h}\right)\right\}$

$= 0 \times$ a finite quantity between -1 and $1 = 0$

$\text{RHL} = \lim_{h\to 0^+} h\sin\frac{1}{h} = 0$

Also, $f(0) = 0$

Thus LHL = RHL = $f(0)$

$\therefore f(x)$ is continuous at $x = 0$

$g(x) = \begin{cases} x^2\sin\frac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$

For $g(x)$

$\text{LHL} = \lim_{h\to 0^-} \left\{-h^2\sin\left(\frac{1}{h}\right)\right\}$

$= 0^2 \times$ a finite quantity between -1 and $1 = 0$

$\text{RHL} = \lim_{h\to 0^+} h^2\sin\left(\frac{1}{h}\right) = 0$

Also $g(0) = 0$

$\therefore g(x)$ is continuous at $x = 0$

55. **(c)** $f(x) = |x-2| = \begin{cases} x-2, & x-2 \ge 0 \\ 2-x, & x-2 \le 0 \end{cases}$

$= \begin{cases} x-2, & x \ge 2 \\ 2-x, & x \le 2 \end{cases}$

Similarly,

$f(x) = |x-5| = \begin{cases} x-5, & x \ge 5 \\ 5-x, & x \le 5 \end{cases}$

$\therefore \quad f(x) = |x-2| + |x-5|$

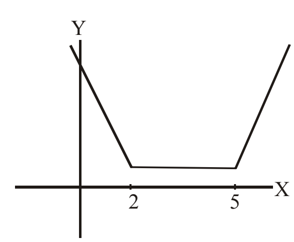
$= \{x - 2 + 5 - x = 3,\ 2 \le x \le 5\}$

Thus $f(x) = 3,\ 2 \le x \le 5$

$f'(x) = 0,\ 2 < x < 5$

$f'(4) = 0$

$\because$ Statement-1 is true



Since $f(x) = 3,\ 2 \le x \le 5$ is constant function.

So, it continuous in 2, 5 and differentiable in (2, 5)

$\because f(2) = 0 + |2-5| = 3$

and $f(5) = |5-2| + 0 = 3$ statement-2 is also true.

56. **(d)** $|\sin x|$ and $e^{|x|}$ are not differentiable at $x = 0$ and $|x|^3$ is differentiable at $x = 0$.

$\therefore$ for $f(x)$ to be differentiable at $x = 0$, we must have $a = 0$, $b = 0$ and c is any real number.

57. **(b)** Given $x + |y| = 2y$

$\Rightarrow x + y = 2y$ or $x - y = 2y$

$\Rightarrow x = y$ or $x = 3y$

This represent a straight line which passes through origin.
Hence, $x+|y|=2y$ is continuous at $x=0$.
Now, we check differentiability at $x=0$

$x+|y|=2y \Rightarrow x+y=2y, y\ge 0$

$x-y=2y,\ \ y<0$

Thus, $f(x)=\begin{cases} x, & y<0 \\ x/3, & y\ge 0 \end{cases}$

Now, $\text{L.H.D.} = \lim\limits_{h\to 0^-} \dfrac{f(x+h)-f(x)}{-h}$

$= \lim\limits_{h\to 0^-} \dfrac{x+h-x}{-h} = -1$

$\text{R.H.D} = \lim\limits_{h\to 0^+} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0^+} \dfrac{\frac{x+h}{3}-\frac{x}{3}}{h} = \lim\limits_{h\to 0^+} \dfrac{1}{3} = \dfrac{1}{3}$

Since, $\text{L.H.D} \ne \text{R.H.D.}$ at $x=0$

$\therefore$ given function is not differentiable at $x=0$

58. **(c)** $\lim\limits_{x\to a} \dfrac{x^2 f(a)-a^2 f(x)}{x-a}$

Applying L-Hospital rule

$= \lim\limits_{x\to a} \dfrac{2xf(a)-a^2 f'(x)}{1} = 2af(a)-a^2 f'(a)$

59. **(c)** Given that, $f(x)=\begin{cases} (x-1)\sin\left(\dfrac{1}{x-1}\right), & \text{if } x\ne 1 \\ 0 & \text{, if } x=1 \end{cases}$

At $x=1$

$\text{R.H.D.} = \lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$

$= \lim\limits_{h\to 0} \dfrac{h\sin\frac{1}{h}-0}{h} = \lim\limits_{h\to 0} \sin\dfrac{1}{h}$ = a finite number

Let this finite number be l

$\text{L.H.D.} = \lim\limits_{h\to 0} \dfrac{f(1-h)-f(1)}{-h}$

$= \lim\limits_{h\to 0} \dfrac{-h\sin\left(\frac{1}{-h}\right)}{-h} = \lim\limits_{h\to 0} \sin\left(\dfrac{1}{-h}\right)$

$= -\lim\limits_{h\to 0} \sin\left(\dfrac{1}{h}\right) = -(\text{a finite number}) = -l$

Thus $\text{R.H.D} \ne \text{L.H.D}$

$\therefore$ f is not differentiable at $x=1$

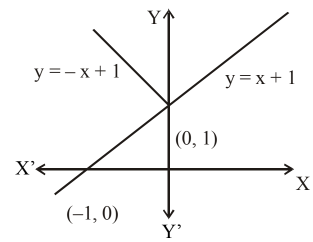
At $x=0$ $f'(0) = \left[\sin\dfrac{1}{(x-1)} - \dfrac{x-1}{(x-1)^2}\cos\left(\dfrac{1}{x-1}\right)\right]_{x=0}$

$= -\sin 1 + \cos 1$

$\therefore$ f is differentiable at $x=0$

60. **(a)** $f(x)=\min\{x+1, |x|+1\}$

$\Rightarrow f(x)=x+1\ \forall\ x\in R$



Since $f(x)=x+1$ is polynomial function
Hence, $f(x)$ is differentiable everywhere for all $x\in R$.

61. **(c)** $f(x)=\begin{cases} \dfrac{x}{1-x}, & x<0 \\ \dfrac{x}{1+x}, & x\ge 0 \end{cases}$

$f(x) = \dfrac{x}{1-x}$ is not define at $x\ne 1$ but here $x<0$ and $f(x)$

$= \dfrac{x}{1+x}$ is not define at $x=-1$ but here $x>0$. So, $f(x)$ is continuous for $x\in R$.

and $f'(x)=\begin{cases} \dfrac{x}{(1-x)^2}, & x<0 \\ \dfrac{x}{(1+x)^2}, & x\ge 0 \end{cases}$

$\therefore$ $f'(x)$ exist at everywhere.

62. **(b)** Given that $|f(x)-f(y)|\le (x-y)^2$, $x, y\in R$ ...(i) and $f(0)=0$

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$|f'(x)| = \lim\limits_{h\to 0}\left|\dfrac{f(x+h)-f(x)}{h}\right| \le \lim\limits_{h\to 0}\left|\dfrac{(h)^2}{h}\right|$

$\Rightarrow |f'(x)|\le 0 \Rightarrow f'(x)=0$

$\Rightarrow f(x)=$ constant

As $f(0)=0$

$\Rightarrow f(1)=0.$

63. **(c)** $f'(1) = \lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$;

Given that function is differentiable so it is continuous also

and $\lim_{h\to 0}\frac{f(1+h)}{h}=5$ and hence $f(1)=0$

Hence, $f'(1)=\lim_{h\to 0}\frac{f(1+h)}{h}=5$

64. (c) Given that $f(0)=0$; $f(x)=xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}$

R.H.L. $=\lim_{h\to 0}(0+h)e^{-2/h}=\lim_{h\to 0}\frac{h}{e^{2/h}}=0$

L.H.L. $=\lim_{h\to 0}(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}=0$

therefore, $f(x)$ is continuous at $x=0$.

Now, R.H.D $=\lim_{h\to 0}\frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-0}{h}=0$

L.H.D. $=\lim_{h\to 0}\frac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}-0}{-h}=1$

therefore, L.H.D. ≠ R.H.D.

$f(x)$ is not differentiable at $x=0$.

65. (d) Let $u=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x=\tan\theta\Rightarrow\theta=\tan^{-1}x$

$\therefore u=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x$

$\therefore\frac{du}{dx}=\frac{1}{2}\times\frac{1}{(1+x^2)}$

Let $v=\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x=\sin\phi\Rightarrow\phi=\sin^{-1}x$

$v=\tan^{-1}\left(\frac{2\sin\phi\cos\phi}{\cos 2\phi}\right)=\tan^{-1}(\tan 2\phi)$

$=2\phi=2\sin^{-1}x$

$\frac{dv}{dx}=2\frac{1}{\sqrt{1-x^2}}$

$\frac{du}{dv}=\frac{du/dx}{dv/dx}=\frac{\sqrt{1-x^2}}{4(1+x^2)}$

$\therefore\left(\frac{du}{dv}\right)_{\left(x=\frac{1}{2}\right)}=\frac{\sqrt{3}}{10}$

66. (c) $(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos y)=a^2-b^2$

Differentiating both sides,

$(-\sqrt{2}b\sin x)(a-\sqrt{2}b\cos y)+(a+\sqrt{2}b\cos x)$

$$(\sqrt{2}b\sin y)\frac{dy}{dx}=0$$

$\Rightarrow\frac{dy}{dx}=\frac{(\sqrt{2}b\sin x)(a-\sqrt{2}b\cos y)}{(a+\sqrt{2}b\cos x)(\sqrt{2}b\sin y)}$

$\therefore\left[\frac{dy}{dx}\right]_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)}=\frac{a-b}{a+b}\Rightarrow\frac{dx}{dy}=\frac{a+b}{a-b}$

67. (91) $y=\sum_{k=1}^{6}k\cos^{-1}\left\{\frac{3}{5}\cos kx-\frac{4}{5}\sin kx\right\}$

Let $\cos a=\frac{3}{5}$ and $\sin a=\frac{4}{5}$

$\therefore y=\sum_{k=1}^{6}k\cos^{-1}\{\cos a\cos kx-\sin a\sin kx\}$

$=\sum_{k=1}^{6}k\cos^{-1}(\cos(kx+a))$

$=\sum_{k=1}^{6}k(kx+a)=\sum_{k=1}^{6}(k^2x+ak)$

$\therefore\frac{dy}{dx}=\sum_{k=1}^{6}k^2=\frac{6(7)(13)}{6}=91.$

68. (Bonus) It is given that

$x=2\sin\theta-\sin 2\theta$ ...(i)

$y=2\cos\theta-\cos 2\theta$ ...(ii)

Differentiating (i) w.r.t. θ, we get

$\frac{dx}{d\theta}=2\cos\theta-2\cos 2\theta$

Differentiating (ii) w.r.t. θ; we get

$\frac{dy}{d\theta}=-2\sin\theta+2\sin 2\theta$

From (ii) ÷ (i), we get

$\therefore\quad\frac{dy}{dx}=\frac{\sin 2\theta-\sin\theta}{\cos\theta-\cos 2\theta}$

$=\frac{2\sin\frac{\theta}{2}.\cos\frac{3\theta}{2}}{2\sin\frac{\theta}{2}.\sin\frac{3\theta}{2}}=\cot\frac{3\theta}{2}$ ...(iii)

Again, differentiating eqn. (iii), we get

$\frac{d^2y}{dx^2}=\frac{-3}{2}\text{cosec}^2\frac{3\theta}{2}.\frac{d\theta}{dx}$

$$\frac{d^2y}{dx^2}=\frac{\frac{-3}{2}\operatorname{cosec}^2\frac{3\theta}{2}}{2(\cos\theta-\cos 2\theta)}$$

$$\frac{d^2y}{dx^2(\theta=\pi)}=-\frac{3}{4(-1-1)}=\frac{3}{8}$$

69. **(a)** $y(\alpha)=\sqrt{\dfrac{\frac{2\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}{\sec^2\alpha}}=\sqrt{\dfrac{2\cos^2\alpha}{\sin\alpha\cos\alpha}+\dfrac{1}{\sin^2\alpha}}$

$=\sqrt{2\cot\alpha+\operatorname{cosec}^2\alpha}=\sqrt{2\cot\alpha+1+\cot^2\alpha}$

$=|1+\cot\alpha|=-1-\cot\alpha \quad \left[\because \alpha\in\left(\frac{3\pi}{4},\pi\right)\right]$

$\frac{dy}{d\alpha}=\operatorname{cosec}^2\alpha \Rightarrow \left(\frac{dy}{d\alpha}\right)_{\alpha=\frac{5\pi}{6}}=4$

70. **(b)** Given, $x=\frac{1}{2}, y=\frac{-1}{4} \Rightarrow xy=\frac{-1}{8}$

$y.\frac{1.(-2x)}{2\sqrt{1-x^2}}+y'\sqrt{1-x^2}$

$=-\left\{1.\sqrt{1-y^2}+\frac{x.(-2y)}{2\sqrt{1-y^2}}y'\right\}$

$\Rightarrow -\frac{xy}{\sqrt{1-x^2}}+y'\sqrt{1-x^2}=-\sqrt{1-y^2}+\frac{xy.y'}{\sqrt{1-y^2}}$

$\Rightarrow y'\left(\sqrt{1-x^2}-\frac{xy}{\sqrt{1-y^2}}\right)=\frac{xy}{\sqrt{1-x^2}}-\sqrt{1-y^2}$

$\Rightarrow y'\left(\frac{\sqrt{3}}{2}+\frac{1}{8.\frac{\sqrt{15}}{4}}\right)=\frac{-1}{8.\sqrt{\frac{3}{2}}}-\frac{\sqrt{15}}{4}$

$\Rightarrow y'\left(\frac{\sqrt{45}+1}{2\sqrt{15}}\right)=-\frac{(1+\sqrt{45})}{4\sqrt{3}}$

$\therefore \quad y'=-\frac{\sqrt{5}}{2}$

71. **(b)** Given, $e^y+xy=e$ ...(i)

Putting $x=0$ in (i), $\Rightarrow$ $e^y=e\Rightarrow y=1$

On differentiating (i) w. r. to x

$e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0$ ...(ii)

Putting $y=1$ and $x=0$ in (ii),

$e\frac{dy}{dx}+0+1=0\Rightarrow \frac{dy}{dx}=-\frac{1}{e}$

On differentiating (ii) w. r. to x,

$e^y\frac{d^2y}{dx^2}+\frac{dy}{dx}.e^y.\frac{dy}{dx}+x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0$ ...(iii)

Putting $y=1, x=0$ and $\frac{dy}{dx}=-\frac{1}{e}$ in (iii),

$e\frac{d^2y}{dx^2}+\frac{1}{e}-\frac{2}{e}=0\Rightarrow \frac{d^2y}{dx^2}=\frac{1}{e^2}$

Hence, $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)\equiv\left(-\frac{1}{e},\frac{1}{e^2}\right)$

72. **(d)** $f(x)=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)$

$=-\tan^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right) \quad \left[\because \frac{\pi}{4}-x\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\right]$

So, $f(x)=-\left(\frac{\pi}{4}-x\right)=x-\frac{\pi}{4}$

Let $y=\Rightarrow f(y)=\ 2y-\frac{\pi}{4}$

Now, differentiate w.r.t. y, $\frac{df(y)}{dy}=2.$

73. **(none)** $2y=\left[\cot^{-1}\left(\dfrac{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}{\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x}\right)\right]^2$

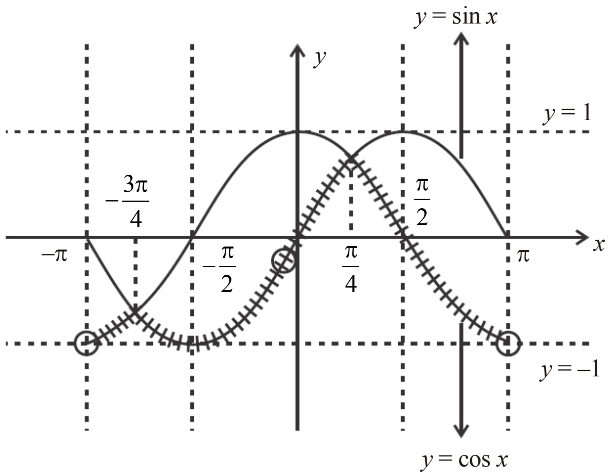
$\Rightarrow 2y=\left[\cot^{-1}\left(\dfrac{\cos\left(\frac{\pi}{6}-x\right)}{\sin\left(\frac{\pi}{6}-x\right)}\right)\right]^2$

$\Rightarrow 2y=\left[\cot^{-1}\left(\cot\left(\frac{\pi}{6}-x\right)\right)\right]^2 \because \frac{\pi}{6}-x\in\left(-\frac{\pi}{3},\frac{\pi}{6}\right)$

$$\Rightarrow 2y=\begin{cases}\left(\frac{7\pi}{6}-x\right)^2, & \text{if } \frac{\pi}{6}-x\in\left(\frac{-\pi}{3},0\right)\\ \left(\frac{\pi}{6}-x\right)^2, & \text{if } \frac{\pi}{6}-x\in\left(0,\frac{\pi}{0}\right)\end{cases}$$

$$\Rightarrow \frac{dy}{dx}=\begin{cases}x-\frac{7\pi}{6} & \text{if } x\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)\\ x-\frac{\pi}{6} & \text{if } x\in\left(0,\frac{\pi}{6}\right)\end{cases}$$

74. **(b)** $f(x) = \min\{\sin x, \cos x\}$



$\because$ $f(x)$ is not differentiable at $x = -\frac{3\pi}{4}, \frac{\pi}{4}$

$\therefore$ $S = \left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$

$\Rightarrow$ $S \subseteq \left\{-\frac{3\pi}{4}, \frac{-\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

75. **(a)** Consider the equation,

$(2x)^{2y} = 4e^{2x-2y}$

Taking log on both sides

$2y\ \ln(2x) = \ln 4 + (2x - 2y) \quad \ldots(1)$

Differentiating both sides w.r.t. x,

$$2y\frac{1}{2x}2 + 2\ln(2x)\frac{dy}{dx} = 0 + 2 - 2\frac{dy}{dx}$$

$$2\frac{dy}{dx}(1 + \ln(2x) = 2 - \frac{2y}{x} = \frac{2x - 2y}{x} \quad \ldots(2)$$

From (1) and (2),

$$\frac{dy}{dx}(1 + \ln 2x) \ = 1 - \frac{1}{x}\left(\frac{\ln 2 + x}{1 + \ln 2x}\right)$$

$$\Rightarrow \quad (1 + \ln 2x)^2\frac{dy}{dx} = 1 + \ln(2x) - \left(\frac{x + \ln 2}{x}\right)$$

$$= \frac{x\ln(2x) - \ln 2}{x}$$

76. **(c)** Let $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b \Rightarrow f'(1) = 3 + 2a + b$

$f''(x) = 6x + 2a \Rightarrow f''(2) = 12 + 2a$

$f'''(x) = 6 \Rightarrow f'''(3) = 6$

$\because$ $f(x) = x^3 + f'(1)x^2 + f''(2)x + f'''(3)$

$\therefore$ $f'(1) = a \Rightarrow 3 + 2a + b = a \Rightarrow a + b = -3 \quad \ldots(1)$

also $f''(2) = b \Rightarrow 12 + 2a = b \Rightarrow 2a - b = -12 \quad \ldots(2)$

and $f''(3) = c \Rightarrow c = 6$

Add (1) and (2)

$3a = -15 \Rightarrow a = -5 \Rightarrow b = 2$

$\Rightarrow$ $f(x) = x^3 - 5x^2 + 2x + 6$

$\Rightarrow$ $f(2) = 8 - 20 + 4 + 6 = -2$

77. **(b)** $\because$ $x = 3\tan t \Rightarrow \frac{dx}{dt} = 3\sec^2 t$

and $y = 3\sec t \Rightarrow \frac{dy}{dt} = 3\sec t \cdot \tan t$

$$\because \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \therefore \frac{dy}{dx} = \frac{\tan t}{\sec t} = \sin t$$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{d}{dt}(\sin t)\cdot\frac{dt}{dx}$$

$$= \cos t \cdot \frac{1}{3\sec^2 t}$$

$$\therefore \quad \frac{d^2y}{dx^2}\left(\text{at } t = \frac{\pi}{4}\right) = \frac{1}{3}\cdot\left(\frac{1}{\sqrt{2}}\right)^3$$

$$= \frac{1}{6\sqrt{2}}$$

78. **(b)** Here, $\frac{dx}{dt} = \frac{1}{2\sqrt{2^{\text{cosec}^{-1}t}}}2^{\text{cosec}^{-1}t}\log 2.\frac{-1}{x\sqrt{x^2-1}}$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{2^{\sec^{-1}t}}}2^{\sec^{-1}t}\log 2.\frac{1}{x\sqrt{x^2-1}}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sqrt{2^{\text{cosec}^{-1}t}}}{\sqrt{2^{\sec^{-1}t}}}\frac{2^{\sec^{-1}t}}{2^{\text{cosec}^{-1}t}}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\sqrt{\frac{2^{\sec^{-1}t}}{2^{\text{cosec}^{-1}t}}} = \frac{-y}{x}$$

79. **(a)** $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$

$=\cos x\,(x^2-2x^2)-x\,(2\sin x-2x\tan x)+1(2x\sin x-x^2\tan x)$

$=-x^2\cos x-2x\sin x+2x^2\tan x+2x\sin x-x^2\tan x$

$=x^2\tan x-x^2\cos x \quad =x^2(\tan x-\cos x)$

$\Rightarrow f'(x)=2x(\tan x-\cos x)+x^2(\sec^2 x+\sin x)$

$\therefore \quad \lim_{x\to 0}\frac{f'(x)}{x}=$

$\lim_{x\to 0}\frac{2x(\tan x-\cos x)+x^2(\sec^2 x+\sin x)}{x}$

$=\lim_{x\to 0}(\tan x-\cos x)+x(\sec^2 x+\sin x)$

$=2(0-1)+0=-2$

So, $\lim_{x\to 0}\frac{f'(x)}{x}=-2$

80. **(a)** Since $f(x)=\sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$

Suppose $3^x=\tan t$

$\Rightarrow f(x)=\sin^{-1}\left(\frac{2\tan t}{1+\tan^2 t}\right)=\sin^{-1}(\sin 2t)=2t$

$=2\tan^{-1}(3x)$

So, $f'(x)=\frac{2}{1+(3^x)^2}\times 3^x.\log_e 3$

$\therefore \quad f'\left(-\frac{1}{2}\right)=\frac{2}{1+\left(3^{-\frac{1}{2}}\right)^2}\times 3^{-\frac{1}{2}}.\log_e 3$

$=\frac{1}{2}\times\sqrt{3}\times\log_e 3 \qquad =\sqrt{3}\times\log_e\sqrt{3}$

81. **(a)** Given, $x^2+y^2+\sin y=4$

After differentiating the above equation w. r. t. x we get

$2x+2y\frac{dy}{dx}+\cos y\frac{dy}{dx}=0$ ... (1)

$\Rightarrow 2x+(2y+\cos y)\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-2x}{2y+\cos y}$

At $(-2,0)$, $\left(\frac{dy}{dx}\right)_{(-2,0)}=\frac{-2\times-2}{2\times 0+\cos 0}$

$\Rightarrow\left(\frac{dy}{dx}\right)_{(-2,0)}=\frac{4}{0+1}$

$\Rightarrow\left(\frac{dy}{dx}\right)_{(-2,0)}=4$ ... (2)

Again differentiating equation (1) w. r. t to x, we get

$2+2\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}-\sin y\left(\frac{dy}{dx}\right)^2+\cos y\frac{d^2y}{dx^2}=0$

$\Rightarrow 2+(2-\sin y)\left(\frac{dy}{dx}\right)^2+(2y+\cos y)\frac{d^2y}{dx^2}=0$

$\Rightarrow(2y+\cos y)\frac{d^2y}{dx^2}=-2-(2-\sin y)\left(\frac{dy}{dx}\right)^2$

$\Rightarrow\frac{d^2y}{dx^2}=\frac{-2-(2-\sin y)\left(\frac{dy}{dx}\right)^2}{2y+\cos y}$

So, at $(-2,0)$,

$\frac{d^2y}{dx^2}=\frac{-2-(2-0)\times 4^2}{2\times 0+1}$

$\Rightarrow\frac{d^2y}{dx^2}=\frac{-2-2\times 16}{1}$

$\Rightarrow\frac{d^2y}{dx^2}=-34$

82. **(b)** Let $F(x)=\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ where $x\in\left(0,\frac{1}{4}\right)$.

$=\tan^{-1}\left(\frac{2.(3x^{3/2})}{1-(3x^{3/2})^2}\right)=2\tan^{-1}(3x^{3/2})$

As $3x^{3/2}\in\left(0,\frac{3}{8}\right)$

$\left[\because 0<x<\frac{1}{4}\Rightarrow 0<x^{3/2}<\frac{1}{8}\Rightarrow 0<3x^{3/2}<\frac{3}{8}\right]$

So $\frac{dF(x)}{dx}=2\times\frac{1}{1+9x^3}\times 3\times\frac{3}{2}\times x^{1/2}=\frac{9}{1+9x^3}\sqrt{x}$

On comparing

$\therefore g(x)=\frac{9}{1+9x^3}$

83. **(d)** $g(x)=f(f(x))$

In the neighbourhood of $x=0$,

$f(x)=|\log 2-\sin x|=(\log 2-\sin x)$

$\therefore g(x)=|\log 2-\sin|\log 2-\sin x||$

$=(\log 2-\sin(\log 2-\sin x))$

$\therefore$ g (x) is differentiable

and $g'(x)=-\cos(\log 2-\sin x)(-\cos x)$

$\Rightarrow g'(0)=\cos(\log 2)$

84. **(c)** $f(x) = y = x^2 - x + 5$

$x^2 - x + \frac{1}{4} - \frac{1}{4} + 5 = y$

$\left(x - \frac{1}{2}\right)^2 + \frac{19}{4} = y$

$\left(x - \frac{1}{2}\right)^2 = y - \frac{19}{4}$

$x - \frac{1}{2} = \pm\sqrt{y - \frac{19}{4}}$

$x = \frac{1}{2} \pm \sqrt{y - \frac{19}{4}}$

As $x > \frac{1}{2}$

$x = \frac{1}{2} + \sqrt{y - \frac{19}{4}}$

$g(x) = \frac{1}{2} + \sqrt{x - \frac{19}{4}}$

$g'(x) = \frac{1}{2\sqrt{x - \frac{19}{4}}}$

$g'(7) = \frac{1}{2\sqrt{7 - \frac{19}{4}}} = \frac{1}{2\frac{\sqrt{28-19}}{2}} = \frac{1}{3}$

85. **(a)** Let $y = \sec(\tan^{-1} x) = \sec\left(\sec^{-1}\sqrt{1+x^2}\right)$

$\Rightarrow y = \sqrt{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \cdot 2x$

At $x = 1$,

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}$.

86. **(b)** $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1 \Rightarrow \frac{2x}{\alpha} + \frac{2y}{4}.\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-4x}{\alpha y}$ ...(i)

$y^3 = 16x \Rightarrow 3y^2.\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{16}{3y^2}$ ...(ii)

Since curves intersects at right angles

$\therefore \frac{-4x}{\alpha y} \times \frac{16}{3y^2} = -1 \Rightarrow 3\alpha y^3 = 64x$

$\Rightarrow \alpha = \frac{64x}{3 \times 16x} = \frac{4}{3}$

87. **(d)** Let $x = \sqrt{a^{\sin^{-1} t}}$

$\Rightarrow x^2 = a^{\sin^{-1} t}$

$\Rightarrow 2 \log x = \sin^{-1} t \,.\, \log a$

$\Rightarrow \frac{2}{x} = \frac{\log a}{\sqrt{1-t^2}}.\frac{dt}{dx}$

$\Rightarrow \frac{2\sqrt{1-t^2}}{x \log a} = \frac{dt}{dx}$ ...(1)

Now, let $y = \sqrt{a^{\cos^{-1} t}}$

$\Rightarrow 2 \log y = \cos^{-1} t.\, \log a$

$\Rightarrow \frac{2}{y}.\frac{dy}{dx} = \frac{-\log a}{\sqrt{1-t^2}}.\frac{dt}{dx}$

$\Rightarrow \frac{2}{y}.\frac{dy}{dx} = \frac{-\log a}{\sqrt{1-t^2}} \times \frac{2\sqrt{1-t^2}}{x \log a}$ (from (1)

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Hence, $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{-y}{x}\right)^2 = \frac{x^2 + y^2}{x^2}$

88. **(b)** Let $y = \frac{x^2 - x}{x^2 + 2x}$

$\Rightarrow (x^2 + 2x)\, y = x^2 - x$

$\Rightarrow x(x+2)y = x(x-1)$

$\Rightarrow x[(x+2)y - (x-1)] = 0$

$\because x \neq 0, \quad \therefore (x+2)y - (x-1) = 0$

$\Rightarrow xy + 2y - x + 1 = 0$

$\Rightarrow x(y-1) = -(2y+1)$

$\therefore x = \frac{2y+1}{1-y} \Rightarrow f^{-1}(x) = \frac{2x+1}{1-x}$

$\frac{d}{dx}(f^{-1}(x)) = \frac{2(1-x) - (2x+1)(-1)}{(1-x)^2}$

$= \frac{2 - 2x + 2x + 1}{(1-x)^2} = \frac{3}{(1-x)^2}$

89. **(c)** Let $f'(x) = \sin[\log x]$ and $y = f\left(\frac{2x+3}{3-2x}\right)$

Now, $\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right).\frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$

$= \sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]\frac{[(6-4x) - (-4x-6)]}{(3-2x)^2}$

$= \frac{12}{(3-2x)^2}.\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

90. **(a)** Given that $g(x)=[f(2f(x))+2]^2$

$\therefore \quad g'(x)=2\left(f(2f(x)+2)\right)\left(\dfrac{d}{dx}\left(f(2f(x)+2)\right)\right)$

$=2f(2f(x)+2)f'(2f(x))+2).(2f'(x))$

$\Rightarrow \quad g'(0)=2f(2f(0)+2.f'(2f(0)+2)$

$.2f'(0)=4f(0)(f'(0))^2=4(-1)(1)^2=-4$

91. **(d)** $x^{2x}-2x^x\cot y-1=0$

$\Rightarrow$ $2\cot y=x^x-x^{-x}$

Let $u=x^x$

$\Rightarrow 2\cot y=u-\dfrac{1}{u}$

Differentiating both sides with respect to x, we get

$-\ 2\text{cosec}^2 y\ \dfrac{dy}{dx}=\left(1+\dfrac{1}{u^2}\right)\dfrac{du}{dx}$

Now $u=x^x$ Taking log both sides

$\Rightarrow \log u=x\log x$

$\Rightarrow \dfrac{1}{u}\dfrac{du}{dx}=1+\log x$

$\Rightarrow \dfrac{du}{dx}=x^x(1+\log x)$

$\therefore$ We get

$-2\text{ cosec}^2 y\ \dfrac{dy}{dx}=(1+x^{-2x}).x^x(1+\log x)$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{\left(x^x+x^{-x}\right)(1+\log x)}{-2(1+\cot^2 y)}$ ...(i)

Put $n=1$ in eqn. $x^{2x}-2x^x\cot y-1=0$, gives

$1-2\cot y-1=0$

$\Rightarrow$ $\cot y=0$

$\therefore$ Putting $x=1$ and $\cot y=0$ in eqn. (i), we get

$y'(1)=\dfrac{(1+1)(1+0)}{-2(1+0)}=-1$

92. **(a)** $x^m.y^n=(x+y)^{m+n}$

taking log both sides

$\Rightarrow$ $m\ ln\,x+n\ ln\,y=(m+n)\ ln\,(x+y)$

Differentiating both sides, we get

$\therefore \quad \dfrac{m}{x}+\dfrac{n}{y}\dfrac{dy}{dx}=\dfrac{m+n}{x+y}\left(1+\dfrac{dy}{dx}\right)$

$\Rightarrow \quad \left(\dfrac{m}{x}-\dfrac{m+n}{x+y}\right)=\left(\dfrac{m+n}{x+y}-\dfrac{n}{y}\right)\dfrac{dy}{dx}$

$\Rightarrow \quad \dfrac{my-nx}{x(x+y)}=\left(\dfrac{my-nx}{y(x+y)}\right)\dfrac{dy}{dx}$

$\Rightarrow \quad \dfrac{dy}{dx}=\dfrac{y}{x}$

93. **(c)** Given that $x=e^{y+e^{y+\cdots\infty}}$ $\Rightarrow x=e^{y+x}$.

Taking log both sides.

$\log x=y+x$ differentiating both side $\Rightarrow \dfrac{1}{x}=\dfrac{dy}{dx}+1$

$\therefore \dfrac{dy}{dx}=\dfrac{1}{x}-1=\dfrac{1-x}{x}$

94. **(b)** $f(x)=ax^2+bx+c$

$f(1)=f(-1)$

$\Rightarrow a+b+c=a-b+c$ or $b=0$

$\therefore f(x)=ax^2+c$ or $f'(x)=2ax$

Now $f'(a);f'(b)$ and $f'(c)$

are $2a(a);2a(b);2a(c)$

i.e. $2a^2, 2ab, 2ac$.

$\Rightarrow$ If a,b,c are in A.P. then $f'(a);f'(b)$ and $f'(c)$ are also in A.P.

95. **(c)** Given that $f(x+y)=f(x)\times f(y)$

Differentiate with respect to x, treating y as constant

$f'(x+y)=f'(x)f(y)$

Putting $x=0$ and $y=x$, we get $f'(x)=f'(0)f(x)$;

$\Rightarrow$ $f'(5)=3f(5)=3\times 2=6$.

96. **(b)** Let $f:R\to R$, with $f(0)=f(1)=0$ and $f'(0)=0$

$\because f(x)$ is differentiable and continuous and $f(0)=f(1)=0$.

Then by Rolle's theorem, $f'(c)=0,\ c\in(0,1)$

Now again

$\because f'(c)=0,\ f'(0)=0$

Then, again by Rolle's theorem,

$f''(x)=0$ for some $x\in(0,1)$

97. **(c)** $y^2+2\log_e(\cos x)=y$ ...(i)

$\Rightarrow 2yy'-2\tan x=y'$ ...(ii)

From (i), $y(0)=0$ or 1

$\therefore y'(0)=0$

Again differentiating (ii) we get,

$2(y')^2+2yy''-2\sec^2 x=y''$

Put $x=0, y(0)=0, 1$ and $y'(0)=0$,

we get, $|y''(0)|=2$.

98. **(b)** Since, Rolle's theorem is applicable

$\therefore$ $f(a)=f(b)$

$f(3)=f(4)\ \Rightarrow\ \alpha=12$

$f'(x)=\dfrac{x^2-12}{x(x^2+12)}$

As $f'(c)=0$ (by Rolle's theorem)

$x=\pm\sqrt{12}$, $\therefore\ c=\sqrt{12}$, $\therefore\ f''(c)=\dfrac{1}{12}$

99. **(c)** $k.x^{k-1}+k.y^{k-1}\dfrac{dy}{dx}=0$

$\Rightarrow \dfrac{dy}{dx}=-\left(\dfrac{x}{y}\right)^{k-1}$

$\Rightarrow \dfrac{dy}{dx}+\left(\dfrac{x}{y}\right)^{k-1}=0$

$\Rightarrow k-1=-\dfrac{1}{3}$

$\Rightarrow k=1-\dfrac{1}{3}=\dfrac{2}{3}$

100. **(b)** Since, $f(x)$ is a polynomial function.

$\therefore$ It is continuous and differentiable in [0, 1]

Here, $f(0)=11, f(1)=1-4+8+11=16$

$f'(x)=3x^2-8x+8$

$\therefore \quad f'(c)=\dfrac{f(1)-f(0)}{1-0}=\dfrac{16-11}{1}$

$=3c^2-8c+8$

$\Rightarrow 3c^2-8c+3=0$

$\Rightarrow c=\dfrac{8\pm2\sqrt{7}}{6}=\dfrac{4\pm\sqrt{7}}{3}$

$\therefore \quad c=\dfrac{4-\sqrt{7}}{3}\in(0,1)$

101. **(b)** $y^{1/5}+y^{-1/5}=2x$

$\Rightarrow \left(\dfrac{1}{5}y^{-4/5}-\dfrac{1}{5}y^{-6/5}\right)\dfrac{dy}{dx}=2$

$\Rightarrow y'\left(y^{1/5}-y^{-1/5}\right)=10y$

$\Rightarrow y^{1/5}+y^{-1/5}=2x$

$\Rightarrow y^{1/5}-y^{-1/5}=\sqrt{4x^2-4}$

$\Rightarrow y'\left(2\sqrt{x^2-1}\right)=10y$

$\Rightarrow y''\left(2\sqrt{x^2-1}\right)+y'2\dfrac{2x}{2\sqrt{x^2-1}}=10y'$

$\Rightarrow y''(x^2-1)+xy'=5\sqrt{x^2-1}\,(y')$

$\Rightarrow \boxed{y''(x^2-1)+xy'-25y=0}$

$\lambda=1, k=-25$

102. **(b)** Let $f(x)=ax^3+bx^2+cx+d$

$\Rightarrow f(3x)=27ax^3+9bx^2+3cx+d$

$\Rightarrow f'(x)=3ax^2+2bx+c$

$\Rightarrow f''(x)=6ax+2b$

$\Rightarrow f(3x)=f'(x)f''(x)$

$\Rightarrow 27a=18a^2$

$\Rightarrow a=\dfrac{3}{2}, b=0, c=0, d=0$

$\Rightarrow f(x)=\dfrac{3}{2}x^3,$

$f'(x)=\dfrac{9}{2}x^2,\ f'(x)=9x$

$\Rightarrow f'(2)=18$

and $f''(2)=18$

$\Rightarrow f''(b)-f'(b)=0$

103. **(d)** $y=\left\{x+\sqrt{x^2-1}\right\}^{15}+\left\{x-\sqrt{x^2-1}\right\}^{15}$

Differentiate w.r.t. 'x'

$$\frac{dy}{dx}=15\left(x+\sqrt{x^2-1}\right)^{14}\left[1+\frac{x}{\sqrt{x^2-1}}\right]$$

$$+15\left(x-\sqrt{x^2-1}\right)^{14}\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{15}{\sqrt{x^2-1}}.y$ ....(i)

$\Rightarrow \sqrt{x^2-1}.\dfrac{dy}{dx}=15y$

Again differentiating both sides w.r.t. x

$$\frac{x}{\sqrt{x^2-1}}.\frac{dy}{dx}+\sqrt{x^2-1}\frac{d^2y}{dx^2}=15\frac{dy}{dx}$$

$\Rightarrow x\dfrac{dy}{dx}+(x^2-1)\dfrac{d^2y}{dx^2}$

$=15\sqrt{x^2-1}.\dfrac{15}{\sqrt{x^2-1}}.y=225y$

104. **(b)** Conduction for Rolls theorem

$f(1)=f(-1)$

and $f'\left(\dfrac{1}{2}\right)=0$

$c=-2$ and $b=\dfrac{1}{2}$

$2b+c=-1$

105. **(b)** Since, f and g both are continuous function on [0, 1] and differentiable on (0, 1) then $\exists c\in(0,1)$ such that

$f'(c)=\dfrac{f(1)-f(0)}{1}=\dfrac{6-2}{1}=4$

and $g'(c)=\dfrac{g(1)-g(0)}{1}=\dfrac{2-0}{1}=2$

Thus, we get $f'(c)=2g'(c)$

106. (c) Let $f(x) = x|x| = x|x|$, $g(x) = \sin x$
and $h(x) = gof(x) = g[f(x)]$

$$\therefore\ h(x)=\begin{cases}\sin x^2, & x\ge 0\\ -\sin x^2, & x<0\end{cases}$$

$$\text{Now, } h'(x)=\begin{cases}2x\cos x^2, & x\ge 0\\ -2x\cos x^2, & x<0\end{cases}$$

Since, L.H.L and R.H.L at $x = 0$ of $h'(x)$ is equal to 0 therefore $h'(x)$ is continuous at $x = 0$
Now, suppose $h'(x)$ is differentiable

$$\therefore\ h''(x)=\begin{cases}2(\cos x^2+2x^2(-\sin x^2), & x\ge 0\\ 2(-\cos x^2+2x^2\sin x^2), & x<0\end{cases}$$

Since, L.H.L and R.H.L at $x = 0$ of $h''(x)$ are different therefore $h''(x)$ is not continuous.
$\Rightarrow h''(x)$ is not differentiable
$\Rightarrow$ our assumption is wrong
Hence $h'(x)$ is not differentiable at $x = 0$.

107. (a) Let $p_1(x) = a_1x^2 + b_1x + c_1$
$p_2(x) = a_2x^2 + b_2x + c_2$
and $p_3(x) = a_3x^2 + b_3x + c_3$
where $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are real numbers.

$$\therefore\quad A(x)=\begin{bmatrix}a_1x^2+b_1x+c_1 & 2a_1x+b_1 & 2a_1\\ a_2x^2+b_2x+c_2 & 2a_2x+b_2 & 2a_2\\ a_3x^2+b_3x+c_3 & 2a_3x+b_3 & 2a_3\end{bmatrix}$$

$$B(x)=\begin{bmatrix}a_1x^2+b_1x+c_1 & a_2x^2+b_2x+c_2 & a_3x^2+b_3x+c_3\\ 2a_1x+b_1 & 2a_2x+b_2 & 2a_3x+b_2\\ 2a_1 & 2a_2 & 2a_3\end{bmatrix}$$

$$\times\begin{bmatrix}a_1x^2+b_1x+c_1 & 2a_1x+b_1 & 2a_1\\ a_2x^2+b_2x+c_2 & 2a_2x+b_2 & 2a_2\\ a_3x^2+b_3x+c_3 & 2a_3x+b_3 & 2a_3\end{bmatrix}$$

It is clear from the above multiplication, the degree of determinant of $B(x)$ can not be less than 4.

108. (b) $f(x) = 2x^3 + ax^2 + bx$
let, $a = -1$, $b = 1$
Given that $f(x)$ satisfy Rolle's theorem in interval $[-1, 1]$
$f(x)$ must satisfy two conditions.
(1) $f(a) = f(b)$
(2) $f'(c) = 0$ (c should be between a and b)

$f(a) = f(1) = 2(1)^3 + a(1)^2 + b(1) = 2 + a + b$
$f(b) = f(-1) = 2(-1)^3 + a(-1)^2 + b(-1)$
$= -2 + a - b$
$f(a) = f(b)$
$2 + a + b = -2 + a - b$
$2b = -4$
$b = -2$

(given that $c=\dfrac{1}{2}$)

$f'(x) = 6x^2 + 2ax + b$

at $x=\dfrac{1}{2}$, $f'(x) = 0$

$$0=6\left(\frac{1}{2}\right)^2+2a\left(\frac{1}{2}\right)+b$$

$$\frac{3}{2}+a+b=0$$

$$\frac{3}{2}+a-2=0$$

$$a=2-\frac{3}{2}=\frac{1}{2}$$

$$2a+b=2\times\frac{1}{2}-2=1-2=-1$$

109. (d) $f(x) = \sin(\sin x)$
$\Rightarrow f'(x) = \cos(\sin x) . \cos x$
$\Rightarrow f''(x) = -\sin(\sin x) . \cos^2 x + \cos(\sin x). (-\sin x)$
$= -\cos^2 x . \sin(\sin x) - \sin x . \cos(\sin x)$
Now $f''(x) + \tan x . f'(x) + g(x) = 0$
$\Rightarrow g(x) = \cos^2 x . \sin(\sin x) + \sin x . \cos(\sin x)$
$- \tan x . \cos x . \cos(\sin x)$
$\Rightarrow g(x) = \cos^2 x . \sin(\sin x)$.

110. (d) Let $g(x)=\dfrac{ax^3}{3}+b.\dfrac{x^2}{2}+cx$

$g'(x) = ax^2 + bx + c$
Given: $ax^2 + bx + c = 0$ and $2a + 3b + 6c = 0$
Statement-2:

(*i*) $g(0) = 0$ and $g(1)=\dfrac{a}{3}+\dfrac{b}{2}+c=\dfrac{2a+3b+6c}{6}$

$$=\frac{0}{6}=0$$

$\Rightarrow g(0) = g(1)$

(*ii*) g is continuous on $[0, 1]$ and differentiable on $(0, 1)$

$\therefore$ By Rolle's theorem $\exists\, k \in (0,1)$ such that $g'(k) = 0$

This holds the statement 2. Also, from statement-2, we can say $ax^2 + bx + c = 0$ has at least one root in $(0, 1)$.
Thus statement-1 and 2 both are true and statement-2 is a correct explanation for statement-1.

111. (c) $\dfrac{d^2x}{dy^2}=\dfrac{d}{dy}\left(\dfrac{dx}{dy}\right)=\dfrac{d}{dx}\left(\dfrac{dx}{dy}\right)\dfrac{dx}{dy}=\dfrac{d}{dx}\left(\dfrac{1}{dy/dx}\right)\dfrac{dx}{dy}$

$$=-\frac{1}{\left(\frac{dy}{dx}\right)^2}\cdot\frac{d^2y}{dx^2}\cdot\frac{1}{\frac{dy}{dx}}\left[\because\frac{d\left(\frac{1}{x}\right)}{dx}=-\frac{1}{x^2}\right]$$

$$=-\frac{1}{\left(\frac{dy}{dx}\right)^3}\frac{d^2y}{dx^2}$$

112. (b) Given that $f(x)=x|x|$ and $g(x)=\sin x$

So that

$gof(x)=g(f(x))=g(x|x|)=\sin x|x|$

$$=\begin{cases}\sin(-x^2), & \text{if } x<0\\ \sin(x^2), & \text{if } x\ge 0\end{cases}=\begin{cases}-\sin x^2, & \text{if } x<0\\ \sin x^2, & \text{if } x\ge 0\end{cases}$$

$$\therefore (gof)'(x)=\begin{cases}-2x\cos x^2, & \text{if } x<0\\ 2x\cos x^2, & \text{if } x\ge 0\end{cases}$$

Here we observe

$L(gof)'(0)=0=R(gof)'(0)$

$\Rightarrow$ gof is differentiable at $x=0$

and $(gof)'$ is continuous at $x=0$

$$\text{Now } (gof)''(x)=\begin{cases}-2\cos x^2+4x^2\sin x^2, & x<0\\ 2\cos x^2-4x^2\sin x^2, & x\ge 0\end{cases}$$

Here

$L(gof)''(0)=-2$ and $R(gof)''(0)=2$

$\because$ $L(gof)''(0)\ne R(gof)''(0)$

$\Rightarrow$ $gof(x)$ is not twice differentiable at $x=0$.

$\therefore$ Statement - 1 is true but statement -2 is false.

113. (c) Using Lagrange's Mean Value Theorem

Let $f(x)$ be a function defined on $[a, b]$

then, $f'(c)=\dfrac{f(b)-f(a)}{b-a}$ ....(i)

$c\in[a, b]$

$\therefore$ Given $f(x)=\log_e x$ $\quad\therefore f'(x)=\dfrac{1}{x}$

$\therefore$ equation (i) become

$$\frac{1}{c}=\frac{f(3)-f(1)}{3-1}$$

$$\Rightarrow\ \frac{1}{c}=\frac{\log_e 3-\log_e 1}{2}=\frac{\log_e 3}{2}$$

$$\Rightarrow\ c=\frac{2}{\log_e 3}\Rightarrow c=2\log_3 e$$

114. (a) As $f(1)=-2$ & $f'(x)\ge 2\ \forall\ x\in[1, 6]$

Applying Lagrange's mean value theorem

$$\frac{f(6)-f(1)}{5}=f'(c)\ge 2$$

$\Rightarrow$ $f(6)\ge 10+f(1)$

$\Rightarrow$ $f(6)\ge 10-2\Rightarrow$ $f(6)\ge 8$.

115. (b) Let $f(x)=a_nx^n+a_{n-1}x^{n-1}+\ ......+a_1x=0$

The other given equation,

$na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+....+a_1=0=f'(x)$

Given $a_1\ne 0\Rightarrow f(0)=0$

Again $f(x)$ has root α, $\Rightarrow f(\alpha)=0$

$\therefore$ $f(0)=f(\alpha)$

$\therefore$ By Rolle's theorem $f'(x)=0$ has root between $(0, \alpha)$

Hence $f'(x)$ has a positive root smaller than α.

116. (d) Let us define a function

$$f(x)=\frac{ax^3}{3}+\frac{bx^2}{2}+cx$$

Being polynomial, it is continuous and differentiable, also,

$f(0)=0$ and $f(1)=\dfrac{a}{3}+\dfrac{b}{2}+c$

$\Rightarrow f(1)=\dfrac{2a+3b+6c}{6}=0$ (given)

$\therefore f(0)=f(1)$

$\therefore f(x)$ satisfies all conditions of Rolle's theorem therefore $f'(x)=0$ has a root in $(0, 1)$

i.e. $ax^2+bx+c=0$ has at lease one root in $(0, 1)$

117. (d) Given that $f(x)=x^n\Rightarrow f(1)=1$

$f'(x)=nx^{n-1}\Rightarrow f'(1)=n$

$f''(x)=n(n-1)x^{n-2}\Rightarrow f''(1)=n(n-1)$

....

$f^n(x)=n!\Rightarrow f^n(1)=n!$

$$f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+...+\frac{(-1)^nf^n(1)}{n!}$$

$$=1-\frac{n}{1!}+\frac{n(n-1)}{2!}-\frac{n(n-1)(n-2)}{3!}+\ldots+(-1)^n\frac{n!}{n!}$$

$$={}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+\ldots+(-1)^n\,{}^nC_n$$

$$=(1-1)^n=0$$

118. **(b)** $\lim\limits_{x\to a}\dfrac{f(a)g'(x)-g(a)f'(x)}{g'(x)-f'(x)}=4$

(By Applying L' Hospital rule)

$\lim\limits_{x\to a}\dfrac{k\ g'(x)-k\ f'(x)}{g'(x)-f'(x)}=4 \quad \therefore k=4.$

119. **(a)** Given that $y=(x+\sqrt{1+x^2})^n$ ...(i)

Differentiating both sides w.r. to x

$$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{1}{2}(1+x^2)^{-1/2}.\ 2x\right)$$

$$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\ \frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}$$

$$=\frac{n(\sqrt{1+x^2}+x)^n}{\sqrt{1+x^2}}$$

or $\sqrt{1+x^2}\dfrac{dy}{dx}=ny$ [from (i)]

$\Rightarrow \sqrt{1+x^2}\,y_1=ny$ $(\because y_1=\dfrac{dy}{dx})$ Squaring both sides,

we get $(1+x^2)y_1^{\,2}=n^2y^2$

Differentiating it w.r. to x,

$(1+x^2)2y_1y_2+y_1^{\,2}.2x=n^2.2yy_1$

$\Rightarrow (1+x^2)y_2+xy_1=n^2y$

120. **(a)** Let $f(x)=\dfrac{ax^3}{3}+\dfrac{bx^2}{2}+cx$

$\Rightarrow f(0)=0$ and

$f(1)=\dfrac{a}{3}+\dfrac{b}{2}+c=\dfrac{2a+3b+6c}{6}=0$

Also $f(x)$ is continuous and differentiable in [0, 1] and [0, 1[. So by Rolle's theorem, $f'(x)=0$.

i.e $ax^2+bx+c=0$ has at least one root in [0, 1].

21

Applications of Derivatives

TOPIC 1 Rate of Change of Quantities

1. The position of a moving car at time t is given by $f(t) = at^2 + bt + c, t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point : **[Sep. 06, 2020 (I)]**
   (a) $(t_2 - t_1)/2$ (b) $a(t_2 - t_1) + b$
   (c) $(t_1 + t_2)/2$ (d) $2a(t_1 + t_2) + b$

2. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec.), when the length of a side of the cube is 10 cm, is : **[Sep. 03, 2020 (II)]**
   (a) 18 (b) 10
   (c) 20 (d) 9

3. If a function $f(x)$ defined by **[Sep. 02, 2020 (I)]**
$$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$$
be continuous for some $a, b, c \in \mathbf{R}$ and $f'(0) + f'(2) = e$, then the value of a is :
   (a) $\frac{1}{e^2 - 3e + 13}$ (b) $\frac{e}{e^2 - 3e - 13}$
   (c) $\frac{e}{e^2 + 3e + 13}$ (d) $\frac{e}{e^2 - 3e + 13}$

4. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is: **[Jan. 9, 2020 (I)]**
   (a) $\frac{5}{6\pi}$ (b) $\frac{1}{54\pi}$
   (c) $\frac{1}{36\pi}$ (d) $\frac{1}{18\pi}$

5. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is: **[April 12, 2019 (I)]**
   (a) $25\sqrt{3}$ (b) $\frac{25}{\sqrt{3}}$
   (c) $\frac{25}{3}$ (d) 25

6. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is : **[April 10, 2019 (II)]**
   (a) $\frac{1}{18\pi}$ (b) $\frac{1}{36\pi}$
   (c) $\frac{5}{6\pi}$ (d) $\frac{1}{9\pi}$

7. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}$. Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10m; is: **[April 09, 2019 (II)]**
   (a) $1/15\pi$ (b) $1/10\pi$
   (c) $2/\pi$ (d) $1/5\pi$

8. If the volume of a spherical ball is increasing at the rate of 4π cc/sec, then the rate of increase of its radius (in cm/sec), when the volume is 288π cc, **[Online April 19, 2014]**
   (a) $\frac{1}{6}$ (b) $\frac{1}{9}$
   (c) $\frac{1}{36}$ (d) $\frac{1}{24}$

9. Two ships A and B are sailing straight away from a fixed point O along routes such that $\angle AOB$ is always 120°. At a certain instance, OA = 8 km, OB = 6 km and the ship A is sailing at the rate of 20 km/hr while the ship B sailing at the rate of 30 km/hr. Then the distance between A and B is changing at the rate (in km/hr): **[Online April 11, 2014]**

(a) $\frac{260}{\sqrt{37}}$ (b) $\frac{260}{37}$

(c) $\frac{80}{\sqrt{37}}$ (d) $\frac{80}{37}$

10. A spherical balloon is being inflated at the rate of 35cc/min. The rate of increase in the surface area (in cm^2/min.) of the balloon when its diameter is 14 cm, is : **[Online April 25, 2013]**

(a) 10 (b) $\sqrt{10}$

(c) 100 (d) $10\sqrt{10}$

11. If the surface area of a sphere of radius r is increasing uniformly at the rate 8 cm^2/s, then the rate of change of its volume is : **[Online April 9, 2013]**

(a) constant (b) proportional to $\sqrt{r}$

(c) proportional to r^2 (d) proportional to r

12. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is: **[2012]**

(a) $\frac{9}{7}$ (b) $\frac{7}{9}$

(c) $\frac{2}{9}$ (d) $\frac{9}{2}$

13. If a metallic circular plate of radius 50 cm is heated so that its radius increases at the rate of 1 mm per hour, then the rate at which, the area of the plate increases (in cm^2/hour) is **[Online May 26, 2012]**

(a) 5π (b) 10π

(c) 100π (d) 50π

14. The weight W of a certain stock of fish is given by $W = nw$, where n is the size of stock and w is the average weight of a fish. If n and w change with time t as $n = 2t^2 + 3$ and $w = t^2 - t + 2$, then the rate of change of W with respect to t at $t = 1$ is **[Online May 19, 2012]**

(a) 1 (b) 8

(c) 13 (d) 5

15. Consider a rectangle whose length is increasing at the uniform rate of 2 *m/sec*, breadth is decreasing at the uniform rate of 3 *m/sec* and the area is decreasing at the uniform rate of 5 m^2/sec. If after some time the breadth of the rectangle is 2 m then the length of the rectangle is **[Online May 12, 2012]**

(a) 2m (b) 4m

(c) 1m (d) 3m

16. If a circular iron sheet of radius 30 cm is heated such that its area increases at the uniform rate of 6π cm^2/hr, then the rate (in mm/hr) at which the radius of the circular sheet increases is **[Online May 7, 2012]**

(a) 1.0 (b) 0.1

(c) 1.1 (d) 2.0

17. Two points A and B move from rest along a straight line with constant acceleration f and f' respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then **[2005]**

(a) $(f - f')m^2 = ff'n$

(b) $(f + f')m^2 = ff'n$

(c) $\frac{1}{2}(f + f')m = ff'n^2$

(d) $(f' - f)n = \frac{1}{2}ff'm^2$

18. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of $2\ \text{cm/s}^2$ and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after **[2005]**

(a) 20 s (b) 1 s

(c) 21 s (d) 24 s

19. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases is **[2005]**

(a) $\frac{1}{36\pi}$ cm/min. (b) $\frac{1}{18\pi}$ cm/min.

(c) $\frac{1}{54\pi}$ cm/min. (d) $\frac{5}{6\pi}$ cm/min

20. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is **[2004]**

(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$

(c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) $(2, 4)$

TOPIC 2 Increasing & Decreasing Functions

21. The function, $f(x) = (3x - 7)x^{2/3}$, $x \in \mathbf{R}$, is increasing for all x lying in : **[Sep. 03, 2020 (I)]**

(a) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$ (b) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(c) $\left(-\infty, \frac{14}{15}\right)$ (d) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

22. Let f be any function continuous on $[a, b]$ and twice differentiable on (a, b). If for all $x \in (a, b), f2(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\dfrac{f(c)-f(a)}{f(b)-f(c)}$ is greater than: **[Jan. 9, 2020 (I)]**

(a) $\dfrac{b+a}{b-a}$ (b) 1

(c) $\dfrac{b-c}{c-a}$ (d) $\dfrac{c-a}{b-c}$

23. Let $f(x) = x\cos^{-1}(-\sin|x|), x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true? **[Jan. 8, 2020 (I)]**

(a) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

(b) $f'(0) = -\frac{\pi}{2}$

(c) f' is not differentiable at $x = 0$

(d) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

24. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $x \in \mathbf{R}$. Then the set of all $x \in \mathbf{R}$, where the function $h(x) = (f o g)(x)$ is increasing, is : **[April 10, 2019 (I)]**

(a) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (b) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

(c) $[0, \infty)$ (d) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$

25. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \dfrac{x^2}{1-x^2}$, is surjective, then A is equal to: **[April 09, 2019 (I)]**

(a) $R - \{-1\}$ (b) [0, “)

(c) $R - [-1, 0)$ (d) $R - (-1, 0)$

26. Let $f: [0 : 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2-x)$, then ϕ is : **[April 08, 2019 (I)]**

(a) increasing on (0, 1) and decreasing on (1, 2).

(b) decreasing on (0, 2)

(c) decreasing on (0, 1) and increasing on (1, 2).

(d) increasing on (0, 2)

27. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\dfrac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is **[Jan. 12, 2019 (II)]**

(a) –7 (b) 5

(c) 7 (d) 6

28. Let $f(x) = \dfrac{x}{\sqrt{a^2+x^2}} - \dfrac{d-x}{\sqrt{b^2+(d-x)^2}}, x \in \mathbf{R}$ where a, b and d are non-zero real constants. Then : **[Jan. 11, 2019 (II)]**

(a) f is an increasing function of x

(b) f is a decreasing function of x

(c) f' is not a continuous function of x

(d) f is neither increasing nor decreasing function of x

29. The function f defined by $f(x) = x^3 - 3x^2 + 5x + 7$, is : **[Online April 9, 2017]**

(a) increasing in R.

(b) decreasing in R.

(c) decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$.

(d) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$.

30. Let $f(x) = \sin^4 x + \cos^4 x$. Then f is an increasing function in the interval :

(a) $\left]\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left]\frac{\pi}{2}, \frac{5\pi}{8}\right]$

(c) $\left]\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left]0, \frac{\pi}{4}\right]$

31. Let f and g be two differentiable functions on R such that $f'(x) > 0$ and $g'(x) < 0$ for all $x \in R$. Then for all x: **[Online April 12, 2014]**

(a) $f(g(x)) > f(g(x-1))$ (b) $f(g(x)) > f(g(x+1))$

(c) $g(f(x)) > g(f(x-1))$ (d) $g(f(x)) < g(f(x+1))$

32. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] **[2013]**

(a) lies between 1 and 2

(b) lies between 2 and 3

(c) lies between .1 and 0

(d) does not exist.

33. **Statement-1:** The function $x^2(e^x + e^{-x})$ is increasing for all $x > 0$.

Statement-2: The functions $x^2 e^x$ and $x^2 e^{-x}$ are increasing for all $x > 0$ and the sum of two increasing functions in any interval (a, b) is an increasing function in (a, b). **[Online April 22, 2013]**

(a) Statement-1 is false; Statement-2 is true.

(b) Statement-1is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.

34. **Statement-1:** The equation $x \log x = 2 - x$ is satisfied by at least one value of x lying between 1 and 2.

Statement-2: The function $f(x) = x \log x$ is an increasing function in [l, 2] and $g(x) = 2 - x$ is a decreasing function in [1, 2] and the graphs represented by these functions intersect at a point in [1, 2] **[Online April 9, 2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** correct explanation for Statement-1.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

35. If $f(x) = xe^{x(1-x)},\ x \in R$, then $f(x)$ is **[Online May 12, 2012]**
(a) decreasing on [–1/2, 1]
(b) decreasing on R
(c) increasing on [–1/2, 1]
(d) increasing on R

36. For real x, let $f(x) = x^3 + 5x + 1$, then **[2009]**
(a) f is onto R but not one-one
(b) f is one-one and onto R
(c) f is neither one-one nor onto R
(d) f is one-one but not onto R

37. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have? **[2008]**
(a) 7 (b) 1
(c) 3 (d) 5

38. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in **[2007]**
(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

39. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched? **[2005]**

	Interval	*Function*
(a)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(b)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(c)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(d)	$(-\infty, -4)$	$x^3 + 6x^2 + 6$

TOPIC 3 Tangents & Normals

40. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point (c, f(c)) is parallel to the line segement joining the points (1, 0) and (e, e), then c is equal to: **[Sep. 06, 2020 (II)]**
(a) $\frac{e-1}{e}$ (b) $e^{\left(\frac{1}{e-1}\right)}$
(c) $e^{\left(\frac{1}{1-e}\right)}$ (d) $\frac{e}{e-1}$

41. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0)? **[Sep. 05, 2020 (II)]**
(a) (2, 2) (b) (2, 6)
(c) (–2, 6) (d) (–2, 4)

42. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to _____. **[NA Sep. 05, 2020 (II)]**

43. If the tangent to the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is _____________. **[NA Sep. 03, 2020 (II)]**

44. If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at $x = 0$ is ___________. **[NA Sep. 02, 2020 (II)]**

45. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to ______. **[NA Jan. 8, 2020 (I)]**

46. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2, 2) is: **[Jan. 8, 2020 (II)]**
(a) $\sqrt{2}$ (b) $4\sqrt{2}$
(c) 2 (d) $2\sqrt{2}$

47. If the tangent to the curve $y = \frac{x}{x^2-3}$, $x \in R, (x \neq \pm\sqrt{3})$, at a point (α, β) (0, 0) on it is parallel to the line $2x + 6y - 11 = 0$, then : **[April 10, 2019 (II)]**
(a) $|6\alpha + 2\beta| = 19$ (b) $|6\alpha + 2\beta| = 9$
(c) $|2\alpha + 6\beta| = 19$ (d) $|2\alpha + 6\beta| = 11$

48. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve? **[April 09, 2019 (I)]**
(a) (–2, 1) (b) (–2, 2)
(c) (2, –1) (d) (2, –2)

49. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (– 1, f(– 1)), then S is equal to: **[April 09, 2019 (I)]**
(a) $\left\{\frac{1}{3}, 1\right\}$ (b) $\left\{-\frac{1}{3}, -1\right\}$
(c) $\left\{\frac{1}{3}, -1\right\}$ (d) $\left\{-\frac{1}{3}, 1\right\}$

50. The tangent and the normal lines at the point ($\sqrt{3}$, 1) to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is : **[April 08, 2019 (II)]**

(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{1}{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{3}}$

51. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is: **[Jan. 12, 2019 (I)]**

(a) 36 (b) $20\sqrt{2}$

(c) 32 (d) $18\sqrt{3}$

52. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point : **[Jan. 12, 2019 (II)]**

(a) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{8}, -7\right)$

(c) $\left(-\frac{1}{8}, 7\right)$ (d) $\left(\frac{1}{4}, \frac{7}{2}\right)$

53. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$, is: **[Jan. 10, 2019 (I)]**

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{3}{2}$ (d) $\frac{5}{4}$

54. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1, e) also passes through the point: **[Jan. 10, 2019 (II)]**

(a) (2, 3e) (b) $\left(\frac{4}{3}, 2e\right)$

(c) $\left(\frac{5}{3}, 2e\right)$ (d) (3, 6e)

55. A helicopter is flying along the curve given by $y - x^{3/2} = 7, (x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is: **[Jan. 10, 2019 (II)]**

(a) $\frac{\sqrt{5}}{6}$ (b) $\frac{1}{3}\sqrt{\frac{7}{3}}$

(c) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (d) $\frac{1}{2}$

56. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan \theta|$ is equal to: **[Jan. 09, 2019 (I)]**

(a) $\frac{4}{9}$ (b) $\frac{8}{15}$

(c) $\frac{7}{17}$ (d) $\frac{8}{17}$

57. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is : **[2018]**

(a) $\frac{7}{2}$ (b) 4

(c) $\frac{9}{2}$ (d) 6

58. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is **[Online April 16, 2018]**

(a) $x + 4y - 2 = 0$ (b) $x + 2y = 0$

(c) $x + y + 1 = 0$ (d) $x - y + 3 = 0$

59. If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is **[Online April 15, 2018]**

(a) $x^2 - 4y^2 + 16x^2y^2 = 0$

(b) $4x^2 - y^2 + 16x^2y^2 = 0$

(c) $4x^2 - y^2 - 16x^2y^2 = 0$

(d) $x^2 - 4y^2 - 16x^2y^2 = 0$

60. If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\in \left(0, \frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to **[Online April 15, 2018]**

(a) $\sqrt{2}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

61. A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram $OABP$ (O being the origin) is formed, then the locus of P is **[Online April 15, 2018]**

(a) $4x^2 - 9y^2 = 121$

(b) $4x^2 + 9y^2 = 121$

(c) $9x^2 - 4y^2 = 169$

(d) $9x^2 + 4y^2 = 169$

62. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point: **[2017]**

(a) $\left(\frac{1}{2},\frac{1}{3}\right)$ (b) $\left(-\frac{1}{2},-\frac{1}{2}\right)$

(c) $\left(\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},-\frac{1}{3}\right)$

63. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directices is $x=-4$, then the equation of the normal to it at $\left(1,\frac{3}{2}\right)$ is : **[2017]**

(a) $x+2y=4$ (b) $2y-x=2$

(c) $4x-2y=1$ (d) $4x+2y=7$

64. A tangent to the curve, $y=f(x)$ at $P(x,y)$ meets x-axis at A and y-axis at B. If $AP:BP=1:3$ and $f(a)=1$, then the curve also passes through the point : **[Online April 9, 2017]**

(a) $\left(\frac{1}{3},24\right)$ (b) $\left(\frac{1}{2},4\right)$

(c) $\left(2,\frac{1}{8}\right)$ (d) $\left(3,\frac{1}{28}\right)$

65. The tangent at the point $(2,-2)$ to the curve, $x^2y^2-2x=4(1-y)$ does not pass through the point : **[Online April 8, 2017]**

(a) $\left(4,\frac{1}{3}\right)$ (b) $(8,5)$

(c) $(-4,-9)$ (d) $(-2,-7)$

66. Consider

$$f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right),x\in\left(0,\frac{\pi}{2}\right).$$ **[2016]**

A normal to $y=f(x)$ at $x=\frac{\pi}{6}$ also passes through the point:

(a) $\left(\frac{\pi}{6},0\right)$ (b) $\left(\frac{\pi}{4},0\right)$

(c) $(0,0)$ (d) $\left(0,\frac{2\pi}{3}\right)$

67. Let C be a curve given by $y(x)=1+\sqrt{4x-3}, x>\frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is : **[Online April 10, 2016]**

(a) $(1,7)$ (b) $(3,-4)$

(c) $(4,-3)$ (d) $(2,3)$

68. If the tangent at a point P, with parameter t, on the curve $x=4t^2+3$, $y=8t^3-1$, $t\in R$, meets the curve again at a point Q, then the coordinates of Q are : **[Online April 9, 2016]**

(a) $(16t^2+3,-64t^3-1)$ (b) $(4t^2+3,-8t^3-2)$

(c) (t^2+3,t^3-1) (d) $(t^2+3,-t^3-1)$

69. The normal to the curve, $x^2+2xy-3y^2=0$, at $(1,1)$ **[2015]**

(a) meets the curve again in the third quadrant.

(b) meets the curve again in the fourth quadrant.

(c) does not meet the curve again.

(d) meets the curve again in the second quadrant.

70. The equation of a normal to the curve, $\sin y=x\sin\left(\frac{\pi}{3}+y\right)$ at $x=0$, is : **[Online April 11, 2015]**

(a) $2x-\sqrt{3}y=0$ (b) $2x+\sqrt{3}y=0$

(c) $2y-\sqrt{3}x=0$ (d) $2y+\sqrt{3}x=0$

71. If the tangent to the conic, $y-6=x^2$ at $(2,10)$ touches the circle, $x^2+y^2+8x-2y=k$ (for some fixed k) at a point (α,β); then (α,β) is : **[Online April 10, 2015]**

(a) $\left(-\frac{7}{17},\frac{6}{17}\right)$ (b) $\left(-\frac{4}{17},\frac{1}{17}\right)$

(c) $\left(-\frac{6}{17},\frac{10}{17}\right)$ (d) $\left(-\frac{8}{17},\frac{2}{17}\right)$

72. The distance, from the origin, of the normal to the curve, $x=2\cos t+2t\sin t$, $y=2\sin t-2t\cos t$ at $t=\frac{\pi}{4}$, is : **[Online April 10, 2015]**

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) $2\sqrt{2}$

73. For the curve $y=3\sin\theta\cos\theta$, $x=e^{\theta}\sin\theta$, $0\le\theta\le\pi$, the tangent is parallel to x-axis when θ is: **[Online April 11, 2014]**

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

74. If an equation of a tangent to the curve, $y-\cos(x+f),-1\ -1\le x\le 1+\pi$, is $x+2y=k$ then k is equal to : **[Online April 25, 2013]**

(a) 1 (b) 2

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

75. The equation of the normal to the parabola, $x^2=8y$ at $x=4$ is **[Online May 19, 2012]**

(a) $x+2y=0$ (b) $x+y=2$

(c) $x-2y=0$ (d) $x+y=6$

76. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is **[2010]**

(a) $y = 1$ (b) $y = 2$
(c) $y = 3$ (d) $y = 0$

77. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is **[2006]**

(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

78. The normal to the curve **[2005]**
$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that

(a) it passes through the origin
(b) it makes an angle $\frac{\pi}{2} + \theta$ with the x- axis
(c) it passes through $\left(a\frac{\pi}{2}, -a\right)$
(d) It is at a constant distance from the origin

79. The normal to the curve $x = a(1 + \cos\theta), y = a\sin\theta$ at 'θ' always passes through the fixed point **[2004]**

(a) (a, a) (b) $(0, a)$
(c) $(0, 0)$ (d) $(a, 0)$

80. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$. If its graph passes through the point (2,1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is **[2004]**

(a) $(x+1)^2$ (b) $(x-1)^3$
(c) $(x+1)^3$ (d) $(x-1)^2$

TOPIC 4 Approximations, Maxima & Minima

81. Let m and M be respectively the minimum and maximum values of **[Sep. 06, 2020 (I)]**

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to :

(a) $(-3, 3)$ (b) $(-3, -1)$
(c) $(-4, -1)$ (d) $(1, 3)$

82. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is ______. **[NA Sep. 06, 2020 (I)]**

83. The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x)\cdot(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly minima, is: **[Sep. 06, 2020 (II)]**

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (b) $\left(-\frac{3}{2}, \frac{3}{2}\right)$
(c) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

84. If $x = 1$ is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then : **[Sep. 05, 2020 (II)]**

(a) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f.
(b) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f.
(c) $x = 1$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.
(d) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

85. The area (in sq. units) of the largest rectangle $ABCD$ whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is : **[Sep. 04, 2020 (II)]**

(a) $\frac{2}{3\sqrt{3}}$ (b) $\frac{1}{3\sqrt{3}}$
(c) $\frac{4}{3}$ (d) $\frac{4}{3\sqrt{3}}$

86. Suppose $f(x)$ is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in \mathbf{R} \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is : **[Sep. 03, 2020 (II)]**

(a) 4 (b) 6
(c) 2 (d) 8

87. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minima at $x =$ ________. **[NA Jan. 8, 2020 (II)]**

88. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If = 4, then which one of the following is not true ? **[Jan. 7, 2020 (II)]**

(a) f is an odd function.
(b) $f(1) - 4f(-1) = 4$.
(c) $x = 1$ is a point of maxima and $x = -1$ is a point of minima of f.
(d) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

89. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0,3] and M is the maximum value of f in [0,3] when k = m, then the ordered pair (m, M) is equal to : **[April 12, 2019 (I)]**

(a) $(4,3\sqrt{2})$ (b) $(4,3\sqrt{3})$

(c) $(3,3\sqrt{3})$ (d) $(5,3\sqrt{6})$

90. Let $a_1, a_2, a_3, \ldots$ be an A. P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1 a_4 a_5$, is : **[April 10, 2019 (II)]**

(a) $\frac{3}{2}$ (b) $\frac{8}{5}$

(c) $\frac{6}{5}$ (d) $\frac{2}{3}$

91. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25, x \in R$, then : **[April 08, 2019 (I)]**

(a) $S_1 = \{-2\}; S_2 = \{0, 1\}$ (b) $S_1 = \{-2, 0\}; S_2 = \{1\}$

(c) $S_1 = \{-2, 1\}; S_2 = \{0\}$ (d) $S_1 = \{-1\}; S_2 = \{0, 2\}$

92. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is : **[April 08, 2019 (II)]**

(a) $\sqrt{6}$ (b) $\frac{2}{3}\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\sqrt{3}$

93. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is: **[Jan. 12, 2019 (I)]**

(a) $\sqrt{19}$ (b) $\frac{\sqrt{79}}{2}$

(c) $\sqrt{34}$ (d) $\sqrt{31}$

94. Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2 = 4x$ and let this X be any point arc POQ of this parabola, where O is vertex of the parabola, such that the area of ΔPXQ is maximum. Then this minimum area (in sq. units) is: **[Jan. 12, 2019 (I)]**

(a) $\frac{75}{2}$ (b) $\frac{125}{4}$

(c) $\frac{625}{4}$ (d) $\frac{125}{2}$

95. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R : x^2 + 30 \le 11x\}$ is : **[Jan. 11, 2019 (I)]**

(a) – 122 (b) – 222

(c) 122 (d) 222

96. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is : **[Jan. 11, 2019 (II)]**

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{m+n}{6mn}$

97. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is: **[Jan. 09, 2019 (I)]**

(a) 6π (b) $3\sqrt{3}\pi$

(c) $\frac{4}{3}\pi$ (d) $2\sqrt{3}\pi$

98. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is : **[2018]**

(a) – 3 (b) $-2\sqrt{2}$

(c) $2\sqrt{2}$ (d) 3

99. Let M and m be respectively the absolute maximum and the absolute minimum values of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0, 3]. Then $M - m$ is equal to **[Online April 16, 2018]**

(a) 1 (b) 5

(c) 4 (d) 9

100. If a right circularcone having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is **[Online April 15, 2018]**

(a) $8\sqrt{3}\pi$ (b) $6\sqrt{2}\pi$

(c) $6\sqrt{3}\pi$ (d) $8\sqrt{2}\pi$

101. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is : **[2017]**

(a) 30 (b) 12.5

(c) 10 (d) 25

102. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then: **[2016]**

(a) x = 2r (b) 2x = r

(c) 2x = (π + 4)r (d) (4 – π)x = πr

103. The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is : **[Online April 9, 2016]**

(a) $\frac{\sqrt{15}}{2}$ (b) $\sqrt{\frac{19}{2}}$

(c) $\sqrt{\frac{15}{2}}$ (d) $\frac{\sqrt{19}}{2}$

104. Let k and K be the minimum and the maximum values of the function $f(x) = \dfrac{(1+x)^{0.6}}{1+x^{0.6}}$ in [0, 1] respectively, then the ordered pair (k, K) is equal to : **[Online April 11, 2015]**

(a) $(2^{-0.4}, 1)$ (b) $(2^{-0.4}, 2^{0.6})$
(c) $(2^{-0.6}, 1)$ (d) $(1, 2^{0.6})$

105. From the top of a 64 metres high tower, a stone is thrown upwards vertically with the velocity of 48 m/s. The greatest height (in metres) attained by the stone, assuming the value of the gravitational acceleration g = 32 m s^2, is: **[Online April 11, 2015]**

(a) 128 (b) 88
(c) 112 (d) 100

106. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then **[2014]**

(a) $\alpha = 2, \beta = -\dfrac{1}{2}$ (b) $\alpha = 2, \beta = \dfrac{1}{2}$
(c) $\alpha = -6, \beta = \dfrac{1}{2}$ (d) $\alpha = -6, \beta = -\dfrac{1}{2}$

107. The minimum area of a triangle formed by any tangent to the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{81} = 1$ and the co-ordinate axes is: **[Online April 12, 2014]**

(a) 12 (b) 18
(c) 26 (d) 36

108. The volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius = $\sqrt{3}$ is: **[Online April 11, 2014]**

(a) $\dfrac{4}{3}\sqrt{3}\pi$ (b) $\dfrac{8}{3}\sqrt{3}\pi$
(c) 4π (d) 2π

109. The cost of running a bus from A to B, is ₹$\left(av + \dfrac{b}{v}\right)$, where v km/h is the average speed of the bus. When the bus travels at 30 km/h, the cost comes out to be ₹ 75 while at 40 km/h, it is ₹ 65. Then the most economical speed (in km/ h) of the bus is : **[Online April 23, 2013]**

(a) 45 (b) 50
(c) 60 (d) 40

110. The maximum area of a right angled triangle with hypotenuse h is : **[Online April 22, 2013]**

(a) $\dfrac{h^2}{2\sqrt{2}}$ (b) $\dfrac{h^2}{2}$
(c) $\dfrac{h^2}{\sqrt{2}}$ (d) $\dfrac{h^2}{4}$

111. Let $a, b \in R$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and $x = 2$

Statement-1 : f has local maximum at $x = -1$ and *at* $x = 2$.

Statement-2 : $a = \dfrac{1}{2}$ and $b = \dfrac{-1}{4}$ **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

112. A line is drawn through the point (1,2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[2012]**

(a) $-\dfrac{1}{4}$ (b) -4
(c) -2 (d) $-\dfrac{1}{2}$

113. Let f: $(-\infty, \infty) \to (-\infty, \infty)$ be defined by $f(x) = x^3 + 1$. **[Online May 26, 2012]**

Statement 1: The function f has a local extremum at $x = 0$

Statement 2: The function f is continuous and differentiable on $(-\infty, \infty)$ and $f'(0) = 0$

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1.
(d) Statement 1 is false, Statement 2 is true.

114. Let f be a function defined by - **[2011RS]**

$$f(x) = \begin{cases} \dfrac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Statement - 1 : $x = 0$ is point of minima of f

Statement - 2 : $f'(0) = 0$.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(c) Statement-1 is true, statement-2 is false.
(d) Statement-1 is false, statement-2 is true.

115. For $x \in \left(0, \dfrac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\sin t\, dt$. Then f has **[2011]**

(a) local minimum at π and 2π
(b) local minimum at π and local maximum at 2π
(c) local maximum at π and local minimum at 2π
(d) local maximum at π and 2π

116. Let f : R $\to$ R be a continuous function defined by

$$f(x)=\frac{1}{e^x+2e^{-x}}$$ **[2010]**

Statement -1 : $f(c)=\frac{1}{3}$, for some $c \in$ R.

Statement -2 : $0<f(x)\le\frac{1}{2\sqrt{2}}$, for all $x \in$ R

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

117. Let f : R $\to$ R be defined by

$$f(x)=\begin{cases} k-2x, & if \;\; x\le -1 \\ 2x+3, & if \;\; x>-1 \end{cases}$$

If f has a local minimum at $x = -1$, then a possible value of k is **[2010]**

(a) 0 (b) $-\frac{1}{2}$
(c) –1 (d) 1

118. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$: **[2009]**

(a) P(–1) is not minimum but P(1) is the maximum of P
(b) P(–1) is the minimum but P(1) is not the maximum of P
(c) Neither P(–1) is the minimum nor P(1) is the maximum of P
(d) P(–1) is the minimum and P(1) is the maximum of P

119. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds? **[2008]**

(a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$
(b) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
(c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
(d) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

120. The function $f(x)=\frac{x}{2}+\frac{2}{x}$ has a local minimum at

(a) $x = 2$ (b) $x = -2$ **[2006]**
(c) $x = 0$ (d) $x = 1$

121. The real number x when added to its inverse gives the minimum value of the sum at x equal to

(a) –2 (b) 2 **[2003]**
(c) 1 (d) –1

122. If the function $f(x)=2x^3-9ax^2+12a^2x+1$, where $a>0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals **[2003]**

(a) $\frac{1}{2}$ (b) 3
(c) 1 (d) 2

123. The maximum distance from origin of a point on the curve $x = a \sin t - b \sin\left(\frac{at}{b}\right)$, $y = a\cos t - b\cos\left(\frac{at}{b}\right)$, both $a, b > 0$ is

(a) $a - b$ (b) $a + b$ **[2002]**
(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

Hints & Solutions

1. **(c)** Average speed $= f'(t) = \dfrac{f(t_2) - f(t_1)}{t_2 - t_1}$

$2at + b = a(t_1 + t_2) + b \Rightarrow t = \dfrac{t_1 + t_2}{2}$

2. **(d)** Let the side of cube be a.

$S = 6a^2 \Rightarrow \dfrac{dS}{dt} = 12a \cdot \dfrac{da}{dt} \Rightarrow 3.6 = 12a \cdot \dfrac{da}{dt}$

$\Rightarrow 12(10)\dfrac{da}{dt} = 3.6 \Rightarrow \dfrac{da}{dt} = 0.03$

$V = a^3 \Rightarrow \dfrac{dV}{dt} = 3a^2 \cdot \dfrac{da}{dt} = 3(10)^2 \cdot \left(\dfrac{3}{100}\right) = 9$

3. **(d)** Since, function $f(x)$ is continuous at $x = 1, 3$

$\therefore f(1) = f(1^+)$

$\Rightarrow ae + be^{-1} = c$ ...(i)

$f(3) = f(3^+)$

$\Rightarrow 9c = 9a + 6c \Rightarrow c = 3a$ ...(ii)

From (i) and (ii),

$b = ae(3 - e)$ ...(iii)

$$f'(x) = \begin{cases} ae^x - be^{-x} & -1 < x < 1 \\ 2cx & 1 < x < 3 \\ 2ax + 2c & 3 < x < 4 \end{cases}$$

$f'(0) = a - b,\ f'(2) = 4c$

Given, $f'(0) + f'(2) = e$

$a - b + 4c = e$ ...(iv)

From eqs. (i), (ii), (iii) and (iv),

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow 13a - 3ae + ae^2 = e$

$\Rightarrow a = \dfrac{e}{e^2 - 3e + 13}$

4. **(d)** Let the thickness of ice layer be $= x$ cm

Total volume $V = \dfrac{4}{3}\pi(10 + x)^3$

$\dfrac{dV}{dt} = 4\pi(10 + x)^2 \dfrac{dx}{dt}$ ...(i)

Since, it is given that

$\dfrac{dV}{dt} = 50\ \text{cm}^3/\text{min}$ ...(ii)

From (i) and (ii), $50 = 4\pi(10 + x)$

$\Rightarrow 50 = 4\pi(10 + 5)^2 \dfrac{dx}{dt}$ [$\because$ thickness of ice $x = 5$]

$\Rightarrow \dfrac{dx}{dt} = \dfrac{1}{18\pi}$ cm / min

5. **(b)** According to the question,

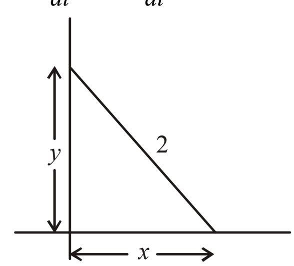
$\dfrac{dy}{dt} = -25$ at $y = 1$

By Pythagoras theorem, $x^2 + y^2 = 4$ ...(i)

When $y = 1 \Rightarrow x = \sqrt{3}$

Diff. equation (i) w. r. t. t,

$2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$



$\Rightarrow x\dfrac{dx}{dt} + y\dfrac{dy}{dt} = 0 \Rightarrow \sqrt{3}\dfrac{dx}{dt} + (-25) = 0$

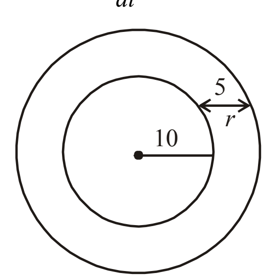
$\Rightarrow \dfrac{dx}{dt} = \dfrac{25}{\sqrt{3}}$ cm/s

6. **(a)** Given that ice melts at a rate of 50 cm^3/min.

$\therefore \dfrac{dV_{\text{ice}}}{dt} = 50$

$V_{\text{ice}} = \dfrac{4}{3}\pi(10 + r)^3 - \dfrac{4}{3}\pi(10)^3$

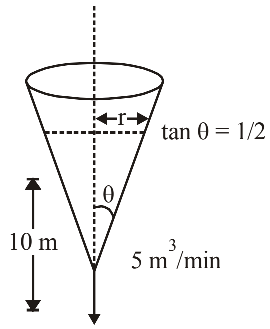
$\Rightarrow \dfrac{dV}{dt} = \dfrac{4}{3}\pi 3(10 + r)^2 \dfrac{dr}{dt} = 4\pi(10 + r)^2 \dfrac{dr}{dt}$



Substitute r = 5,

$50 = 4\pi(225)\dfrac{dr}{dt} \Rightarrow \dfrac{dr}{dt} = \dfrac{50}{4\pi(225)} = \dfrac{1}{18\pi}$ cm/min

7. **(d)**



Given that water is poured into the tank at a constant rate of 5 m^3/minute.

$\therefore \ \dfrac{dv}{dt} = 5\,\text{m}^3/\text{min}$

Volume of the tank is,

$V = \dfrac{1}{3}\pi r^2 h$ ...(i)

where r is radius and h is height at any time.

By the diagram,

$\tan\theta = \dfrac{r}{h} = \dfrac{1}{2}$

$\Rightarrow \ h = 2r \Rightarrow \dfrac{dh}{dt} = \dfrac{2dr}{dt}$ ...(ii)

Differentiate eq. (i) w.r.t. 't', we get

$$\frac{dV}{dt} = \frac{1}{3}\left(\pi 2r\frac{dr}{dt}h + \pi r^2\frac{dh}{dt}\right)$$

Putting $h = 10$, $r = 5$ and $\dfrac{dV}{dt} = 5$ in the above equation.

$$5 = \frac{75\pi}{3}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}\text{m/min.}$$

8. **(c)** Volume of sphere V = $\dfrac{4}{3}\pi r^3$ ...(i)

$\dfrac{dv}{dt} = \dfrac{4}{3}.3\pi r^2.\dfrac{dr}{dt}$

$4\pi = 4\pi r^2.\dfrac{dr}{dt}$

$\dfrac{1}{r^2} = \dfrac{dr}{dt}$

Since, V = 288π, therefore from (i), we have

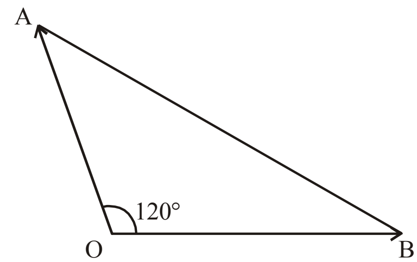
$288\pi = \dfrac{4}{3}\pi(r^3) \Rightarrow \dfrac{288\times 3}{4} = r^3$

$\Rightarrow 216 = r^3$

$\Rightarrow r = 6$

Hence, $\dfrac{dr}{dt} = \dfrac{1}{36}$.

9. **(a)**



Let OA = x km, OB = y km, AB = R

$(AB)^2 = (OA)^2 + (OB)^2 - 2\,(OA)\,(OB)\cos 120°$

$R^2 = x^2 + y^2 - 2xy\left(-\dfrac{1}{2}\right) = x^2 + y^2 + xy$ ...(i)

R at $x = 6$ km, and $y = 8$ km

$R = \sqrt{6^2 + 8^2 + 6\times 8} = 2\sqrt{37}$

Differentiating equation (i) with respect to t

$$2R\frac{dR}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + \left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$$

$$= \frac{1}{2R}[2\times 8\times 20 + 2\times 6\times 30 + (8\times 30 + 6\times 20)]$$

$$\frac{dR}{dt} = \frac{1}{2\times 2\sqrt{37}}[1040] = \frac{260}{\sqrt{37}}$$

10. **(a)** Volume of sphere $V = \dfrac{4}{3}\pi r^3$

$\dfrac{dV}{dt} = \dfrac{4}{3}.\pi.3r^2.\dfrac{dr}{dt}$

$35 = 4\pi r^2.\dfrac{dr}{dt}$ or $\dfrac{dr}{dt} = \dfrac{35}{4\pi r^2}$ ...(i)

Surface area of sphere = S = $4\pi r^2$

$\dfrac{dS}{dt} = 4\pi\times 2r\times\dfrac{dr}{dt} = 8\pi r.\dfrac{dr}{dt}$

$\dfrac{dS}{dt} = \dfrac{70}{r}$ (By using (i))

Now, diameter = 14 cm, $r = 7$

$\therefore \ \dfrac{dS}{dt} = 10$

11. **(d)** $V = \dfrac{4}{3}\pi r^3 \Rightarrow \dfrac{dV}{dt} = 4\pi r^2.\dfrac{dr}{dt}$ ...(i)

$S = 4\pi r^2 \Rightarrow \dfrac{dS}{dt} = 8\pi r.\dfrac{dr}{dt}$

$\Rightarrow 8 = 8\pi r\dfrac{dr}{dt} \Rightarrow \dfrac{dr}{dt} = \dfrac{1}{\pi r}$

Putting the value of $\frac{dr}{dt}$ in (i), we get

$$\frac{dV}{dt} = 4\pi r^2 \times \frac{1}{\pi r} = 4r$$

$\Rightarrow \frac{dV}{dt}$ is proportional to r.

12. **(c)** Volume of spherical balloon = $V = \frac{4}{3}\pi r^3$

Differentiate both the side, w.r.t 't' we get,

$$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right) \quad ...(i)$$

$\therefore$ After 49 min,

Volume = $(4500 - 49 \times 72)\pi = (4500 - 3528)\pi = 972\,\pi\,m^3$

$\Rightarrow V = 972\,\pi\,m^3$

$\therefore \quad 972\pi = \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = 3 \times 243 = 3 \times 3^5 = 3^6 = (3^2)^3$

$\Rightarrow r = 9$

Given $\frac{dV}{dt} = 72\pi$

Putting $\frac{dV}{dt} = 72\pi$ and $r = 9$, we get

$\therefore \quad 72\pi = 4\pi \times 9 \times 9\left(\frac{dr}{dt}\right)$

$\Rightarrow \frac{dr}{dt} = \left(\frac{2}{9}\right)$

13. **(b)** Let $A = \pi r^2$ be area of metalic circular plate of $r = 50$ cm.

Also, given $\frac{dr}{dt} = 1\text{mm} = \frac{1}{10}\text{cm}$

$\therefore A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi.50.\frac{1}{10} = 10\pi$

Hence, area of plate increases in 10π cm^2/hour.

14. **(c)** Let W = nw

$\Rightarrow \frac{dW}{dt} = n\frac{dw}{dt} + w.\frac{dn}{dt} \quad ...(i)$

Given : $w = t^2 - t + 2$ and $n = 2t^2 + 3$

$\Rightarrow \frac{dw}{dt} = 2t - 1$ and $\frac{dn}{dt} = 4t$

$\therefore$ Equation (i)

$\Rightarrow \frac{dw}{dt} = (2t^2 + 3)(2t - 1) + (t^2 - t + 2)(4t)$

Thus, $\left.\frac{dW}{dt}\right|_{t=1} = (2 + 3)(2 - 1) + (2)(4)$

$= 5(1) + 8 = 13$

15. **(d)** Let A be the area, b be the breadth and ℓ be the length of the rectangle.

Given : $\frac{dA}{dt} = -5$, $\frac{d\ell}{dt} = 2$, $\frac{db}{dt} = -3$

We know, $A = \ell \times b$

$\Rightarrow \frac{dA}{dt} = \ell.\frac{db}{dt} + b.\frac{d\ell}{dt} = -3\ell + 2b$

$\Rightarrow -5 = -3\ell + 2b$.

When $b = 2$, we have

$-5 = -3\ell + 4 \Rightarrow \ell = \frac{9}{3} = 3m$

16. **(b)** Let $A = \pi r^2$.

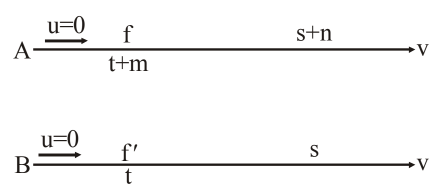
$\frac{dA}{dt} = 2\pi r.\frac{dr}{dt}$

$6\pi = 2\pi(30).\frac{dr}{dt}$

$\Rightarrow \frac{3}{30} = \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{10} = 0.1$

Thus, the rate at which the radius of the circular sheet increases is 0.1

17. **(d)**



As per question if point B moves s distance in t time then point A moves $(s + n)$ distance in time $(t + m)$ after which both have same velocity v.

Then using equation $v = u + at$ we get

$$v = f(t+m) = f't \Rightarrow t = \frac{fm}{f'-f} \quad ....(i)$$

Using equation $v^2 = u^2 + 2$, as we get

$$v^2 = 2f(s+n) = 2f's \quad \Rightarrow s = \frac{fn}{f'-f} \quad ....(ii)$$

Also for point B using the eqn $s = ut + \frac{1}{2}at^2$, we get

$$s = \frac{1}{2}f't^2$$

Substituting values of t and s from equations (i) and (ii) in the above relation, we get

$$\frac{fn}{f'-f} = \frac{1}{2}f'\frac{f^2m^2}{(f'-f)^2}$$

$$\Rightarrow (f'-f)n = \frac{1}{2}ff'm^2$$

18. **(c)** Let the lizard catches the insect after time t then distance covered by lizard = 21cm + distance covered by insect

$$\Rightarrow \frac{1}{2}ft^2 = 4\times t + 21$$

$$\Rightarrow \frac{1}{2}\times 2\times t^2 = 20\times t + 21$$

$$\Rightarrow t^2 - 20t - 21 = 0 \Rightarrow t = 21\,\text{sec}$$

19. **(b)** Given that

Total radius $r = 10 + 5 = 15$ cm

$$\frac{dv}{dt} = 50\ \text{cm}^3/\text{min} \Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 50$$

$$\Rightarrow 4\pi r^2\frac{dr}{dt} = 50$$

$$\Rightarrow \frac{dr}{dt} = \frac{50}{4\pi(15)^2} = \frac{1}{18\pi}\,\text{cm/min}$$

20. **(a)** Given $y^2 = 18x \Rightarrow 2y\frac{dy}{dx} = 18 \Rightarrow \frac{dy}{dx} = \frac{9}{y}$

ATQ $\frac{dy}{dt} = \frac{2dx}{dt} \Rightarrow \frac{dy}{dx} = 2$

$$\Rightarrow \frac{9}{y} = 2 \Rightarrow y = \frac{9}{2}$$

Putting in $y^2 = 18x \Rightarrow x = \frac{9}{8}$

$\therefore$ Required point is $\left(\frac{9}{8},\frac{9}{2}\right)$

21. **(a)** $f(x) = (3x-7)\cdot x^{2/3}$

$$f'(x) = 3x^{2/3} + (3x-7)\cdot\frac{2}{3}x^{-1/3}$$

$$= \frac{15x-14}{3x^{1/3}}$$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & 0 & & \frac{14}{15} & \end{array}$$

For increasing function

$f'(x) > 0$ then $x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

22. **(d)** Since, function $f\ (x)$ is twice differentiable and continuous in $x \in [a, b]$. Then, by LMVT for $x \in [a, c]$

$$\frac{f(c)-f(a)}{c-a} = f'(\alpha), \alpha \in (a,c)$$

Again by LMVT for $x \in [c, b]$

$$\frac{f(b)-f(c)}{b-c} = f'(\beta), \beta \in (c,b)$$

$\because f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$$f'(\alpha) > f'(\beta) \Rightarrow \frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c} \quad (\because f\ (x) \text{ is increasing})$$

23. **(d)** $f'(x) = x\,(\pi - \cos^{-1}(\sin|x|))$

$$= x\left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right) = x\left(\frac{\pi}{2} + |x|\right)$$

$$f(x) = \begin{cases} x\left(\frac{\pi}{2}+x\right), & x \ge 0 \\ x\left(\frac{\pi}{2}-x\right), & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{\pi}{2}+2x, & x \ge 0 \\ \frac{\pi}{2}-2x, & x < 0 \end{cases}$$

Hence, $f'(x)$ is increasing in $\left(0,\frac{\pi}{2}\right)$ and decreasing in $\left(\frac{-\pi}{2},0\right)$.

24. **(b)** Given functions are, $f(x) = e^x - x$ and $g(x) = x^2 - x$

$$f(g(x)) = e^{(x^2-x)} - (x^2-x)$$

Given $f(g(x))$ is increasing function.

$$\therefore (f(g(x)))' = e^{(x^2-x)}\times(2x-1) - 2x + 1$$

$$= (2x-1)e^{(x^2-x)} + 1 - 2x = (2x-1)[e^{(x^2-x)} - 1] \ge 0$$

For $(f(g(x)))' \ge 0$,

$(2x-1)\ \&\ [e^{(x^2-x)} - 1]$ are either both positive or negative

$$\begin{array}{ccccccc} - & & +\text{ve} & & -\text{ve} & & + \\ \hline & 0 & & \frac{1}{2} & & 1 & \end{array}$$

$$x \in \left[0,\frac{1}{2}\right] \cup [1,\infty)$$

25. **(c)** $f(x)=\dfrac{x^2}{1-x^2}$

$\Rightarrow f(-x)=\dfrac{x^2}{1-x^2}=f(x)$

$f'(-x)=\dfrac{2x}{(1-x^2)^2}$

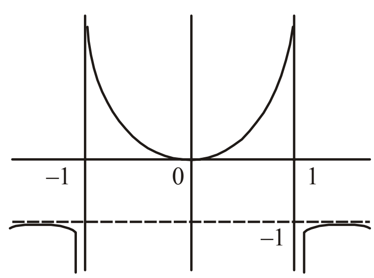
f(x) increases in x $\in (10, \infty)$

Also f(0) = 0 and

$\lim\limits_{x\to\pm\infty} f(x)=-1$ and f(x) is even function

Set A = R – [–1, 0)

And the graph of function f(x) is



26. **(c)** $f(x)=f(x)+f(2-x)$

Now, differentiate w.r.t. x,

$f'(x)=f'(x)-f'(2-x)$

For $f(x)$ to be increasing $f'(x)>0$

$\Rightarrow \quad f'(x)-f'(2-x)>0$

$\Rightarrow \quad f'(x)>f'(2-x)$

But $f''(x)>0 \Rightarrow f'(x)$ is an increasing function

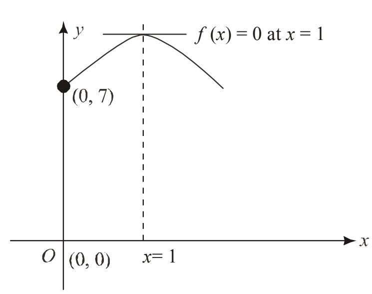
Then, $f'(x)>f'(2-x)>0$

$\Rightarrow \quad x>2-x$

$\Rightarrow \quad x>1$

Hence, $f(x)$ is increasing on (1, 2) and decreasing on (0, 1).

27. **(c)** $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7, f(0) = 7$



$\Rightarrow \quad f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(1) = 0$

$\Rightarrow \quad 1 - 2a + 4 + a = 0$

$\Rightarrow \quad a = 5$

Then, $f(x) = x^3 - 9x^2 + 15x + 7$

Now,

$\dfrac{f(x)-14}{(x-1)^2}=0$

$\Rightarrow \quad \dfrac{x^3-9x^2+15x+7-14}{(x-1)^2}=0$

$\Rightarrow \quad \dfrac{(x-1)^2(x-7)}{(x-1)^2}=0 \Rightarrow x=7$

28. **(a)** $f(x)=\dfrac{x}{\sqrt{a^2+x^2}}-\dfrac{(d-x)}{\sqrt{b^2+(d-x)^2}}$

$=\dfrac{x}{\sqrt{a^2+x^2}}+\dfrac{(x-d)}{\sqrt{b^2+(x-d)^2}}$

$$f'(x)=\frac{\sqrt{a^2+x^2}-\dfrac{x(2x)}{2\sqrt{a^2+x^2}}}{(a^2+x^2)}+\frac{\sqrt{b^2+(x-d)^2}-\dfrac{(x-d)2(x-d)}{2\sqrt{b^2+(x-d)^2}}}{\left(b^2+(x-d)^2\right)}$$

$$=\frac{a^2+x^2-x^2}{(a^2+x^2)^{3/2}}+\frac{b^2+(x-d)^2-(x-d)^2}{\left(b^2+(x-d)^2\right)^{3/2}}$$

$$=\frac{a^2}{\left(a^2+x^2\right)^{3/2}}+\frac{b^2}{\left(b^2+(x-d)^2\right)^{3/2}}>0$$

$\Rightarrow \quad f'(x)>0, \forall x \in R$

$\Rightarrow \quad f(x)$ is increasing function.

Hence, $f(x)$ is increasing function.

29. **(a)** $f(x)=x^3-3x^2+5x+7$

For increasing

$f'(x)=3x^2-6x+5>0$

$\Rightarrow x\in R$

For decreasing

$f'(x)=3x^2-6x+5<0$

30. **(c)** $f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x \cos x + 4\cos^3 x(-\sin x)$

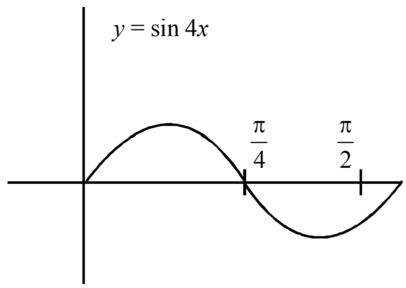
$= 4\sin x \cos x (\sin^2 x - \cos^2 x)$

$= -2\sin 2x \cos 2x = -\sin 4x$

$f(x)$ is increasing when $f'(x) > 0$

$\Rightarrow\ -\sin 4x > 0 \Rightarrow\ \sin 4x < 0$

$\Rightarrow\ x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right]$



31. **(b)** Since $f'(x) > 0$ and $g'(x) < 0$, therefore $f(x)$ is increasing function and $g(x)$ is decreasing function.

$\Rightarrow f(x+1) > f(x)$ and $g(x+1) < g(x)$

$\Rightarrow g[f(x+1)] < g[f(x)]$ and $f[g(x+1)] < f[g(x)]$

Hence option (b) is correct.

32. **(d)** $f(x) = 2x^3 + 3x + k$

$f'(x) = 6x^2 + 3 > 0\ \forall x \in R \quad (\because\ x^2 > 0)$

$\Rightarrow\ f(x)$ is strictly increasing function

$\Rightarrow\ f(x) = 0$ has only one real root, so two roots are not possible.

33. **(c)** Let $y = x^2 \cdot e^{-x}$

For increasing function,

$\frac{dy}{dx} > 0 \Rightarrow x[(2-x)e^{-x}] > 0$

$\because\ x > 0,\ \therefore\ (2-x)e^{-x} > 0$

$\Rightarrow (2-x)\frac{1}{e^x} > 0$

For $0 < x < 2$, $(2-x) < 0$

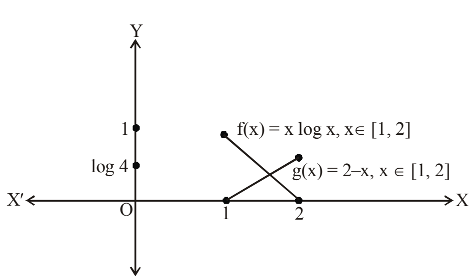
$\therefore\ \frac{1}{e^x} < 0$, but it is not possible

Hence the statement-2 is false.

34. **(a)** $f(x) = x \log x, f(1) = 0, f(2) = 4$

$g(x) = 2 - x, g(1) = 1, g(2) = 0$

$\log 10 > \log 4 \Rightarrow 1 > \log 4$



Thus statement -1 and 2 both are true and statement-2 is a correct explanation of statement 1.

35. **(c)** $f(x) = xe^{x(1-x)}, x \in R$

$f'(x) = e^{x(1-x)}.\left[1 + x - 2x^2\right]$

$= -e^{x(1-x)}.\left[2x^2 - x - 1\right]$

$= -2e^{x(1-x)}.\left[\left(x + \frac{1}{2}\right)(x-1)\right]$

$f'(x) = -2e^{x(1-x)}.A$

where $A = \left(x + \frac{1}{2}\right)(x-1)$

Now, exponential function is always +ve and $f'(x)$ will be opposite to the sign of A which is –ve in $\left[-\frac{1}{2}, 1\right]$

Hence, $f'(x)$ is +ve in $\left[-\frac{1}{2}, 1\right]$

$\therefore\ f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$

36. **(b)** Given that $f(x) = x^3 + 5x + 1$

$\therefore\ f'(x) = 3x^2 + 5 > 0,\ \forall x \in R$

$\Rightarrow f(x)$ is strictly increasing on R

$\Rightarrow f(x)$ is one one

$\therefore$ Being a polynomial $f(x)$ is continuous and increasing.

on R with $\lim\limits_{x\to\infty} f(x) = -\infty$

and $\lim\limits_{x\to\infty} f(x) = \infty$

$\therefore$ Range of $f = (-\infty, \infty) = R$

Hence f is onto also. So, f is one one and onto R.

37. **(b)** Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$\Rightarrow f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \forall\ x \in R$ ...(i)

$\Rightarrow f$ is an increasing function on R

Also $\lim\limits_{x\to\infty} f(x) = \infty$ and $\lim\limits_{x\to-\infty} f(x) = -\infty$ ...(ii)

From (i) and (ii) clear that the curve $y = f(x)$ crosses x-axis only once.

$\therefore f(x) = 0$ has exactly one real root.

38. **(d)** Given that $f(x) = \tan^{-1}(\sin x + \cos x)$

Differentiate w.r. to x

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}.(\cos x - \sin x)$$

$$= \frac{\sqrt{2}.\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}{1+(\sin x+\cos x)^2}$$

$$= \frac{\sqrt{2}\left(\cos\frac{\pi}{4}.\cos x - \sin\frac{\pi}{4}.\sin x\right)}{1+(\sin x+\cos x)^2}$$

$$\therefore \quad f'(x) = \frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{1+(\sin x+\cos x)^2}$$

Given that $f(x)$ is increasing

$\therefore f'(x) > 0 \Rightarrow \cos\left(x+\frac{\pi}{4}\right) > 0$

$\Rightarrow -\frac{\pi}{2} < x+\frac{\pi}{4} < \frac{\pi}{2}$

$\Rightarrow -\frac{3\pi}{4} < x < \frac{\pi}{4}$

Hence, $f(x)$ is increasing when

$n \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

39. **(c)** From option (c), $f(x) = 3x^2 - 2x + 1$ is increasing when $f'(x) = 6x - 2 \geq 0$

$\Rightarrow \quad x \in [1/3, \infty)$

$\therefore f(x)$ is incorrectly matched with $\left(-\infty, \frac{1}{3}\right]$

40. **(b)** The given tangent to the curve is,

$y = x\log_e x \qquad (x > 0)$

$\Rightarrow \frac{dy}{dx} = 1+\log_e x$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=c} = 1+\log_e c \qquad$ (slope)

$\because$ The tangent is parallel to line joining (1, 0), (e, e)

$\therefore 1+\log_e c = \frac{e-0}{e-1}$

$\Rightarrow \log_e c = \frac{e}{e-1} - 1 \Rightarrow \log_e c = \frac{1}{e-1}$

$\Rightarrow c = e^{\frac{1}{e-1}}$

41. **(c)** The given curve is, $x^4 \cdot e^y + 2\sqrt{y+1} = 3$

Differentiating w.r.t. x, we get

$(4x^3 + x^4 \cdot y')e^y + \frac{y'}{\sqrt{1+y}} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{-4x^3e^y}{\left(\frac{1}{\sqrt{y+1}} + e^y x^4\right)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,0)} = -2$

$\therefore$ Equation of tangent;

$y - 0 = -2(x-1) \Rightarrow 2x + y = 2$

Only point (–2, 6) lies on the tangent.

42. **(0.50)**

The given curve $y = (x-1)(x-2)$, intersects the x-axis at $A(1, 0)$ and $B(2, 0)$.

$\therefore \frac{dy}{dx} = 2x - 3; \left(\frac{dy}{dx}\right)_{(x=1)} = -1$ and $\left(\frac{dy}{dx}\right)_{(x=2)} = 1$

Equation of tangent at $A(1, 0)$,

$y = -1(x-1) \Rightarrow x + y = 1$

Equation of tangent at $B(2, 0)$,

$y = 1(x-2) \Rightarrow x - y = 2$

So $a = 1$ and $b = 2$

$\Rightarrow \frac{a}{b} = \frac{1}{2} = 0.5.$

43. **(4)**

For (1, 2) of $y^2 = 4x \Rightarrow t = 1, a = 1$

Equation of normal to the parabola

$\Rightarrow tx + y = 2at + at^3$

$\Rightarrow x + y = 3$ intersect x-axis at (3, 0)

$y = e^x \Rightarrow \frac{dy}{dx} = e^x$

Equation of tangent to the curve

$\Rightarrow y - e^c = e^c(x-c)$

$\because$ Tangent to the curve and normal to the parabola intersect at same point.

$\therefore 0 - e^c = e^c(3-c) \Rightarrow c = 4.$

44. **(91)**

$$y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$$

Let $\cos a = \frac{3}{5}$ and $\sin a = \frac{4}{5}$

$$\therefore y = \sum_{k=1}^{6} k\cos^{-1}\{\cos a\cos kx - \sin a\sin kx\}$$

$$= \sum_{k=1}^{6} k\cos^{-1}(\cos(kx+a))$$

$$= \sum_{k=1}^{6} k(kx+a) = \sum_{k=1}^{6}(k^2x+ak)$$

$$\therefore \frac{dy}{dx} = \sum_{k=1}^{6} k^2 = \frac{6(7)(13)}{6} = 91.$$

45. **(4.0)** $P \equiv (x_1, y_1)$

$2yy' - 6x + y' = 0$

$$\Rightarrow \quad y' = \left(\frac{6x_1}{1+2y_1}\right)$$

$$\left(\frac{\frac{3}{2}-y_1}{-x_1}\right) = -\left(\frac{1+2y_1}{6x_1}\right)$$

[By point slope form, $y - y_1 = m(x - x_1)$]

$\Rightarrow \quad 9 - 6y_1 = 1 + 2y_1$

$\Rightarrow \quad y_1 = 1$

$\therefore \quad x_1 = \pm 2$

$\therefore \quad$ Slope of tangent $(m) = \left(\frac{\pm 12}{3}\right) = \pm 4$

$\therefore \quad |m| = 4$

46. **(d)** Given equation of curve is

$x^2 + 2xy - 3y^2 = 0$

$\Rightarrow \quad 2x + 2y + 2xy' - 6yy' = 0$

$\Rightarrow \quad x + y + xy' - 3yy' = 0$

$\Rightarrow \quad y'(x - 3y) = -(x + y)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y}{3y-x}$$

Slope of normal $= \frac{-dx}{dy} = \frac{x-3y}{x-3y}$

Normal at point (2, 2) $= \frac{2-6}{2+2} = -1$

Equation of normal to curve $= y - 2 = -1\ (x - 2)$

$\Rightarrow \quad x + y = 4$

$\therefore \quad$ Perpendicular distance from origin

$$= \left|\frac{0+0-4}{\sqrt{2}}\right| = 2\sqrt{2}$$

47 **(a)** Given curve is, $y = \frac{x}{x^2-3}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2-3)-x(2x)}{(x^2-3)^2} = \frac{-x^2-3}{(x^2-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{\alpha^2-3}{(\alpha^2-3)^2} = -\frac{2}{6} = -\frac{1}{3}$$

$3(\alpha^2+3) = (\alpha^2-3)^2 \Rightarrow \alpha^2 = 9$

And, $\beta = \frac{x}{a^2-3} \quad \Rightarrow \alpha^2 - 3 = \frac{\alpha}{\beta} \Rightarrow \frac{\alpha}{\beta} = 6$

$$\Rightarrow a = \pm 3, \beta = \pm\frac{1}{2}$$

These values of α and β satisfies $|6\alpha + 2\beta| = 19$

48. **(d)** $y = x^3 + ax - b$

Since, the point (1, –5) lies on the curve.

$\Rightarrow 1 + a - b = -5$

$\Rightarrow a - b = -6$ ...(i)

$$\frac{dy}{dx} = 3x^2 + a$$

$$\left(\frac{dy}{dx}\right)_{at\ x=1} = 3 + a$$

Since, required line is perpendicular to $y = x - 4$, then slope of tangent at the point P (1, –5) = –1

$3 + a = -1$

$a = -4$

$b = 2$

the equation of the curve is $y = x^3 - 4x - 2$

(2, –2) lies on the curve

49. **(d)** $y = f(x) = x^3 - x^2 - 2x$

$$\frac{dy}{dx} - 3x^2 - 2x - 2$$

$f(1) = 1 - 1 - 2 = -2, \qquad f(-1) = -1 - 1 + 2 = 0$

Since the tangent to the curve is parallel to the line segment joining the points (1, –2) (–1, 0)

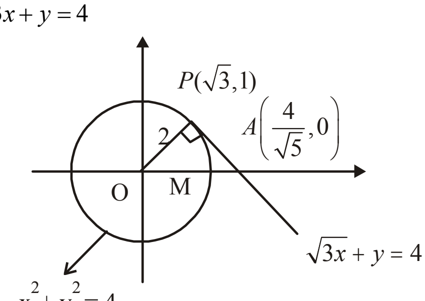
Since their slopes are equal

$$\Rightarrow 3x^2 - 2x - 2 = \frac{-2-0}{2} \qquad \Rightarrow x = 1, \frac{-1}{3}$$

Hence, the required set $S = \left\{\frac{-1}{3}, 1\right\}$

50. **(c)** Equation of tangent to circle at point $(\sqrt{3}, 1)$ is

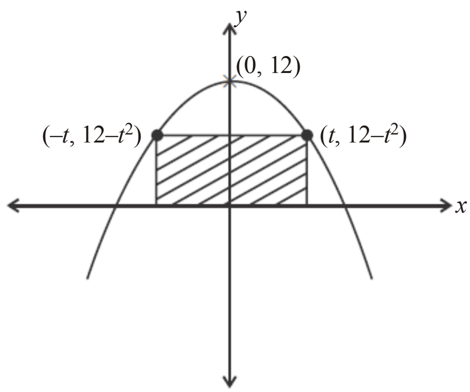
$\sqrt{3}x + y = 4$



coordinates of the point $A = \left(\frac{4}{\sqrt{3}}, 0\right)$

Area = $\frac{1}{2} \times OA \times PM = \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}}$ sq. units

51. **(c)** Given, the equation of parabola is,
$x^2 = 12 - y$

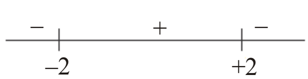


Area of the rectangle = $(2t)(12 - t^2)$
$A = 24t - 2t^3$

$\frac{dA}{dt} = 24 - 6t^2$

Put $\frac{dA}{dt} = 0 \Rightarrow 24 - 6t^2 = 0$

$\Rightarrow \quad t = \pm 2$



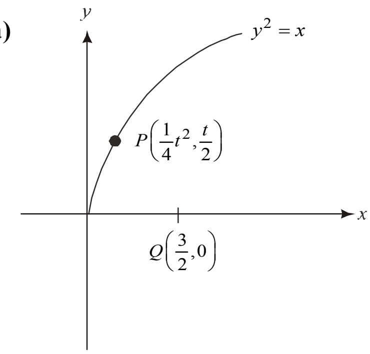
At $t = 2$, area is maximum $= 24(2) - 2(2)^3$
$= 48 - 16 = 32$ sq. units

52. **(b)** $\because$ Tangent to the given curve is parallel to line $2y = 4x + 1$
$\therefore$ Slope of tangent (m) = 2
Then, the equation of tangent will be of the form
$y = 2x + c$ ...(i)
$\because$ Line (i) and curve $y = x^2 - 5x + 5$ has only one point of intersection.
$\therefore \quad 2x + c = x^2 - 5x + 5$
$x^2 - 7x + (5 - c) = 0$
$\therefore \quad D = 49 - 4(5 - c) = 0$

$\Rightarrow \quad c = -\frac{29}{4}$

Hence, the equation of tangent: $y = 2x - \frac{29}{4}$

53. **(a)**



Here the curve is parabola with $a = \frac{1}{4}$.

Let P$(at^2, 2at)$ or $P\left(\frac{t^2}{4}, \frac{t}{2}\right)$ be a point on the curve.

Now, $y^2 = x$

$\Rightarrow 2y\frac{dy}{dx} = 1 = \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{at\ p} = \frac{1}{t}$

$\therefore$ equation of normal at P to $y^2 = x$ is,

$\left(y - \frac{t}{2}\right) = -t\left(x - \frac{1}{4}t^2\right)$

$\Rightarrow y = -tx + \frac{1}{2}t + \frac{1}{4}t^3$ ...(i)

For minimum PQ, (i) passes through $Q\left(\frac{3}{2}, 0\right)$

$\frac{-3}{2}t + \frac{t}{2} + \frac{t^3}{4} = 0 \Rightarrow -4t + t^3 = 0$

$\Rightarrow \quad t(t^2 - 4) = 0 \Rightarrow t = -2, 0, 2$

$\because \quad t \geq 0 \Rightarrow t = 0, 2$

If $t = 0$, $P(0, 0) \Rightarrow AP = \frac{3}{2}$

If $t = 2$, $P(1, 1) \Rightarrow AP = \frac{\sqrt{5}}{2}$

Shortest distance $\left(\frac{3}{2}, 0\right)$ and $y = \sqrt{x}$ is $\frac{\sqrt{5}}{2}$

54. **(b)** The equation of curve $y = xe^{x^2}$

$\Rightarrow \quad \frac{dy}{dx} = e^{x^2}.1 + x.e^{x^2}.2x$

Since (1, e) lies on the curve $y = xe^{x^2}$, then equation of tangent at (1, e) is

$$y - e = \left(e^{x^2}(1+2x^2)\right)_{x=1}(x-1)$$

$y - e = 3e(x - 1)$

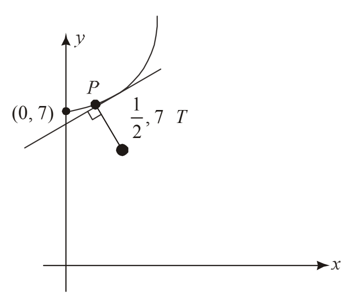
$3ex - y = 2e$

So, equation of tangent to the curve passes through the point $\left(\frac{4}{3}, 2e\right)$

55. **(c)** $f(x) = y = x^{3/2} + 7$

$\Rightarrow \frac{dy}{dx} \Rightarrow \frac{3}{2}\sqrt{x} > 0$

$\Rightarrow$ $f(x)$ is increasing function $\forall\ x > 0$



Let $P\left(x_1, x_1^{3/2}+7\right)$

$m_{TP} = m_{\text{at P}} = -1$

$$\Rightarrow \left(\frac{x_1^{3/2}}{x_1 - \frac{1}{2}}\right) \times \frac{3}{2}x_1^{\frac{1}{2}} = -1$$

$$\Rightarrow -\frac{2}{3} = \frac{x_1^2}{x_1 - \frac{1}{2}}$$

$\Rightarrow -3x_1^2 = 2x_1 - 1 \Rightarrow 3x_1^2 + 2x_1 - 1 = 0$

$\Rightarrow 3x_1^2 + 3x_1 - x_1 - 1 = 0$

$\Rightarrow 3x_1(x_1 + 1) - 1(x_1 + 1) = 0$

$\Rightarrow x_1 = \frac{1}{3}$ $\qquad (\because x_1 > 0)$

$$\Rightarrow P\left(\frac{1}{3}, 7 + \frac{1}{3\sqrt{3}}\right)$$

$$TP = \sqrt{\frac{1}{27} + \frac{1}{36}} = \frac{1}{6}\sqrt{\frac{7}{3}}$$

56. **(b)** Since, the equation of curves are

$y = 10 - x^2$ ...(i)

$y = 2 + x^2$ ...(ii)

Adding eqn (i) and (ii), we get

$2y = 12 \Rightarrow y = 6$

Then, from eqn (i)

$x = \pm 2$

Differentiate equation (i) with respect to x

$$\frac{dy}{dx} = -2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = -4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = 4$$

Differentiate equation (ii) with respect to x

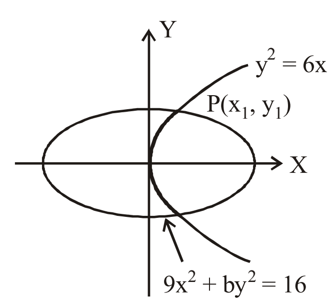
$$\frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = 4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = -4$$

$$\text{At } (2, 6) \tan\theta = \left(\frac{(-4)-(4)}{1+(-4)\times(4)}\right) = \frac{8}{15}$$

$$\text{At } (-2, 6), \tan\theta = \frac{(4)-(-4)}{1+(4)(-4)} = \frac{8}{-15} \Rightarrow |\tan\theta| = \frac{8}{15}$$

$$\therefore |\tan\theta| = \frac{8}{15}$$

57. **(c)** Let curve intersect each other at point P(x_1, y_1)



Since, point of intersection is on both the curves, then

$y_1^2 = 6x_1$ ...(i)

and $9x_1^2 + by_1^2 = 16$ ...(ii)

Now, find the slope of tangent to both the curves at the point of intersection P(x_1, y_1)

For slope of curves:

Curve (i):

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = m_1 = \frac{3}{y_1}$$

Curve (ii):

and $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = m_2 = -\frac{9x_1}{by_1}$

Since, both the curves intersect each other at right angle then,

$m_1m_2 = -1 \Rightarrow \frac{27x_1}{by_1^2} = 1 \Rightarrow b = 27\frac{x_1}{y_1^2}$

$\therefore$ from equation (i), $b = 27 \times \frac{1}{6} = \frac{9}{2}$

58. **(c)** Let $P(2t, t^2)$ be any point on the parabola.
Centre of the given circle $C = (-g, -f) = (-3, 0)$
For PC to be minimum, it must be the normal to the parabola at P.

Slope of line $PC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{t^2 - 0}{2t + 3}$

Also, slope of tangent to parabola at $P = \frac{dy}{dx} = \frac{x}{2} = t$

$\therefore$ Slope of normal $= \frac{-1}{t}$

$\therefore \frac{t^2 - 0}{2t + 3} = \frac{-1}{t}$

$\Rightarrow t^3 + 2t + 3 = 0$

$\Rightarrow (t + 1)(t^2 - t + 3) = 0$

$\therefore$ Real roots of above equation is
$t = -1$

Coordinate of $P = (2t, t^2) = (-2, 1)$
Slope of tangent to parabola at $P = t = -1$
Therefore, equation of tangent is:
$(y - 1) = (-1)(x + 2)$
$\Rightarrow x + y + 1 = 0$

59. **(d)** Equation of hyperbola is :
$4y^2 = x^2 + 1$

$\Rightarrow -x^2 + 4y^2 = 1$

$\Rightarrow -\frac{x^2}{1^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$

$\therefore a = 1, b = \frac{1}{2}$

Now, tangent to the curve at point (x_1, y_1) is given by

$4 \times 2y_1\frac{dy}{dx} = 2x_1$

$\Rightarrow \frac{dy}{dx} = \frac{2x_1}{8y_1} = \frac{x_1}{4y_1}$

Equation of tangent at (x_1, y_1) is
$y = mx + c$

$\Rightarrow y = \frac{x_1}{4y_1} \cdot x + c$

As tangent passes through (x_1, y_1)

$\therefore y_1 = \frac{x_1 x_1}{4y_1} + c$

$\Rightarrow C = \frac{4y_1^2 - x_1^2}{4y_1} = \frac{1}{4y_1}$

Therefore, $y = \frac{x_1}{4y_1}x + \frac{1}{4y_1} \quad \Rightarrow 4y_1y = x_1x + 1$

which intersects x axis at $A\left(\frac{-1}{x_1}, 0\right)$ and y axis at $B\left(0, \frac{1}{4y_1}\right)$

Let midpoint of AB is (h, k)

$\therefore h = \frac{-1}{2x_1}$

$\Rightarrow x_1 = \frac{-1}{2h}$ & $y_1 = \frac{1}{8k}$

Thus, $4\left(\frac{1}{8k}\right)^2 = \left(\frac{-1}{2h}\right)^2 + 1$

$\Rightarrow \frac{1}{16k^2} = \frac{1}{4h^2} + 1$

$\Rightarrow 1 = \frac{16k^2}{4h^2} + 16k^2$

$\Rightarrow h^2 = 4k^2 + 16h^2k$.

So, required equation is
$x^2 - 4y^2 - 16x^2y^2 = 0$

60. **(b)** Since, $x^2 + 3y^2 = 9$

$\Rightarrow 2x + 6y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{3y}$

Slope of normal is $-\frac{dx}{dy} = \frac{3y}{x}$

$$\Rightarrow \left(-\frac{dx}{dy}\right)_{(3\cos\theta, \sqrt{3}\sin\theta)} = \frac{3\sqrt{3}\sin\theta}{3\cos\theta} = \sqrt{3}\tan\theta = m_1$$

$$\& \left(-\frac{dx}{dy}\right)_{(-3\sin\theta, \sqrt{3}\cos\theta)}$$

$$= \frac{3\sqrt{3}\cos\theta}{-3\sin\theta} = -\sqrt{3}\cot\theta = m_2$$

As, β is the anagle between the normals to the given ellipse then

$$\tan\beta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1 - 3\tan\theta\cot\theta}\right| = \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1-3}\right|$$

So, $\tan\beta = \frac{\sqrt{3}}{2}|\tan\theta + \cot\theta|$

$$\Rightarrow \frac{1}{\cot\beta} = \frac{\sqrt{3}}{2}\left|\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right|$$

$$\Rightarrow \frac{1}{\cot\beta} = \frac{\sqrt{3}}{2}\left|\frac{1}{\sin\theta\cos\theta}\right|$$

$$\Rightarrow \frac{1}{\cot\beta} = \frac{\sqrt{3}}{\sin 2\theta} \Rightarrow \frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$$

61. **(c)** Given, $4x^2 - 9y^2 = 36$

After differentiating w.r.t. x, we get

$$4.2.x - 9.2.y.\frac{dy}{dx} = 0$$

$\Rightarrow$ Slope of tangent $= \frac{dy}{dx} = \frac{4x}{9y}$

So, slope of normal $= \frac{-9y}{4x}$

Now, equation of normal at point (x_0, y_0) is given by

$$y - y_0 = \frac{-9y_0}{4x_0}(x - x_0)$$

As normal intersects X axis at A, Then

$A \equiv \left(\frac{13x_0}{9}, 0\right)$ and $B \equiv \left(0, \frac{13y_0}{4}\right)$

As $OABP$ is a parallelogram

$\therefore$ midpoint of $OB \equiv \left(0, \frac{13y_0}{8}\right) \equiv$ Midpoint of AP

So, $P(x, y) \equiv \left(\frac{-13x_0}{9}, \frac{13y_0}{4}\right)$ ...(i)

$\because (x_0, y_0)$ lies on hyperbola, therefore

$4(x_0)^2 - 9(y_0)^2 = 36$ ...(ii)

From equation (i): $x_0 = \frac{-9x}{13}$ and $y_0 = \frac{4y}{13}$

From equation (ii), we get

$9x^2 - 4y^2 = 169$

Hence, locus of point P is : $9x^2 - 4y^2 = 169$

62. **(c)** We have $y = \frac{x+6}{(x-2)(x-3)}$

At y-axis, $x = 0 \Rightarrow y = 1$

On differentiating, we get

$$\frac{dy}{dx} = \frac{(x^2 - 5x + 6)(1) - (x+6)(2x-5)}{(x^2 - 5x + 6)^2}$$

$\frac{dy}{dx} = 1$ at point $(0, 1)$

$\therefore$ Slope of normal $= -1$

Now equation of normal is $y - 1 = -1(x - 0)$

$\Rightarrow y - 1 = -x$

$x + y = 1$

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right)$ satisfy it.

63. **(c)** Eccentricity of ellipse $= \frac{1}{2}$

Now, $-\frac{a}{e} = -4 \Rightarrow a = 4 \times \frac{1}{2} = 2 \Rightarrow a = 2$

We have $b^2 = a^2(1 - e^2) = a^2\left(1 - \frac{1}{4}\right)$

$= 4 \times \frac{3}{4} = 3$

$\therefore$ Equation of ellipse is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Now differentiating, we get

$$\Rightarrow \frac{x}{2} + \frac{2y}{3} \times y' = 0 \Rightarrow y' = -\frac{3x}{4y}$$

$$y'\Big|_{(1,3/2)} = -\frac{3}{4} \times \frac{2}{3} = -\frac{1}{2}$$

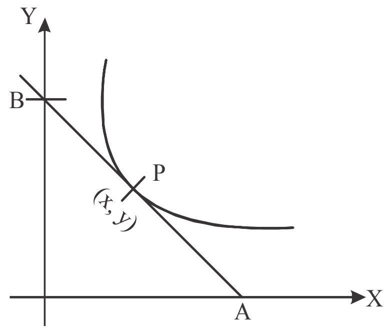
Slope of normal = 2

$\therefore$ Equation of normal at $\left(1,\frac{3}{2}\right)$ is

$y-\frac{3}{2}=2(x-1) \Rightarrow 2y-3=4x-4$

$\therefore \quad 4x-2y=1$

64. **(c)**



Let y = f (x) be a curve
slope of tangent = f ′ (x)
Equation of tangent (Y – y) = f ′ (x) (X – x)
Put Y = 0

$\Rightarrow \quad X=\left(x-\frac{y}{f'(x)}\right)$

Put X = 0

$\Rightarrow \quad Y=y-x f'(x)$

$\Rightarrow \quad A=\left(x-\frac{y}{f'(x)},0\right)$

and B = (0, y – x f ′ (x))

$\therefore \quad AP : PB = 1 : 3$

$\Rightarrow \quad x=\frac{3}{4}\left(x-\frac{y}{f'(x)}\right)$

$\Rightarrow \quad x=\frac{-3y}{f'(x)} \Rightarrow \frac{dy}{dx}=\frac{-3y}{x}$

$\frac{dy}{y}=\frac{-3\,dx}{x} \Rightarrow y=\frac{C}{x^3}$

$\because \quad f(a)=1 \Rightarrow C=1$

$\therefore \quad y=\frac{1}{x^3}$ is required curve and $\left(2,\frac{1}{8}\right)$ passing through $y=\frac{1}{x^3}$

65. **(d)** $x^2y^2-2x=4-4y$

Differentiate w.r.t. 'x'

$2xy^2+2y\,.\,x^2\,.\,\frac{dy}{dx}-2=-4\,.\,\frac{dy}{dx}$

$\Rightarrow \quad \frac{dy}{dx}(2y\,.\,x^2+4)=2-2x\,.\,y^2$

$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{2,-2}=\frac{2-2\times2\times4}{2(-2)\times4+4}=\frac{-14}{-12}=\frac{7}{6}$

$\therefore \quad$ Equation of tangent is

$(y+2)=\frac{7}{6}(x-2)$ or $7x-6y=26$

$\therefore$ (–2, –7) does not passes through the required tangent.

66. **(d)** $f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$

$=\tan^{-1}\left(\sqrt{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{x}-\cos\frac{x}{2}\right)^2}}\right)=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$\Rightarrow \quad y=\frac{\pi}{4}+\frac{x}{2} \Rightarrow \frac{dy}{dx}=\frac{1}{2}$

Slope of normal $=\frac{-1}{\left(\frac{dy}{dx}\right)}=-2$

Equation of normal at $\left(\frac{\pi}{6},\frac{\pi}{4}+\frac{\pi}{12}\right)$

$y-\left(\frac{\pi}{4}+\frac{\pi}{12}\right)=-2\left(x-\frac{\pi}{6}\right)$

$y-\frac{4\pi}{12}=-2x+\frac{2\pi}{6}$

$y-\frac{\pi}{3}=-2x+\frac{\pi}{3}$

$y=-2x+\frac{2\pi}{3}$

This equation is satisfied only by the point $\left(0,\frac{2\pi}{3}\right)$

67. **(a)** $\frac{dy}{dx}=\frac{1}{2\sqrt{4x-3}}\times4=\frac{2}{3}$

$\Rightarrow \quad 4x-3=9$

$\Rightarrow \quad x=3$

So, $y = 4$

Equation of normal at $P(3, 4)$ is

$$y - 4 = -\frac{3}{2}(x - 3)$$

i.e. $2y - 8 = -3x + 9$

$\Rightarrow \quad 3x + 2y - 17 = 0$

This line is satisfied by the point (1, 7)

68. **(d)** $P(4t^2 + 3, 8t^3 - 1)$

$$\frac{dy/dt}{dt/dt} = \frac{dy}{dx} = 3t \text{ (slope of tangent at } P)$$

Let $Q = (4\lambda^2 + 3, 8\lambda^3 - 1)$

slope of PQ = $3t$

$$\frac{8t^3 - 8\lambda^3}{4t^2 - 4\lambda^2} = 3t$$

$\Rightarrow \quad t^3 - 3\lambda^2 t + 2\lambda^3 = 0$

$(t - \lambda) . (t^2 + t\lambda - 2\lambda^2) = 0$

$(t - \lambda)^2 . (t + 2\lambda) = 0$

$t = \lambda$ (or) $\lambda = \dfrac{-t}{2}$

$\therefore \quad Q[t^2 + 3, -t^3 - 1]$.

69. **(b)** Given curve is

$x^2 + 2xy - 3y^2 = 0$ ...(i)

Differentiatew.r.t. x

$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 1$$

Equation of normal at (1, 1) is

$y = 2 - x$ ...(ii)

Solving eqs. (i) and (ii), we get

$x = 1, 3$

Point of intersection (1, 1), (3, –1)

Normal cuts the curve again in 4th quadrant.

70. **(b)** Given curve is $\sin y = x \sin\left(\dfrac{\pi}{3} + y\right)$

Diff with respect to x, we get

$$\cos y \frac{dy}{dx} = \sin\left(\frac{\pi}{3} + y\right) + x\cos\left(\frac{\pi}{3} + y\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin\left(\frac{\pi}{3} + y\right)}{\cos y - x\cos\left(\frac{\pi}{3} + y\right)}$$

$\dfrac{dy}{dx}$ at (0, 0) = $\dfrac{\sqrt{3}}{2}$

$\Rightarrow$ Equation of normal is $y - 0 = -\dfrac{2}{\sqrt{3}}(x - 0)$

$\Rightarrow 2x + \sqrt{3}\, y = 0$

71. **(d)** $x^2 - y + 6 = 0$

$$2x - \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 2x$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,10)} = 4$$

equation of tangent

$y - 10 = 4(x - z)$

$4x - y + z = 0$

tangent passes through (α, β)

$4\alpha - \beta + z = 0 \Rightarrow \beta = 4\alpha + z$ ...(i)

and $2x + 2yy' + 8 - 2y' = 0$

$$y' = \frac{2x + 8}{2 - 2y} = \frac{2\alpha + 8}{2 - 2\beta} = 4 \quad \text{...(ii)}$$

from (i) and (ii)

$$\alpha = \frac{-8}{17}, \beta = \frac{2}{17}$$

$$\left(\frac{-8}{17}, \frac{2}{17}\right)$$

72. **(a)** Given that

$x = 2\cos t + 2t\sin t$

so, $\dfrac{dx}{dt} = -2\sin t + 2[t\cos t + \sin t]$

$$\frac{dy}{dt} = 2\cos t - 2[-t\sin t + \cos t]$$

$$\frac{dy}{dx} = 2t\sin t$$

$$\frac{dy}{dx} = \frac{2t\sin t}{2t\cos t}$$

$$\frac{dy}{dx} = \tan t$$

$$\left(\frac{dy}{dx}\right)_{t=\pi/4} = 1$$

so the slope of the normal is – 1

At $t = \pi/4$ $x = \sqrt{2} + \dfrac{\pi}{2\sqrt{2}}$ and

$y = \sqrt{2} - \pi/2\sqrt{2}$

the equation of normal is

$$\left[y - \left(\sqrt{2} - \pi/2\sqrt{2}\right)\right] = -1\left[\left(x - \left(\sqrt{2} + \pi/2\sqrt{2}\right)\right)\right]$$

$$y - \sqrt{2} + \frac{\pi}{2\sqrt{2}} = -x + \sqrt{2} + \pi/2\sqrt{2}$$

$x + y = 2\sqrt{2}$, so the distance from the origin is 2

73. **(c)** Given, $y = 3 \sin \theta . \cos \theta$

$$\frac{dy}{d\theta} = 3[\sin\theta(-\sin\theta)+\cos\theta(\cos\theta)]$$

$$\frac{dy}{d\theta} = 3[\cos^2\theta-\sin^2\theta] = 3 \cos 2\theta \quad ...(i)$$

and $x = e^{\theta} \sin \theta$

$$\frac{dx}{d\theta} = e^{\theta} \cos \theta + \sin \theta\, e^{\theta}$$

$$\frac{dx}{d\theta} = e^{\theta} (\sin \theta + \cos \theta) \quad ...(ii)$$

Dividing (i) by (i)

$$\frac{dy}{dx} = \frac{3\cos 2\theta}{e^{\theta}(\sin\theta+\cos\theta)} = \frac{3(\cos^2\theta-\sin^2\theta)}{e^{\theta}(\sin\theta+\cos\theta)}$$

$$\frac{dy}{dx} = \frac{3(\cancel{\cos\theta+\sin\theta})(\cos\theta-\sin\theta)}{e^{\theta}(\cancel{\sin\theta+\cos\theta})}$$

$$\frac{dy}{dx} = \frac{3(\cos\theta-\sin\theta)}{e^{\theta}}$$

Given tangent is parallel to x-axis then $\frac{dy}{dx} = 0$

$$0 = \frac{3(\cos\theta-\sin\theta)}{e^{\theta}}$$

or $\cos \theta - \sin \theta = 0 \Rightarrow \cos \theta = \sin \theta$

$$\Rightarrow \tan \theta = 1 \Rightarrow \tan \theta = \frac{\tan \pi}{4} \Rightarrow \theta = \frac{\pi}{4}$$

74. **(d)** Let $y = \cos (x + y)$

$$\Rightarrow \frac{dy}{dx} = -\sin (x+y)\left(1+\frac{dy}{dx}\right) \quad ...(i)$$

Now, given equation of tangent is

$x + 2y = k$

$$\Rightarrow \text{Slope} = \frac{-1}{2}$$

So, $\frac{dy}{dx} = \frac{-1}{2}$ put this value in (i), we get

$$\frac{-1}{2} = -\sin (x+y)\left(1-\frac{1}{2}\right)$$

$\Rightarrow \sin (x + y) = 1$

$$\Rightarrow x+y = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2}-x$$

Now, $\frac{\pi}{2} - x = \cos (x+y)$

$$\Rightarrow x = \frac{\pi}{2} \text{ and } y = 0$$

Thus $x + 2y = k \Rightarrow \frac{\pi}{2} = k$

75. **(d)** $x^2 = 8y \quad ...(i)$

When, $x = 4$, then $y = 2$

Now $\frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4}, \left.\frac{dy}{dx}\right]_{x=4} = 1$

Slope of normal $= -\frac{1}{\frac{dy}{dx}} = -1$

Euqation of normal at $x = 4$ is

$y - 2 = - 1 (x - 4)$

$\Rightarrow y = - x + 4 + 2 = - x + 6$

$\Rightarrow x + y = 6$

76. **(c)** Since the tangent is parallel to x-axis,

$$\therefore \frac{dy}{dx} = 0 \Rightarrow 1-\frac{8}{x^3} = 0 \Rightarrow x = 2 \Rightarrow y = 3$$

Equation of the tangent is $y - 3 = 0 (x - 2)$

$\Rightarrow y = 3$

77. **(b)** $\frac{dy}{dx} = 2x-5 \therefore m_1 = (2x-5)_{(2,0)} = -1,$

$m_2 = (2x-5)_{(3,0)} = 1 \Rightarrow m_1 m_2 = -1$

i.e. the tangents are perpendicular to each other.

78. **(d)** Given $x = a(\cos\theta + \theta\sin\theta)$

$$\Rightarrow \frac{dx}{d\theta} = a(-\sin\theta+\sin\theta+\theta\cos\theta)$$

$$\Rightarrow \frac{dx}{d\theta} = a\theta\cos\theta \quad .....(i)$$

$y = a(\sin\theta - \theta\cos\theta)$

$$\frac{dy}{d\theta} = a[\cos\theta-\cos\theta+\theta\sin\theta]$$

$$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta \quad .....(ii)$$

From equations (i) and (ii) we get

$\frac{dy}{dx} = \tan \theta \Rightarrow$ Slope of normal $= - \cot \theta$

Equation of normal at 'θ' is

$y - a (\sin \theta - \theta \cos \theta) = - \cot \theta (x - a (\cos \theta + \theta \sin \theta))$

$\Rightarrow y \sin\theta - a\sin^2\theta + a\,\theta\cos\theta\sin\theta$

$= - x \cos\theta + a\cos^2\theta + a\,\theta\sin\theta\cos\theta$

$\Rightarrow x \cos\theta + y \sin\theta = a$

Clearly this is an equation of straight line which is at a constant distance 'a' from origin.

79. **(d)** Since, $x = a(1 + \cos\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = -a\sin\theta$ and $y = a\sin\theta$

$\Rightarrow \dfrac{dy}{d\theta} = a\cos\theta$

$\therefore \dfrac{dy}{dx} = -\cot\theta.$

$\therefore$ The slope of the normal at $\theta = \tan\theta$

$\therefore$ The equation of the normal at θ is

$y - a\sin\theta = \tan\theta(x - a - a\cos\theta)$

$\Rightarrow y\cos\theta - a\sin\theta\cos\theta = x\sin\theta - a\sin\theta - a\sin\theta\cos\theta$

$\Rightarrow x\sin\theta - y\cos\theta = a\sin\theta$

$\Rightarrow y = (x-a)\tan\theta$

which always passes through $(a, 0)$

80. **(b)** $f''(x) = 6(x-1)$. Inegrating, we get

$f'(x) = 3x^2 - 6x + c$

Slope at (2, 1) $= f'(2) = c = 3$

[$\because$ slope of tangent at (2,1) is 3]

$\therefore f'(x) = 3x^2 - 6x + 3 = 3(x-1)^2$

Inegrating again, we get $f(x) = (x-1)^3 + D$

The curve passes through (2, 1)

$\Rightarrow 1 = (2-1)^3 + D \Rightarrow D = 0$

$\therefore f(x) = (x-1)^3$

81. **(b)** $C_1 \to C_1 + C_2$

Let $f(x) = \begin{vmatrix} 2 & 1+\sin^2 x & \sin 2x \\ 2 & \sin^2 x & \sin 2x \\ 1 & \sin^2 x & 1+\sin 2x \end{vmatrix}$

$R_1 \to R_1 - 2R_3;\ R_2 \to R_2 - 2R_3$

$= \begin{vmatrix} 0 & \cos^2\theta & -(2+\sin 2x) \\ 0 & -\sin^2 x & -(2+\sin 2x) \\ 1 & \sin^2 x & 1+\sin 2x \end{vmatrix} = -2 - 2\sin 2x$

$f'(x) = -2\cos 2x = 0$

$\Rightarrow \cos 2x = 0 \Rightarrow x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$

$f''(x) = 4\sin 2x$

So, $f''\left(\dfrac{\pi}{4}\right) = 4 > 0$ (minima)

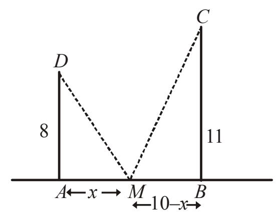
$m = f\left(\dfrac{\pi}{4}\right) = -2 - 1 = -3$

$f''\left(\dfrac{3\pi}{4}\right) = -4 < 0$ (maxima)

$M = f\left(\dfrac{3\pi}{4}\right) = -2 + 1 = -1$

So, $(m, M) = (-3, -1)$

82. **(5)**



Let $AM = x$ m

$\therefore (MD)^2 + (MC)^2 = 64 + x^2 + 121 + (10-x)^2 = f(x)$ (say)

$f'(x) = 2x - 2(10-x) = 0$

$\Rightarrow 4x = 20 \Rightarrow x = 5$

$f''(x) = 2 - 2(-1) > 0$

$\therefore f(x)$ is minimum at $x = 5$ m.

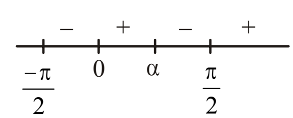
83. **(d)** $f(x) = (1-\cos^2 x)(\lambda + \sin x) = \sin^2 x(\lambda + \sin x)$

$\Rightarrow f(x) = \lambda\sin^2 x + \sin^3 x$ ...(i)

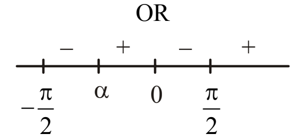
$\Rightarrow f'(x) = \sin x\cos x[2\lambda + 3\sin x] = 0$

$\Rightarrow \sin x = 0$ and $\sin x = -\dfrac{2\lambda}{3} \Rightarrow x = \alpha$ (let)

So, $f(x)$ will change its sign at $x = 0, \alpha$ because there is exactly one maxima and one minima in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$



OR



Now, $\sin x = -\dfrac{2\lambda}{3}$

$\Rightarrow -1 \le -\dfrac{2\lambda}{3} \le 1 \Rightarrow -\dfrac{3}{2} \le \lambda \le \dfrac{3}{2} - \{0\}$

$\because$ If $\lambda = 0 \Rightarrow f(x) = \sin^3 x$ (from (i))

Which is monotonic, then no maxima/minima

So, $\lambda \in \left(-\dfrac{3}{2}, \dfrac{3}{2}\right) - \{0\}$

84. **(d)** The given function

$$f(x)=(3x^2+ax-2-a)e^x$$

$$f'(x)=(6x+a)e^x+(3x^2+ax-2-a)e^x$$

$$f'(x)=[3x^2+(a+6)x-2]e^x$$

$\because$ $x = 1$ is critical point :

$\therefore f'(1)=0$

$$\Rightarrow (3+a+6-2)\cdot e=0$$

$$\Rightarrow a=-7 \qquad (\because e>0)$$

$$\therefore f'(x)=(3x^2-x-2)e^x$$

$$=(3x+2)(x-1)e^x$$

+ $-2/3$ − 1 +

$\therefore x=-\dfrac{2}{3}$ is point of local maxima.

and $x = 1$ is point of local minima.

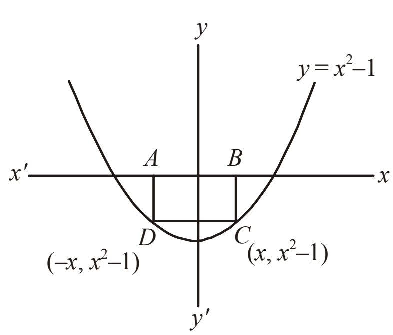
85. **(d)** Area of rectangle $ABCD$

$$A=2x\cdot(x^2-1)=2x^3-2x$$

$$\therefore \quad \frac{dA}{dx}=6x^2-2$$

For maximum area $\dfrac{dA}{dx}=0\Rightarrow x=\pm\dfrac{1}{\sqrt{3}}$

$$\frac{d^2A}{dx^2}=(12x)\Rightarrow\left(\frac{d^2A}{dx^2}\right)_{x=\frac{-1}{\sqrt{3}}}=\frac{-12}{\sqrt{3}}<0$$



$\therefore$ Maximum area $=\left|\dfrac{2}{3\sqrt{3}}-\dfrac{2}{\sqrt{3}}\right|=\dfrac{4}{3\sqrt{3}}$

86. **(a)** $\because$ The critical points are $-1, 0, 1$

$$\therefore f'(x)=k\cdot x(x+1)(x-1)=k(x^3-x)$$

$$\Rightarrow f(x)=k\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C$$

$$\Rightarrow f(0)=C$$

$$\because f(x)=f(0)$$

$$\Rightarrow k\frac{(x^4-2x^2)}{4}+C=C$$

$$\Rightarrow x^2(x^2-2)=0$$

$$\Rightarrow x=0,\sqrt{2},-\sqrt{2}$$

$$\Rightarrow T=\{0,\sqrt{2},-\sqrt{2}\}$$

87. **(3)** Let $f(x) = ax^3 + bx^2 + cx + d$

$f(-1) = 10$ and $f(1) = -6$

$-a + b - c + d = 10$ ...(i)

$a + b + c + d = -6$ ...(ii)

Solving equations (i) and (ii), we get

$$a=\frac{1}{4},\ d=\frac{35}{4}$$

$$b=\frac{-3}{4},\ c=-\frac{9}{4}$$

$\Rightarrow \quad f(x) = a(x^3 - 3x^2 - 9x) + d$

$f'(x)=\dfrac{3}{4}(x^2-2x-3)=0$

$\Rightarrow \quad x = 3, -1$

+ -1 − 3

Local minima exist at $x = 3$

88. **(d)** $f(x) = ax^5 + bx^4 + cx^3$

$$\lim_{x\to 0}\left(2+\frac{ax^5+bx^4+cx^3}{x^3}\right)=4$$

$\Rightarrow \quad 2 + c = 4 \quad \Rightarrow \quad c = 2$

$f'(x) = 5ax^4 + 4bx^3 + 6x^2$

$= x^2(5ax^2 + 4bx + 6)$

Since, $x = \pm 1$ are the critical points,

$\therefore \quad f'(1) = 0 \quad \Rightarrow \quad 5a + 4b + 6 = 0$ ...(i)

$f'(-1) = 0 \quad \Rightarrow \quad 5a - 4b + 6 = 0$ ...(ii)

From eqns. (i) and (ii),

$b = 0$ and $a=-\dfrac{6}{5}$

$$f(x)=\frac{-6}{5}x^5+2x^3$$

$f'(x) = -6x^4 + 6x^2 = 6x^2(-x^2 + 1)$

$= -6x^2(x + 1)(x - 1)$

− -1 + 1 −

$\therefore \quad f(x)$ has minima at $x = -1$ and maxima at $x = 1$

89. **(b)** Given function $f(x) = x\sqrt{kx - x^2} = \sqrt{kx^3 - x^4}$

Differentiating w. r. t. x,

$$f'(x) = \frac{(3kx^2 - 4x^3)}{2\sqrt{kx^3 - x^4}} \geq 0 \text{ for } x \in [0, 3]$$

[$\because f(x)$ is increasing in [0, 3]]

$\Rightarrow 3k - 4x \geq 0 \Rightarrow 3k \geq 4x$

i.e., $3k \geq 4x$ for $x \in [0, 3]$

$\therefore k \geq 4$ i.e., $m = 4$

Putting $k = 4$ in the function, $f(x) = x\sqrt{4x - x^2}$

For max. value, $f'(x) = 0$

i.e. $\dfrac{12x^2 - 4x^3}{2\sqrt{4x^3 - x^4}} = 0 \Rightarrow x = 3$

$y = 3\sqrt{3}$ i.e., $M = 3\sqrt{3}$

90. **(b)** $a_6 = a + 5d = 2$

Here, a is first term of A.P and d is common difference

Let $A = a_1 a_4 a_5 = a(a + 3d)(a + 4d)$

$= a(2 - 2d)(2 - d)$

$A = (2 - 5d)(4 - 6d + 2d^2)$

By $\dfrac{dA}{dd} = 0$

$(2 - 5d)(-6 + 4d) + (4 - 6d + 2d^2)(-5) = 0$

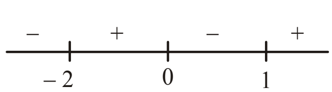
$-15d^2 + 34d - 16 = 0 \Rightarrow d = \dfrac{8}{5}, \dfrac{2}{3}$

For $d = \dfrac{8}{5}, \dfrac{d^2A}{dd^2} < 0$.

Hence $d = \dfrac{8}{5}$

91. **(c)** $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36[x^3 + x^2 - 2x] = 36x(x - 1)(x + 2)$



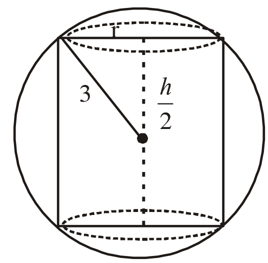
Here at -2 & 1, $f'(x)$ changes from negative value to positive value.

$\Rightarrow -2$ & 1 are local minimum points. At 0, $f'(x)$ changes from positive value to negative value.

$\Rightarrow$ 0 is the local maximum point.

Hence, $S_1 = \{-2, 1\}$ and $S_2 = \{0\}$

92. **(c)** Let radius of base and height of cylinder be r and h respectively.



$$\therefore r^2 + \frac{h^2}{4} = 9 \quad \text{...(i)}$$

Now, volume of cylinder, $V = \pi r^2 h$

Substitute the value of r^2 from equation (i),

$$V = \pi h\left(9 - \frac{h^2}{4}\right) \Rightarrow V = 9\pi h - \frac{\pi}{4}h^3$$

Differentiating w.r.t. h,

$$\frac{dV}{dh} = 9\pi - \frac{3}{4}\pi h^2$$

For maxima/minima,

$$\frac{dV}{dh} = 0 \Rightarrow h = \sqrt{12}$$

and $\dfrac{d^2V}{dh^2} = -\dfrac{3}{2}\pi h$

$$\left(\frac{d^2V}{dh^2}\right)_{h=\sqrt{12}} < 0$$

Volume is maximum when $h = 2\sqrt{3}$

93. **(a)** Let, the functions is,

$$f(\theta) = 3\cos\theta + 5\sin\theta \cdot \cos\frac{\pi}{6} - 5\sin\frac{\pi}{6}\cos\theta$$

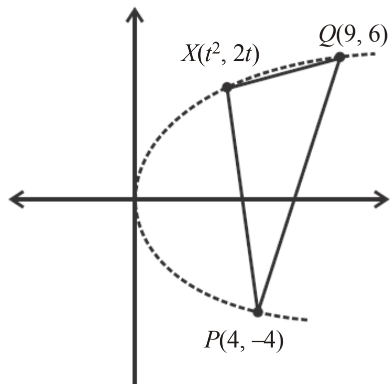
$$= 3\cos\theta + 5 \times \frac{\sqrt{3}}{2}\sin\theta - 5 \times \frac{1}{2}\cos\theta$$

$$= \left(3 - \frac{5}{2}\right)\cos\theta + 5 \times \frac{\sqrt{3}}{2}\sin\theta$$

$$= \frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$$

$$\max f(\theta) = \sqrt{\frac{1}{4} + \frac{25}{4} \times 3} = \sqrt{\frac{76}{4}} = \sqrt{19}$$

94. (b)



Parametric equations of the parabola $y^2 = 4x$ are, $x = t^2$ and $y = 2t$.

$$\text{Area } \Delta PXQ = \frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 4 & -4 & 1 \\ 9 & 6 & 1 \end{vmatrix}$$

$= -5t^2 + 5t + 30$

$= -5(t^2 - t - 6)$

$$= -5\left[\left(t - \frac{1}{2}\right)^2 - \frac{25}{4}\right]$$

For maximum area $t = \dfrac{1}{2}$

$\therefore$ maximum area $= 5\left(\dfrac{25}{4}\right) = \dfrac{125}{4}$

95. (c) Consider the function,

$f(x) = 3x(x - 3)^2 - 40$

Now $S = \{x \in R : x^2 + 30 \le 11x\}$

So $x^2 - 11x + 30 \le 0 \quad \Rightarrow \quad x \in [5, 6]$

$\therefore f(x)$ will have maximum value for $x = 6$

The maximum value of function is,

$f(6) = 3 \times 6 \times 3 \times 3 - 40 = 122.$

96. (c) $A = \dfrac{x^m y^n}{(1+x^{2m})(1+y^{2n})} = \dfrac{1}{(x^{-m}+x^m)(y^{-n}+y^n)}$

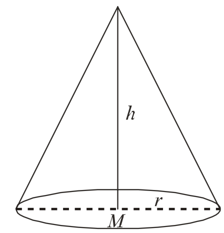
$\dfrac{x^m + y^{-m}}{2} \ge (x^m . x^{-m})^{\frac{1}{2}} \Rightarrow x^m + x^{-m} \ge 2$

In the same way, $y^{-n} + y^n \ge 2$

Then, $(x^m + x^{-m})(y^{-n} + y^n) \ge 4$

$\Rightarrow \quad \dfrac{1}{(x^m + x^{-m})(y^{-n} + y^n)} \le \dfrac{1}{4}$

97. (d)



$h^2 + r^2 = \ell^2 = 9$ ...(i)

Volume of cone

$V = \dfrac{1}{3}\pi r^2 h$ ...(ii)

From (i) and (ii),

$\Rightarrow V = \dfrac{1}{3}\pi(9 - h^2)h$

$\Rightarrow V = \dfrac{1}{3}\pi(9h - h^3) \Rightarrow \dfrac{dv}{dh} = \dfrac{1}{3}\pi\left(9 - 3h^2\right)$

For maxima/minima,

$\dfrac{dV}{dh} = 0 \Rightarrow \dfrac{1}{3}\pi(9 - 3h^2) = 0$

$\Rightarrow h = \pm\sqrt{3} \Rightarrow h = \sqrt{3} \qquad (\because h > 0)$

Now; $\dfrac{d^2V}{dh^2} = \dfrac{1}{3}\pi(-6h)$

Here, $\left(\dfrac{d^2V}{dh^2}\right)_{\text{at } h=\sqrt{3}} < 0$

Then, $h = \sqrt{3}$ is point of maxima

Hence, the required maximum volume is,

$V = \dfrac{1}{3}\pi(9-3)\sqrt{3} = 2\sqrt{3}\pi$

98. (c) Here, $h(x) = \dfrac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \left(x - \dfrac{1}{x}\right) + \dfrac{2}{x - \frac{1}{x}}$

When $x - \dfrac{1}{x} < 0$

$\therefore \quad x - \dfrac{1}{x} + \dfrac{2}{x - \frac{1}{x}} \le -2\sqrt{2}$

Hence, $-2\sqrt{2}$ will be local maximum value of h(x).

When $x - \dfrac{1}{x} > 0$

$\therefore \quad x-\frac{1}{x}+\frac{2}{x-\frac{1}{x}} \geq 2\sqrt{2}$

Hence, $2\sqrt{2}$ will be local minimum value of h(x).

99. **(a)** Here, $f(x) = 2x^3 - 9x^2 + 12x + 5$

$\Rightarrow \quad f'(x) = 6x^2 - 18x + 12 = 0$

For maxima or minima put $f'(x) = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow x = 1$ or $x = 2$

Now, $f''(x) = 12x - 18$

$\Rightarrow \quad f''(1) = 12(1) - 18 = -6 < 0$

Hence, $f(x)$ has maxima at $x = 1$

$\therefore$ maximum value $= M = f(1) = 2 - 9 + 12 + 5 = 10.$

And, $f''(2) = 12(2) - 18 = 6 > 0.$

Hence, $f(x)$ has minima at $x = 2.$

$\therefore$ minimum value $= m = f(2)$

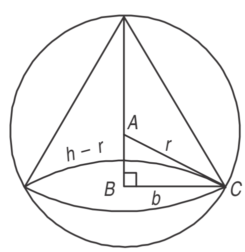
$= 2(8) - 9(4) + 12(2) + 5 = 9$

$\therefore M - m = 10 - 9 = 1$

100. **(a)** Sphere of radius $r = 3\ cm$

Let b, h be base radius and height of cone respectively.

So, volume of cone $= \frac{1}{2}\pi b^2 h$



In right angled ΔABC by Pythagoras theorem

$(h-r)^2 + b^2 = r^2$ ...(i)

$\Rightarrow b^2 = r^2 - (h-r)^2 = r^2 - (h^2 - 2hr + r^2) = 2hr - h^2$

$\therefore$ Volume $(v) = \frac{1}{3}\pi h\left[2hr - h^2\right] = \frac{1}{3}\left[2h^2 r - h^3\right]$

$$\frac{dv}{dh} = \frac{1}{3}\left[4hr - 3h^2\right] = 0 \Rightarrow h(4r - 3h) = 0$$

$$\frac{d^2v}{dh^2} = \frac{1}{3}[4r - 6h]$$

At $h = \frac{4r}{3}, \frac{d^2v}{dh^2} = \frac{1}{3}\left[4r - \frac{4r}{3}\times 6\right] = \frac{1}{3}[4r - 8r] < 0$

$\Rightarrow$ maximum volume ocurs at $h = \frac{4r}{3} = \frac{4}{3}\times 3 = 4\ cm$

As from (i),

$(h-r)^2 + b^2 = r^2$

$\Rightarrow b^2 = 2hr - h^2 = 2\cdot\frac{4r}{3}r - \frac{16r^2}{9} = \frac{8r^2}{3} - \frac{16r^2}{9}$

$= \frac{(24-16)\,r^2}{9} = \frac{8r^2}{9}$

$\Rightarrow b = \frac{2\sqrt{2}}{3}r = 2\sqrt{2}\ cm$

Therefore curved surface area $= \pi bl$

$= \pi b\sqrt{h^2 + r^2} = \pi 2\sqrt{2}\sqrt{4^2 + 8} = 8\sqrt{3}\pi\ cm^2$

101. **(d)** We have

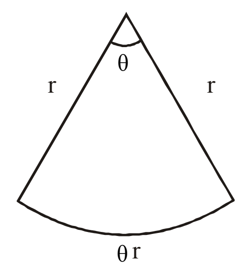
Total length = r + r + rθ = 20

$\Rightarrow 2r + r\theta = 20$

$\Rightarrow \theta = \frac{20 - 2r}{r}$ ...(i)

$A = \text{Area} = \frac{\theta}{2\pi}\times\pi r^2$

$= \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{20-2r}{r}\right)$



$A = 10r - r^2$

For A to be maximum

$\frac{dA}{dr} = 0 \Rightarrow 10 - 2r = 0$

$\Rightarrow r = 5$

$\frac{d^2A}{dr^2} = -2 < 0$

$\therefore$ For r = 5 A is maximum

From (i)

$\theta = \frac{20 - 2(5)}{5} = \frac{10}{5} = 2$

$A = \frac{2}{2\pi}\times\pi(5)^2 = 25$ sq. m

102. **(a)** $4x + 2\pi r = 2 \quad \Rightarrow 2x + \pi r = 1$

$S = x^2 + \pi r^2$

$$S = \left(\frac{1-\pi r}{2}\right)^2 + \pi r^2$$

$$\frac{dS}{dr} = 2\left(\frac{1-\pi r}{2}\right)\left(\frac{-\pi}{2}\right) + 2\pi r$$

$$\Rightarrow \frac{-\pi}{2} + \frac{\pi^2 r}{2} + 2\pi r = 0 \quad \Rightarrow r = \frac{1}{\pi + 4}$$

$$\Rightarrow x = \frac{2}{\pi + 4} \quad \Rightarrow x = 2r$$

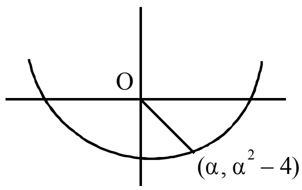
103. (a) $D = \sqrt{\alpha^2 + (\alpha^2 - 4)^2}$

$D^2 = \alpha^2 + \alpha^4 + 16 - 8\alpha^2 = \alpha^4 - 7\alpha^2 + 16$

$\frac{dD^2}{d\alpha} = 4\alpha^3 - 14\alpha = 0$

$2\alpha(2\alpha^2 - 7) = 0$

$\alpha^2 = \frac{7}{2}$



$D^2 = \frac{49}{4} - \frac{49}{2} + 16 = -\frac{49}{4} + 16 = \frac{15}{4}$

$D = \frac{\sqrt{15}}{2}$

104. (a) Let $f(x) = \frac{(1+x)^{\frac{3}{5}}}{1+x^{\frac{3}{5}}}$ and $x \in [0, 1]$

$$\Rightarrow f'(x) = \frac{(1+x^{\frac{3}{5}})\frac{3}{5}(1+x)^{-\frac{2}{5}} - \frac{3}{5}(1+x)^{\frac{3}{5}}(x^{\frac{-2}{5}})}{(1+x^{\frac{3}{5}})^2}$$

$$= \frac{3}{5}\left[\left(1+x^{\frac{3}{5}}\right)(1+x)^{-\frac{2}{5}} - (1+x)^{\frac{3}{5}}x^{\frac{-2}{5}}\right]$$

$$= \frac{3}{5}\left[\frac{1+x^{\frac{3}{5}}}{(1+x)^{\frac{2}{5}}} - \frac{(1+x)^{\frac{3}{5}}}{x^{\frac{2}{5}}}\right]$$

$$= \frac{x^{\frac{2}{5}} + x - 1 - x}{x^{\frac{2}{5}}(1+x)^{\frac{2}{5}}} = \frac{x^{\frac{2}{5}} - 1}{x^{\frac{2}{5}}(1+x)^{\frac{2}{5}}} < 0$$

Also, $f(0) = 1 \Rightarrow f(x) \in [2^{-0.4}, 1]$

$f(a) = 2^{-0.4}$

105. (d) Let 'u' be the velocity

$\therefore$ u = 4 8 m/s, Given, g = 32

At maximum height v = 0

Now, we know $v^2 = u^2 - 2gh$

$\Rightarrow 0 = (48)^2 - 2(32)h \Rightarrow h = 36$

Maximum height = 36 + 64 = 100 mt

106. (a) Let $f(x) = \alpha \log|x| + \beta x^2 + x$

Differentiate both side,

$f'(x) = \frac{\alpha}{x} + 2\beta x + 1$

Since $x = -1$ and $x = 2$ are extreme points therefore $f'(x) = 0$ at these points.

Put $x = -1$ and $x = 2$ in $f'(x)$, we get

$-\alpha - 2\beta + 1 = 0 \Rightarrow \alpha + 2\beta = 1$ ...(i)

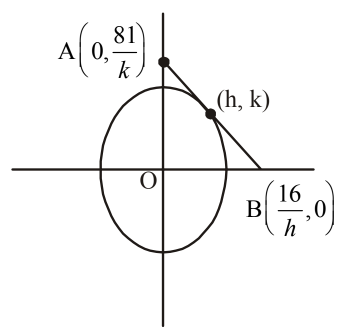
$\frac{\alpha}{2} + 4\beta + 1 = 0 \Rightarrow \alpha + 8\beta = -2$ ...(ii)

On solving (i) and (ii), we get

$6\beta = -3 \Rightarrow \beta = -\frac{1}{2}$

$\therefore \alpha = 2$

107. (d)



Let (h, k) be the point on ellipse through which tangent is passing.

Equation of tangent at (h, k) = $\frac{xh}{16} + \frac{yk}{81} = 1$

at $y = 0$, $x = \frac{16}{h}$

at $x = 0$, $y = \frac{81}{k}$

Area of AOB = $\frac{1}{2} \times \left(\frac{16}{h}\right) \times \left(\frac{81}{k}\right) = \frac{648}{hk}$

$A^2 = \frac{(648)^2}{h^2k^2}$ ...(i)

(h, k) must satisfy equation of ellipse

$\frac{h^2}{16} + \frac{k^2}{81} = 1$

$h^2 = \frac{16}{81}(81 - k^2)$

Putting value of h^2 in equation (i)

$$A^2 = \frac{81(648)^2}{16\times k^2(81-k^2)} = \frac{\alpha}{81k^2-k^4}$$

differentiating w.r. to k

$$2AA' = \alpha\left(\frac{-1}{81k^2-k^4}\right)(162k-4k^3)$$

$2AA' = -2A\ (81k - 4k^3) \Rightarrow A' = -81k - 4k^3$

Put $A' = 0$

$\Rightarrow\ 162k - 4k^3 = 0,\ k\ (162 - 4k^2) = 0$

$$\Rightarrow\ k = 0,\ k = \pm\frac{9}{\sqrt{2}}$$

$A'' = -(81 - 12k^2)$

For both value of k, $A'' = 405 > 0$

Area will be minimum for $k = \pm\dfrac{9}{\sqrt{2}}$

$$h^2 = \frac{16}{81}(81-k^2) = 8$$

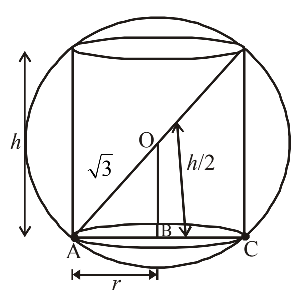
$$h = \pm 2\sqrt{2}$$

Area of triangle AOB $= \dfrac{648\times\sqrt{2}}{2\sqrt{2}\times 9} = 36$ sq unit

108. (c) Given, radius of sphere $= \sqrt{3}$

Now, In ΔOAB, by Pythagoras theorem

$(OA)^2 = (OB)^2 + (AB)^2$



$$(\sqrt{3})^2 = \left(\frac{h}{2}\right)^2 + r^2$$

$$3 = \frac{h^2}{4} + r^2 \Rightarrow \boxed{r^2 = 3 - \frac{h^2}{4}} \quad \text{...(i)}$$

Now, volume of cylinder $= \pi r^2 h$

$$V = \pi\left(3-\frac{h^2}{4}\right)h \quad \text{(using eq. (i))}$$

$$V = 3\pi h - \frac{\pi h^3}{4} \quad \text{...(ii)}$$

Now, for largest possible right circular cylinder the volume must be maximum

$\therefore$ For maximum volume, $\dfrac{dV}{dh} = 0$

Now, Differentiating eq. (2) w.r.t. h

$$\frac{dV}{dh} = 3\pi - \frac{3}{4}\pi h^2$$

or $3\pi - \dfrac{3}{4}\pi h^2 = 0 \Rightarrow \cancel{3}\cancel{\pi} = \dfrac{\cancel{3}}{4}\cancel{\pi} h^2$

$\Rightarrow h^2 = 4 \Rightarrow h = 2$

Now, volume (V) of the cylinder

$$= \pi\left(3-\frac{h^2}{4}\right)h = \pi(6-2) = 4\pi$$

109. (c) Let cost $C = av + \dfrac{b}{v}$

According to given question,

$$30a + \frac{b}{30} = 75 \quad \text{... (i)}$$

$$40a + \frac{b}{40} = 65 \quad \text{... (ii)}$$

On solving (i) and (ii), we get

$a = \dfrac{1}{2}$ and $b = 1800$

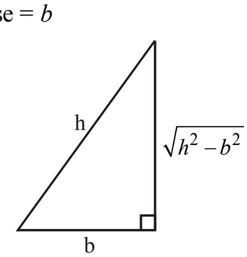
Now, $C = av + \dfrac{b}{v}$

$$\Rightarrow \frac{dC}{dv} = a - \frac{b}{v^2}$$

$$\frac{dC}{dv} = 0 \Rightarrow a - \frac{b}{v^2} = 0$$

$$\Rightarrow v = \sqrt{\frac{b}{a}} = \sqrt{3600} \Rightarrow v = 60 \text{ kmph}$$

110. (d) Let base $= b$



Altitude (or perpendicular) $= \sqrt{h^2-b^2}$

Area, $A = \dfrac{1}{2}\times$ base $\times$ altitude $= \dfrac{1}{2}\times b\times\sqrt{h^2-b^2}$

$$\Rightarrow \frac{dA}{db} = \frac{1}{2}\left[\sqrt{h^2-b^2} + b\cdot\frac{-2b}{2\sqrt{h^2-b^2}}\right]$$

$$=\frac{1}{2}\left[\frac{h^2-2b^2}{\sqrt{h^2-b^2}}\right]$$

Put $\frac{dA}{db}=0$, $\Rightarrow \quad b=\frac{h}{\sqrt{2}}$

Maximum area $=\frac{1}{2}\times\frac{h}{\sqrt{2}}\times\sqrt{h^2-\frac{h^2}{2}}=\frac{h^2}{4}$

111. (b) Given that, $f(x)=\ln|x|+bx^2+ax$

$\therefore \quad f'(x)=\frac{1}{x}+2bx+a$

At $x=-1$, $f'(x)=-1-2b+a=0$

$\Rightarrow \quad a-2b=1$ ...(i)

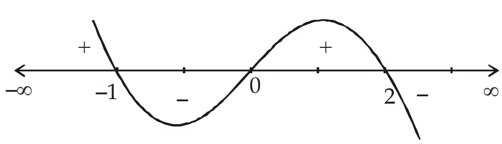
At $x=2$, $f'(x)=\frac{1}{2}+4b+a=0$

$\Rightarrow \quad a+4b=-\frac{1}{2}$ ...(ii)

On solving (i) and (ii) we get $a=\frac{1}{2}, b=-\frac{1}{4}$

Thus, $f'(x)=\frac{1}{x}-\frac{x}{2}+\frac{1}{2}=\frac{2-x^2+x}{2x}$

$$=\frac{-x^2+x+2}{2x}=\frac{-(x^2-x-2)}{2x}=\frac{-(x+1)(x-2)}{2x}$$



So maxima at $x=-1, 2$

112. (c) Equation of a line passing through (x_1,y_1) having slope m is given by $y-y_1=m(x-x_1)$

Since the line PQ is passing through (1,2) therefore its equation is $(y-2)=m(x-1)$

where m is the slope of the line PQ.

Now, point $P(x,0)$ will also satisfy the equation of PQ

$\therefore \quad y-2=m(x-1) \Rightarrow 0-2=m(x-1)$

$\Rightarrow \quad -2=m(x-1) \Rightarrow \quad x-1=\frac{-2}{m}$

$\Rightarrow \quad x=\frac{-2}{m}+1$

Also, $OP=\sqrt{(x-0)^2+(0-0)^2}=x=\frac{-2}{m}+1$

Similarly, point $Q(0,y)$ will satisfy equation of PQ

$\therefore \quad y-2=m(x-1)$

$\Rightarrow \quad y-2=m(-1)$

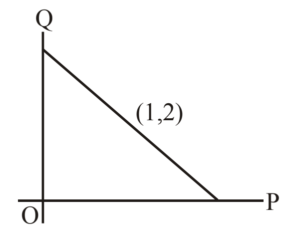
$\Rightarrow y=2-m$ and $OQ=y=2-m$

Area of $\Delta POQ=\frac{1}{2}(OP)(OQ)=\frac{1}{2}\left(1-\frac{2}{m}\right)(2-m)$

($\because$ Area of $\Delta=\frac{1}{2}\times\text{base}\times\text{height}$)

$$=\frac{1}{2}\left[2-m-\frac{4}{m}+2\right]=\frac{1}{2}\left[4-\left(m+\frac{4}{m}\right)\right]$$

$$=2-\frac{m}{2}-\frac{2}{m}$$



Let Area $=f(m)=2-\frac{m}{2}-\frac{2}{m}$

Now, $f'(m)=\frac{-1}{2}+\frac{2}{m^2}$

Put $f'(m)=0$

$\Rightarrow m^2=4 \Rightarrow m=\pm 2$

Now, $f''(m)=\frac{-4}{m^3}$

$f''(m)\big|_{m=2}=-\frac{1}{2}<0$

$f''(m)\big|_{m=-2}=\frac{1}{2}>0$

Area will be least at $m=-2$

Hence, slope of PQ is -2.

113. (d) Let $f:(-\infty,\infty)\rightarrow(-\infty,\infty)$ be defined by $f(x)=x^3+1$.

Clearly, $f(x)$ is symmetric along $y=1$ and it has neither maxima nor minima.

$\therefore$ Statement-1 is false.

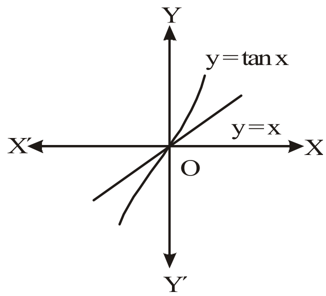
Hence, option (d) is correct.

114. (b) $f(x)=\begin{cases}\frac{\tan x}{x}, & x\neq 0\\ 1, & x=0\end{cases}$

For $x>0$

$\tan x>x$

$\frac{\tan x}{x}>1$



For $x < 0 \Rightarrow \tan x < x$

$\Rightarrow \dfrac{\tan x}{x} > 1$

$f(0) = 1$ at $x = 0$

$\Rightarrow \quad x = 0$ is the point of minima

So, Statement 1 is true. Statement 2 is also true.

115. (c) $f'(x) = \sqrt{x}\sin x$

$f'(x) = 0$

$\Rightarrow x = 0$ or $\sin x = 0$

$\Rightarrow x = 2\pi, \pi$

$f''(x) = \sqrt{x}\cos x + \dfrac{1}{2\sqrt{x}}\sin x$

$= \dfrac{1}{2\sqrt{x}}(2x\cos x + \sin x)$

At $x = \pi$, $f''(x) < 0$

Hence, local maxima at $x = \pi$

At $x = 2\pi$, $f''(x) > 0$

Hence local minima at $x = 2\pi$

116. (d) Given $f(x) = \dfrac{1}{e^x + 2e^{-x}} = \dfrac{e^x}{e^{2x} + 2}$

$f'(x) = \dfrac{(e^{2x}+2)e^x - 2e^{2x}.e^x}{(e^{2x}+2)^2}$

$f'(x) = 0 \Rightarrow e^{2x} + 2 = 2e^{2x}$

$\Rightarrow e^{2x} = 2 \quad \Rightarrow e^x = \sqrt{2}$

$\because f''(\sqrt{2}) = +ve$

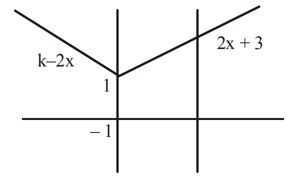
$\therefore$ Maximum values of $f(x) = \dfrac{\sqrt{2}}{4} = \dfrac{1}{2\sqrt{2}}$

$\Rightarrow 0 < f(x) \le \dfrac{1}{2\sqrt{2}} \quad \forall x \in R$

Since, $0 < \dfrac{1}{3} < \dfrac{1}{2\sqrt{2}}$

$\Rightarrow$ for some $c \in R$, $f(c) = \dfrac{1}{3}$

117. (c) $f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$



Clear that $f(x)$ is minimum at (–1, 1)

$\therefore f(-1) = 1$

$1 = k + 2 \Rightarrow k = -1$

118. (a) Given that $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$\Rightarrow \quad P'(x) = 4x^3 + 3ax^2 + 2bx + c$

But given $P'(0) = 0 \Rightarrow c = 0$

$\therefore \quad P(x) = x^4 + ax^3 + bx^2 + d$

Again given that $P(-1) < P(1)$

$\Rightarrow \quad 1 - a + b + d < 1 + a + b + d$

$\Rightarrow \quad a > 0$

Now $P'(x) = 4x^3 + 3ax^2 + 2bx = x(4x^2 + 3ax + 2b)$

As $P'(x) = 0$, there is only one solution $x = 0$, therefore $4x^2 + 3ax + 2b = 0$ should not have any real roots i.e. $D < 0$

$\Rightarrow \quad 9a^2 - 32b < 0 \Rightarrow b > \dfrac{9a^2}{32} > 0$

Hence $a, b > 0$

$\Rightarrow P'(x) = 4x^3 + 3ax^2 + 2bx > 0 \qquad \forall x > 0$

$\therefore \quad P(x)$ is an increasing function on (0,1)

$\therefore \quad P(0) < P(a)$

Similarly we can prove $P(x)$ is decreasing on (– 1, 0)

$\therefore \quad P(-1) > P(0)$

So we can conclude that

Max $P(x) = P(1)$ and Min $P(x) = P(0)$

$\Rightarrow P(-1)$ is not minimum but $P(1)$ is the maximum of P.

119. (a) Let $y = x^3 - px + q \Rightarrow \dfrac{dy}{dx} = 3x^2 - p$

For maxima and minima

$\dfrac{dy}{dx} = 0 \Rightarrow 3x^2 - p = 0 \Rightarrow x = \pm\sqrt{\dfrac{p}{3}}$

$\dfrac{d^2y}{dx^2} = 6x \quad \left.\dfrac{d^2y}{dx^2}\right|_{x=\sqrt{\frac{p}{3}}} = +ve$ and $\left.\dfrac{d^2y}{dx^2}\right|_{x=-\sqrt{\frac{p}{3}}} = -ve$

$\therefore y$ has minimum at $x = \sqrt{\dfrac{p}{3}}$ and maximum at

$x = -\sqrt{\dfrac{p}{3}}$

120. **(a)** Given $f(x)=\dfrac{x}{2}+\dfrac{2}{x}\Rightarrow f'(x)=\dfrac{1}{2}-\dfrac{2}{x^2}=0$

$\Rightarrow x^2 = 4 \Rightarrow x = 2, -2;$

Now, $f''(x)=\dfrac{4}{x^3}$

$f''(x)\big]_{x=2}=+ve\Rightarrow f(x)$ has local min at $x = 2$.

121. **(c)** ATQ, $y=x+\dfrac{1}{x}\Rightarrow\dfrac{dy}{dx}=1-\dfrac{1}{x^2}$

For maxima. or minima.,

$1-\dfrac{1}{x^2}=0\Rightarrow x=\pm 1$

$\dfrac{d^2y}{dx^2}=\dfrac{2}{x^3}\Rightarrow\left(\dfrac{d^2y}{dx^2}\right)_{x=1}=2>0$

$\therefore$ y is minimum at $x = 1$

122. **(d)** $f(x)=2x^3-9ax^2+12a^2x+1$

$f'(x)=6x^2-18ax+12a^2;$

For maxima or minima.

$6x^2-18ax+12a^2=0\Rightarrow x^2-3ax+2a^2=0$

$\Rightarrow x = a$ or $x = 2a$.

$f''(x)=12x-18a$

$f''(a) = -6a < 0$ $\therefore f(x)$ is max. at $x = a$,

$f''(2a) = 6a > 0$

$\therefore$ $f(x)$ is min. at $x = 2a$

$\therefore$ $p = a$ and $q = 2a$

ATQ, $p^2 = q$

$\therefore a^2 = 2a \Rightarrow a = 2$ or $a = 0$

but $a > 0$, therefore, $a = 2$.

123. **(b)** We know that distance of origin from

$(x, y)=\sqrt{x^2+y^2}$

$=\sqrt{a^2+b^2-2ab\cos\left(t-\dfrac{at}{b}\right)};$

$\le\sqrt{a^2+b^2+2ab}$

$\left[\left\{\cos\left(t-\dfrac{at}{b}\right)\right\}_{\min}=-1\right]=a+b$

$\therefore$ Maximum distance from origin $= a + b$

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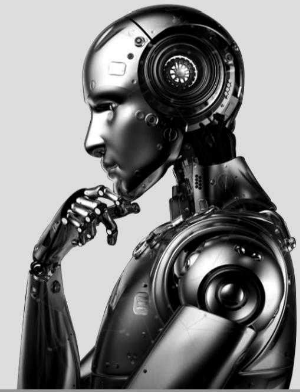


Integrals

TOPIC 1 Standard Integrals, Integration by Substitution, Integration by Parts

1. $\lim_{x\to 1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$ **[Sep. 06, 2020 (I)]**

(a) is equal to $\frac{1}{2}$ (b) is equal to 1

(c) is equal to $-\frac{1}{2}$ (d) does not exist

2. If $\int (e^{2x}+2e^x-e^{-x}-1)e^{(e^x+e^{-x})}\,dx = g(x)e^{(e^x+e^{-x})}+c$, where c is a constant of integeration, then $g(0)$ is equal to: **[Sep. 05, 2020 (I)]**

(a) e (b) e^2 (c) 1 (d) 2

3. If $\int \dfrac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|B(\theta)|+C$, where C is a constant of integration, then $\dfrac{B(\theta)}{A}$ can be : **[Sep. 05, 2020 (II)]**

(a) $\dfrac{2\sin\theta+1}{\sin\theta+3}$ (b) $\dfrac{2\sin\theta+1}{5(\sin\theta+3)}$

(c) $\dfrac{5(\sin\theta+3)}{2\sin\theta+1}$ (d) $\dfrac{5(2\sin\theta+1)}{\sin\theta+3}$

4. The integral $\int\left(\dfrac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to (where C is a constant of integration) : **[Sep. 04, 2020 (I)]**

(a) $\tan x-\dfrac{x\sec x}{x\sin x+\cos x}+C$

(b) $\sec x+\dfrac{x\tan x}{x\sin x+\cos x}+C$

(c) $\sec x-\dfrac{x\tan x}{x\sin x+\cos x}+C$

(d) $\tan x+\dfrac{x\sec x}{x\sin x+\cos x}+C$

5. Let $f(x)=\int\dfrac{\sqrt{x}}{(1+x)^2}dx\ (x\ge 0)$. Then $f(3)-f(1)$ is equal to : **[Sep. 04, 2020 (I)]**

(a) $-\dfrac{\pi}{12}+\dfrac{1}{2}+\dfrac{\sqrt{3}}{4}$ (b) $\dfrac{\pi}{6}+\dfrac{1}{2}-\dfrac{\sqrt{3}}{4}$

(c) $-\dfrac{\pi}{6}+\dfrac{1}{2}+\dfrac{\sqrt{3}}{4}$ (d) $\dfrac{\pi}{12}+\dfrac{1}{2}-\dfrac{\sqrt{3}}{4}$

6. If $\int\sin^{-1}\left(\sqrt{\dfrac{x}{1+x}}\right)dx = A(x)\tan^{-1}(\sqrt{x})+B(x)+C$, where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be : **[Sep. 03, 2020 (II)]**

(a) $(x+1, -\sqrt{x})$ (b) $(x+1, \sqrt{x})$

(c) $(x-1, -\sqrt{x})$ (d) $(x-1, \sqrt{x})$

7. The integral $\int\dfrac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to: (where C is a constant of integration) **[Jan. 9, 2020 (I)]**

(a) $\left(\dfrac{x-3}{x+4}\right)^{1/7}+C$ (b) $-\left(\dfrac{x-3}{x+4}\right)^{-1/7}+C$

(c) $\dfrac{1}{2}\left(\dfrac{x-3}{x+4}\right)^{3/7}+C$ (d) $-\dfrac{1}{13}\left(\dfrac{x-3}{x+4}\right)^{-13/7}+C$

8. If $\int\dfrac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)} = \lambda\tan\theta+2\log_e|f(\theta)|+C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to: **[Jan. 9, 2020 (II)]**

(a) $(1, 1-\tan\theta)$ (b) $(-1, 1-\tan\theta)$

(c) $(-1, 1+\tan\theta)$ (d) $(1, 1+\tan\theta)$

9. If $\int\dfrac{\cos x\,dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda}+c$ where c is a constant of integration, then $\lambda f\left(\dfrac{\pi}{3}\right)$ is equal to: **[Jan. 8, 2020 (II)]**

(a) $-\dfrac{9}{8}$ (b) 2 (c) $\dfrac{9}{8}$ (d) –2

10. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :
(Here C is a constant of integration) **[April 12, 2019 (I)]**

(a) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$ (b) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|}+C$

(c) $\log_e\left|\frac{x^3+1}{x}\right|+C$ (d) $\log_e\frac{|x^3+1|}{x^2}+C$

11. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B(x) are respectively : **[April 12, 2019 (II)]**

(a) $x+\alpha$ and $\log_e|\sin(x+\alpha)|$

(b) $x-\alpha$ and $\log_e|\sin(x-\alpha)|$

(c) $x-\alpha$ and $\log_e|\cos(x-\alpha)|$

(d) $x+\alpha$ and $\log_e|\sin(x-\alpha)|$

12. If $\int \frac{dx}{(x^2-2x+10)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$ where C is a constant of integration, then : **[April 10, 2019 (I)]**

(a) $A=\frac{1}{54}$ and $f(x)=3(x-1)$

(b) $A=\frac{1}{81}$ and $f(x)=3(x-1)$

(c) $A=\frac{1}{27}$ and $f(x)=9(x-1)$

(d) $A=\frac{1}{54}$ and $f(x)=9(x-1)^2$

13. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set $S = \{x \in R : f(x) = f(0)\}$ contains exactly: **[April 09, 2019 (I)]**
(a) four irrational numbers.
(b) four rational numbers.
(c) two irrational and two rational numbers.
(d) two irrational and one rational number.

14. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to: **[April 09, 2019 (I)]**

(a) $-3\tan^{-1/3}x+C$ (b) $-\frac{3}{4}\tan^{-4/3}x+C$

(c) $-3\cot^{-1/3}x+C$ (d) $3\tan^{-1/3}x+C$
(Here C is a constant of integration)

15. If $\int e^{\sec x}(\sec x\tan x\, f(x)+(\sec x\tan x+\sec^2 x))\,dx$ $= e^{\sec x}f(x)+C$, then a possible choice of $f(x)$ is: **[April 09, 2019 (II)]**

(a) $\sec x+\tan x+\frac{1}{2}$ (b) $\sec x-\tan x-\frac{1}{2}$

(c) $\sec x+x\tan x-\frac{1}{2}$ (d) $x\sec x+\tan x+\frac{1}{2}$

16. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :
(where c is a constant of integration.) **[April 08, 2019 (I)]**
(a) $2x+\sin x+2\sin 2x+c$ (b) $x+2\sin x+2\sin 2x+c$
(c) $x+2\sin x+\sin 2x+c$ (d) $2x+\sin x+\sin 2x+c$

17. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}}+C$, where C is a constant of integration, then the function $f(x)$ is equal to : **[April 08, 2019 (II)]**

(a) $\frac{3}{x^2}$ (b) $-\frac{1}{6x^3}$ (c) $-\frac{1}{2x^2}$ (d) $-\frac{1}{2x^3}$

18. The integral $\int \cos(\log_e x)dx$ is equal to :
(where C is a constant of integration) **[Jan. 12, 2019 (I)]**

(a) $\frac{x}{2}\left[\sin(\log_e x)-\cos(\log_e x)\right]+C$

(b) $x\left[\cos(\log_e x)+\sin(\log_e x)\right]+C$

(c) $\frac{x}{2}\left[\cos(\log_e x)+\sin(\log_e x)\right]+C$

(d) $x\left[\cos(\log_e x)-\sin(\log_e x)\right]+C$

19. The integral $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to:
(where C is a constant of integration) **[Jan. 12, 2019 (II)]**

(a) $\frac{x^4}{6(2x^4+3x^2+1)^3}+C$ (b) $\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

(c) $\frac{x^4}{(2x^4+3x^2+1)^3}+C$ (d) $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

20. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m+C$, for a suitable chosen integer m and a function A (x), where C is a constant of integration, then $(A(x))^m$ equals : **[Jan. 11, 2019 (I)]**

(a) $\frac{-1}{27x^9}$ (b) $\frac{-1}{3x^3}$ (c) $\frac{1}{27x^6}$ (d) $\frac{1}{9x^4}$

21. If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then $f(x)$ is equal to: **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{3}(x+1)$ (b) $\frac{2}{3}(x+2)$

(c) $\frac{2}{3}(x-4)$ (d) $\frac{1}{3}(x+4)$

22. Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$

Then $\int \frac{(\sin^n \theta + \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is equal to: **[Jan 10, 2019(I)]**

(a) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

(b) $\frac{n}{n^2+1}\left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

(c) $\frac{n}{n^2-1}\left(1 + \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

(d) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n+1} \theta}\right)^{\frac{n+1}{n}} + C$

(where C is a constant of integration)

23. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral **[Jan. 09, 2019(I)]**

$\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}} dx$ is equal to:

(a) $\log_e \left|\frac{1}{2}\sec^2(x^2-1)\right| + c$

(b) $\frac{1}{2}\log_e |\sec(x^2-1)| + c$

(c) $\frac{1}{2}\log_e \left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

(d) $\log_e \left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$

(where c is a constant of integration)

24. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2+1+2x^7)^2} dx, (x \geq 0)$, and $f(0) = 0$, then the value of $f(1)$ is: **[Jan. 09, 2019 (II)]**

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

25. The integral

$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to: **[2018]**

(a) $\frac{-1}{3(1+\tan^3 x)} + C$ (b) $\frac{1}{1+\cot^3 x} + C$

(c) $\frac{-1}{1+\cot^3 x} + C$ (d) $\frac{1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

26. If

$\int \frac{\tan x}{1+\tan x+\tan^2 x} dx = x - \frac{K}{\sqrt{A}}\tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right) + C,$

(C is a constant of integration), then the ordered pair (K, A) is euqal to **[Online April 16, 2018]**

(a) (2, 3) (b) (2, 1) (c) (–2, 1) (d) (–2, 3)

27. If $f\left(\frac{x-4}{x+2}\right) = 2x+1$, $(x \in R = \{1, -2\})$, then $\int f(x)\,dx$ is equal to (where C is a constant of integration) **[Online April 15, 2018]**

(a) $12\log_e|1-x| - 3x + c$
(b) $-12\log_e|1-x| - 3x + c$
(c) $-12\log_e|1-x| + 3x + c$
(d) $12\log_e|1-x| + 3x + c$

28. $\int \frac{2x+5}{\sqrt{7-6x-x^2}} dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$

(where C is a constant of integration), then the ordered pair (A, B) is equal to **[Online April 15, 2018]**

(a) (–2, –1) (b) (2, –1)
(c) (–2, 1) (d) (2, 1)

29. If $f\left(\frac{3x-4}{3x+4}\right) = x+2$, $x \neq -\frac{4}{3}$, and

$\int f(x)\,dx = A\log|1-x| + Bx + C$, then the ordered pair (A, B) is equal to : **[Online April 9, 2017]**

(where C is a constant of integration)

(a) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (b) $\left(-\frac{8}{3}, \frac{2}{3}\right)$

(c) $\left(-\frac{8}{3}, -\frac{2}{3}\right)$ (d) $\left(\frac{8}{3}, -\frac{2}{3}\right)$

30. The integral $\int \sqrt{1+2\cot x(\operatorname{cosec} x + \cot x)}\,dx$

$\left(0 < x < \frac{\pi}{2}\right)$ is equal to : **[Online April 8, 2017]**

(where C is a constant of integration)

(a) $2\log\left|\sin\frac{x}{2}\right| + C$ (b) $4\log\left|\sin\frac{x}{2}\right| + C$

(c) $2\log\left|\cos\frac{x}{2}\right| + C$ (d) $4\log\left|\cos\frac{x}{2}\right| + C$

31. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then A + B + C equals : **[Online April 9, 2016]**

(a) $\frac{16}{5}$ (b) $\frac{27}{10}$ (c) $\frac{7}{10}$ (d) $\frac{21}{5}$

32. If $\int \frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}}dt = \frac{1}{2}(g(t))^2 + C$, where C is a constant, then g(b) is equal to : **[Online April 11, 2015]**

(a) $\frac{1}{\sqrt{5}}\log(2+\sqrt{5})$ (b) $\frac{1}{2}\log(2+\sqrt{5})$

(c) $2\log(2+\sqrt{5})$ (d) $\log(2+\sqrt{5})$

33. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to **[2014]**

(a) $(x+1)e^{x+\frac{1}{x}}+c$ (b) $-xe^{x+\frac{1}{x}}+c$

(c) $(x-1)e^{x+\frac{1}{x}}+c$ (d) $xe^{x+\frac{1}{x}}+c$

34. The integral $\int \frac{\sin^2 x\cos^2 x}{\left(\sin^3 x+\cos^3 x\right)^2}dx$ is equal to: **[Online April 12, 2014]**

(a) $\frac{1}{\left(1+\cot^3 x\right)}+c$ (b) $-\frac{1}{3\left(1+\tan^3 x\right)}+c$

(c) $\frac{\sin^3 x}{\left(1+\cos^3 x\right)}+c$ (d) $-\frac{\cos^3 x}{3\left(1+\sin^3 x\right)}+c$

35. The integral $\int x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx\ (x>0)$ is equal to: **[Online April 11, 2014]**

(a) $-x+(1+x^2)\tan^{-1}x+c$
(b) $x-(1+x^2)\cot^{-1}x+c$
(c) $-x+(1+x^2)\cot^{-1}x+c$
(d) $x-(1+x^2)\tan^{-1}x+c$

36. $\int \frac{\sin^8 x-\cos^8 x}{\left(1-2\sin^2 x\cos^2 x\right)}dx$ is equal to: **[Online April 9, 2014]**

(a) $\frac{1}{2}\sin 2x+c$ (b) $-\frac{1}{2}\sin 2x+c$

(c) $-\frac{1}{2}\sin x+c$ (d) $-\sin^2 x+c$

37. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to **[2013]**

(a) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$

(b) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(c) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

(d) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

38. If the integral

$$\int \frac{\cos 8x+1}{\cot 2x-\tan 2x}dx = A\cos 8x+k,$$

where k is an arbitrary constant, then A is equal to : **[Online April 25, 2013]**

(a) $-\frac{1}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

39. If the $\int \frac{5\tan x}{\tan x-2}dx = x + a\ln|\sin x-2\cos x|+k$, then a is equal to : **[2012]**

(a) -1 (b) -2 (c) 1 (d) 2

40. If $f(x) = \int\left(\frac{x^2+\sin^2 x}{1+x^2}\right)\sec^2 x\,dx$ and $f(0)=0$, then $f(1)$ equals **[Online May 19, 2012]**

(a) $\tan 1-\frac{\pi}{4}$ (b) $\tan 1+1$

(c) $\frac{\pi}{4}$ (d) $1-\frac{\pi}{4}$

41. The integral of $\frac{x^2-x}{x^3-x^2+x-1}$ w.r.t. x is **[Online May 12, 2012]**

(a) $\frac{1}{2}\log\left(x^2+1\right)+C$ (b) $\frac{1}{2}\log\left|x^2-1\right|+C$

(c) $\log\left(x^2+1\right)+C$ (d) $\log\left|x^2-1\right|+C$

42. Let $f(x)$ be an indefinite integral of $\cos^3 x$.
Statement 1: $f(x)$ is a periodic function of period π.
Statement 2: $\cos^3 x$ is a periodic function. **[Online May 7, 2012]**

(a) Statement 1 is true, Statement 2 is false.
(b) Both the Statements are true, but Statement 2 is not the correct explanation of Statement 1.
(c) Both the Statements are true, and Statement 2 is correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true.

43. The value of $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is **[2008]**

(a) $x+\log|\cos\left(x-\frac{\pi}{4}\right)|+c$

(b) $x-\log|\sin\left(x-\frac{\pi}{4}\right)|+c$

(c) $x+\log|\sin\left(x-\frac{\pi}{4}\right)|+c$

(d) $x-\log|\cos\left(x-\frac{\pi}{4}\right)|+c$

44. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ equals **[2007]**

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

45. $\int \frac{dx}{\cos x-\sin x}$ is equal to **[2004]**

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

46. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax+B\log\sin(x-\alpha),+C,$ then value of (A,B) is **[2004]**

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

47. $f(x)$ and $g(x)$ are two differentiable functions on $[0, 2]$ such that $f''(x)-g''(x)=0,\ f'(1)=2g'(1)=4f(2)=3g(2)=9$ then $f(x)-g(x)$ at $x=3/2$ is **[2002]**

(a) 0 (b) 2 (c) 10 (d) 5

TOPIC 2 Integration of the Forms: $\int e^x(f(x)+f'(x))dx$, $\int e^{kx}(df(x)+f'(x))dx$, Integration by Partial Fractions, Integration of Some Special Irrational Algebraic Functions, Integration of Different Expressions of e^x

48. The integral $\int_1^2 e^x \cdot x^x(2+\log_e x)\,dx$ equals: **[Sep. 06, 2020 (II)]**

(a) $e(4e+1)$ (b) $4e^2-1$

(c) $e(4e-1)$ (d) $e(2e-1)$

49. A value of α such that

$$\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$$ is : **[April 12, 2019 (II)]**

(a) -2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 2

50. If $\int x^5 e^{-x^2}dx = g(x)e^{-x^2}+c$, where c is a constant of integration, then $g(-1)$ is equal to : **[April 10, 2019 (II)]**

(a) -1 (b) 1 (c) $-\frac{5}{2}$ (d) $-\frac{1}{2}$

51. If $\int x^5 e^{-4x^3}dx=\frac{1}{48}e^{-4x^3}f(x)+C$, where C is a constant of integration, then $f(x)$ is equal to: **[Jan. 10, 2019 (II)]**

(a) $-2x^3-1$ (b) $-4x^3-1$

(c) $-2x^3+1$ (d) $4x^3+1$

52. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to :

(where C is a constant of integration) **[Online April 10, 2016]**

(a) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$ (b) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(c) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

53. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals : **[2015]**

(a) $-(x^4+1)^{\frac{1}{4}}+c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (d) $(x^4+1)^{\frac{1}{4}}+c$

54. The integral $\int \frac{dx}{(x+1)^{\frac{3}{4}}(x-2)^{\frac{5}{4}}}$ is equal to : **[Online April 10, 2015]**

(a) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}}+C$ (b) $4\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}}+C$

(c) $4\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}}+C$ (d) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}}+C$

55. If m is a non-zero number and

$$\int \frac{x^{5m-1}+2x^{4m-1}}{\left(x^{2m}+x^{m}+1\right)^{3}}dx = f(x)+c,$$

then f(x) is: **[Online April 19, 2014]**

(a) $\frac{x^{5m}}{2m\left(x^{2m}+x^{m}+1\right)^{2}}$ (b) $\frac{x^{4m}}{2m\left(x^{2m}+x^{m}+1\right)^{2}}$

(c) $\frac{2m\left(x^{5m}+x^{4m}\right)}{\left(x^{2m}+x^{m}+1\right)^{2}}$ (d) $\frac{\left(x^{5m}-x^{4m}\right)}{2m\left(x^{2m}+x^{m}+1\right)^{2}}$

56. The integral $\int \frac{x\,dx}{2-x^2+\sqrt{2-x^2}}$ equals : **[Online April 23, 2013]**

(a) $\log\left|1+\sqrt{2+x^2}\right|+c$ (b) $-\log\left|1+\sqrt{2-x^2}\right|+c$

(c) $-x\log\left|1-\sqrt{2-x^2}\right|+c$ (d) $x\log\left|1-\sqrt{2+x^2}\right|+c$

57. If $\int \frac{x^2-x+1}{x^2+1}e^{\cot^{-1}x}dx = A(x)e^{\cot^{-1}x}+C$, then $A(x)$ is equal to : **[Online April 22, 2013]**

(a) $-x$ (b) x (c) $\sqrt{1-x}$ (d) $\sqrt{1+x}$

58. If $\int \frac{dx}{x+x^7} = p(x)$ then, $\int \frac{x^6}{x+x^7}dx$ is equal to: **[Online April 9, 2013]**

(a) $\ln|x|-p(x)+c$ (b) $\ln|x|+p(x)+c$
(c) $x-p(x)+c$ (d) $x+p(x)+c$

59. $\int \left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to **[2005]**

(a) $\frac{\log x}{(\log x)^2+1}+C$ (b) $\frac{x}{x^2+1}+C$

(c) $\frac{xe^x}{1+x^2}+C$ (d) $\frac{x}{(\log x)^2+1}+C$

TOPIC 3 Evaluation of Definite Integral by Substitution, Properties of Definite Integrals

60. If $I_1 = \int_0^1(1-x^{50})^{100}dx$ and $I_2 = \int_0^1(1-x^{50})^{101}dx$ such that $I_2 = \alpha I_1$ then α equals to : **[Sep. 06, 2020 (I)]**

(a) $\frac{5049}{5050}$ (b) $\frac{5050}{5049}$ (c) $\frac{5050}{5051}$ (d) $\frac{5051}{5050}$

61. The value of $\int_{-\pi/2}^{\pi/2}\frac{1}{1+e^{\sin x}}dx$ is: **[Sep. 05, 2020 (I)]**

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

62. Let $f(x)=|x-2|$ and $g(x)=f(f(x))$, $x\in[0,4]$.

Then $\int_0^3(g(x)-f(x))\,dx$ is equal to : **[Sep. 04, 2020 (I)]**

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

63. The integral

$$\int_{\pi/6}^{\pi/3}\tan^3 x\cdot\sin^2 3x(2\sec^2 x\cdot\sin^2 3x+3\tan x\cdot\sin 6x)\,dx$$

is equal to : **[Sep. 04, 2020 (II)]**

(a) $\frac{7}{18}$ (b) $-\frac{1}{9}$ (c) $-\frac{1}{18}$ (d) $\frac{9}{2}$

64. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\le x$ respectively of a real number x. If $\int_0^n\{x\}\,dx$, $\int_0^n[x]\,dx$ and $10(n^2-n)$, $(n\in\mathbf{N}, n>1)$ are three consecutive terms of a G.P., then n is equal to ________________. **[NA Sep. 04, 2020 (II)]**

65. $\int_{-\pi}^{\pi}|\pi-|x||\,dx$ is equal to : **[Sep. 03, 2020 (I)]**

(a) $\sqrt{2}\pi^2$ (b) $2\pi^2$ (c) π^2 (d) $\frac{\pi^2}{2}$

66. If the value of the integral $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to : **[Sep. 03, 2020 (II)]**

(a) $2\sqrt{3}-\pi$ (b) $2\sqrt{3}+\pi$
(c) $3\sqrt{2}+\pi$ (d) $3\sqrt{2}-\pi$

67. The integral $\int_0^2\left||x-1|-x\right|dx$ is equal to ________. **[NA Sep. 02, 2020 (I)]**

68. Let $[t]$ denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is ___________.

[NA Sep. 02, 2020 (II)]

69. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x)dx$ is equal to: **[Jan. 9, 2020 (I)]**

(a) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$ (b) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

(c) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$ (d) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

70. The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ is equal to:

[Jan. 9, 2020 (I)]

(a) 2π (b) $2\pi^2$ (c) π^2 (d) 4π

71. If $I=\int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then: **[Jan. 8, 2020 (II)]**

(a) $\frac{1}{8}<I^2<\frac{1}{4}$ (b) $\frac{1}{9}<I^2<\frac{1}{8}$

(c) $\frac{1}{16}<I^2<\frac{1}{9}$ (d) $\frac{1}{6}<I^2<\frac{1}{2}$

72. If $f(a+b+1-x)=f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x)+f(x+1))dx$ is equal to: **[Jan. 7, 2020 (I)]**

(a) $\int_{a+1}^{b+1} f(x)dx$ (b) $\int_{a-1}^{b-1} f(x)dx$

(c) $\int_{a-1}^{b-1} f(x+1)dx$ (d) $\int_{a+1}^{b+1} f(x+1)dx$

73. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx=5$, is:

[Jan. 7, 2020 (II)]

(a) $\log_e 2$ (b) $\log_e\left(\frac{3}{2}\right)$ (c) $\log_e\sqrt{2}$ (d) $\log_e\left(\frac{4}{3}\right)$

74. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0,2\pi)-\{\pi\}$ which satisfy the equation, $2\cot^2\theta-\frac{5}{\sin\theta}+4=0$, then $\int_{\theta_1}^{\theta_2}\cos^2 3\theta\,d\theta$ is equal to:

[Jan. 7, 2020 (II)]

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3}+\frac{1}{6}$ (d) $\frac{\pi}{9}$

75. Let $f: R \to R$ be a continuously differentiable function such that $f(2)=6$ and $f'(2)=\frac{1}{48}$.

If $\int_6^{f(x)} 4t^3 dt=(x-2)\,g(x)$, then $\lim_{x\to 2} g(x)$ is equal to:

[April 12, 2019 (I)]

(a) 18 (b) 24 (c) 12 (d) 36

76. If $\int_0^{\frac{x}{2}} \frac{\cot x}{\cot x+\operatorname{cosec} x}dx=m(\pi+n)$, then $m.n$ is equal to:

[April 12, 2019 (I)]

(a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) -1

77. The value of $\int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$, where [t] denotes the greatest integer function, is: **[April 10, 2019 (I)]**

(a) π (b) $-\pi$ (c) -2π (d) 2π

78. The integral $\int_{\pi/6}^{\pi/3}\sec^{2/3}x\operatorname{cosec}^{4/3}x\;dx$ is equal to:

[April 10, 2019 (II)]

(a) $3^{5/6}-3^{2/3}$ (b) $3^{4/3}-3^{1/3}$

(c) $3^{7/6}-3^{5/6}$ (d) $3^{5/3}-3^{1/3}$

79. The value of $\int_0^{\pi/2}\frac{\sin^3 x}{\sin x+\cos x}dx$ is: **[April 9, 2019 (I)]**

(a) $\frac{\pi-2}{8}$ (b) $\frac{\pi-1}{4}$ (c) $\frac{\pi-2}{4}$ (d) $\frac{\pi-1}{2}$

80. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is:

[April 09, 2019 (II)]

(a) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$ (b) $\frac{\pi}{4}-\log_e 2$

(c) $\frac{\pi}{2}-\log_e 2$ (d) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$

81. If $f: R\to R$ is a differentiable function and $f(2)=6$, then $\lim_{x\to 2}\int_6^{f(x)}\frac{2t\,dt}{(x-2)}$ is: **[April 09, 2019 (II)]**

(a) $24f'(2)$ (b) $2f'(2)$ (c) 0 (d) $12f'(2)$

82. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and $g(x)=\log_e x$, $(x>0)$ then the value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))dx$ is:

[April 8, 2019 (I)]

(a) $\log_e 3$ (b) $\log_e e$ (c) $\log_e 2$ (d) $\log_e 1$

83. Let $f(x)=\int_0^x g(t)\,dt$, where g is a non-zero even function. If $f(x+5)=g(x)$, then $\int_0^x f(t)\,dt$ equals : **[April 08, 2019 (II)]**

(a) $\int_{x+5}^{5} g(t)\,dt$ (b) $\int_{5}^{x+5} g(t)\,dt$

(c) $2\int_{5}^{x+5} g(t)\,dt$ (d) $5\int_{x+5}^{5} g(t)\,dt$

84. Let f and g be continuous functions on [0, a] such that $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$, then $\int_0^a f(x)g(x)\,dx$ is equal to: **[Jan. 12, 2019 (I)]**

(a) $4\int_0^a f(x)\,dx$ (b) $\int_0^a f(x)\,dx$

(c) $2\int_0^a f(x)\,dx$ (d) $-3\int_0^a f(x)\,dx$

85. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{x}\right\}\log_e x\,dx$ is equal to : **[Jan. 12, 2019 (II)]**

(a) $\frac{1}{2}-e-\frac{1}{e^2}$ (b) $-\frac{1}{2}+\frac{1}{e}-\frac{1}{2e^2}$

(c) $\frac{3}{2}-\frac{1}{e}-\frac{1}{2e^2}$ (d) $\frac{3}{2}-e-\frac{1}{2e^2}$

86. The value of the integral $\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}\,dx$ (where [x] denotes the greatest integer less than or equal to x) is : **[Jan. 11, 2019 (I)]**

(a) 0 (b) sin 4 (c) 4 (d) 4 – sin 4

87. The integral $\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$ equals : **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$ (b) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(c) $\frac{\pi}{40}$ (d) $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

88. Let $I=\int_a^b (x^4-2x^2)\,dx$. If I is minimum then the ordered pair (a, b) is: **[Jan 10, 2019 (I)]**

(a) $\left(0,\sqrt{2}\right)$ (b) $\left(-\sqrt{2},0\right)$

(c) $\left(\sqrt{2},-\sqrt{2}\right)$ (d) $\left(-\sqrt{2},\sqrt{2}\right)$

89. If $\int_0^x f(t)\,dt = x^2+\int_x^1 t^2 f(t)\,dt$, then $f'(1/2)$ is: **[Jan. 10, 2019 (II)]**

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{4}{5}$ (d) $\frac{6}{25}$

90. The value of $\int_{-\pi/2}^{\pi/2}\frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is: **[Jan. 10, 2019 (II)]**

(a) $\frac{1}{12}(7\pi+5)$ (b) $\frac{1}{12}(7\pi-5)$

(c) $\frac{3}{20}(4\pi-3)$ (d) $\frac{3}{10}(4\pi-3)$

91. The value of $\int_0^{\pi}|\cos x|^3\,dx$ is: **[Jan 9, 2019 (I)]**

(a) 0 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

92. Let f be a differentiable function from **R** to **R** such that $|f(x)-f(y)|\le 2|x-y|^{3/2}$, for all $x, y, \in$ **R**. If $f(0)=1$ then $\int_0^1 f^2(x)\,dx$ is equal to : **[Jan. 09, 2019 (II)]**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

93. If $\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{2k\sec\theta}}\,d\theta = 1-\frac{1}{\sqrt{2}},(k>0)$ then the value of k is: **[Jan. 09, 2019 (II)]**

(a) 4 (b) $\frac{1}{2}$ (c) 1 (d) 2

94. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x}{1+2^x}\,dx$ is : **[2018]**

(a) $\frac{\pi}{2}$ (b) 4π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

95. If $f(x)=\int_0^x t(\sin x-\sin t)\,dt$ then **[Online April 16, 2018]**

(a) $f'''(x)+f'(x)=\cos x-2x\sin x$
(b) $f'''(x)+f''(x)-f'(x)=\cos x$
(c) $f'''(x)-f''(x)=\cos x-2x\sin x$
(d) $f'''(x)+f''(x)=\sin x$

96. The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$ is **[Online April 15, 2018]**

(a) $\frac{\pi}{2}(\sqrt{2}+1)$ (b) $\pi(\sqrt{2}-1)$

(c) $2\pi(\sqrt{2}-1)$ (d) $\pi\sqrt{2}$

97. If $I_1=\int_0^1 e^{-x}\cos^2 x\,dx$; $I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$ and $I_3=\int_0^1 e^{-x^3} dx$; then **[Online April 15, 2018]**

(a) $I_2>I_3>I_1$ (b) $I_3>I_1>I_2$

(c) $I_2>I_1>I_3$ (d) $I_3>I_2>I_1$

98. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx$ is **[Online April 15, 2018]**

(a) $\frac{3}{16}\pi$ (b) 0 (c) $\frac{3}{8}\pi$ (d) $\frac{3}{4}$

99. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to : **[2017]**

(a) –1 (b) –2 (c) 2 (d) 4

100. Let $I_n=\int \tan^n x\,dx, (n>1)$. $I_4+I_6=a\tan^5 x+bx^5+C$, where C is constant of integration, then the ordered pair (a, b) is equal to : **[2017]**

(a) $\left(-\frac{1}{5},0\right)$ (b) $\left(-\frac{1}{5},1\right)$ (c) $\left(\frac{1}{5},0\right)$ (d) $\left(\frac{1}{5},-1\right)$

101. If $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}}=\frac{k}{k+5}$ then k is equal to : **[Online April 9, 2017]**

(a) 1 (b) 2 (c) 3 (d) 4

102. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x+\cot x)^3}dx$ equals : **[Online April 8, 2017]**

(a) $\frac{15}{128}$ (b) $\frac{15}{64}$ (c) $\frac{13}{32}$ (d) $\frac{15}{256}$

103. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to : **[2016]**

(a) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$ (b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$ (d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

104. For $x\in R, x\neq 0$, if y(x) is a differentiable function such that $x\int_1^x y(t)dt=(x+1)\int_1^x ty(t)dt$, then y(x) equals : (where C is a constant) **[Online April 10, 2016]**

(a) $Cx^3 e^{\frac{1}{x}}$ (b) $\frac{C}{x^2}e^{-\frac{1}{x}}$ (c) $\frac{C}{x}e^{-\frac{1}{x}}$ (d) $\frac{C}{x^3}e^{-\frac{1}{x}}$

105. The value of the integral

$$\int_4^{10} \frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$$, where [x] denotes the greatest integer less than or equal to x, is : **[Online April 10, 2016]**

(a) $\frac{1}{3}$ (b) 6 (c) 7 (d) 3

106. If $2\int_0^1 \tan^{-1}x\,dx=\int_0^1 \cot^{-1}(1-x+x^2)dx$, then $\int_0^1 \tan^{-1}(1-x+x^2)dx$ is equal to : **[Online April 9, 2016]**

(a) $\frac{\pi}{2}+\log 2$ (b) log2

(c) $\frac{\pi}{2}-\log 4$ (d) log4

107. The integral **[2015]**

$$\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$$ is equal to :

(a) 1 (b) 6 (c) 2 (d) 4

108. Let f : R → R be a function such that $f(2-x)=f(2+x)$ and $f(4-x)=f(4+x)$, for all $x\in R$ and $\int_0^2 f(x)\,dx=5$. Then the value of $\int_{10}^{50} f(x)\,dx$ is : **[Online April 11, 2015]**

(a) 125 (b) 80 (c) 100 (d) 200

109. Let f : (–1, 1) → R be a continuous function. If $\int_0^{\sin x} f(t)dt=\frac{\sqrt{3}}{2}x$, then $f\left(\frac{\sqrt{3}}{2}\right)$ is equal to : **[Online April 11, 2015]**

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{3}$

110. For $x>0$, let $f(x)=\int_1^x \frac{\log t}{1+t}dt$. Then $f(x)+f\left(\frac{1}{x}\right)$ is equal to: **[Online April 10, 2015]**

(a) $\frac{1}{4}(\log x)^2$ (b) $\log x$

(c) $\frac{1}{2}(\log x)^2$ (d) $\frac{1}{4}\log x^2$

111. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals: **[2014]**

(a) $4\sqrt{3}-4$ (b) $4\sqrt{3}-4-\frac{\pi}{3}$

(c) $\pi-4$ (d) $\frac{2\pi}{3}-4-4\sqrt{3}$

112. Let function F be defined as $F(x)=\int_1^x \frac{e^t}{t}dt, x>0$ then the value of the integral $\int_1^x \frac{e^t}{t+a}dt$, where $a>0$, is: **[Online April 19, 2014]**

(a) $e^a\left[F(x)-F(1+a)\right]$

(b) $e^{-a}\left[F(x+a)-F(a)\right]$

(c) $e^a\left[F(x+a)-F(1+a)\right]$

(d) $e^{-a}\left[F(x+a)-F(1+a)\right]$

113. If for a continuous function f(x), $\int_{-\pi}^{t}(f(x)+x)dx=\pi^2-t^2$, for all $t\geq-\pi$, then $f\left(-\frac{\pi}{3}\right)$ is equal to: **[Online April 12, 2014]**

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

114. If [ ] denotes the greatest integer function, then the integral $\int_0^{\pi}[\cos x]dx$ is equal to: **[Online April 12, 2014]**

(a) $\frac{\pi}{2}$ (b) 0 (c) -1 (d) $-\frac{\pi}{2}$

115. If for $n\geq1$, $P_n=\int_1^e(\log x)^n dx$, then $P_{10}-90P_8$ is equal to: **[Online April 11, 2014]**

(a) -9 (b) 10e (c) -9 e (d) 10

116. The integral $\int_0^{\frac{1}{2}}\frac{\ln(1+2x)}{1+4x^2}dx$, equals: **[Online April 9, 2014]**

(a) $\frac{\pi}{4}\ln2$ (b) $\frac{\pi}{8}\ln2$ (c) $\frac{\pi}{16}\ln2$ (d) $\frac{\pi}{32}\ln2$

117. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x|t|\,dt, x\in\mathbb{R}$, which are parallel to the line $y=2x$, are equal to : **[2013]**

(a) ±1 (b) ±2 (c) ±3 (d) ±4

118. **Statement-1 :** The value of the integral $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$

Statement-2 : $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx.$ **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

119. For $0\leq x\leq\frac{\pi}{2}$, the value of

$\int_0^{\sin^2 x}\sin^{-1}(\sqrt{t})dt+\int_0^{\cos^2 x}\cos^{-1}(\sqrt{t})\,dt$ equals : **[Online April 25, 2013]**

(a) $\frac{\pi}{4}$ (b) 0 (c) 1 (d) $-\frac{\pi}{4}$

120. The value of $\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$ is : **[Online April 23, 2013]**

(a) π (b) $\frac{\pi}{2}$ (c) 4π (d) $\frac{\pi}{4}$

121. The integral $\int_{7\pi/4}^{7\pi/3}\sqrt{\tan^2 x}\,dx$ is equal to : **[Online April 22, 2013]**

(a) $\log 2\sqrt{2}$ (b) $\log 2$

(c) $2\log 2$ (d) $\log\sqrt{2}$

122. If $x=\int_0^y\frac{dt}{\sqrt{1+t^2}}$, then $\frac{d^2y}{dx^2}$ is equal to : **[Online April 9, 2013]**

(a) y (b) $\sqrt{1+y^2}$

(c) $\frac{x}{\sqrt{1+y^2}}$ (d) y^2

123. If $g(x)=\int_0^x\cos 4t\,dt$, then $g(x+\pi)$ equals **[2012]**

(a) $\frac{g(x)}{g(\pi)}$ (b) $g(x)+g(\pi)$

(c) $g(x)-g(\pi)$ (d) $g(x).g(\pi)$

124. If $[x]$ is the greatest integer $\le x$, then the value of the integral $\int_{-0.9}^{0.9}\left(\left[x^2\right]+\log\left(\frac{2-x}{2+x}\right)\right)dx$ is **[Online May 26, 2012]**

(a) 0.486 (b) 0.243 (c) 1.8 (d) 0

125. The value of the integral $\int_0^{0.9}\left[x-2[x]\right]dx$, where [.] denotes the greatest integer function is **[Online May 19, 2012]**

(a) 0.9 (b) 1.8 (c) -0.9 (d) 0

126. If $\frac{d}{dx}G(x)=\frac{e^{\tan x}}{x}$, $x\in(0,\pi/2)$, then $\int_{1/4}^{1/2}\frac{2}{x}.e^{\tan(\pi x^2)}dx$ is equal to **[Online May 12, 2012]**

(a) $G(\pi/4)-G(\pi/16)$ (b) $2[G(\pi/4)-G(\pi/16)]$

(c) $\pi[G(1/2)-G(1/4)]$ (d) $G(1/\sqrt{2})-G(1/2)$

127. If $\int_e^x t\,f(t)\,dt=\sin x-x\cos x-\frac{x^2}{2}$, for all $x\in R-\{0\}$, then the value of $f\left(\frac{\pi}{6}\right)$ is **[Online May 7, 2012]**

(a) 1/2 (b) 1 (c) 0 (d) $-1/2$

128. Let [.] denote the greatest integer function then the value of $\int_0^{1.5}x\left[x^2\right]dx$ is :. **[2011 RS]**

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$

129. The value of $\int_0^1\frac{8\log(1+x)}{1+x^2}dx$ is **[2011]**

(a) $\frac{\pi}{8}\log 2$ (b) $\frac{\pi}{2}\log 2$ (c) $\log 2$ (d) $\pi\log 2$

130. Let $p(x)$ be a function defined on **R** such that $p'(x)=p'(1-x)$, for all $x\in[0,1]$, $p(0)=1$ and $p(1)=41$. Then $\int_0^1 p(x)\,dx$ equals **[2010]**

(a) 21 (b) 41 (c) 42 (d) $\sqrt{41}$

131. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to : **[2009]**

(a) 1 (b) -1 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

132. Let $I=\int_0^1\frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true? **[2008]**

(a) $I>\frac{2}{3}$ and $J>2$ (b) $I<\frac{2}{3}$ and $J<2$

(c) $I<\frac{2}{3}$ and $J>2$ (d) $I>\frac{2}{3}$ and $J<2$

133. The solution for x of the equation $\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{2}$ is **[2007]**

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$

(c) 2 (d) None of these

134. Let $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where $f(x)=\int_l^x\frac{\log t}{1+t}dt$, Then $F(e)$ equals **[2007]**

(a) 1 (b) 2 (c) 1/2 (d) 0,

135. The value of $\int_1^a[x]f'(x)dx$, $a>1$ where $[x]$ denotes the greatest integer not exceeding x is **[2006]**

(a) $af(a)-\{f(1)+f(2)+.............f([a])\}$

(b) $[a]f(a)-\{f(1)+f(2)+.............f([a])\}$

(c) $[a]f([a])-\{f(1)+f(2)+.............f(a)\}$

(d) $af([a])-\{f(1)+f(2)+.............f(a)\}$

136. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}}[(x+\pi)^3+\cos^2(x+3\pi)]dx$ is equal to **[2006]**

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}-1$

137. $\int_0^{\pi}xf(\sin x)dx$ is equal to **[2006]**

(a) $\pi\int_0^{\pi}f(\cos x)dx$ (b) $\pi\int_0^{\pi}f(\sin x)dx$

(c) $\frac{\pi}{2}\int_0^{\pi/2}f(\sin x)dx$ (d) $\pi\int_0^{\pi/2}f(\cos x)dx$

138. The value of integral, $\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is **[2006]**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 1

139. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0, is **[2005]**

(a) $a\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{a}$ (d) 2π

140. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then **[2005]**

(a) $I_2 > I_1$ (b) $I_1 > I_2$ (c) $I_3 = I_4$ (d) $I_3 > I_4$

141. Let $f: R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals **[2005]**

(a) 24 (b) 36 (c) 12 (d) 18

142. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$

and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$,

then the value of $\frac{I_2}{I_1}$ is **[2004]**

(a) 1 (b) –3 (c) –1 (d) 2

143. If $\int_0^{\pi} xf(\sin x)dx = A \int_0^{\pi/2} f(\sin x)dx$, then A is **[2004]**

(a) 2π (b) π (c) $\frac{\pi}{4}$ (d) 0

144. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ is **[2004]**

(a) 3 (b) 1 (c) 2 (d) 0

145. The value of $\int_{-2}^{3} |1-x^2| dx$ is **[2004]**

(a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

146. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is **[2003]**

(a) $\frac{1}{n+1} + \frac{1}{n+2}$ (b) $\frac{1}{n+1}$

(c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1} - \frac{1}{n+2}$.

147. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$, is **[2003]**

(a) $e + \frac{e^2}{2} + \frac{5}{2}$ (b) $e - \frac{e^2}{2} - \frac{5}{2}$

(c) $e + \frac{e^2}{2} - \frac{3}{2}$ (d) $e - \frac{e^2}{2} - \frac{3}{2}$.

148. If $f(a+b-x) = f(x)$ then $\int_a^b xf(x)dx$ is equal to **[2003]**

(a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b-x)dx$

(c) $\frac{a+b}{2}\int_a^b f(x)dx$ (d) $\frac{b-a}{2}\int_a^b f(x)dx$.

149. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x \sin x}$ is **[2003]**

(a) 0 (b) 3 (c) 2 (d) 1

150. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y)g(y)\,dy$, then **[2003]**

(a) $F(t) = te^{-t}$ (b) $F(t) = 1 - te^{-t}(1+t)$

(c) $F(t) = e^t - (1+t)$ (d) $F(t) = te^t$.

151. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is **[2002]**

(a) $\frac{\pi^2}{4}$ (b) π^2 (c) zero (d) $\frac{\pi}{2}$

152. $\int_0^2 [x^2] dx$ is **[2002]**

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$

(c) $\sqrt{2} - 1$ (d) $-\sqrt{2} - \sqrt{3} + 5$

153. $I_n = \int_0^{\pi/4} \tan^n x\, dx$ then $\lim_{n \to \infty} n[I_n + I_{n+2}]$ equals **[2002]**

(a) $\frac{1}{2}$ (b) 1 (c) ∞ (d) zero

154. $\int_0^{10\pi} |\sin x|\, dx$ is **[2002]**

(a) 20 (b) 8 (c) 10 (d) 18

TOPIC 4 Reduction Formulae for Definite Integration, Gamma & Beta Function, Walli's Formula, Summation of Series by Integration

155. Let a function $f:[0,5]\to\mathbf{R}$ be continuous, $f(1)=3$ and F be defined as:

$$F(x)=\int_1^x t^2 g(t)dt,\ \text{where } g(t)=\int_1^x f(u)du$$

Then for the function F, the point $x=1$ is : **[Jan. 9, 2020 (II)]**

(a) a point of local minima.
(b) not a critical point.
(c) a point of local maxima.
(d) a point of inflection.

156. $\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+.....+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to : **[April 10, 2019 (I)]**

(a) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$ (b) $\frac{4}{3}(2)^{4/3}$
(c) $\frac{3}{2}(2)^{4/3}-\frac{4}{3}$ (d) $\frac{4}{3}(2)^{3/4}$

157. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+.....+\frac{1}{5n}\right)$ is equal to : **[Jan. 12, 2019 (II)]**

(a) $\frac{\pi}{4}$ (b) $\tan^{-1}(3)$ (c) $\frac{\pi}{2}$ (d) $\tan^{-1}(2)$

158. If $\lim\limits_{n\to\infty}\frac{1^a+2^a+.........+n^a}{(n+1)^{a-1}[(na+2)+.....(na+n)]}=\frac{1}{60}$

for some positive real number a, then a is equal to : **[Online April 9, 2017]**

(a) 7 (b) 8 (c) $\frac{15}{2}$ (d) $\frac{17}{2}$

159. $\lim\limits_{n\to\infty}\left(\frac{(n+1)(n+2)...3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to: **[2016]**

(a) $\frac{9}{e^2}$ (b) $3\log 3-2$
(c) $\frac{18}{e^4}$ (d) $\frac{27}{e^2}$

160. $f(x)=\int\frac{dx}{\sin^6 x}$ is a polynomial of degree **[Online May 26, 2012]**

(a) 5 in $\cot x$ (b) 5 in $\tan x$
(c) 3 in $\tan x$ (d) 3 in $\cot x$

161. $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}...........+\frac{1}{n}\sec^2 1\right]$ equals **[2005]**

(a) $\frac{1}{2}\sec 1$ (b) $\frac{1}{2}\operatorname{cosec} 1$
(c) $\tan 1$ (d) $\frac{1}{2}\tan 1$

162. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ is **[2004]**

(a) $e+1$ (b) $e-1$ (c) $1-e$ (d) e

163. $\lim\limits_{n\to\infty}\frac{1+2^4+3^4+...n^4}{n^5}-\lim\limits_{n\to\infty}\frac{1+2^3+3^3+...n^3}{n^5}$ **[2003]**

(a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$

164. $\lim\limits_{n\to\infty}\frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}$ is **[2002]**

(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$
(c) $\frac{1}{p}-\frac{1}{p-1}$ (d) $\frac{1}{p+2}$

Hints & Solutions

1. **(Bonus)**

$$\lim_{x\to 1}\frac{\frac{1}{2}\sin(x-1)^4}{(x-1)\sin(x-1)}$$

Let $x-1=h$ when $x\to 1$ then $h\to 0$

$$\lim_{h\to 0}\frac{\sin h^4}{h^4}\times\frac{h}{\sin h}\times h^2=1\times 1\times 0=0$$

(No any option is correct)

2. **(d)** $\int (e^{2x}+2e^x-e^{-x}-1)\cdot e^{(e^x+e^{-x})}dx$

$$I=\int (e^{2x}+e^x-1)\cdot e^{(e^x+e^{-x})}dx+\int (e^x-e^{-x})e^{(e^x+e^{-x})}dx$$

$$=\int e^x(e^x+1-e^{-x})\cdot e^{(e^x+e^{-x})}dx+e^{(e^x+e^{-x})}$$

$$=\int (e^x-e^{-x}+1)e^{(e^x+e^{-x}+x)}dx+e^{(e^x+e^{-x})}$$

Let $e^x+e^{-x}+x=t \Rightarrow (e^x+e^{-x}+1)dx=dt$

$$=\int e^t dt+e^{(e^x+e^{-x})}=e^t+e^{(e^x+e^{-x})}+C$$

$$=e^{(e^x+e^{-x}+x)}+e^{(e^x+e^{-x})}+C$$

$$=(e^x+1).e^{(e^x+e^{-x})}+C$$

So, $g(x)=1+e^x$ and $g(0)=2$

3. **(d)** Let $\sin\theta=t\Rightarrow\cos\theta\,d\theta=dt$

$$\int\frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta=\frac{dt}{5+7t-2+2t^2}$$

$$\Rightarrow\frac{1}{2}\int\frac{dt}{\left(t+\frac{7}{4}\right)^2-\left(\frac{5}{4}\right)^2}=\frac{1}{5}\ln\left|\frac{t+\frac{1}{2}}{t+3}\right|+C$$

$$=\frac{1}{5}\ln\left|\frac{2t+1}{t+3}\right|+C=\frac{1}{5}\ln\left|\frac{2\sin\theta+1}{\sin\theta+3}\right|+C$$

$$\therefore B(\theta)=\frac{2\sin\theta+1}{2(\sin\theta+3)}\text{ and }A=\frac{1}{5}$$

$$\Rightarrow\frac{B(\theta)}{A}=\frac{5(2\sin\theta+1)}{(\sin\theta+3)}$$

4. **(a)** $\int\frac{x^2}{(x\sin x+\cos x)^2}dx$

$$\because\frac{d}{dx}(x\sin x+\cos x)=x\cos x$$

$$=\int\underset{\text{II}}{\frac{x\cos x}{(x\sin x+\cos x)^2}}\underset{\text{I}}{\left(\frac{x}{\cos x}\right)}dx$$

$$=\frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right]-\int\frac{x\sin x+\cos x}{\cos^2 x}\left[\frac{-1}{x\sin x+\cos x}\right]dx$$

$$=\frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right]+\int\sec^2 x\,dx$$

$$=\frac{-x\sec x}{x\sin x+\cos x}+\tan x+C$$

5. **(d)** $\int\frac{\sqrt{x}}{(1+x)^2}dx\ (x>0)$

Put $x=\tan^2\theta\Rightarrow 2x\,dx=2\tan\theta\sec^2\theta\,d\theta$

$$I=\int\frac{2\tan^2\theta\cdot\sec^2\theta}{\sec^4\theta}d\theta=\int 2\sin^2\theta d\theta$$

$$=\theta-\frac{\sin 2\theta}{2}+C$$

$$\Rightarrow f(x)=\theta-\frac{1}{2}\times\frac{2\tan\theta}{1+\tan^2\theta}+C$$

$$f(x)=\theta-\frac{\tan\theta}{1+\tan^2\theta}+C=\tan^{-1}\sqrt{x}-\frac{\sqrt{x}}{1+x}+C$$

Now $f(3)-f(1)=\tan^{-1}(\sqrt{3})-\frac{\sqrt{3}}{1+3}-\tan^{-1}(1)+\frac{1}{2}$

$$=\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$$

6. **(a)** $I=\int\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx=\int\underset{\text{I}}{\tan^{-1}\sqrt{x}}\cdot\underset{\text{II}}{1}\,dx$

$$= x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}.x\,dx + C$$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{t\cdot 2t\,dt}{1+t^2} + C$$

(Put $x = t^2 \Rightarrow dx = 2t\,dt$)

$$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}dt + C$$

$$= x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + C$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$$

$$\Rightarrow A(x) = x+1 \Rightarrow B(x) = -\sqrt{x}$$

7. **(a)** $I = \int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$

$$= \int \left(\frac{x-3}{x+4}\right)^{\frac{-6}{7}} \frac{1}{(x+4)^2}dx$$

Let $\frac{x-3}{x+4} = t^7$,

Differentiate on both sides, we get

$$\frac{7}{(x+4)^2}dx = 7t^6 dt$$

Hence, $I = \int t^{-6}t^6 dt = t + C = \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

8. **(c)** $I = \int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)}$

$$= \int \frac{\sec^2\theta}{\frac{1+\tan^2\theta}{1-\tan^2\theta} + \frac{2\tan\theta}{1-\tan^2\theta}}d\theta$$

$$= \int \frac{\sec^2\theta(1-\tan^2\theta)}{(1+\tan\theta)^2}d\theta$$

$$= \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta}d\theta$$

Let $\tan\theta = t \;\Rightarrow\; \sec^2\theta\, d\theta = dt$, then

$$I = \int\left(\frac{1-t}{1+t}\right)dt = \int\left(-1+\frac{2}{1+t}\right)dt$$

$= -t + 2\log(1+t) + C$

$= -\tan\theta + 2\log(1+\tan\theta) + C$

Hence, by comparison $\lambda = -1$ and $f(x) = 1 + \tan\theta$

9. **(d)** Let $I = \int \frac{\cos x\,dx}{\sin^3 x(1+\sin^6 x)^{2/3}}$

$= f(x)(1+\sin^6 x)^{1/\lambda} + c$ ...(i)

If $\sin x = t$

then, $\cos x\,dx = dt$

$$I = \int \frac{dt}{t^3\left(1+t^6\right)^{\frac{2}{3}}} = \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Put $1+\frac{1}{t^6} = r^3 \;\Rightarrow\; \frac{dt}{t^7} = \frac{-1}{2}r^2 dr$

$$-\frac{1}{2}\int\frac{r^2 dr}{r^2} = -\frac{1}{2}r + c$$

$$= -\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{\frac{1}{3}} + c = -\frac{1}{2\sin^2 x}(1+\sin^6 x)^{\frac{1}{3}} + c$$

$f(x) = -\frac{1}{2}\operatorname{cosec}^2 x$ and $\lambda = 3$ [from eqn. (i)]

$$\therefore\; \lambda f\left(\frac{\pi}{3}\right) = -2$$

10. **(c)** Given integral, $I = \int \frac{(2x^3-1)dx}{x^4+x} = \int \frac{(2x-x^{-2})dx}{x^2+x^{-1}}$

Put $x^2 + x^{-1} = u \Rightarrow (2x - x^{-2})dx = du$

$$\Rightarrow\; I = \int \frac{du}{u} = \log|u| + c = \log|x^2+x^{-1}| + c$$

$$= \log\left|\frac{x^3+1}{x}\right| + c$$

11. **(b)** Given integral

$$\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha}dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$$

Let $x - \alpha = t \Rightarrow dx = dt$

$$= \int \frac{\sin(t+2\alpha)}{\sin t}dt = \int[\cos 2\alpha + \sin 2\alpha.\cot t]dt$$

$= \cos 2\alpha\,.\,t + \sin 2\alpha\,.\,\log|\sin t| + c$

$= (x-\alpha)\,.\,\cos 2\alpha + \sin 2\alpha\,.\,\log|\sin(x-\alpha)| + c$

12. **(a)** Let $I = \int \frac{dx}{(x^2-2x+10)^2} = \int \frac{dx}{((x-1)^2+9)^2}$

Let $(x-1)^2 = 9\tan^2\theta$ ...(i)

$$\Rightarrow \tan\theta = \frac{x-1}{3}$$

After differentiating equation ...(i), we get

$2(x-1)\,dx = 18\tan\theta\sec^2\theta\,d\theta$

$$\therefore\ I = \int \frac{18\tan\theta\sec^2\theta d\theta}{2\times 3\tan\theta\times 81\sec^4\theta}$$

$$I = \frac{1}{27}\int \cos^2\theta d\theta = \frac{1}{27}\times\frac{1}{2}\int(1+\cos 2\theta)d\theta$$

$$I = \frac{1}{54}\left\{\theta + \frac{\sin 2\theta}{2}\right\} + c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{1}{2}\times\frac{2\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right] + c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + c$$

Compare it with $A\left[\tan^{-1}\left(\frac{x-1}{b}\right) + \frac{f(x)}{x^2-2x+10}\right] + c$,

we get: $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

13. **(d)** Since, function f(x) have local extreem points at $x = -1, 0, 1$. Then

$f(x) = K(x+1)x(x-1)$

$= K(x^3 - x)$

$$\Rightarrow f(x) = K\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C \quad \text{(using integration)}$$

$\Rightarrow f(0) = C$

$$\because f(x) = f(0) \Rightarrow K\left(\frac{x^4}{4} - \frac{x^2}{2}\right) = 0$$

$$\Rightarrow \frac{x^2}{2}\left(\frac{x^2}{2} - 1\right) = 0 \quad \Rightarrow x = 0, 0, \sqrt{2}, -\sqrt{2}$$

$$\therefore S = \{0, -\sqrt{2}, \sqrt{2}\}$$

14. **(a)** $I = \int \sec^{\frac{2}{3}} x.\text{cosec}^{\frac{4}{3}} dx$

$$I = \int \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$$

Put $\tan x = z$

$\Rightarrow \sec^2 x\, dx = dz$

$$\Rightarrow I = \int z^{-\frac{4}{3}}\cdot dz = \frac{z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)} + C \Rightarrow I = -3(\tan x)^{\frac{-1}{3}} + C$$

15. **(a)** Given,

$$\int e^{\sec x}\left(\sec x\tan x\, f(x) + \left(\sec x + \tan x + \sec^2 x\right)\right)dx$$

$$= e^{\sec x} f(x) + C \quad ...(i)$$

$$\because \int e^{g(x)}\left(\left(g'(x)f(x)\right) + f'(x)\right)dx = e^{g(x)}\times f(x) + C$$

Our comparing above equation by equation (i),

$$f(x) = \int\left(\left(\sec x\tan x\right) + \sec^2 x\right)dx$$

$\therefore f(x) = \sec x + \tan x + C$

16. **(c)** $$\int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}dx = \int \frac{2\cos\frac{x}{2}.\sin\frac{5x}{2}}{2\cos\frac{x}{2}.\sin\frac{x}{2}}dx$$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x}dx$$

$$= \int (3 - 4\sin^2 x + 2\cos x)dx$$

[$\because \sin 2x = 2\sin x\cos x$ and $\sin 3x = 3\sin x - 4\sin^3 x$]

$$= \int (3 - 2(1-\cos 2x) + 2\cos x)dx$$

$$= \int (1 + 2\cos x + 2\cos 2x)dx$$

$= x + 2\sin x + \sin 2x + c$

17. **(d)** Let, $$\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = \int \frac{dx}{x^7(1+x^{-6})^{\frac{2}{3}}}$$

Put $1 + x^{-6} = t^3 \Rightarrow -6^{-7}dx = 3t^2 dt \Rightarrow \frac{dx}{x^7} = \left(-\frac{1}{2}\right)t^2 dt$

Now, $$I = \int\left(-\frac{1}{2}\right)\frac{t^2 dt}{t^2} = -\frac{1}{2}t + C$$

$$= -\frac{1}{2}(1+x^{-6})^{\frac{1}{3}} + C = -\frac{1}{2}\frac{(1+x^6)^{\frac{1}{3}}}{x^2} + C$$

$$= -\frac{1}{2x^3}x(1+x^6)^{\frac{1}{3}} + C$$

Hence, $f(x) = -\frac{1}{2x^3}$

18. **(c)** Let the integral, $I = \int \cos(\ln x)\,dx$

$\Rightarrow \; I = \cos(\ln x).x - \int \frac{-\sin(\ln x)}{x}.x\,dx$

$= x\cos(\ln x) + \int \sin(\ln x)dx$

$= x\cos(\ln x) + \sin(\ln x).x - \int \frac{\cos(\ln x)}{x}.x\,dx$

$= x\cos(\ln x) + \sin(\ln x)\,.\,x - I$

$\Rightarrow \; 2I = x(\cos(\ln x) + \sin(\ln x)) + C$

$\Rightarrow \; I = \frac{x}{2}[\cos(\ln x) + \sin(\ln x)] + C$

19. **(b)** $I = \int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}\,dx = \int \frac{3x^{13}+2x^{11}}{x^{16}\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}\,dx$

$I = \int \frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}\,dx$

Let $2+\frac{3}{x^2}+\frac{1}{x^4} = t$, $-2\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx = dt$

Then, $I = \int \frac{-\frac{dt}{2}}{t^4} = -\frac{1}{2}\frac{t^{-4+1}}{-4+1} + C$

$I = \frac{-1}{2} \times \frac{1}{(-3)} \frac{1}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3} + C$

$I = \frac{1}{6}\frac{x^{12}}{(2x^4+3x^2+1)^3} + C$

20. **(a)** $A(x)\left(\sqrt{1-x^2}\right)^m + C = \int \frac{\sqrt{1-x^2}}{x^4}\,dx$

$= \int \frac{\sqrt{\frac{1}{x^2}-1}}{x^3}\,dx$

Let $\frac{1}{x^2} - 1 = u^2$

$\Rightarrow \; -\frac{2}{x^3} = \frac{2u\,du}{dx}$

$\frac{dx}{x^3} = -u\,du$

$A(x)\left(\sqrt{1-x^2}\right)^m + C = \int(-u^2)du = -\frac{u^3}{3} + C$

$= -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} + C$

$= -\frac{1}{3}\cdot\frac{1}{x^3}\cdot(1-x^2)^{\frac{3}{2}} + C$

$= \frac{-1}{3x^3}\left(\sqrt{1-x^2}\right)^3 + C$

Compare both sides,

$\Rightarrow \; A(x) = -\frac{1}{3x^3}$ and $m = 3$

$\Rightarrow \; (A(x))^3 = \frac{-1}{27x^9}$

21. **(c)** Let $I = \int \frac{x+1}{\sqrt{2x-1}}dx$

Put $\sqrt{2x-1} = t$

$\therefore \; 2x - 1 = t^2 \Rightarrow dx = t\,dt$

$I = \int \frac{(t^2+3)}{2}dt = \frac{t^3}{6} + \frac{3t}{2} + C$

$= \frac{(2x-1)^{\frac{3}{2}}}{6} + \frac{3}{2}(2x-1)^{\frac{1}{2}} + C$

$= \sqrt{2x-1}\left(\frac{x+4}{3}\right) + C$

$= f(x).\sqrt{2x-1} + C$

Hence, $f(x) = \frac{x+4}{3}$

22. **(a)** Let, $I = \int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$

Let $\sin\theta = u$

$\Rightarrow \; \cos\theta\,d\theta = du$

$\therefore \; I = \int \frac{(u^n - u)^{\frac{1}{n}}}{u^{n+1}}du$

$= \int \frac{\left(1-\frac{1}{u^{n-1}}\right)^{\frac{1}{n}}}{u^n}du = \int u^{-n}(1-u^{1-n})^{\frac{1}{n}}du$

Let $1 - u^{1-n} = v$

$\Rightarrow \; -(1-n)u^{-n}\,du = dv$

$\Rightarrow \; u^{-n}\,du = \frac{dv}{n-1}$

$$\therefore\quad I=\int v^{\frac{1}{n}}\cdot\frac{dv}{n-1}=\frac{1}{n-1}\cdot\frac{v^{\frac{1}{n}+1}}{\frac{1}{n}+1}$$

$$=\frac{n}{n^2-1}v^{\frac{n+1}{n}}+C=\frac{n}{n^2-1}\left(1-\frac{1}{u^{n-1}}\right)^{\frac{n+1}{n}}+C$$

$$=\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$$

23. **(c, d)** Consider the given integral

$$I=\int x\sqrt{\frac{2\sin(x^2-1)-2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1)+2\sin(x^2-1)\cos(x^2-1)}}dx$$

$(\therefore \sin 2\theta = 2\sin\theta\cos\theta)$

$$\Rightarrow\quad I=\int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}dx$$

$$\Rightarrow\quad I=\int x\left|\tan\left(\frac{x^2-1}{2}\right)\right|dx,$$

Now let $\frac{x^2-1}{2}=t\ \Rightarrow\ \frac{2x}{2}dx=dt$

$$\therefore\quad I=\int|\tan(t)|\,dt=\ln|\sec t|+C$$

or $\quad I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c=\frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

24. **(a)** $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx,\ x\geq 0$

$$=\int\frac{5x^8+7x^6}{x^{14}(x^{-5}+x^{-7}+2)^2}dx$$

$$=\int\frac{5x^{-6}+7x^{-8}}{(2+x^{-7}+x^{-5})^2}dx$$

Let $2+x^{-7}+x^{-5}=t$

$\Rightarrow\ (-7x^{-8}-5x^{-6})dx=dt$

$$\Rightarrow\ f(x)=\int\frac{-dt}{t^2}=\int -t^{-2}dt=t^{-1}+c$$

$$\Rightarrow\ f(x)=\frac{1}{2+x^{-7}+x^{-5}}+c,\ f(0)=0\ \Rightarrow\ c=0$$

$$\therefore\ f(1)=\frac{1}{4}$$

25. **(a)** Let I

$$=\int\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$$

$$=\int\frac{\sin^2 x\cos^2 x}{[(\sin^2 x+\cos^2 x)(\sin^3 x+\cos^3 x)]^2}dx$$

$$=\int\frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx=\int\frac{\tan^2 x\cdot\sec^2 x}{(1+\tan^3 x)^2}dx$$

Now, put $(1+\tan^3 x)=t$

$\Rightarrow\ 3\tan^2 x\sec^2 x\,dx=dt$

$$\therefore\quad I=\frac{1}{3}\int\frac{dt}{t^2}=-\frac{1}{3t}+C=\frac{-1}{3(1+\tan^3 x)}+C$$

26. **(a)** Let $I=\int\frac{\tan x}{1+\tan x+\tan^2 x}dx$

$$\Rightarrow I=\int\frac{\tan x+1+\tan^2 x}{\tan x+1+\tan^2 x}dx-\int\frac{(1+\tan^2 x)}{1+\tan x+\tan^2 x}$$

$$\Rightarrow I=x-\int\frac{\sec^2 x\,dx}{1+\tan x+\tan^2 x}$$

Put $\tan x=t\ \Rightarrow\ \sec^2 x\,.\,dx=dt$

$$\therefore I=x-\int\frac{dt}{t^2+t+\frac{1}{4}+1-\frac{1}{4}}$$

$$=x-\int\frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I=x-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+C$$

$$\Rightarrow I=x-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{3}}\right)+C$$

$\therefore\ A=3$ and $K=2$

27. **(b)** Suppose, $\frac{x-4}{x+2}=y\Rightarrow x-4=y\,(x+2)$

$$\Rightarrow x\,(1-y)=2y+4\Rightarrow x=\frac{2y+4}{1-y}$$

So, $f(y)=2\left(\frac{2y+4}{1-y}\right)+1$

Now, $f(x)=2\left(\frac{2x+4}{1-x}\right)+1=\frac{3x+9}{1-x}$

$$= \frac{3(x+3)}{1-x} = \frac{3(x-1+4)}{1-x} = -3 + \frac{12}{1-x}$$

$$\therefore \int f(x)\,dx = -12\log_e|1-x| - 3x + c$$

28. **(a)** $\because 7-6x-x^2 = 16-(x+3)^2$

and $\frac{d}{dx}(7-6x-x^2) = -2x-6$

So, $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = \int \frac{2x+6}{\sqrt{7-6x-x^2}}dx$

$$-\int \frac{1}{\sqrt{16-(x+3)^2}}dx$$

$$= -2\sqrt{7-6x-x^2} - \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

Therefore, $A = -2, \& B = -1.$

29. **(b)** $f\left(\frac{3x-4}{3x+4}\right) = x+2,\ x \neq -\frac{4}{3}$

Consider $\frac{3x-4}{3x+4} = t$

$\Rightarrow 3x-4 = 3tx+4t$

$\Rightarrow x = \frac{4t+4}{3-3t} + 2$

$\Rightarrow f(t) = \frac{10-2t}{3-3t}$

$\Rightarrow f(x) = \frac{2x-10}{3x-3}$

$\therefore \int f(x)dx = \int \frac{2x-10}{3x-3}dx$

$$= \int \frac{2x}{3x-3}dx - 10\int \frac{dx}{3x-3}$$

$$= \frac{2}{3}\int \frac{x-1}{x-1}dx + \frac{2}{3}\int \frac{dx}{x-1} - \frac{10}{3}\int \frac{dx}{x-1}$$

$$= \frac{2x}{3} - \frac{8}{3}\ln(x-1) + C$$

Here, $A = -\frac{8}{3}, B = \frac{2}{3}$

$\therefore (A, B) = \left(-\frac{8}{3}, \frac{2}{3}\right)$

30. **(a)** Let, $I = \int \sqrt{1+2\cot x \operatorname{cosec} x + 2\cot^2 x}\,.dx$

$\Rightarrow I = \int \sqrt{\frac{\sin^2 x + 2\cos x + 2\cos^2 x}{\sin^2 x}}\,.dx$

$\Rightarrow I = \int \sqrt{\frac{1+2\cos x + \cos^2 x}{\sin x}}\,.dx$

$\Rightarrow I = \int \left|\frac{1+\cos x}{\sin x}\right|.dx$

$\Rightarrow I = \int |\operatorname{cosec} x + \cot x|\,.dx$

$\Rightarrow I = \log|\operatorname{cosec} x - \cot x| + \log|\sin x| + C$

$\Rightarrow I = \log|1-\cos x| + C$

$\Rightarrow I = \log\left|2\sin^2\frac{x}{2}\right| + C$

$\Rightarrow I = \log\left|\sin^2\frac{x}{2}\right| + \log 2 + C$

$\Rightarrow I = 2\log\left|\sin\frac{x}{2}\right| + C_1$

31. **(a)** $\int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}} = \int \frac{dx}{2\cos^4 x\sqrt{\tan x}}$

Let $\tan x = t^2 \Rightarrow \sec^2 x = 1 + t^4$

$\sec^2 x\,dx = 2t\,dt$

$$= \int \frac{\sec^4 x\,dx}{2\sqrt{\tan x}} = \int \frac{\sec^2 x(\sec^2 x\,dx)}{2\sqrt{\tan x}}$$

$$= \int \frac{(1+t^4)2t\ dt}{2t} = \int (1+t^4)\,dt = t + \frac{t^5}{5} + k$$

$$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x + k\left[t = \sqrt{\tan x}\right]$$

$A = \frac{1}{2}, B = \frac{5}{2}, C = \frac{1}{5}$

$A + B + C = \frac{16}{5}$

32. **(d)** Let $I = \int \frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}}dt$

put $u = \log(t+\sqrt{1+t^2})$

$$du = \frac{1}{t+\sqrt{1+t^2}}\cdot\left[\frac{t+\sqrt{1+t^2}}{\sqrt{1+t^2}}\right] = \frac{1}{\sqrt{1+t^2}}dt$$

$\therefore I = \int u\,du = \frac{u^2}{2} + c$

Since, $I = \frac{1}{2}[g(t)]^2 + c$

$\therefore \quad g(t) = \log\ (t + \sqrt{1+t^2})$

Put t = 2

$g(b) = \log\ (2+\sqrt{5})$

33. **(d)** Let $I = \int\left(1+x-\frac{1}{x}\right)e^{x+1/x}dx$

$= \int e^{x+1/x}dx + \int\left(x-\frac{1}{x}\right)e^{x+1/x}dx$

$= x.e^{x+1/x} - \int x\left(1-\frac{1}{x^2}\right)e^{x+1/x}dx + \int\left(x-\frac{1}{x}\right)e^{x+1/x}dx$

$= x.e^{x+\frac{1}{x}} - \int\left(x-\frac{1}{x}\right)e^{x+1/x}dx + \int\left(x-\frac{1}{x}\right)e^{x+1/x}dx$

$= xe^{x+\frac{1}{x}} + C$

34. **(b)** Let $I = \int \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$

$I = \int\left(\frac{\sin x.\cos x}{\sin^3 x+\cos^3 x}\right)^2 dx$

$I = \int\left(\frac{\sin x.\cancel{\cos x}}{\cancel{\cos^3} x\,(1+\tan^3 x)}\right)^2 dx$

$= \int\left(\frac{\sin x.\sec^2 x}{(1+\tan^3 x)}\right)^2 dx$

Put $1 + \tan^3 x = t$

$dt = 3\tan^2 x\sec^2 x\, dx$ or $dx = \frac{dt}{3\tan^2 x\sec^2 x}$

$\therefore\ I = \int\frac{\sin^2 x.\sec^4 x}{t^2}\times\frac{dt}{3\tan^2 x\sec^2 x}$

$I = \frac{1}{3}\int\frac{\sin^2 x.\sec^4 x}{t^2}\times\frac{dt}{\frac{\sin^2 x}{\cos^2 x}\times\sec^2 x}$

$= \frac{1}{3}\int\frac{\cancel{\sin^2 x.\sec^4 x}}{t^2}\times\frac{dt}{\cancel{\sin^2 x\sec^4 x}}$

$\therefore\ I = \frac{1}{3}\int\frac{dt}{t^2} = \frac{1}{3}\int t^{-2}dt$

$I = \frac{1}{3}\left[\frac{t^{-2+1}}{-2+1}\right]+c = \frac{-1}{3}\left[\frac{1}{t}\right]+c$

or $I = -\frac{1}{3(1+\tan^3 x)}+c$

35. **(a)** Let $I = \int x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

$\therefore\ I = 2\int \underset{\text{II}}{x}.\underset{\text{I}}{\tan^{-1} x}\, dx$

Applying Integration by parts

$I = 2\left[\tan^{-1} x\int xdx - \int\left(\frac{d}{dx}(\tan^{-1} x)\int xdx\right)dx\right]$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \int\frac{1}{1+x^2}\times\frac{x^2}{2}dx\right]+c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\frac{x^2+1-1}{x^2+1}dx\right]+c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\frac{\cancel{x^2+1}}{\cancel{x^2+1}}dx + \frac{1}{2}\int\frac{1}{1+x^2}dx\right]+c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int 1.dx + \frac{1}{2}\tan^{-1} x\right]+c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x\right]+c$

$I = x^2\tan^{-1} x + \tan^{-1} x - x + c$

or $\boxed{I = -x+(x^2+1)\tan^{-1} x + c}$

36. **(b)** Let $I = \int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$

$= \int\frac{(\sin^4 x)^2-(\cos^4 x)^2}{1-2\sin^2 x\cos^2 x}dx$

$= \int\frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{1-2\sin^2 x\cos^2 x}dx$

$= \int\frac{[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x]\ [(\sin^2 x+\cos^2 x][\sin^2 x-\cos^2 x]}{1-2\sin^2 x\cos^2 x}dx$

$= -\int\cos 2x\, dx = \frac{-\sin 2x}{2}+c = -\frac{1}{2}\sin 2x + c$

37. **(c)** Let $\int f(x)dx = \psi(x)$

Let $I = \int x^5 f(x^3)dx$

put $x^3 = t$

$\Rightarrow\ 3x^2dx = dt$

$I = \frac{1}{3}\int 3\cdot x^2\cdot x^3\cdot f(x^3)\cdot dx$

$= \frac{1}{3}\int tf(t)dt = \frac{1}{3}\left[t\int f(t)dt - \int f(t)dt\right]$

$$= \frac{1}{3}\left[t\psi(t) - \int \psi(t)dt\right]$$

$$= \frac{1}{3}\left[x^3\psi(x^3) - 3\int x^2\psi(x^3)dx\right] + c$$

$$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$$

38. **(a)** Let $I = \int \frac{\cos 8x + 1}{\cot 2x - \tan 2x} dx$

Now, $D^r = \cot 2x - \tan 2x = \frac{\cos 2x}{\sin 2x} - \frac{\sin 2x}{\cos 2x}$

$$= \frac{\cos^2 2x - \sin^2 2x}{\sin 2x \cos 2x} = \frac{2\cos 4x}{\sin 4x}$$

$$\therefore \ I = \int \frac{2\cos^2 4x}{\frac{2\cos 4x}{\sin 4x}} dx = \int \frac{2\cos^2 4x \,.\, \sin 4x}{2\cos 4x} dx$$

$$= \frac{1}{2}\int \sin 8x\, dx = -\frac{1}{2}\frac{\cos 8x}{8} + k = -\frac{1}{16}.\cos 8x + k$$

Now, $-\frac{1}{16}.\cos 8x + k = A\cos 8x + k$

$\Rightarrow A = -\frac{1}{16}$

39. **(d)** $\int \frac{5\tan x}{\tan x - 2} dx = \int \frac{5\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} - 2} dx$

$$= \int\left(\frac{5\sin x}{\cos x} \times \frac{\cos x}{\sin x - 2\cos x}\right)dx$$

$$= \int \frac{5\sin x\, dx}{\sin x - 2\cos x}$$

$$= \int\left(\frac{4\sin x + \sin x + 2\cos x - 2\cos x}{\sin x - 2\cos x}\right)dx$$

$$= \int \frac{(\sin x - 2\cos x) + (4\sin x + 2\cos x)}{\sin x - 2\cos x} dx$$

$$= \int \frac{(\sin x - 2\cos x) + 2(\cos x + 2\sin x)}{(\sin x - 2\cos x)} dx$$

$$= \int \frac{\sin x - 2\cos x}{\sin x - 2\cos x} dx + 2\int\left(\frac{\cos x + 2\sin x}{\sin x - 2\cos x}\right)dx$$

$$= \int dx + 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x} dx = I_1 + I_2$$

where, $I_1 = \int dx$ and $I_2 = 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x} dx$

Let $\sin x - 2\cos x = t$

$\Rightarrow$ $(\cos x + 2\sin x)\, dx = dt$

$\therefore$ $I_2 = 2\int \frac{dt}{t} = 2\ln t + C = 2\,ln\,(\sin x - 2\cos x) + C$

Hence, $I_1 + I_2 = \int dx + 2\ln(\sin x - 2\cos x) + c$

$= x + 2ln\,|(\sin x - 2\cos x)| + k \Rightarrow a = 2$

40. **(a)** Let $f(x) = \int\left(\frac{x^2 + \sin^2 x}{1 + x^2}\right)\sec^2 x\, dx$

$$= \int \frac{x^2\sec^2 x + \frac{\sin^2 x}{\cos^2 x}}{1 + x^2} dx$$

$$= \int \frac{x^2\sec^2 x + \tan^2 x}{1 + x^2} dx$$

$$= \int \frac{x^2(1 + \tan^2 x) + \tan^2 x}{1 + x^2} dx$$

$$= \int \frac{x^2 + \tan^2 x(1 + x^2)}{1 + x^2} dx$$

$$= \int \frac{x^2}{1 + x^2} dx + \int \tan^2 x\, dx$$

$$= \int \frac{x^2 + 1 - 1}{1 + x^2} dx + \int(\sec^2 x - 1)dx$$

$$= \int 1\, dx - \int \frac{dx}{1 + x^2} + \int sec^2 x dx - \int dx$$

$= -\tan^{-1}x + \tan x + c$

Given : $f(0) = 0$

$\Rightarrow f(0) = -\tan^{-1}0 + \tan 0 + c \ \Rightarrow \ c = 0$

$\therefore f(x) = -\tan^{-1}x + \tan x$

Now, $f(1) = -\tan^{-1}(1) + \tan 1 = \tan 1 - \frac{\pi}{4}$

41. **(a)** Let $I = \int \frac{x^2 - x}{x^3 - x^2 + x - 1} dx$

$$= \int \frac{x(x-1)}{x^2(x-1) + (x-1)} dx = \int \frac{x\,dx}{x^2 + 1} = \frac{1}{2}\int \frac{2x\,dx}{(x^2 + 1)}$$

Let $x^2 + 1 = t \Rightarrow 2x\,dx = dt$

$\therefore$ $I = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log t + c$

$$= \frac{1}{2}\log(x^2 + 1) + c$$

where 'c' is the constant of integration.

42. **(d)** Statement - 2: $\cos^3 x$ is a periodic function.

It is a true statement.

Statement - 1

Given $f(x) = \int \cos^3 x\, dx = \int\left(\frac{\cos 3x}{4} + \frac{3\cos x}{4}\right)dx$

$$= \frac{1}{4}\frac{\sin 3x}{3} + \frac{3}{4}\sin x = \frac{1}{12}\sin 3x + \frac{3}{4}\sin x$$

Now, period of $\frac{1}{12}\sin 3x = \frac{2\pi}{3}$

Period of $\frac{3}{4}\sin x = 2\pi$

Hence period of $f(x) = \frac{\text{L.C.M.}(2\pi, 2\pi)}{\text{HCF of }(1,3)} = \frac{2\pi}{1} = 2\pi$

Thus, $f(x)$ is a periodic function of period 2π.
Hence, Statement - 1 is false.

43. **(c)** Let $I = \sqrt{2}\int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)}$

Let $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$\Rightarrow I = \sqrt{2}\int \frac{\sin\left(t + \frac{\pi}{4}\right)}{\sin t}dt = \frac{\sqrt{2}}{\sqrt{2}}\int\left(\frac{\sin t + \cos t}{\sin t}\right)dt$

$\Rightarrow I = \int(1 + \cot t)dt = t + \log|\sin t| + c_1$

$= x - \frac{\pi}{4} + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c_1$

$= x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c \quad \left(\text{where } c = c_1 - \frac{\pi}{4}\right)$

44. **(c)** $I = \int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$\Rightarrow I = \int \frac{dx}{2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right]}$

$= \frac{1}{2}\int \frac{dx}{\left[\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x\right]} = \frac{1}{2}.\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$

$\Rightarrow I = \frac{1}{2}.\int \operatorname{cosec}\left(x + \frac{\pi}{6}\right)dx$

We know that

$\int \operatorname{cosec} x\, dx = \log|(\tan x/2)| + C$

$\therefore \quad I = \frac{1}{2}.\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

45. **(a)** $\int \frac{dx}{\cos x - \sin x} = \int \frac{dx}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}$

$= \int \frac{dx}{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}\int \sec\left(x + \frac{\pi}{4}\right)dx$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8}\right)\right| + C$

$\left[\because \int \sec x\, dx = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|\right]$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$

46. **(b)** $\int \frac{\sin x}{\sin(x - \alpha)}dx = \int \frac{\sin(x - \alpha + \alpha)}{\sin(x - \alpha)}dx$

$= \int \frac{\sin(x - \alpha)\cos\alpha + \cos(x - \alpha)\sin\alpha}{\sin(x - \alpha)}dx$

$= \int\{\cos\alpha + \sin\alpha\cot(x - \alpha)\}dx$

$= (\cos\alpha)x + (\sin\alpha)\log\sin(x - \alpha) + C$

Comparing with $Ax + B\log\sin(x - \alpha) + c$

$\therefore A = \cos\alpha,\ B = \sin\alpha$

47. **(d)** $\because\ f''(x) - g''(x) = 0$

Integrating, $f'(x) - g'(x) = c$;

$\Rightarrow f'(1) - g'(1) = c \Rightarrow 4 - 2 = c \Rightarrow c = 2.$

$\therefore\ f'(x) - g'(x) = 2$;

Integrating, $f(x) - g(x) = 2x + c_1$

$\Rightarrow f(2) - g(2) = 4 + c_1 \Rightarrow 9 - 3 = 4 + c_1$;

$\Rightarrow c_1 = 2 \ \therefore\ f(x) - g(x) = 2x + 2$

At $x = 3/2$, $f(x) - g(x) = 3 + 2 = 5$.

48. **(c)** $I = \int_1^2 e^x x^x(2 + \log_e x)dx$

$I = \int_1^2 e^x x^x[1 + (1 + \log_e x)]dx$

$= \int_1^2 e^x[x^x + x^x(1 + \log_e x)]dx$

$\because \int e^x(f(x) + f'(x))dx = e^x f(x) + c$

$\therefore I = \left[e^x x^x\right]_1^2$

$= e^2 \times 4 - e \times 1 = 4e^2 - e = e(4e - 1)$

49. **(a)** $\int\limits_{\alpha}^{\alpha+1} \frac{dx}{(x + \alpha)(x + \alpha + 1)}$

$= \int\limits_{\alpha}^{\alpha+1}\left[\frac{1}{x + \alpha} - \frac{1}{x + \alpha + 1}\right]dx$ [Using partial fraction]

$= \log\left(\frac{(x + \alpha)}{(x + \alpha + 1)}\right)\Bigg|_{\alpha}^{\alpha+1} = \log\left(\frac{2\alpha + 1}{2\alpha + 2}.\frac{2\alpha + 1}{2\alpha}\right)$

$= \log\frac{9}{8}$ (Given)

So, $\frac{(2\alpha+1)^2}{\alpha(\alpha+1)}=\frac{9}{2} \Rightarrow \quad 8\alpha^2+8\alpha+2=9\alpha^2+9\alpha$

$\Rightarrow \alpha^2+\alpha-2=0 \Rightarrow \alpha=1,-2$

50. **(c)** Let, $1=\int x^2.e^{-x^2}dx$

Put $-x^2=t \Rightarrow -2x\,dx=dt$

$1=\int \frac{t^2.e^t dt}{(-2)}=\frac{-1}{2}e^t(t^2-2t+2)c$

$\therefore g(x)=\frac{-1}{2}(x^4+2x^2+2) \Rightarrow g(-1)=\frac{-5}{2}$

51. **(b)** $I=\int x^5 e^{-4x^3}dx$

Put $-4x^3=\theta$

$\Rightarrow -12x^2\,dx=d\theta$

$\Rightarrow x^2\,dx=-\frac{d\theta}{12}$

$I=\int \frac{1}{48}\theta e^{\theta}\,d\theta=\frac{1}{48}[\theta\, e^{\theta}-e^{\theta}]+C$

$I=\frac{1}{48}e^{-4x^3}(-4x^3-1)+C$

Then, by comparison

$f(x)=-4x^3-1$

52. **(c)** $I=\int \frac{dx}{(1+\sqrt{x}).\sqrt{x}\sqrt{1-x}}$

Put $1+\sqrt{x}=t \Rightarrow \frac{1}{2\sqrt{x}}\,dx=dt$

$\Rightarrow I=\int \frac{2dt}{t\sqrt{2t-t^2}}$

Again put $t=\frac{1}{z} \Rightarrow dt=\frac{-1}{2^2}dz$

$\Rightarrow I=2\int \frac{\frac{-1}{z^2}dz}{\frac{1}{z}\sqrt{\frac{2}{z}-\frac{1}{z^2}}}=2\int \frac{-dz}{\sqrt{2z-1}}$

$=-2\sqrt{2z-1}+c=-2\sqrt{\frac{2}{t}-1}+c$

$=-2\sqrt{\frac{2-t}{t}}+c=-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+c$

53. **(b)** $I=\int \frac{dx}{x^2(x^4+1)^{3/4}}=\int \frac{dx}{x^3(1+x^{-4})^{3/4}}$

Let $x^{-4}=y$

$\Rightarrow -4x^{-3}\,dx=dy \Rightarrow dx=\frac{-1}{4}x^3dy$

$\therefore \quad I=\frac{-1}{4}\int \frac{x^3dy}{x^3(1+y)^{3/4}}=\frac{-1}{4}\int \frac{dy}{(1+y)^{3/4}}$

$=\frac{-1}{4}\times 4(1+y)^{1/4}=-(1+x^{-4})^{1/4}+C$

$=-\left(\frac{x^4+1}{x^4}\right)^{1/4}+C$

54. **(b)** $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$

$\int \frac{dx}{\left(\frac{x+1}{x-2}\right)^{3/4}(x-2)^2}$, put $\frac{x+1}{x-2}=t$

$\frac{-3}{(x-2)^2}=\frac{dt}{dx}$

$\frac{dx}{(x-2)^2}=-\frac{dt}{3}=\frac{-1}{3}\int \frac{dt}{t^{3/4}}=-\frac{1}{3}\int t^{\frac{-3}{4}}dt$

$=\frac{1}{3}\left[\frac{t^{\frac{-3}{4}+1}}{\frac{-3}{4}+1}\right]=\frac{-4}{3}\left[\frac{x+1}{x-2}\right]^{1/4}+c$

55. **(b)** $\int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3}dx=\int \frac{x^{5m-1}+2x^{4m-1}}{x^{6m}(1+x^{-m}+x^{-2m})^3}dx$

$=\int \frac{x^{-m-1}+2x^{-2m-1}}{(1+x^{-m}+x^{-2m})^3}dx$

Put $t=1+x^{-m}+x^{-2m}$

$\therefore \quad \frac{dt}{dx}=-mx^{-m-1}-2mx^{-2m-1}$

$\Rightarrow \frac{dt}{-m}=(x^{-m-1}+2x^{-2m-1})dx$

$\therefore \quad \int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3}dx=\frac{1}{-m}\int t^{-3}dt=\frac{1}{2mt^2}+C$

$=\frac{1}{2m(1+x^{-m}+x^{-2m})^2}+C$

$=\frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}+C$

$\therefore \quad f(x)=\frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}$

56. **(b)** $I=\int \frac{x\,dx}{2-x^2+\sqrt{2-x^2}}$

Put $t=\sqrt{2-x^2}, \frac{dt}{dx}=\frac{1}{2\sqrt{2-x^2}}.(-2x)$

$\Rightarrow -t\,dt = x\,dx$

$\therefore \quad I = \int \frac{(-t)\,dt}{t^2+t} = -\int \frac{1}{t+1}\,dt = -\log|t+1|$

$= -\log\left|\sqrt{2-x^2}+1\right| + c$

57. **(b)** Let $I = \int \frac{x^2-x+1}{x^2+1} \cdot e^{\cot^{-1}x}\,dx$

Put $x = \cot t \Rightarrow -\text{cosec}^2 t\,dt = dx$

Now, $1+\cot^2 t = \text{cosec}^2 t$

$\therefore \quad I = \int \frac{e^t(\cot^2 t - \cot t + 1)}{(1+\cot^2 t)}(-\text{cosec}^2 t)\,dt$

$= -\int e^t(\text{cosec}^2 t - \cot t)\,dt$

$= \int e^t(\cot t - \text{cosec}^2 t)\,dt = e^t \cot t + C$

$= e^{\cot^{-1}x}(x) + C \equiv A(x) \cdot e^{\cot^{-1}x} + C$

$\Rightarrow A(x) = x$

58. **(a)** $\int \frac{x^6}{x+x^7}\,dx = \int \frac{x^6}{x(1+x^6)}\,dx = \int \frac{(1+x^6)-1}{x(1+x^6)}\,dx$

$= \int \frac{1}{x}\,dx - \int \frac{1}{x+x^7}\,dx = \ln|x| - p(x) + c$

59. **(d)** $\int \frac{(\log x - 1)^2}{(1+(\log x)^2)^2}\,dx = \int \frac{1+(\log x)^2 - 2\log x}{\left[1+(\log x)^2\right]^2}\,dx$

$= \int \left[\frac{1}{(1+(\log x)^2)} - \frac{2\log x}{(1+(\log x)^2)^2}\right]dx$

$\therefore \quad I = \int \left[\frac{e^t}{1+t^2} - \frac{2t\,e^t}{(1+t^2)^2}\right]dt$

$= \int e^t \left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2}\right]dt$

$\left[\text{Which is of the form } \int e^x\left(f(x) + f'(x)\right)dx = f(x)\cdot e^x + c\right]$

$= \frac{e^t}{1+t^2} + c = \frac{x}{1+(\log x)^2} + c$

60. **(c)** $I_2 = \int_0^1 (1-x^{50})^{101}\,dx = \int_0^1 (1-x^{50})(1-x^{50})^{100}\,dx$

$I_2 = \int_0^1 (1-x^{50})^{100}\,dx - \int_0^1 \underset{\text{I}}{x}\,\underbrace{x^{49}(1-x^{50})^{100}}_{\text{II}}\,dx$

$I_2 = I_1 + \left[\frac{x}{5050}(1-x^{50})^{101}\right]_0^1 - \int_0^1 \frac{(1-x^{50})^{101}}{5050}\,dx$

$I_2 = I_1 + 0 - \frac{I_2}{5050}$

$\Rightarrow \frac{5051}{5050} I_2 = I_1 \Rightarrow I_2 = \frac{5050}{5051} I_1$

$\Rightarrow \alpha = \frac{5050}{5051}$

61. **(c)** $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}\,dx$

$= \int_{-\pi/2}^{0} \frac{1}{1+e^{\sin x}}\,dx + \int_0^{\pi/2} \frac{1}{1+e^{\sin x}}\,dx$

$= \int_0^{\pi/2} \left(\frac{1}{1+e^{\sin x}} + \frac{1}{1+e^{-\sin x}}\right)dx$

$= \int_0^{\pi/2} \frac{1+e^{\sin x}}{1+e^{\sin x}}\,dx = \frac{\pi}{2}$

62. **(a)** $f(x) = |x-2| = \begin{cases} 2-x, & x<2 \\ x-2, & x \ge 2 \end{cases}$

$g(x) = f(f(x)) = \begin{cases} 2-f(x), & f(x)<2 \\ f(x)-2, & f(x) \ge 2 \end{cases}$

$= \begin{cases} 2-(2-x), & 2-x<2, \quad x<2 \\ (2-x)-2, & 2-x \ge 2, \quad x<2 \\ 2-(x-2), & x-2<2, \quad x \ge 2 \\ (x-2)-2, & x-2 \ge 2, \quad x \ge 2 \end{cases}$

$= \begin{cases} -x & 0 < x \le 0 \\ x & 0 < x < 2 \\ 4-x & 2 \le x < 4 \\ x-4 & x \ge 4 \end{cases}$

$\therefore \int_0^3 [g(x) - f(x)]\,dx$

$= \int_0^2 x\,dx + \int_2^3 (4-x)\,dx - \int_0^3 |x-2|\,dx = 1$

63. **(c)** $\int_{\pi/6}^{\pi/3} \left[\frac{1}{2}\frac{d(\tan^4 x)}{dx}\cdot \sin^4 3x + \frac{1}{2}\tan^4 x \cdot \frac{d(\sin^4 3x)}{dx}\right]dx$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} d(\tan^4 x \cdot \sin^4 3x)\,dx$

$= \left[\frac{\tan^4 x \sin^4 3x}{2}\right]_{\pi/6}^{\pi/3} = \frac{9\cdot 0}{2} - \frac{\frac{1}{9}\cdot 1}{2} = \frac{-1}{18}$

64. **(21)**

$\int_0^n \{x\}\,dx = n\int_0^1 x\cdot dx = \frac{n}{2}$

$$\Rightarrow \int_0^n [x]dx = \int_0^n (x - \{x\})dx = \frac{n^2}{2} - \frac{n}{2}$$

According to the questions,

$\frac{n}{2}, \frac{n^2-n}{2}, 10(n^2-n)$ are in GP

$$\therefore \left(\frac{n^2-n}{2}\right)^2 = \frac{n}{2} \times 10(n^2-n)$$

$$\Rightarrow n^2 = 21n \Rightarrow n = 21.$$

65. (c) $I = \int_{-\pi}^{\pi} |\pi - |x|| \, dx \quad [\because |\pi - |x|| \text{ is even}]$

$$= 2\int_0^{\pi} |\pi - |x|| \, dx$$

$$= 2\int_0^{\pi} (\pi - x)\, dx$$

$$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = 2\left(\pi^2 - \frac{\pi^2}{2}\right) = \pi^2.$$

66. (a) $\frac{k}{6} = \int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx$

Let $x = \sin\theta;\ dx = \cos\theta\, d\theta,$

then $\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta\cos\theta}{\cos^3\theta} d\theta$

$$\therefore \frac{k}{6} = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos^3\theta} \cdot \cos\theta\, d\theta$$

$$\Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \tan^2\theta\, d\theta = \int_0^{\frac{\pi}{6}} (\sec^2\theta - 1)\, d\theta$$

$$\Rightarrow \frac{k}{6} = (\tan\theta - \theta)_0^{\pi/6} = \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) = \frac{2\sqrt{3} - x}{6}$$

$$\Rightarrow k = 2\sqrt{3} - \pi$$

67. (1.50)

$$\int_0^2 \big||x-1| - x\big| dx = \int_0^1 |1 - x - x| dx + \int_1^2 \big| x - 1 - x \big| dx$$

$$= \int_0^1 (1-2x)dx + \int_{1/2}^1 (2x-1)dx + \int_1^2 dx$$

$$= [x - x^2]_0^{\frac{1}{2}} + [x^2 - x]_{\frac{1}{2}}^1 + [x]_1^2$$

$$= \frac{1}{2} - \frac{1}{4} + (1-1) - \left(\frac{1}{4} - \frac{1}{2}\right) + 2 - 1 = \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$$

68. (1)

$$\int_1^2 |2x - [3x]| \, dx$$

$$= \int_1^2 |3x - [3x] - x| \, dx$$

$$= \int_1^2 |\{3x\} - x| \, dx = \int_1^2 (x - \{3x\})dx$$

$$= \int_1^2 x dx - \int_1^2 \{3x\}\, dx$$

$$= \left[\frac{x^2}{2}\right]_1^2 - 3\int_0^{1/3} 3x\ dx$$

$$= \frac{(4-1)}{2} - 9\left[\frac{x^2}{2}\right]_0^{1/3} = \frac{3}{2} - \frac{1}{2} = 1$$

69. (d) $\int_0^1 (a + bx + cx^2)dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}\Big|_0^1 = a + \frac{b}{2} + \frac{c}{3}$

Now, $f(1) = a + b + c, f(0) = a$ and $f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

Now, $\frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right)$

$$= \frac{1}{6}\left(a + b + c + a + 4\left(a + \frac{b}{2} + \frac{c}{4}\right)\right)$$

$$= \frac{1}{6}(6a + 3b + 2c) = a + \frac{b}{2} + \frac{c}{3}$$

Hence, $\int_0^1 f(x) = \frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\}$

70. (c) $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$$= \int_0^{\pi}\left[\frac{x\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}\right] dx$$

$$\left[\because \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx\right]$$

$$= \int_0^{\pi} \frac{2\pi \sin^8 x}{\sin^8 x + \cos^8 x} dx$$

$$= 2\pi \int_0^{\pi/2} \left[\frac{\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{\cos^8 x}{\sin^8 x + \cos^8 x} \right] dx$$

$$= 2\pi \int_0^{\pi/2} 1 dx = 2\pi \times \frac{\pi}{2} = \pi^2$$

71. **(b)** $f(x) = \dfrac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

$$f'(x) = \frac{-1}{2}\left(\frac{(6x^2 - 18x + 12)}{(2x^3 - 9x^2 - 12x + 4)^{3/2}} \right)$$

$$= \frac{-6(x-1)(x-2)}{2(2x^3 - 9x^2 + 12x + 4)^{3/2}}$$

$f(1) = \dfrac{1}{3}$ and $f(2) = \dfrac{1}{\sqrt{8}}$

It is increasing function

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$$

$$\frac{1}{9} < I^2 < \frac{1}{8}$$

72. **(c)** $I = \dfrac{1}{(a+b)} \displaystyle\int_a^b x[f(x) + f(x+1)]dx$ ...(i)

$x \to a + b - x$

$$I = \frac{1}{(a+b)} \int_a^b (a+b-x)[f(a+b-x) + f(a+b+1-x)]dx$$

$I = \dfrac{1}{(a+b)} \displaystyle\int_a^b (a+b-x)[f(x+1) + f(x)]dx$ ...(ii)

[$\because$ put $x \to x+1$ in $f(a+b+1-x) = f(x)$]

Add (i) and (ii)

$$2I = \int_a^b [f(x+1) + f(x)]dx$$

$$2I = \int_a^b f(x+1)dx + \int_a^b f(x)dx$$

$$= \int_a^b f(a+b+1-x)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$\therefore \displaystyle\int_{a-1}^{b-1} f(x+1)dx$ [$\because$ Put $x \to x+1$]

73. **(a)** $4\alpha \left\{ \displaystyle\int_{-1}^{0} e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx \right\} = 5$

$$\Rightarrow 4\alpha \left\{ \frac{e^{\alpha x}}{\alpha}\Big|_{-1}^{0} + \frac{e^{-\alpha x}}{-\alpha}\Big|_{0}^{2} \right\} = 5$$

$$\Rightarrow 4\alpha \left\{ \left(\frac{1 - e^{-\alpha}}{\alpha} \right) - \left(\frac{e^{-2\alpha} - 1}{\alpha} \right) \right\} = 5$$

$$\Rightarrow 4(2 - e^{-\alpha} - e^{-2\alpha}) = 5$$

Put $e^{-\alpha} = t$

$\Rightarrow 4t^2 + 4t - 3 = 0$ $\Rightarrow (2t+3)(2t-1) = 0$

$\Rightarrow e^{-\alpha} = \dfrac{1}{2} \Rightarrow \alpha = \log_e 2$

74. **(a)** $2\cot^2\theta - \dfrac{5}{\sin\theta} + 4 = 0$

$$\frac{2\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$$

$\Rightarrow 2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0, \sin\theta \neq 0$

$\Rightarrow 2\sin^2\theta - 5\sin\theta + 2 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) = 0$

$\therefore \sin\theta = \dfrac{1}{2}$ $\Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$$\therefore \int_{\pi/6}^{5\pi/6} \cos^2 3\theta \, d\theta = \int_{\pi/6}^{5\pi/6} \frac{1 + \cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 6\theta}{6} \right]_{\pi/6}^{5\pi/6} = \frac{1}{2}\left[\frac{5\pi}{6} - \frac{\pi}{6} + \frac{1}{6}(0-0) \right]$$

$$= \frac{1}{2} \cdot \frac{4\pi}{6} = \frac{\pi}{3}$$

75. **(a)** Given, $\displaystyle\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$

Differentiating both sides,

$4(f(x))^3 . f'(x) = g'(x)(x-2) + g(x)$

Putting $x = 2$, $\dfrac{4(6)^3 . 1}{48} = g(2)$ $\Rightarrow \displaystyle\lim_{x \to 2} g(x) = 18$

76. **(d)** $\int_0^{\pi/2} \frac{\cot x\, dx}{\cot x + \operatorname{cosec} x}$

$= \int_0^{\pi/2} \frac{\cot x\, dx}{1+\cos x} = \int_0^{\pi/2} \left(1 - \frac{1}{1+\cos x}\right) dx$

$= [x]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx = \frac{\pi}{2} - \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{x}{2} dx$

$= \frac{\pi}{2} - \left(\tan \frac{x}{2}\right)_0^{\pi/2} = \frac{\pi}{2} - [1] = \left(\frac{\pi}{2} - 1\right) = m\pi + mn$

$\therefore\ m = ,\ n = -2$, Hence, $mn = -1$

77. **(b)** $I = \int_0^{2\pi} [\sin 2x(1+\cos 3x)]dx$ ...(1)

$\because \int_0^a f(x) = \int_0^a f(a-x)dx$

$\therefore I = \int_0^{2\pi} [-\sin 2x(1+\cos 3x)]dx$ ...(2)

From (1) + (2), we get;

$2I = \int_0^{2\pi} (-1)dx \Rightarrow 2I = -(x)_0^{2\pi} \Rightarrow I = -\pi$

78. **(c)** Let, $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x . \operatorname{cosec}^{\frac{4}{3}} x\, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1.dx}{\cos^{\frac{2}{3}} x . \sin^{\frac{4}{3}} x}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1dx}{\cos^2 x . \tan^{\frac{4}{3}} x} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$

Let $\tan x = u$

$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} u^{-\frac{4}{3}} du = \frac{3\left[u^{-\frac{1}{3}}\right]_{\frac{1}{\sqrt{3}}}}{-1}$

$= -3\left[3^{-\frac{1}{6}} - \frac{1}{3^{\frac{-1}{6}}}\right] = -3(3^{\frac{-1}{6}} - 3^{\frac{1}{6}})$

$= 3(3^{\frac{1}{6}} - 3^{\frac{1}{6}}) = (3^{\frac{7}{6}} - 3^{\frac{5}{6}})$

79. **(b)** Let $I = \int_0^{\pi/2} \frac{\sin^3 x dx}{\sin x + \cos x}$ ...(1)

Use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\cos^3 x\, dx}{\sin x + \cos x}$ ...(2)

Adding equation (1) and (2), we get

$\Rightarrow\ 2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin(2x)\right) dx$

$\Rightarrow I = \frac{1}{2}\left[x + \frac{1}{4}\cos 2x\right]_0^{\pi/2}$

$\Rightarrow\ I = \frac{\pi - 1}{4}$

80. **(0)** $\int_0^1 x \cot^{-1}\left(1 - x^2 + x^4\right) dx = \int_0^1 x \tan^{-1}\left(\frac{1}{1+x^4-x^2}\right)$

$= \int_0^1 x \tan^{-1}\left(\frac{x^2 - \left(x^2-1\right)}{1+x^2\left(x^2-1\right)}\right) dx$

$= \frac{1}{2}\int_0^1 1 \tan^{-1} t^2 dt - \frac{1}{2}\int_{-1}^0 1 \tan^{-1} k\ dk$

Put $x^2 = t \Rightarrow 2xdx = dt$ in the first integral
and $x^2 - 1 = k \Rightarrow 2xdx = dk$ in the second integral.

$= \frac{1}{2}\int_0^1 1 \tan^{-1} t dt - \frac{1}{2}\int_0^1 1 \tan^{-1} k dk$

$= \frac{1}{2}\left(t \tan^{-1} t\Big|_0^1 - \int_0^1 \frac{t}{1+t^2} dt\right)$

$-\frac{1}{2}\left(k \tan^{-1} k\Big|_0^1 - \int_{-1}^0 \frac{k}{1+k^2} dk\right)$

$= \frac{1}{2}\left(\frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+t^2\right)\Big|_0^1\right)\right) - \frac{1}{2}\left(-\frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+k^2\right)\Big|_{-1}^0\right)\right)$

$= \left(\frac{\pi}{8} - \frac{1}{4}\ln 2\right) - \left(\frac{-\pi}{8} - \frac{1}{4}10 - \ln 2\right) = \frac{\pi}{4} - \frac{1}{2}\ln 2$

81. **(d)** Using L' Hospital rule and Leibnitz theorem, we get

$$\lim_{x\to 2}\frac{\int_6^{f(x)} 2t\,dt}{(x-2)}=\lim_{x\to 2}\frac{2f(x)f'(x)-0}{1}$$

Putting $x=2$, $2f(2)f'(2)=12f'(2)$ $\quad [\because f(2)=6]$

82. **(d)** $g(f(x))=\log\left(\frac{2-x\cos x}{2+x\cos x}\right), x>0$

Let $I=\int_{-\pi/4}^{\pi/4}\log\left(\frac{2-x\cos x}{2+x\cos x}\right)dx$ ...(i)

Use the property $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

Then, equation (i) becomes,

$I=\int_{-\pi/4}^{\pi/4}\log\left(\frac{2+x\cos x}{2-x\cos x}\right)dx$ ...(ii)

Adding (i) and (ii)

$$2I=\int_{-\pi/4}^{\pi/4}\log\left(\frac{2-x\cos x}{2+x\cos x}\cdot\frac{2+x\cos x}{2-x\cos x}\right)dx$$

$$2I=\int_{-\pi/2}^{\pi/2}\log(1)dx=0$$

$\Rightarrow I=0=\log 1$

83. **(a)** $f(x)=\int_0^x g(g)dt,$ ...(i)

$\because$ g is a non-zero even function.

$\therefore g(-x)=g(x),$ ...(ii)

Given, $f(x+5)=g(x)$ ...(iii)

From (i) $f'(x)=g(x)$

Let, $I=\int_0^x f(t)dt,$

Put $t=\lambda-5\Rightarrow I=\int_5^{x+5} f(\lambda-5)d\lambda$

$\because f(x+5)=g(x)$

$\Rightarrow f(-x+5)=g(-x)=g(x)$ ...(iv)

$I=\int_5^{x+5} f(\lambda-5)d\lambda$

$f(0)=0$, $g(x)$ is even $\Rightarrow f(x)$ is odd

$\therefore I=\int_5^{x+5}-f(5-\lambda)d\lambda$

$\Rightarrow I=\int_5^{x+5} g(\lambda)d\lambda=\int_{x+5}^{5} g(t)dt$ (from (iv))

84. **(c)** $f(x)=f(a-x)$

$g(x)+g(a-x)=4$

Let, the integral,

$I=\int_0^a f(x)g(x)dx$

$=\int_0^a f(a-x)\cdot g(a-x)dx$

$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$

$\Rightarrow I=\int_0^a f(x)[4-g(x)]dx$

$\Rightarrow I=\int_0^a 4f(x)dx-\int_0^a f(x)\cdot g(x)dx$

$\Rightarrow I=\int_0^a 4f(x)dx-I$

$\Rightarrow 2I=\int_0^a 4f(x)dx$

$\Rightarrow I=2\int_0^a f(x)dx$

85. **(d)** $I=\int_1^e\left\{\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{x}\right\}\log_e x\,dx$

Let $\left(\frac{x}{e}\right)^x=t$

$\Rightarrow x\ln\left(\frac{x}{e}\right)=\ln t$

$\Rightarrow x(\ln x-1)=\ln t$

On differentiating both sides w.r. tx we get

$\ln x.dx=\frac{dt}{t}$

When $x=e$ then $t=1$ and when $x=1$ then $t=\frac{1}{e}$.

$$I=\int_{\frac{1}{e}}^{1}\left(t^2-\frac{1}{t}\right)\cdot\frac{dt}{t}=\int_{\frac{1}{e}}^{1}\left(t-\frac{1}{t^2}\right)dt$$

$$=\left(\frac{t^2}{2}+\frac{1}{t}\right)_{\frac{1}{e}}^{1}=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2e^2}+e\right)=\frac{3}{2}-e-\frac{1}{2e^2}$$

86. **(a)** Let $f(x)=\dfrac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}$

So, $f(-x)=\dfrac{\sin^2(-x)}{\left[\frac{-x}{\pi}\right]+\frac{1}{2}}$ $\quad\because [-x]=-1-[x]$

$\Rightarrow f(-x)=\dfrac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}=\dfrac{\sin^2 x}{-\frac{1}{2}-\left[\frac{x}{\pi}\right]}=-f(x)$

$\Rightarrow f(x)$ is odd function

Hence, $\int_{-2}^{2} f(x)\,dx=0$

87. **(b)** $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\dfrac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$

$=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\dfrac{\tan^5 x.\sec^2 x}{2\frac{\sin x}{\cos x}\left(\left(\tan^5 x\right)^2+1\right)}$

$=\dfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\dfrac{\tan^4 x.\sec^2 x}{\left(\tan^5 x\right)^2+1}dx.$

Let $\tan^4 x=t$.

$5\tan^4 x\,.\,\sec^2 x\,dx=dt$.

When $x\to\dfrac{\pi}{4}$ then $t\to 1$

and $x\to\dfrac{\pi}{6}$ then $t\to\left(\dfrac{1}{\sqrt{3}}\right)^5$

$\therefore\ I=\dfrac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1}\dfrac{dt}{t^2+1}$

$=\dfrac{1}{10}\left(\dfrac{\pi}{4}-\tan^{-1}\left(\dfrac{1}{9\sqrt{3}}\right)\right)$

88. **(d)** $I=\int_a^b (x^4-2x^2)\,dx$

$\Rightarrow \dfrac{dI}{dx}=x^4-2x^2=0$ (for minimum)

$\Rightarrow x=0,\pm\sqrt{2}$

Also, $I=\left[\dfrac{x^5}{5}-\dfrac{2x^3}{3}\right]_a^b$

For a = 0, b = $\sqrt{2}$

$I=\dfrac{-8\sqrt{2}}{15}$

For a = $-\sqrt{2}$, b = 0

$I=\dfrac{-8\sqrt{2}}{15}$.

For a = $\sqrt{2}$, b = $-\sqrt{2}$

$I=\dfrac{16\sqrt{2}}{15}$.

For a = $-\sqrt{2}$, b = $\sqrt{2}$

$I=\dfrac{-16\sqrt{2}}{15}$.

$\therefore\ I$ is minimum when $(a,b)=\left(-\sqrt{2},\sqrt{2}\right)$

89. **(a)** $\int_0^x f(t)dt=x^2+\int_x^1 t^2 f(t)dt$

$\Rightarrow f(x)=2x-x^2 f(x)$

$\Rightarrow f(x)=\dfrac{2x}{1+x^2}$

$\Rightarrow f'(x)=\dfrac{2(1-x^2)}{(1+x^2)^2}$

Then,

$f'(1/2)=\dfrac{2\left(1-\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2}=\dfrac{3}{2}\times\dfrac{16}{25}=\dfrac{24}{25}$

90 **(c)** $I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\dfrac{dx}{[x]+[\sin x]+4}$

$=\int_{\frac{-\pi}{2}}^{-1}\dfrac{dx}{-2-1+4}+\int_{-1}^{0}\dfrac{dx}{-1-1+4}+\int_{0}^{1}\dfrac{dx}{0+0+4}+\int_{1}^{\frac{\pi}{2}}\dfrac{dx}{1+0+4}$

$=\left(-1+\dfrac{\pi}{2}\right)+\dfrac{1}{2}(0+1)+\dfrac{1}{4}(1-0)+\dfrac{1}{5}\left(\dfrac{\pi}{2}-1\right)$

$=\dfrac{3\pi}{5}-\dfrac{9}{20}=\dfrac{3}{20}(4\pi-3)$

91. **(b)** $I=\int_0^{\pi}|\cos x|^3\,dx$

$=2\int_0^{\pi/2}\cos^3 x\,dx$

$=\frac{2}{4}\int_0^{\pi/2}(3\cos x+\cos 3x)dx$

$[\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta]$

$=\frac{1}{2}\left[3\sin x+\frac{\sin 3x}{3}\right]_0^{\pi/2}$

$=\frac{1}{2}\left(3-\frac{1}{3}\right)=\frac{4}{3}$

92. **(a)** $\because f:R\to R$

and $|f(x)-f(y)|\le 2\cdot|x-y|^{3/2}$

$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right|\le 2\sqrt{x-y}$

$\Rightarrow \lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right|\le \lim_{x\to y}2\sqrt{x-y}$

$\Rightarrow |f'(x)|=0$

$\therefore$ $f(x)$ is a constant function.

Given $f(0)=1 \Rightarrow f(x)=1$

Hence, the integral

$\int_0^1 f^2(x)\,dx=\int_0^1 1\,dx=[x]_0^1=1$

93. **(d)** Let, $I=\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta$

$=\frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta$

Let $\cos\theta = t^2$

$\therefore$ $\sin\theta\,d\theta=-2t\,dt$

Hence, integral becomes,

$I=\frac{1}{\sqrt{2k}}\int_1^{\sqrt{\frac{1}{2}}}\frac{-2t\,dt}{t}$

$=\sqrt{\frac{2}{k}}\int_{\frac{1}{\sqrt{2}}}^{1}dt$

$=\sqrt{\frac{2}{k}}\left(1-\frac{1}{\sqrt{2}}\right)$

$=\frac{\sqrt{2}-1}{\sqrt{k}}$

$=1-\frac{1}{\sqrt{2}}$ (Given)

$\therefore$ $k=2$

94. **(c)** Let, $I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$ ...(i)

Using, $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, we get :

$I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^{-x}}dx$ ...(ii)

Adding (i) and (ii), we get;

$2I=\int_{-\pi/2}^{\pi/2}\sin^2 x\,dx \Rightarrow 2I=2\cdot\int_0^{x/2}\sin^2 x\,dx$

$\Rightarrow 2I=2\times\frac{\pi}{4}\Rightarrow I=\frac{\pi}{4}$

95. **(a)** $f(x)=\int_0^x t(\sin x-\sin t)\cdot dt$

$=\sin x\int_0^x t\cdot dt-\int_0^x t\sin t\cdot dt$

$=\frac{x^2}{2}\sin x+[t\cos t]_0^x+\sin x$

$\Rightarrow f(x)=\frac{x^2}{2}\sin x+x\cos x+\sin x$

$f'(x)=\frac{x^2}{2}\cos x+2\cos x$

$f''(x)=x\cos x-\frac{x^2}{2}\sin x-2\sin x$

$f'''(x)=\cos x-2x\sin x-\frac{x^2}{2}\cos x-2\cos x$

$\therefore$ $f'''(x)+f'(x)=\cos x-2x\sin x$

96. **(a)** Let $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x}{1+\sin x}dx$

also let $K=\frac{x}{1+\sin x}$

Multiplying numerator and denominator by $(1-\sin x)$, we get;

$$K=\frac{x(1-\sin x)}{1-(\sin x)^2}=\frac{x(1-\sin x)}{(\cos x)^2}$$
$= x(1-\sin x)\sec^2 x$
$= x\sec^2 x - x\sin x\sec^2 x = x\sec^2 x - x\tan x\sec x$

Now, $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\sec^2 x dx-\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\sec x\tan x dx$

$$=\left[x\tan x-\int\frac{dx}{dx}\tan x dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}-\left[x\sec x-\int\frac{dx}{dx}\sec x dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$=\left[x\tan x-\ln|\sec x|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$-\left[x\sec x-\ln|\sec x+\tan x|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}+c$$

$$\Rightarrow I=\left\{\left[\frac{3\pi}{4}\tan\frac{3\pi}{4}-\ln\left|\frac{3\pi}{4}\right|\right.\right.$$

$$\left.\left.-\left[\frac{3\pi}{4}\sec\frac{3\pi}{4}-\ln\left|\sec\frac{3\pi}{4}+\tan\frac{3\pi}{4}\right|\right]\right]\right\}$$

$$-\left\{\left[\frac{\pi}{4}\tan\frac{\pi}{4}-\ln\left|\frac{\pi}{4}\right|\right.\right.$$

$$\left.\left.-\left[\frac{\pi}{4}\sec\frac{\pi}{4}-\ln\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|\right]\right]\right\}$$

$$=\frac{\pi}{2}(\sqrt{2}+1)$$

97. **(d)** Given:

$I_1=\int_0^1 e^{-x}\cos^2 x\,dx;$

$I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$ and

$I_3=\int_0^1 e^{-x^3}dx$

For $x\in(0,1)$
$\Rightarrow x>x^2$ or $-x<-x^2$
and $x^2>x^3$ or $-x^2<-x^3$

$\therefore\ e^{-x^2}<e^{-x^3}$ and $e^{-x}<e^{-x^2}$

$\Rightarrow e^{-x}<e^{-x^2}<e^{-x^3}$

$\Rightarrow e^{-x^3}>e^{-x^2}>e^{-x}$
$\Rightarrow I_3>I_2>I_1$

98. **(c)** Let

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx \text{ ..... (1)}$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4(-x))\left(1+\log\left(\frac{2+\sin(-x)}{2-\sin(-x)}\right)\right)\cdot dx$$

$$=\left[\because\int_a^b f(x).dx=\int_a^b f(a+b-x).dx\right]$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin^4 x)\left(1+\log\left(\frac{2-\sin x}{2+\sin x}\right)\right).dx$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x\left(1-\log\left(\frac{2+\sin x}{2-\sin x}\right)\right).dx \text{ .....(2)}$$

After adding equation (1) and (2) we get,

$$2I=2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x\,.dx$$

$$2I=4\int_0^{\frac{\pi}{2}}\sin^4 x\,.dx$$

$$I=2\int_0^{\frac{\pi}{2}}\sin^4 x\,.dx=\frac{2\times\frac{3}{2}\times\frac{1}{2}\times\pi}{2\times2}=\frac{3\pi}{8}$$

[By Gamma function]

99. **(c)** $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1+\cos x}$ ...(i)

$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1-\cos x}$ ...(ii)

Using $\int_a^b f(x)dx=\int_a^b f(a+b-x)\,dx$

Adding (i) and (ii)

$$2I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{2}{\sin^2 x}dx;\quad I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\operatorname{cosec}^2 x\,dx$$

$$I=-(\cot x)_{\pi/4}^{3\pi/4}=-\left[\cot\frac{3\pi}{4}-\cot\frac{\pi}{4}\right]=2$$

100. **(c)** $I_n=\int\tan^n x\,dx, n>1$

Let $I=I_4+I_6$

$$=\int(\tan^4 x+\tan^6 x)dx=\int\tan^4 x\sec^2 x\,dx$$

Let $\tan x = t$

$\Rightarrow \sec^2 x\, dx = dt$

$\therefore I = \int t^4 dt = \frac{t^5}{5} + C$

$= \frac{1}{5}\tan^5 x + C \Rightarrow$ On comparing, we have

$a = \frac{1}{5}, b = 0$

101. (a) Let $I = \int_1^2 \frac{dx}{((x-1a)^2+3)^{3/2}}$

Let; $x - 1 = \sqrt{3}\tan\theta$

$\Rightarrow dx = \sqrt{3}\sec^2 . d\theta$

$$\Rightarrow I = \int_0^{\pi/6} \frac{\sqrt{3}\sec^2\theta\, d\theta}{\left(\left(\sqrt{3}\tan\theta\right)^2 + \left(\sqrt{3}\right)^2\right)^{3/2}}$$

$$= \frac{1}{3}\int_0^{\pi/6} \frac{\sec^2\theta}{\sec^3\theta} d\theta = \frac{1}{3}\int_0^{\pi/6} \cos\theta\, d\theta$$

$$= \frac{1}{3}[\sin\theta]_0^{\pi/6} = \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}$$

$$= \frac{1}{6} = \frac{k}{k+5} \Rightarrow k+5 = 6k$$

$$\Rightarrow \boxed{k=1}$$

102. (a) $$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{\cos 2x}{\left(\frac{1}{\sin 2x}\right)^3} = \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos 2x \times \sin 2x . \sin^2(2x)\, dx$$

$$= \frac{1}{4}\int_{\pi/12}^{\pi/4} \sin 4x . (1-\cos 4x)\, dx$$

$$= \frac{1}{4}\left[\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin 4x - \frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin 8x\right]$$

$$= \frac{1}{4}\left[-\frac{\cos 4x}{4} + \frac{\cos 8x}{16}\right]_{\pi/12}^{\pi/4} = \frac{1}{4}\left[\frac{15}{32}\right] = \frac{15}{128}$$

103. (d) $$\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3} dx$$

Dividing by x^{15} in numerator and denominator

$$\int \frac{\frac{2}{x^3}+\frac{5}{x^6}\, dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}$$

Let $1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$$\Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6}\right)dx = dt \Rightarrow \left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx = -dt$$

This gives,

$$\int \frac{\frac{2}{x^3}+\frac{5}{x^6}\, dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3} = \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2} + C = \frac{x^{10}}{2(x^5+x^3+1)^2} + C$$

104. (d) $$x\int_1^x y(t)\, dt = x\int_1^x ty(t)\, dt + \int_1^x ty(t)\, dt$$

Differentiate *w.r.* to x.

$$\int_1^x y(t)dt + x[y(x) - y(1)]$$

$$= \int_1^x ty(t)dt + x[xy(x) - y(1)] + xy(x) - y(1)$$

$$\int_1^x y(t)dt = \int_1^x ty(t)dt + x^2 y(x) - y(1)$$

Diff. again *w.r.* to x

$y(x) - y(a) = xy(x) - y(a) + 2x\,y(x) + x^2 y^1(x)$

$(1-3x)\,y(x) = x^2 y^1(x)$

$$\frac{y^1(x)}{y(x)} = \frac{1-3x}{x^2}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1-3x}{x^2} \Rightarrow \ln y = -\frac{1}{x} - 3\ln x$$

$$\ln(yx^3) = -\frac{1}{x}$$

$$yx^3 = -e^{-1/x}$$

$$y = \frac{e^{-\frac{1}{x}}}{x^3} \text{ or } y = \frac{ce^{-\frac{1}{x}}}{x^3}$$

105. (d) $$I = \int_4^{10} \frac{[x^2]}{[x^2-28x+196]+[x^2]} dx \quad .....(a)$$

Use $$\int_a^b f(a+b-x)\, dx = \int_a^b f(x)\, dx$$

$$I=\int_4^{10}\frac{\left[(x-14)^2\right]}{\left[x^2\right]+\left[(x-14)^2\right]}dx \quad .....(b)$$

(a) + (b)

$$2I=\int_4^{10}\frac{\left[(x-14)^2\right]+\left[x^2\right]}{\left[x^2\right]+\left[(x-14)^2\right]}dx$$

$$2I=\int_4^{10}dx \Rightarrow 2I=6 \qquad \Rightarrow I=3$$

106. (b) $2\int_0^1 \tan^{-1}x\,dx=\int_0^1\left(\frac{\pi}{2}-\tan^{-1}(1-x+x^2)\right)dx$

$$2\int_0^1 \tan^{-1}x\,dx=\int_0^1\frac{\pi}{2}dx-\int_0^1\tan^{-1}(1-x+x^2)\,dx$$

$$\int_0^1\tan^{-1}(1-x+x^2)\,dx=\frac{\pi}{2}-2\int_0^1\tan^{-1}x\,dx \quad .....(a)$$

Let, $I_1=\int_0^1\tan^{-1}x\,dx$

$$=\left[(\tan^{-1}x)x\right]_0^1-\int_0^1\frac{1}{1+x^2}x\,dx$$

$$=\frac{\pi}{4}-\int_0^1\frac{x}{1+x^2}dx=\frac{\pi}{4}-\frac{1}{2}\log 2$$

By equation (a)

$$\frac{\pi}{2}-2\left[\frac{\pi}{4}-\frac{1}{2}\log 2\right]=\log 2$$

107. (a) $I=\int_2^4\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$

$$I=\int_2^4\frac{\log x^2}{\log x^2+\log(6-x)^2}dx. \quad ...(i)$$

$$I=\int_2^4\frac{\log(6-x)^2}{\log(6-x)^2+\log x^2}dx \quad ...(ii)$$

Adding (i) and (ii)

$$2I=\int_2^4 dx \quad =[x]_2^4=2$$

$I=1$

108. (c) Let $f:R\to R$ be a function such that $f(2-x)=f(e+x)$

Put $x=2+x$ we get

$f(-x)=f(4+x)=f(4-x)$

$\Rightarrow \; f(x)=f(x+4)$

Hence period is 4

Consider $\int_{10}^{50}f(x)dx=10\int_{10}^{14}f(x)dx=10\,[5+5]=100$

109. (d) Let f : (–1, 1) → R be a continuous function

Let $\int_0^{\sin x}f(t)dt=\frac{\sqrt{3}}{2}x$

$$f(\sin x).\frac{d}{dx}(\sin x)=\frac{\sqrt{3}}{2}$$

$$\Rightarrow f(\sin x).\cos x=\frac{\sqrt{3}}{2}$$

put $x=\frac{\pi}{3}$

$$f\left(\sin\frac{\pi}{3}\right).\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right).\frac{1}{2}=\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right)=\sqrt{3}$$

110. (c) $f\left(\frac{1}{x}\right)=\int_1^{1/x}\frac{\ln t}{1+t}dt$

Let $t=\frac{1}{z}$

$$dt=-\frac{1}{z^2}dz$$

$$f(x)=\int_1^x\frac{\ln z}{z(z+1)}dz$$

$$f(x)+f\left(\frac{1}{x}\right)=\int_1^x\frac{\ln x}{z}dz=\left[\frac{(\ln z)^2}{2}\right]_1^x=\frac{(\ln x)^2}{2}$$

111. (b) Let $I=\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx=\int_0^{\pi}\left|2\sin\frac{x}{2}-1\right|dx$

$$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$$

$$\left[\because\ \sin\frac{x}{2}=\frac{1}{2}\Rightarrow\frac{x}{2}=\frac{\pi}{6}\right.$$

$$\left.\Rightarrow x=\frac{\pi}{3},\frac{x}{2}=\frac{5\pi}{6}\Rightarrow x=\frac{5\pi}{3}>\pi\right]$$

$$=\left[x+4\cos\frac{x}{2}\right]_0^{\pi/3}+\left[-4\cos\frac{x}{2}-x\right]_{\pi/3}^{\pi}$$

$$=\frac{\pi}{3}+4\frac{\sqrt{3}}{2}-4+\left(0-\pi+4\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)$$

$$=4\sqrt{3}-4-\frac{\pi}{3}$$

112. (d) $F(x)=\int_1^x \frac{e^t}{t}dt,\ x>0$

Let $I=\int_1^x \frac{e^t}{t+a}dt$

Put $t+a=z \Rightarrow t=z-a;\ dt=dz$
for $t=1,\ z=1+a$
for $t=x,\ z=x+a$

$$\therefore\ I=\int_{1+a}^{x+a}\frac{e^{z-a}}{z}dz$$

$$=e^{-a}\int_{1+a}^{x+a}\frac{e^z}{z}dz\equiv e^{-a}\int_{1+a}^{x+a}\frac{e^t}{t}dt$$

$$I=e^{-a}\left[\int_{1+a}^{1}\frac{e^t}{t}dt+\int_1^{x+a}\frac{e^t}{t}dt\right]$$

$$=e^{-a}\left[-\int_1^{1+a}\frac{e^t}{t}dt+\int_1^{x+a}\frac{e^t}{t}dt\right]$$

$$=e^{-a}[-F(1+a)+F(x+a)]$$

(By the definition of F(x))

$$=e^{-a}[F(x+a)-F(1+a)]$$

113. (a) Let $\int_{-\pi}^{t}(f(x)+x)dx=\pi^2-t^2$

$$\Rightarrow \int_{-\pi}^{t}f(x)dx+\int_{-\pi}^{t}xdx=\pi^2-t^2$$

$$\Rightarrow \int_{-\pi}^{t}f(x)dx+\left(\frac{t^2}{2}-\frac{\pi^2}{2}\right)=\pi^2-t^2$$

$$\Rightarrow \int_{-\pi}^{t}f(x)dx=\frac{3}{2}(\pi^2-t^2)$$

differentiating with respect to t

$$\frac{d}{dt}\left[\int_{-\pi}^{t}f(x)dx\right]=\frac{3}{2}\frac{d}{dt}(\pi^2-t^2)$$

$$f(t).\frac{dt}{dt}-f(-\pi)\frac{d}{dt}(-\pi)=-3t$$

$$f(t)=-3t$$

$$f\left(-\frac{\pi}{3}\right)=-3\left(-\frac{\pi}{3}\right)=\pi$$

114. (d) Let $I=\int_0^{\pi}[\cos x]dx$ ...(1)

$$I=\int_0^{\pi}[\cos(\pi-x)]dx=\int_0^{\pi}[-\cos x]dx \quad ...(2)$$

On adding (1) and (2), we get

$$2I=\int_0^{\pi}[\cos x]dx+\int_0^{\pi}[-\cos x]dx$$

$$2I=\int_0^{\pi}[\cos x]+[-\cos x]dx$$

$$2I=\int_0^{\pi}-1dx \qquad (\because [x]+[-x]=-1\ \text{ if } x\notin Z)$$

$$2I=-x\big|_0^{\pi}=-\pi$$

$$\Rightarrow I=\frac{-\pi}{2}$$

115. (c) $P_n=\int_1^e(\log x)^n dx$

put $\log x=t$ then $x=e^t$ and $dx=e^t\,dt$
Also, when $x=1$, then $t=\log 1=0$
and when $x=e$, then $t=\log_e e=1$

$$\therefore\ P_n=\int_0^1 t^n.e^t\,dt$$

$$\therefore\ P_{10}=\int_0^1 t^{10}e^t dt \text{ and } P_8=\int_0^1 t^8e^t dt$$

$$\text{Now, } P_{10}-90P_8=\int_0^1 \underset{\mathrm{I}}{t^{10}}\,\underset{\mathrm{II}}{e^t}\,dt-90\int_0^1 t^8e^t dt$$

$$P_{10}-90\,P_8=\left[t^{10}e^t\right]_0^1-10\int_0^1 t^9e^t dt-90\int_0^1 t^8e^t dt$$

$P_{10}-90\,P_8$

$$=e-10\left[t^9\int_0^1 e^t dt-\int_0^1\frac{d}{dt}(t^9)\int e^t dt\right]-90\int_0^1 t^8e^t dt$$

$$P_{10}-90\,P_8=e-10\left[e-9\int_0^1 t^8e^t dt\right]-90\int_0^1 t^8e^t dt$$

$$P_{10}-90\,P_8=e-10e+90\int t^8\cancel{e^t}\,dt-90\int_0^1 t^8\cancel{e^t}\,dt$$

$$\therefore\ P_{10}-90P_8=-9e$$

116. (c) Let $I=\int_0^{1/2}\frac{\ln(1+2x)}{1+4x^2}dx$ or $\int_0^{1/2}\frac{\ln(1+2x)}{1+(2x)^2}dx$

Put $2x=\tan\theta$

$$\therefore\ \frac{2dx}{d\theta}=\sec^2\theta \text{ or } dx=\frac{\sec^2\theta d\theta}{2}$$

also when $x = 0 \Rightarrow \theta = 0$

and when $x = \frac{1}{2} \Rightarrow \theta = 45°$ or $\frac{\pi}{4}$

$$\therefore \text{ I} = \int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{1+\tan^2\theta} \times \frac{\sec^2\theta d\theta}{2}$$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{1+\tan^2\theta} \times \sec^2\theta d\theta$$

$(\because\ 1 + \tan^2\theta = \sec^2\theta)$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta \quad ...(i)$$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

(Using the property of definite integral)

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[1+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\times\tan\theta}\right]d\theta$$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta$$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[\frac{1+\cancel{\tan\theta}+1-\cancel{\tan\theta}}{1+\tan\theta}\right]d\theta$$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[\frac{2}{1+\tan\theta}\right]d\theta$$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} [\ln 2 - \ln(1+\tan\theta)]d\theta$$

$$\text{I} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln 2 . d\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta$$

$$\text{I} = \frac{1}{2}\ln 2\theta\Big|_0^{\pi/4} - I \qquad \text{(from eq. (i))}$$

$$\text{I} + \text{I} = \frac{1}{2}\ln 2\left(\frac{\pi}{4}-0\right)$$

$$2\text{I} = \frac{1}{2}\times\frac{\pi}{4}\times\ln 2$$

$$2\text{I} = \frac{\pi}{8}\ln 2 \text{ or } I = \frac{\pi}{16}\ln 2$$

117. (a) Since, $y = \int_0^x |t|dt,\ x \in R$

therefore $\frac{dy}{dx} = |x|$

But from $y = 2x$, $\therefore \frac{dy}{dx} = 2$

$\Rightarrow |x| = 2 \Rightarrow x = \pm 2$

Points $y = \int_0^{\pm 2} |t|dt = \pm 2$

$\therefore$ Equation of tangent is

$y - 2 = 2(x-2)$ or $y + 2 = 2(x+2)$

$\Rightarrow$ x-intercept $= \pm 1$.

118. (d) Let, $\text{I} = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$$= \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}\,dx}{1+\sqrt{\tan x}} \quad \ldots(i)$$

Also, given

$$\text{I} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}\,dx}{1+\sqrt{\tan x}} \quad \ldots(ii)$$

By adding (i) and (ii), we get

$$2\text{I} = \int_{\pi/6}^{\pi/3} dx$$

$$\Rightarrow \text{I} = \frac{1}{2}\left[\frac{\pi}{3}-\frac{\pi}{6}\right] = \frac{\pi}{12},$$

Statement-1 is false

$$\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

It is fundamental property.

Statement -2 is true.

119. (a) Consider

$$\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})\,dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})\,dt$$

Let $\text{I} = f(x)$ after integrating and putting the limits.

$$f'(x) = \sin^{-1}\sqrt{\sin^2 x}\,(2\sin x\cos x) - 0$$
$$+\cos^{-1}\sqrt{\cos^2 x}\,(-2\cos x\sin x) - 0$$

$\therefore f'(x) = 0 \Rightarrow f(x) = \text{C}$ (constant)

Now, we find $f(x)$ at $x = \frac{\pi}{4}$

$$\therefore \text{ I} = \int_0^{1/2} \sin^{-1}\sqrt{t}\,dt + \int_0^{1/2} \cos^{-1}\sqrt{t}\,dt$$

$$= \int_0^{1/2} (\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t})\, dt = \int_0^{1/2} \frac{\pi}{2}\, dt = \frac{\pi}{4} = C$$

$\therefore\ f(x) = \frac{\pi}{4}$

$\therefore$ Required integration $= \frac{\pi}{4}$

120. (d) $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}\, dx$ ...(i)

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}}\, dx$, by replacing x by $\left(\frac{\pi}{2} - \frac{\pi}{2} - x\right)$

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{2^x.\sin^2 x}{1+2^x}\, dx$ ...(ii)

Adding equations (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = \frac{1}{2}\int_{-\pi/2}^{\pi/2} (1-\cos 2x)\, dx$$

$$\Rightarrow I = \frac{1}{4}\left[x + \frac{\sin 2x}{2}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{4}\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2}\right)\right]$$

$$\Rightarrow I = \frac{1}{4}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \frac{\pi}{4}$$

121. (d) Let $I = \int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x}\, dx$

$$= \int_{7\pi/4}^{7\pi/3} \tan x\, dx = -\log\cos x \Big|_{7\pi/4}^{7\pi/3}$$

$$= -\left[\log\cos\frac{7\pi}{3} - \log\cos\frac{7\pi}{4}\right]$$

$$= \log\cos\frac{7\pi}{4} - \log\cos\frac{7\pi}{3}$$

$$= \log\left[\frac{\cos\frac{7\pi}{4}}{\cos\frac{7\pi}{3}}\right] = \log\left[\frac{\cos\left(2\pi - \frac{\pi}{4}\right)}{\cos\left(2\pi + \frac{\pi}{3}\right)}\right]$$

$$= \log\left(\frac{\cos\frac{\pi}{4}}{\cos\frac{\pi}{3}}\right) = \log\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}\right)$$

$$= \log\left(\frac{2}{\sqrt{2}}\right) = \log\sqrt{2}.$$

122. (a) $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$

$\Rightarrow 1 = \frac{1}{\sqrt{1+y^2}} \cdot \frac{dy}{dx}$

$\left[\because \text{ If } I(x) = \int_{\phi(x)}^{\psi(x)} f(t)\, dt, \text{ then } \frac{dI(x)}{dx} = f\{\psi(x)\}.\left\{\frac{d}{dx}\psi(x)\right\} - f\{\phi(x)\}.\left\{\frac{d}{dx}\phi(x)\right\}\right]$

$\frac{dy}{dx} = \sqrt{1-y^2}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{2\sqrt{1+y^2}}.2y.\frac{dy}{dx} = \frac{y}{\sqrt{1+y^2}}.\sqrt{1+y^2} = y$$

123. (b, c) $g(x+\pi) = \int_0^{x+\pi} \cos 4t\, dt$

$$= \int_0^x \cos 4t\, dt + \int_x^{\pi+x} \cos 4t\, dt = g(x) + \int_0^{\pi} \cos 4t\, dt$$

(it is clear from graph of cos 4t)

$$\int_x^{\pi+x} \cos 4t\, dt = \int_0^{\pi} \cos 4t\, dt = g(x) + g(\pi) = g(x) - g(\pi)$$

($\because$ From graph of cos 4t, $g(\pi) = 0$)

124. (d) $\int_{-0.9}^{0.9} \left\{[x^2] + \log\left(\frac{2-x}{2+x}\right)\right\} dx$

$$= \int_{-0.9}^{0.9} [x^2]dx + \int_{-0.9}^{0.9} \log\left(\frac{2-x}{2+x}\right) dx$$

$$= 0 + \int_{-0.9}^{0.9} \log\left(\frac{2-x}{2+x}\right) dx$$

Put $x = -x \Rightarrow f(x) = \log\frac{2-x}{2+x}$

and $f(-x) = \log\frac{2+x}{2-x} = -\log\frac{(2-x)}{2+x} = -f(x)$

So, it is an odd function, hence Required integral = 0.

125. (d) Since $\int_0^a [x] = 0$ where $0 \le a \le 1$

$$\therefore \int_0^{0.9} [x - 2[x]]dx = 0$$

126. (a) Let $\frac{d}{dx}G(x)=\frac{e^{\tan x}}{x}, x\in\left(0,\frac{\pi}{2}\right)$

Now, $I=\int_{1/4}^{1/2}\frac{2}{x}e^{\tan \pi x^2}\cdot dx=\int_{1/4}^{1/2}\frac{2\pi x}{\pi x^2}e^{\tan \pi x^2}.dx$

Let $\pi x^2=t\Rightarrow 2\pi x\,dx=dt$

When $x=\frac{1}{2}, t=\frac{\pi}{4}$ and $x=\frac{1}{4}, t=\frac{\pi}{16}$

$\therefore\ I=\int_{\pi/16}^{\pi/4}\frac{e^{\tan t}}{t}dt=G(t)\Big|_{\frac{\pi}{16}}^{\frac{\pi}{4}}=G\left(\frac{\pi}{4}\right)-G\left(\frac{\pi}{16}\right)$

127. (d) Let $\int_e^x t\,f(t)\,dt=\sin x-x\cos x-\frac{x^2}{2}$

By using Leibnitz rule, we get

$$\frac{d}{dx}\left[\int_e^x t\,f(t)\,dt\right]=\frac{d}{dx}\left[\sin x-x\cos x-\frac{x^2}{2}\right]$$

$\Rightarrow\ x f(x)-e f(e).\,0=x\sin x-x$

Now, put $x=\frac{\pi}{6}$, we get

$\frac{\pi}{6}.f\left(\frac{\pi}{6}\right)=\frac{\pi}{6}.\sin\frac{\pi}{6}-\frac{\pi}{6}$

$\Rightarrow f\left(\frac{\pi}{6}\right)=\sin\frac{\pi}{6}-1=\frac{1}{2}-1=-\frac{1}{2}$

128. (c) $\int_0^{1.5}x\left[x^2\right]dx=\int_0^1 x[x^2]dx+\int_1^{\sqrt{2}}x\left[x^2\right]dx+\int_{\sqrt{2}}^{1.5}x\left[x^2\right]dx$

$=\int_0^1 x.0\,dx+\int_1^{\sqrt{2}}x dx+\int_{\sqrt{2}}^{1.5}2x dx=0+\left[\frac{x^2}{2}\right]_1^{\sqrt{2}}+\left[x^2\right]_{\sqrt{2}}^{1.5}$

$=\frac{1}{2}(2-1)+(2.25-2)\ =\frac{1}{2}+0.25$

$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

129. (d) $I=\int_0^1\frac{8\log(1+x)}{1+x^2}dx$

Put $x=\tan\theta$,

$\therefore\ dx=\sec^2\theta d\theta$

$\therefore I=8\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{1+\tan^2\theta}.\sec^2\theta\,d\theta$

$I=8\int_0^{\pi/4}\log(1+\tan\theta)d\theta$ ...(i)

Applying $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

$=8\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$

$=8\int_0^{\pi/4}\log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta=8\int_0^{\pi/4}\log\left[\frac{2}{1+\tan\theta}\right]d\theta$

$=8\int_0^{\pi/4}[\log 2-\log(1+\tan\theta)]d\theta$

$=8\log 2\int_0^{\pi 14}1d\theta-8\int_0^{\pi 14}\log(1+\tan\theta)\,d\theta$

$\mathrm{I}=8\cdot(\log 2)[x]_0^{\pi/4}-8\int_0^{\pi/4}\log(1+\tan\theta)d\theta$

$\mathrm{I}=8\cdot\frac{\pi}{4}\cdot\log 2-\mathrm{I}$ [From equation (i)]

$\Rightarrow\ 2\mathrm{I}=2\pi\log 2$

$\therefore\ \mathrm{I}=\pi\log 2$

130. (a) $p'(x)=p'(1-x)$

$\Rightarrow\ p(x)=-p(1-x)+c$

at $x=0$

$p(0)=-p(1)+c\ \Rightarrow\ 42=c$

Now, $p(x)=-p(1-x)+42$

$\Rightarrow p(x)+p(1-x)=42$

Let $I=\int_0^1 p(x)dx$ ...(i)

$\Rightarrow\ I=\int_0^1 p(1-x)dx$ ...(ii)

Adding eqn. (i) and (ii),

$2I=\int_0^1(42)\,dx\ \Rightarrow I=21$

131. (c) Let $\mathrm{I}=\int_0^{\pi}[\cot x]\ dx$ ....(i)

$=\int_0^{\pi}[\cot\ (\pi-x)]\ dx\ =\int_0^{\pi}[-\cot x]\ dx$ ....(ii)

Adding eq^n s (i) & (ii),

We get

$2\mathrm{I}=\int_0^{\pi}([\cot x]+[-\cot x])dx$

$=\int_0^{\pi}(-1)dx$

$[\because[x]+[-x]=-1,\ \text{if}\ x\notin z\ \text{and}\ [x]+[-x]\ =0,\ \text{if}\ x\in z]$

$=[-x]_0^{\pi}=-\pi\qquad\Rightarrow\ \mathrm{I}=-\frac{\pi}{2}$

132. (b) We know that $\frac{\sin x}{x} < 1$, for x ∈ (0, 1)

$\Rightarrow \frac{\sin x}{\sqrt{x}} < \sqrt{x}$ on x ∈ (0, 1)

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \sqrt{x} dx = \left[\frac{2x^{3/2}}{3}\right]_0^1$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \frac{2}{3} \Rightarrow I < \frac{2}{3}$

Also $\frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$ for $x \in (0, 1)$

$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-1/2} dx = \left[2\sqrt{x}\right]_0^1 = 2$

$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < 2 \Rightarrow J < 2$

133. (d) $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$\therefore \left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$ $\left[\because \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x\right]$

$\Rightarrow \sec^{-1} x - \sec^{-1} \sqrt{2} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sec^{-1} x = \frac{\pi}{2} + \frac{\pi}{4}$

$\Rightarrow \sec^{-1} x = \frac{3\pi}{4} \Rightarrow x = \sec \frac{3\pi}{4} = \sec\left(\pi - \frac{\pi}{4}\right)$

$\Rightarrow x = -\sec\frac{\pi}{4} \Rightarrow x = -\sqrt{2}$

134. (c) Given that $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where

$f(x) = \int_1^x \frac{\log t}{1+t} dt$

$\therefore F(e) = f(e) + f\left(\frac{1}{e}\right)$

$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$ ...(1)

Let $I = \int_1^{1/e} \frac{\log t}{1+t} dt$

$\therefore$ Put $\frac{1}{t} = z \Rightarrow -\frac{1}{t^2} dt = dz \Rightarrow dt = -\frac{dz}{z^2}$

when $t = 1 \Rightarrow z = 1$ and when $t = 1/e$
$\Rightarrow z = e$

$\therefore I = \int_1^e \frac{\log\left(\frac{1}{z}\right)}{1+\frac{1}{z}} \left(-\frac{dz}{z^2}\right)$

$= \int_1^e \frac{(\log 1 - \log z).z}{z+1} \left(-\frac{dz}{z^2}\right)$

$= \int_1^e -\frac{\log z}{(z+1)} \left(-\frac{dz}{z}\right)$ $[\because \log 1 = 0]$

$= \int_1^e \frac{\log z}{z(z+1)} dz$

$\therefore I = \int_1^e \frac{\log t}{t(t+1)} dt$

[By property $\int_a^b f(t)dt = \int_a^b f(x)dx$]

Now from eqn. (1)

$F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$

$= \int_1^e \frac{t.\log t + \log t}{t(1+t)} dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)} dt$

$\Rightarrow F(e) = \int_1^e \frac{\log t}{t} dt$

Let $\log t = x$ $\therefore \frac{1}{t} dt = dx$

[when $t = 1, x = 0$ and when $t = e, x = \log e = 1$]

$\therefore F(e) = \int_0^1 x\, dx$ $F(e) = \left[\frac{x^2}{2}\right]_0^1$

$\Rightarrow F(e) = \frac{1}{2}$

135. (b) Let $a = k + h$ where k is an integer such that and $0 \le h < 1$

$\Rightarrow [a] = k$

$\therefore \int_1^a [x] f'(x) dx = \int_1^2 1 f'(x) dx + \int_2^3 2 f'(x) dx +$

$\cdots \int_{k-1}^{k} (k-1) f'(x) dx + \int_k^{k+h} k f'(x) dx$

$= \{f(2) - f(1)\} + 2\{f(3) - f(2)\} + 3\{f(4) - f(3)\}$
$+ \ldots\ldots + (k-1)\{f(k) - f(k-1)\} + k\{f(k+h) - f(k)\}$

$= -f(1) - f(2) - f(3) \ldots\ldots - f(k) + kf(k+h)$

$= [a] f(a) - \{f(1) + f(2) + f(3) + \ldots f([a])\}$

136. (c) $I = \int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)] dx$

Put $x + \pi = t$

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[t^3+\cos^2 t]dt=2\int_0^{\frac{\pi}{2}}\cos^2 t\,dt$$

[$\because t^3$ is odd and $\cos^2 t$ is even function]

$$=\int_0^{\frac{\pi}{2}}(1+\cos 2t)dt=\frac{\pi}{2}+0$$

137. (d) $I=\int_0^{\pi}xf(\sin x)dx=\int_0^{\pi}(\pi-x)f(\sin x)dx$

$$=\pi\int_0^{\pi}f(\sin x)dx-I \Rightarrow 2I=\pi\int_0^{\pi}f(\sin x)dx$$

$$I=\frac{\pi}{2}\int_0^{\pi}f(\sin x)dx=\pi\int_0^{\pi/2}f(\sin x)dx$$

[$\because \sin(\pi-x)=\sin x$]

$$=\pi\int_0^{\pi/2}f(\cos x)dx$$

138. (b) $I=\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ ...(1)

$$I=\int_3^6\frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx \quad \text{...(2)}$$

[$\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx$]

Adding equation (1) and (2)

$$2I=\int_3^6 dx=[x]_3^6=3 \Rightarrow I=\frac{3}{2}$$

139. (b) Let $I=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$ ....(1)

$$=\int_{-\pi}^{\pi}\frac{\cos^2(-x)}{1+a^{-x}}dx$$

$$\left[\text{Using}\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

$$=\int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{1+a^x}dx \quad \text{...(2)}$$

Adding equations (1) and (2) we get

$$2I=\int_{-\pi}^{\pi}\cos^2 x\left(\frac{1+a^x}{1+a^x}\right)dx=\int_{-\pi}^{\pi}\cos^2 x\,dx$$

$$=2\int_0^{\pi}\cos^2 x\,dx \quad [\because f(\pi-x)=f(x)]$$

$$=2\times2\int_0^{\frac{\pi}{2}}\cos^2 x\,dx=4\int_0^{\frac{\pi}{2}}\sin^2 x\,dx \quad \left[\because f\left(\frac{\pi}{2}-x\right)=f(x)\right]$$

$$\Rightarrow I=2\int_0^{\frac{\pi}{2}}\sin^2 x\,dx=2\int_0^{\frac{\pi}{2}}\left(1-\cos^2 x\right)dx$$

$$\Rightarrow I=2\int_0^{\frac{\pi}{2}}dx-2\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$$

$$\Rightarrow I+I=2\left(\frac{\pi}{2}\right)=\pi \Rightarrow I=\frac{\pi}{2}$$

140. (b) $I_1=\int_0^1 2^{x^2}dx,\ I_2=\int_0^1 2^{x^3}dx,$

$$I_3=\int_0^1 2^{x^2}dx,\ I_4=\int_0^1 2^{x^3}dx$$

$\because\ 2^{x^3}<2^{x^2},\ 0<x<1$

$$\Rightarrow \int_0^1 2^{x^2}dx>\int_0^1 2^{x^3}dx \Rightarrow I_1>I_2$$

and $2^{x^3}>2^x,\ x>1$

$\Rightarrow I_4>I_3$

141. (d) $\lim\limits_{x\to2}\int_0^{f(x)}\frac{4t^3}{x-2}dt=\lim\limits_{x\to0}\frac{\int_0^{f(x)}4t^3\,dt}{x-2}$

Applying L Hospital rule

$$\lim_{x\to2}\frac{[4f(x)^3f'(x)]}{1}=4(f(2))^3f'(2)$$

$$=4\times6^3\times\frac{1}{48}=18$$

142. (d) $f(x)=\frac{e^x}{1+e^x} \Rightarrow f(-x)=\frac{e^{-x}}{1+e^{-x}}=\frac{1}{e^x+1}$

$\therefore f(x)+f(-x)=1\ \forall\ x\in R$

Now $I_1=\int_{f(-a)}^{f(a)}xg\{x(1-x)\}dx$

$$=\int_{f(-a)}^{f(a)}(1-x)g\{x(1-x)\}dx$$

$$\left[\text{using}\int_a^b f(x)\,dx\ a=\int_a^b f(a+b-x)dx\right]$$

$$\Rightarrow \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx - \int_{f(-a)}^{f(a)} xg(x(1-x)dx$$

$$= I_2 - I_1 \Rightarrow 2I_1 = I_2$$

143. (b) Let $I = \int_0^{\pi} xf(\sin x)dx$ ...(i)

We know that

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx = \int_0^{\pi} (\pi - x)f(\sin x)dx \quad ...(ii)$$

Adding (i) and (ii)

$$\therefore 2I = \pi \int_2^{\pi} f(\sin x)dx = \pi.2\int_0^{\frac{\pi}{2}} f(\sin x)dx$$

$[\because \sin(\pi - x) = \sin x]$

$$\therefore I = \pi \int_0^{\frac{\pi}{2}} f(\sin x)dx \Rightarrow A = \pi$$

Let $\log x = t \Rightarrow e^t = x$

$$\Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = xdt \Rightarrow e^t dt.$$

144. (c) $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}dx$

$$\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{(\sin x + \cos x)}dx = \int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx$$

$$\left[\because \sin x + \cos x > 0 \text{ if } 0 < x < \frac{\pi}{2}\right]$$

or $I = [-\cos x + \sin x]_0^{\frac{\pi}{2}} = 2$

145. (d) $\int_{-2}^{3} |1-x^2|\,dx = \int_{-2}^{3} |x^2-1|\,dx$

Now $|x^2-1| = \begin{cases} x^2-1 & if \quad x \le -1 \\ 1-x^2 & if \quad -1 \le x \le 1 \\ x^2-1 & if \quad x \ge 1 \end{cases}$

$\therefore$ Integral is $\int_{-2}^{-1} (x^2-1)dx + \int_{-1}^{1} (1-x^2)dx + \int_{1}^{3} (x^2-1)dx$

$$= \left[\frac{x^3}{3} - x\right]_{-2}^{-1} + \left[x - \frac{x^3}{3}\right]_{-1}^{1} + \left[\frac{x^3}{3} - x\right]_{1}^{3}$$

$$= \left(-\frac{1}{3}+1\right) - \left(-\frac{8}{3}+2\right) + \left(2 - \frac{2}{3}\right) + \left(\frac{27}{3} - 3\right) - \left(\frac{1}{3} - 1\right)$$

$$= \frac{2}{3} + \frac{2}{3} + \frac{4}{3} + 6 + \frac{2}{3} = \frac{28}{3}$$

146. (d) $I = \int_0^1 x(1-x)^n\,dx = \int_0^1 (1-x)(1-1+x)^n\,dx$

$$= \int_0^1 (1-x)x^n dx = \int_0^1 (x^n - x^{n+1})dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2}$$

147. (d) Given that $f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating both side we get

$$\log f(x) = x + c \Rightarrow f(x) = e^{x+c}$$

$$f(0) = 1 \Rightarrow f(x) = e^x$$

$$\therefore g(x) = x^2 - f(x) = x^2 - e^x$$

$$\therefore \int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2 - e^x)dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= \left[x^2 e^x\right]_0^1 - 2\left[xe^x - e^x\right]_0^1 - \frac{1}{2}\left[e^{2x}\right]_0^1$$

$$= e - \left[\frac{e^2}{2} - \frac{1}{2}\right] - 2[e - e + 1] = e - \frac{e^2}{2} - \frac{3}{2}$$

148. (c) $I = \int_a^b xf(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$

We know that

$$\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$= (a+b)\int_a^b f(a+b-x)dx - \int_a^b xf(a+b-x)dx$$

$$= (a+b)\int_a^b f(x)\,dx - \int_a^b xf(x)dx$$

$[\because$ Given that $f(a+b-x) = f(x)]$

$$2I = (a+b)\int_a^b f(x)dx$$

$$\Rightarrow \quad I = \frac{(a+b)}{2}\int_a^b f(x)dx$$

149. (d) $\lim\limits_{x\to 0} \dfrac{\dfrac{d}{dx}\int_0^{x^2} \sec^2 t\,dt}{\dfrac{d}{dx}(x\sin x)} = \lim\limits_{x\to 0} \dfrac{\sec^2 x^2 .2x}{\sin x + x\cos x}$

(by L' Hospital rule)

$$\lim_{x\to 0} \frac{2\sec^2 x^2}{\left(\dfrac{\sin x}{x} + \cos x\right)} = \frac{2\times 1}{1+1} = 1$$

150. (c) $F(t) = \int_0^t f(t-y)g(y)dy$

$$= \int_0^t e^{t-y} y dy = e^t \int_0^t e^{-y} y dy$$

$$= e^t\left[-ye^{-y} - e^{-y}\right]_0^t = -e^t\left[ye^{-y} + e^{-y}\right]_0^t$$

$$= -e^t\left[t e^{-t} + e^{-t} - 0 - 1\right] = -e^t\left[\frac{t+1-e^t}{e^t}\right]$$

$$= e^t - (1+t)$$

151. (b) $\int_{-\pi}^{\pi} \dfrac{2x(1+\sin x)}{1+\cos^2 x}dx$

$$= \int_{-\pi}^{\pi} \frac{2x\,dx}{1+\cos^2 x} + 2\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$$

$$= 0 + 4\int_0^{\pi} \frac{x\sin x\,dx}{1+\cos^2 x};$$

We know that

$\because \int_{-a}^{a} f(\mathrm{x})\mathrm{dx} = 0$, if $f(x)$ is odd.

$= 2\int_0^a f(x)dx$, if $f(x)$ is even

$$I = 4\int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$$

$$\mathrm{I} = 4\int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \mathrm{I} = 4\pi\int_0^{\pi} \frac{\sin x\,dx}{1+\cos^2 x} - 4\int \frac{x\sin x\,dx}{1+\cos^2 x}$$

$$\Rightarrow 2\mathrm{I} = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

put $\cos x = t \Rightarrow -\sin x\,dx = dt$

when $x = 0, t = 1$ and when $x = \pi, t = -1$

$$\therefore \mathrm{I} = -2\pi\int_1^{-1} \frac{1}{1+t^2}dt = 2\pi\int_{-1}^{1} \frac{1}{1+t^2}dt$$

$$= 2\pi\left[\tan^{-1} t\right]_{-1}^{1} = 2\pi\left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= 2\pi\left[\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)\right] = 2\pi.\frac{\pi}{2} = \pi^2$$

152. (d) We know that $[x]$ is greatest integer function less than equal to x

$$\therefore \int_0^2 \left[x^2\right]dx = \int_0^1 \left[x^2\right]dx + \int_1^{\sqrt{2}} \left[x^2\right]dx + \int_{\sqrt{2}}^{\sqrt{3}} \left[x^2\right] + \int_{\sqrt{3}}^{2} \left[x^2\right]dx$$

$$= \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$$

$$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$$

$$= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$

$$= 5 - \sqrt{3} - \sqrt{2}$$

153. (b) $I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x(1+\tan^2 x)dx$

$$= \int_0^{\pi/4} \tan^n x \sec^2 x\,dx = \left[\frac{\tan^{n+1} x}{n+1}\right]_0^{\pi/4}$$

$$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$= \frac{1-0}{n+1} = \frac{1}{n+1}$$

$$\therefore I_n + I_{n+2} = \frac{1}{n+1} \Rightarrow \lim_{n\to\infty} n\left[I_n + I_{n+2}\right]$$

$$= \lim_{n\to\infty} n.\frac{1}{n+1} = \lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{n}{n\left(1+\dfrac{1}{n}\right)} = 1$$

154. (a) $I = \int_0^{10\pi} |\sin x|\,dx = 10\int_0^{\pi} |\sin x|\,dx$

$[\because \sin(10\pi - x) = \sin x]$

$$= 10\int_0^{\pi} \sin x\,dx$$

$\because \sin x > 0$, for $0 < x < \pi$.

as $\sin(\pi - x) = \sin x$

$$I = 20\int_0^{\pi/2} \sin x\,dx = 20\left[-\cos x\right]_0^{\pi/2} = 20$$

155. (a) $F(x)=\int_1^x t^2 g(t)dt$

Differentiate by using Leibnitz's rule, we get

$$F'(x)=x^2g(x)=x^2\int_1^x f(u)du \quad ...(i)$$

At $x=1$,

$$F'(1)=1\int_1^1 f(u)du=0$$

Now, differentiate eqn (i)

$$F''(x)=x^2 f(x)-2x\int_1^x f(u)du$$

At $x=1$,

$$F''(1)=1.f(1)-2\times 1.\int_1^1 f(u)du$$

$=f(1)-2\times 0=f(1)$

$F''(1)=3$

Then, for $F'(1)=0, F''(1)=3>0$

Hence, $x=1$ is a point of local minima.

156. (a) $\lim_{n\to\infty}\dfrac{(n+1)^{\frac{1}{3}}+(n+2)^{\frac{1}{3}}+...+(n+n)^{\frac{1}{3}}}{n(n)^{\frac{1}{3}}}$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{(n+r)^{\frac{1}{3}}}{n.n^{\frac{1}{3}}}$$

$$=\int_0^1 (1+x)^{\frac{1}{3}}dx \qquad \left[\because \frac{r}{n}\to x \text{ and } \frac{1}{n}\to \frac{d}{x}\right]$$

$$=\left[\frac{3}{4}(1+x)^{\frac{4}{3}}\right]_0^1=\frac{3}{4}(2)^{\frac{4}{3}}-\frac{3}{4}$$

157. (d) Let $L=\lim_{n\to\infty}\sum_{r=1}^{2n}\dfrac{n}{n^2+r^2}=\int_0^2\dfrac{dx}{1+x^2}$

$$\left[\because \frac{r}{n}\to x, \frac{1}{r}\to dx\right]$$

$=\left[\tan^{-1}x\right]_0^2$

$=\tan^{-1}2$

158. (a) $\lim_{n\to\infty}\dfrac{\frac{1}{(a+1)}.n^{a+1}+a_1n^a+a_2n^{a-1}+.......}{(n+1)^{a-1}.n^2\left(a+\frac{1+\frac{1}{n}}{2}\right)}=\dfrac{1}{60}$

$$\Rightarrow \lim_{n\to\infty}\frac{\left(\frac{1}{n}\right)^a+\left(\frac{2}{n}\right)^a+.......+\left(\frac{n}{n}\right)^a}{(n+1)^{a-1}\left[n^2a+\frac{n(n+1)}{2}\right]}=\frac{1}{60}$$

$$=\frac{\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^a}{\left(1+\frac{1}{n}\right)^{a-1}\left[a+\frac{1}{2}\left(1+\frac{1}{n}\right)\right]}=\frac{1}{60}$$

$$=\frac{\int_0^1 x^a dx}{\left(a+\frac{1}{2}\right)}=\frac{1}{60}=\frac{\frac{1}{a+1}}{a+\frac{1}{2}}=\frac{1}{60}$$

$$\Rightarrow \frac{\frac{1}{a+1}}{\left(a+\frac{1}{2}\right)}=\frac{1}{60}$$

$\Rightarrow (a+1)(2a+1)=120$

$\Rightarrow 2a^2+3a-119=0$

$\Rightarrow 2a^2+17a-14a-119=0$

$\Rightarrow (a-7)(2a+17)=0$

$\Rightarrow a=7, -\dfrac{17}{2}$

159. (d) $y=\lim_{n\to\infty}\left(\dfrac{(n+1)(n+2)...3n}{n^{2n}}\right)^{\frac{1}{n}}$

$$\ln y=\lim_{n\to\infty}\frac{1}{n}\ln\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)....\left(1+\frac{2n}{n}\right)$$

$$\ln y=\lim_{n\to\infty}\frac{1}{n}\left[\ln\left(1+\frac{1}{n}\right)+\ln\left(1+\frac{2}{n}\right)+....+\ln\left(1+\frac{2n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\ln\left(1+\frac{r}{n}\right)=\int_0^2\ln(1+x)dx$$

Let $1+x=t\Rightarrow dx=dt$

when $x=0,\ t=1$

$x=2,\ t=3$

$$\ln y=\int_1^3 \ln t\, d\,t=[t\ln t-t]_1^3=\ln\left(\frac{3^3}{e^2}\right)=\ln\left(\frac{27}{e^2}\right)$$

$$\Rightarrow y=\frac{27}{e^2}$$

160. (a) Let $f(x) = \int \frac{dx}{\sin^6 x}$

$f(x) = \int \text{cosec}^6 x\, dx$

From reduction formula, we have

$$I_n = \int \text{cosec}^n x\, dx = -\frac{\text{cosec}^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

$$\therefore f(x) = -\frac{\text{cosec}^4 x \cot x}{5} + \frac{4}{5}\left[\frac{-\text{cosec}^2 x \cot x}{3} + \frac{2}{3} I_2\right]$$

$$= -\frac{\text{cosec}^4 x \cot x}{5} - \frac{4}{15}\text{cosec}^2 x.\cot x + \frac{8}{15}[-\cot x]$$

$$= \frac{-(1+\cot^2 x)^2.\cot x}{5} - \frac{4}{15}(1+\cot^2 x)\cot x$$

$$-\frac{8}{15}(-\cot x) \quad (\because \text{cosec}^2 x = 1+\cot^2 x)$$

$$= \frac{-1}{5}\left[1+\cot^4 x + 2\cot^2 x\right]\cot x - \frac{4}{15}\left[\cot x + \cot^3 x\right]$$

$$-\frac{8}{15}\cot x$$

$$= \frac{-1}{5}[\cot x + \cot^5 x + 2\cot^3 x]$$

$$\frac{-4}{15}\cot x - \frac{4}{15}\cot^3 x - \frac{8}{15}\cot x$$

$$= \frac{-15}{15}\cot x - \frac{\cot^5 x}{5} - \frac{10}{15}\cot^3 x$$

$$= \frac{-\cot^5 x}{5} - \frac{2}{3}\cot^3 x - \cot x$$

It is a polynomial of degree 5 in cot x.

161. (d) $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \frac{3}{n^2}\sec^2\frac{9}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ is equal to

$$\lim_{n\to\infty}\frac{r}{n^2}\sec^2\frac{r^2}{n^2} = \lim_{n\to\infty}\frac{1}{n}.\frac{r}{n}\sec^2\frac{r^2}{n^2}$$

$\Rightarrow$ Given limit is equal to value of integral

$$\int_0^1 x\sec^2 x^2 dx$$

or $\frac{1}{2}\int_0^1 2x\sec\ x^2 dx = \frac{1}{2}\int_0^1 \sec^2 t dt$ [put $x^2 = t$]

$$= \frac{1}{2}(\tan t)_0^1 = \frac{1}{2}\tan 1.$$

162. (b) $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ [Using definite integrals as limit of sum]

$$= \int_0^1 e^x dx = e - 1$$

163. (a) $\lim_{n\to\infty}\frac{1^4+2^4+3^4+.......n^4}{n^5} - \lim_{n\to\infty}\frac{1^3+2^3+3^3+.......+n^3}{n^5}$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^4 - \lim_{n\to\infty}\frac{1}{n}.\lim_{n\to\infty}\frac{1}{n}\left(\frac{r}{n}\right)^3$$

$$= \int_0^1 x^4\,dx - \lim_{n\to\infty}\frac{1}{n}\times\int_0^1 x^3\,dx = \left[\frac{x^5}{5}\right]_0^1 - 0 = \frac{1}{5}$$

164. (a) We have $\lim_{n\to\infty}\frac{1^p+2^p+....+n^p}{n^{p+1}}$;

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^p}{n^p\cdot n} = \int_0^1 x^p dx = \left[\frac{x^{p+1}}{p+1}\right]_0^1 = \frac{1}{p+1}$$

23

Applications of Integrals

TOPIC 1 Curve & X-axis Between two Ordinates, Area of the Region Bounded by a Curve & Y-axis Between two Abscissa

1. The area (in sq. units) of the region $A = \{(x, y) : (x-1)[x] \le y \le 2\sqrt{x},\ 0 \le x \le 2\}$, where $[t]$ denotes the greatest integer function, is : **[Sep. 05, 2020 (II)]**

(a) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (b) $\frac{4}{3}\sqrt{2} + 1$

(c) $\frac{8}{3}\sqrt{2} - 1$ (d) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$

2. The area (in sq. units) of the region

$\{(x, y) : 0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ \frac{1}{2} \le x \le 2\}$ is : **[Sep. 03, 2020 (I)]**

(a) $\frac{23}{16}$ (b) $\frac{79}{24}$ (c) $\frac{79}{16}$ (d) $\frac{23}{6}$

3. Given: $f(x) = \begin{cases} x & , \quad 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \quad x = \frac{1}{2} \\ 1 - x & , \quad \frac{1}{2} < x \le 1 \end{cases}$

and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is : **[Jan. 9, 2020 (II)]**

(a) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (c) $\frac{1}{2} - \frac{\sqrt{3}}{4}$ (d) $\frac{1}{2} + \frac{\sqrt{3}}{4}$

4. The area (in sq. units) of the region
A = {(x, y) ∈R × R|0 d" x d"3, 0 d" y d" 4, y d" $x^2 + 3x$} is : **[April 8, 2019 (I)]**

(a) $\frac{53}{6}$ (b) 8 (c) $\frac{59}{6}$ (d) $\frac{26}{3}$

5. The area of the region
$A = \{(x, y): 0 \le y \le x\,|x| + 1 \text{ and } -1 \le x \le 1\}$ in sq. units is: **[Jan. 09, 2019 (II)]**

(a) $\frac{2}{3}$ (b) 2 (c) $\frac{4}{3}$ (d) $\frac{1}{3}$

6. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is : **[2018]**

(a) $\frac{1}{2}(\sqrt{3} + 1)$ (b) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$

(c) $\frac{1}{2}(\sqrt{2} - 1)$ (d) $\frac{1}{2}(\sqrt{3} - 1)$

7. Let $f : [-2, 3] \to [0, \infty)$ be a continuous function such that $f(1-x) = f(x)$ for all $x \in [-2, 3]$.
If R_1 is the numerical value of the area of the region bounded by $y = f(x)$, $x = -2$, $x = 3$ and the axis of x and

$R_2 = \int_{-2}^{3} x f(x)\,dx$, then : **[Online April 25, 2013]**

(a) $3R_1 = 2R_2$ (b) $2R_1 = 3R_2$
(c) $R_1 = R_2$ (d) $R_1 = 2R_2$

8. Let $f(x)$ be a non – negative continuous function such that the area bounded by the curve $y = f(x)$, x - axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is

$\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is **[2005]**

(a) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (b) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

(c) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (d) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

9. The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is **[2005]**

(a) 1 (b) 2 (c) 3 (d) 4

10. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit with the axes then $\int_0^2 xf'(x)dx$ is **[2002]**

(a) 3/2 (b) 1 (c) 5/4 (d) –3/4

TOPIC 2 Different Cases of Area Bounded Between the Curves

11. The area (in sq. units) of the region $A = \{(x, y) : |x| + |y| \le 1, 2y^2 \ge |x|\}$ is: **[Sep. 06, 2020 (I)]**

(a) $\frac{1}{3}$ (b) $\frac{7}{6}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$

12. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to: **[Sep. 06, 2020 (II)]**

(a) $\frac{4}{3}$ (b) $\frac{8}{3}$ (c) $\frac{7}{2}$ (d) $\frac{16}{3}$

13. Consider a region $R = \{(x, y) \in \mathbf{R}^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is **true**? **[Sep. 02, 2020 (II)]**

(a) $\alpha^3 - 6\alpha^2 + 16 = 0$ (b) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

(c) $3\alpha^2 - 8\alpha + 8 = 0$ (d) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

14. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is: **[Jan. 7, 2020 (I)]**

(a) $(24\pi - 1)$ (b) $(6\pi - 1)$

(c) $(12\pi - 1)$ (d) $(12\pi - 1)/6$

15. The area (in sq. units) of the region $\{(x, y) \in R^2 | 4x^2 \le y \le 8x + 12\}$ is: **[Jan. 7, 2020 (II)]**

(a) $\frac{125}{3}$ (b) $\frac{128}{3}$ (c) $\frac{124}{3}$ (d) $\frac{127}{3}$

16. For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b$ $(0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation: **[Jan. 8, 2020 (I)]**

(a) $x^6 - 6x^3 + 4 = 0$ (b) $x^6 - 12x^3 + 4 = 0$

(c) $x^6 + 6x^3 - 4 = 0$ (d) $x^6 - 12x^3 - 4 = 0$

17. The area (in sq. units) of the region $\{(x, y) \in R^2 : x^2 \le y \le |3 - 2x|$, is: **[Jan. 8, 2020 (II)]**

(a) $\frac{32}{3}$ (b) $\frac{34}{3}$ (c) $\frac{29}{3}$ (d) $\frac{31}{3}$

18. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to : **[April 12, 2019 (I)]**

(a) $\frac{10}{3}$ (b) 6 (c) $\frac{8}{3}$ (d) $-\frac{2}{3}$

19. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to : **[April 12, 2019 (II)]**

(a) $2\sqrt{6}$ (b) 48 (c) 24 (d) $4\sqrt{3}$

20. The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a : **[April 10, 2019 (I)]**

(a) square of side length $2\sqrt{2}$ units

(b) rhombus of side length 2 units

(c) square of area 16 sq. units

(d) rhombus of area $8\sqrt{2}$ sq. units

21. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is : **[April 10, 2019 (II)]**

(a) $\log_e 2 + \frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{1}{2}$ (d) $\frac{3}{2} - \frac{1}{\log_e 2}$

22. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is: **[April 9, 2019 (I)]**

(a) $\frac{10}{3}$ (b) $\frac{9}{2}$ (c) $\frac{31}{6}$ (d) $\frac{13}{6}$

23. The area (in sq. units) of the region $A = \{(x, y) : \frac{y^2}{2} \le x \le y + 4\}$ is: **[April 09, 2019 (II)]**

(a) $\frac{53}{3}$ (b) 30 (c) 16 (d) 18

24. Let $S(\alpha) = \{(x, y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(\alpha) = 2 : 5$, then λ equals : **[April 08, 2019 (II)]**

(a) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (b) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$

(c) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (d) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

25. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is : **[Jan. 12, 2019 (I)]**

(a) $\frac{15}{4}$ (b) $\frac{21}{2}$ (c) $\frac{17}{4}$ (d) $\frac{15}{2}$

26. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is : **[Jan. 11, 2019 (I)]**

(a) $\frac{5}{4}$ (b) $\frac{9}{8}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$

27. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is : **[Jan. 11, 2019 (II)]**

(a) $\frac{8}{3}$ (b) $\frac{37}{24}$ (c) $\frac{187}{24}$ (d) $\frac{14}{3}$

28. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is: **[Jan. 10, 2019 (I)]**

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$

29. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is: **[Jan. 9, 2019 (I)]**

(a) $\frac{8}{3}$ (b) $\frac{32}{3}$ (c) $\frac{56}{3}$ (d) $\frac{14}{3}$

30. If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines $y = 0$ and $x = t\ (t > 1)$ is 1 sq. unit, then t is equal to **[Online April 16, 2018]**

(a) $\frac{4}{3}$ (b) $e^{2/3}$ (c) $\frac{3}{2}$ (d) $e^{3/2}$

31. The area (in sq. units) of the region $\left\{x \in R : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\right\}$, is **[Online April 15, 2018]**

(a) $\frac{13}{3}$ (b) $\frac{10}{3}$ (c) $\frac{5}{3}$ (d) $\frac{8}{3}$

32. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is : **[2017]**

(a) $\frac{5}{2}$ (b) $\frac{59}{12}$ (c) $\frac{3}{2}$ (d) $\frac{7}{3}$

33. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is : **[Online April 8, 2017]**

(a) $\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$ (b) $\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$

(c) $\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}$ (d) $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$

34. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is : **[2016]**

(a) $\pi - \frac{4\sqrt{2}}{3}$ (b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

(c) $\pi - \frac{4}{3}$ (d) $\pi - \frac{8}{3}$

35. The area (in sq. units) of the region described by $A = \{(x, y) | y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0\}$ is: **[Online April 9, 2016]**

(a) $\frac{19}{6}$ (b) $\frac{17}{6}$ (c) $\frac{7}{2}$ (d) $\frac{13}{6}$

36. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is **[2015]**

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$

37. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to : **[Online April 10, 2015]**

(a) $\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

38. The area of the region described by $A = \left\{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\right\}$ is: **[2014]**

(a) $\frac{\pi}{2} - \frac{2}{3}$ (b) $\frac{\pi}{2} + \frac{2}{3}$ (c) $\frac{\pi}{2} + \frac{4}{3}$ (d) $\frac{\pi}{2} - \frac{4}{3}$

39. The area of the region above the x-axis bounded by the curve $y = \tan x$, $0 \leq x \leq \frac{\pi}{2}$ and the tangent to the curve at $x = \frac{\pi}{4}$ is: **[Online April 19, 2014]**

(a) $\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$ (b) $\frac{1}{2}\left(\log 2 + \frac{1}{2}\right)$

(c) $\frac{1}{2}(1 - \log 2)$ (d) $\frac{1}{2}(1 + \log 2)$

40. Let $A = \{(x, y) : y^2 \leq 4x, y - 2x \geq -4\}$. The area (in square units) of the region A is: **[Online April 9, 2014]**

(a) 8 (b) 9 (c) 10 (d) 11

41. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : **[2013]**

(a) 9 (b) 36 (c) 18 (d) $\frac{27}{4}$

42. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$, and above x-axis is : **[Online April 23, 2013]**

(a) $2\sqrt{2}$ (b) $2\sqrt{2}-2$
(c) $2\sqrt{2}+2$ (d) 0

43. The area of the region (in sq. units), in the first quadrant bounded by the parabola $y = 9x^2$ and the lines $x = 0$, $y = 1$ and $y = 4$, is : **[Online April 22, 2013]**

(a) 7/9 (b) 14/3 (c) 7/3 (d) 14/9

44. The area bounded by the curve $y = \ln(x)$ and the lines $y = 0$, $y = \ln(c)$ and $x = 0$ is equal to : **[Online April 9, 2013]**

(a) 3 (b) $3\ln(c) - 2$
(c) $3\ln(c) + 2$ (d) 2

45. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is : **[2012]**

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$ (c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

46. The area bounded by the parabola $y^2 = 4x$ and the line $2x - 3y + 4 = 0$, in square unit, is **[Online May 26, 2012]**

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{2}$

47. The area of the region bounded by the curve $y = x^3$, and the lines, $y = 8$, and $x = 0$, is **[Online May 19, 2012]**

(a) 8 (b) 12 (c) 10 (d) 16

48. If a straight line $y - x = 2$ divides the region $x^2 + y^2 \le 4$ into two parts, then the ratio of the area of the smaller part to the area of the greater part is **[Online May 12, 2012]**

(a) $3\pi - 8 : \pi + 8$ (b) $\pi - 3 : 3\pi + 3$
(c) $3\pi - 4 : \pi + 4$ (d) $\pi - 2 : 3\pi + 2$

49. The area enclosed by the curves $y = x^2$, $y = x^3$, $x = 0$ and $x = p$, where $p > 1$, is 1/6. The p equals **[Online May 12, 2012]**

(a) 8/3 (b) 16/3 (c) 2 (d) 4/3

50. The parabola $y^2 = x$ divides the circle $x^2 + y^2 = 2$ into two parts whose areas are in the ratio **[Online May 7, 2012]**

(a) $9\pi + 2 : 3\pi - 2$ (b) $9\pi - 2 : 3\pi + 2$
(c) $7\pi - 2 : 2\pi - 3$ (d) $7\pi + 2 : 3\pi + 2$

51. The area bounded by the curves **[2011 RS]**

$y^2 = 4x$ and $x^2 = 4y$ is:

(a) $\frac{32}{3}$ sq units (b) $\frac{16}{3}$ sq units
(c) $\frac{8}{3}$ sq. units (d) 0 sq. units

52. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x-axis is **[2011]**

(a) 1 square unit (b) $\frac{3}{2}$ square units
(c) $\frac{5}{2}$ square units (d) $\frac{1}{2}$ square unit

53. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is **[2010]**

(a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-1$
(c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-2$

54. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent of the parabola at the point (2, 3) and the x-axis is: **[2009]**

(a) 6 (b) 9 (c) 12 (d) 3

55. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to **[2008]**

(a) $\frac{5}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

56. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[2007]**

(a) 1/6 (b) 1/3 (c) 2/3 (d) 1

57. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is **[2005]**

(a) 1 : 2 : 1 (b) 1 : 2 : 3 (c) 2 : 1 : 2 (d) 1 : 1 : 1

58. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is **[2004]**

(a) 4 (b) 2 (c) 3 (d) 1

59. The area of the region bounded by the curves $y = |x - 1|$ *and* $y = 3 - |x|$ is **[2003]**

(a) 6 sq. units (b) 2 sq. units
(c) 3 sq. units (d) 4 sq. units.

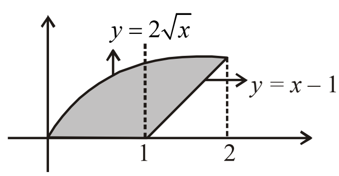
60. The area bounded by the curves $y = \ln x$, $y = \ln|x|$, $y = |\ln x|$ and $y = |\ln|x||$ is **[2002]**

(a) 4sq. units (b) 6 sq. units
(c) 10 sq. units (d) none of these

Hints & Solutions

1. **(a)** $[x]=0$ when $x\in[0,1)$ and $[x]=1$ when $x\in[1,2)$

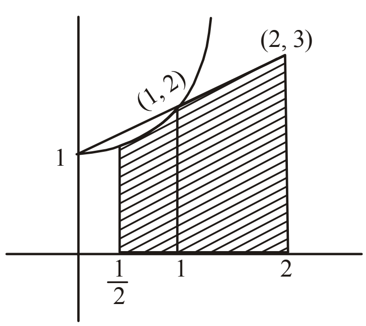
$$y=\begin{cases}0 & 0\le x<1\\ x-1 & 1\le x<2\end{cases}$$



$$\therefore A=\int_0^2 2\sqrt{x}\,dx-\frac{1}{2}(1)(1)$$

$$=\left.\frac{4x^{3/2}}{3}\right|_0^2-\frac{1}{2}=\frac{8\sqrt{2}}{3}-\frac{1}{2}$$

2. **(b)**



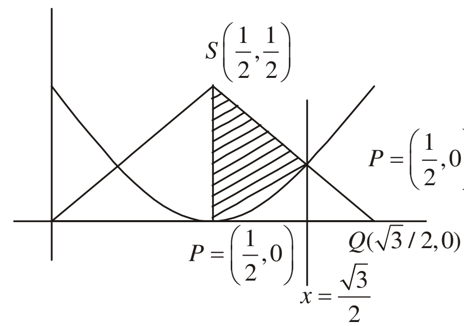
$$\text{Required area}=\int_{\frac{1}{2}}^{1}(x^2+1)dx+\int_1^2(x+1)dx$$

$$=\left[\frac{x^3}{3}+x\right]_{\frac{1}{2}}^{1}+\left[\frac{x^2}{2}+x\right]_1^2$$

$$=\left[\frac{4}{3}-\frac{13}{24}\right]+\frac{5}{2}=\frac{79}{24}.$$

3. **(b)** Coordinates of $P\left(\frac{1}{2},0\right),Q\left(\frac{\sqrt{3}}{2},0\right),R\left(\frac{\sqrt{3}}{2},1-\frac{\sqrt{3}}{2}\right)$

and $S\left(\frac{1}{2},\frac{1}{2}\right)$



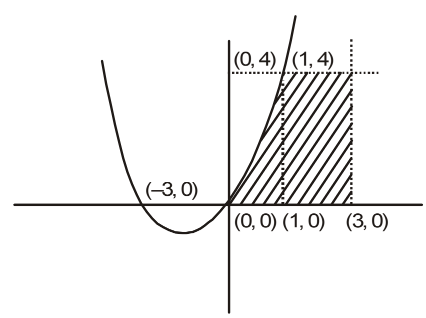
Required area = Area of trapezium $PQRS$

$$-\int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2dx$$

$$=\frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right)-\frac{1}{3}\left(\left(x-\frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2}$$

$$=\frac{\sqrt{3}}{4}-\frac{1}{3}$$

4. **(c)** Since, the relation $y\le x^2+3x$ represents the region below the parabola in the 1st quadrant



$\because\ y=4$

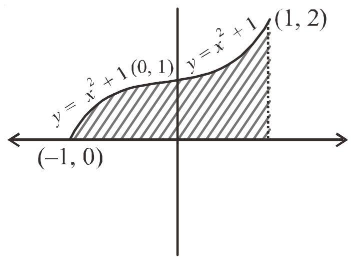
$\Rightarrow\ x^2+3x=4\Rightarrow x=1,-4$

$\therefore$ the required area = area of shaded region

$$=\int_0^1(x^2+3x)dx+\int_1^3 4.dx=\left[\frac{x^3}{3}+\frac{3x^2}{2}\right]_0^1+[4x]_1^3$$

$$=\frac{1}{3}+\frac{3}{2}+8=\frac{59}{6}$$

5. **(b)** Given $A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\}$



$\therefore$ Area of shaded region

$= \int_{-1}^{0}(-x^2+1)dx + \int_0^1 (x^2+1)dx$

$= \left(-\frac{x^3}{3}+x\right)_{-1}^{0} + \left(\frac{x^3}{3}+x\right)_0^1$

$= 0 - \left(\frac{1}{3}-1\right) + \left(\frac{1}{3}+1\right) - (0+0)$

$= \frac{2}{3}+\frac{4}{3} = \frac{6}{3} = 2$ square units

6. **(d)** Here, $18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow (3x-\pi)(6x-\pi) = 0$

$\Rightarrow \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

Also, $gof(x) = \cos x$

$\therefore$ Req. area $= \int_{\pi/6}^{\pi/3} \cos x\, dx = \frac{\sqrt{3}-1}{2}$

7. **(d)** We have

$R_2 = \int_{-2}^{3} x\, f(x)\, dx = \int_{-2}^{3} (1-x)\, f(1-x)\, dx$

$\left[\text{Using} \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

$\Rightarrow R_2 = \int_{-2}^{3} (1-x)\, f(x)\, dx$

$(\because f(x) = f(1-x) \text{ on } [-2, 3])$

$\therefore R_2 + R_2 = \int_{-2}^{3} x\, f(x)\, dx + \int_{-2}^{3} (1-x)\, f(x)\, dx$

$= \int_{-2}^{3} f(x)\, dx = R_1$

$\Rightarrow 2R_2 = R_1$

8. **(d)** From given condition

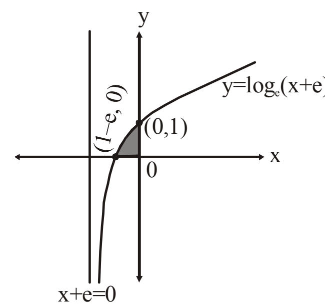
$\int_{\pi/4}^{\beta} f(x)dx = \beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$

Differentiating w. r. t β, we get

$f(\beta) = \beta\cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$

$f\left(\frac{\pi}{2}\right) = \beta\cdot 0 + \left(1-\frac{\pi}{4}\right)\sin\frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$

9. **(a)**



Required area $A = \int_{1-e}^{0} y\,dx = \int_{1-e}^{0} \log_e(x+e)dx$

put $x+e = t \Rightarrow dx = dt$ also when $x = 1-e, t = 1$ and when $x = 0, t = e$

$\therefore A = \int_1^e \log_e t\,dt = [t\log_e t - t]_1^e$

$e - e - 0 + 1 = 1$

Hence the required area is 1 square unit.

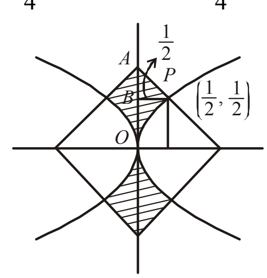
10. **(d)** Given that $\int_0^2 f(x)dx = \frac{3}{4}$; Now,

$\int_0^2 xf'(x)dx = x\int_0^2 f'(x)dx - \int_0^2 f(x)dx$

$= [x\, f(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4}$

$= 0 - \frac{3}{4}\ (\because f(2) = 0) = -\frac{3}{4}$.

11. **(d)**

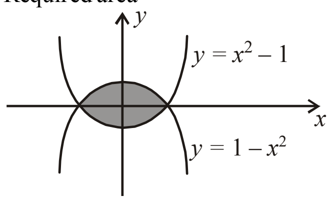


Required area $= 4\left[\int_0^{\frac{1}{2}} 2y^2 dy + \frac{1}{2}\text{area}(\Delta PAB)\right]$

$= 4\left[\frac{2}{3}\left[y^3\right]_0^{\frac{1}{2}} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right] = 4\left[\frac{2}{3}\times\frac{1}{8}+\frac{1}{8}\right]$

$= 4\times\frac{5}{24} = \frac{5}{6}$.

12. **(b)** Required area

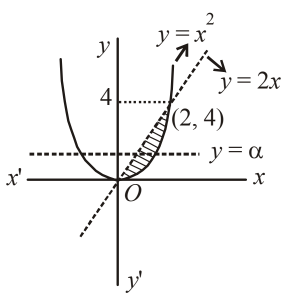


Area $= 2\int_0^1 \left((1-x^2)-(x^2-1)\right)dx$

$= 4\int_0^1 (1-x^2)\,dx$

$= 4\left(x-\dfrac{x^3}{3}\right)\Bigg|_0^1 = 4\left(1-\dfrac{1}{3}\right) = 4\cdot\dfrac{2}{3} = \dfrac{8}{3}$ sq. units

13. **(b)** Let $y = x^2$ and $y = 2x$



According to question

$\therefore \int_0^\alpha \left(\sqrt{y}-\dfrac{y}{2}\right)dy = \int_\alpha^4 \left(\sqrt{y}-\dfrac{y}{2}\right)dy$

$\Rightarrow \left[\dfrac{y^{3/2}}{\frac{3}{2}}\right]_0^\alpha - \left[\dfrac{y^2}{4}\right]_0^\alpha = \left[\dfrac{y^{3/2}}{\frac{3}{2}}\right]_\alpha^4 - \left[\dfrac{y^2}{4}\right]_\alpha^4$

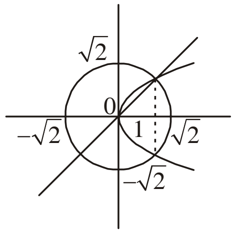
$\Rightarrow \dfrac{2}{3}\alpha^{3/2} - \dfrac{\alpha^2}{4} = \dfrac{2}{3}(8-\alpha^{3/2}) - \dfrac{1}{4}(16-\alpha^2)$

$\Rightarrow \dfrac{4}{3}\alpha^{3/2} - \dfrac{\alpha^2}{2} = \dfrac{4}{3}$

$\Rightarrow 8\alpha^{3/2} - 3\alpha^2 = 8$

$\therefore 3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

14. **(d)** Total area – enclosed area between line and parabola

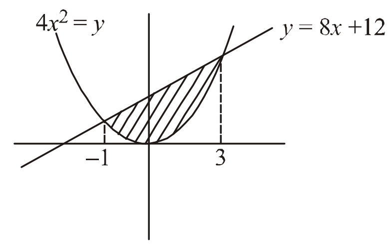


$= 2\pi - \int_0^1 \sqrt{x} - x\,dx$

$= 2\pi - \left(\dfrac{2x^{3/2}}{3} - \dfrac{x^2}{2}\right)_0^1$

$= 2\pi - \left(\dfrac{2}{3}-\dfrac{1}{2}\right) = 2\pi - \left(\dfrac{1}{6}\right) = \dfrac{12\pi-1}{6}$

15. **(b)**



Given curves are

$4x^2 = y$ ...(i)

$y = 8x + 12$ ...(ii)

From eqns. (i) and (ii),

$4x^2 = 8x + 12$

$\Rightarrow x^2 - x - 3 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow (x+1)(x-3) = 0$

$\Rightarrow x = -1, 3$

Required area bounded by curves is given by

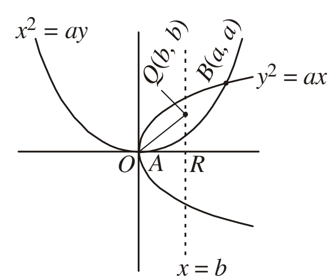
$A = \int_{-1}^{3} (8x + 12 - 4x^2)dx$

$A = \dfrac{8x^2}{2} + 12x - \dfrac{4x^3}{3}\Bigg|_{-1}^{3}$

$= (4(9) + 36 - 36) - \left(4 - 12 + \dfrac{4}{3}\right)$

$= 36 + 8 - \dfrac{4}{3} = 44 - \dfrac{4}{3} = \dfrac{132-4}{3} = \dfrac{128}{3}$

16. **(b)** Given eqns. are, $x^2 = ay$ and $y^2 = ax$



After solving, we get $x = a, y = a$
Now, coordinates of B is (a, a) and A is $(0, 0)$
Now, coordinates of Q is (b, b)

$\therefore \quad \frac{1}{2}b^2 = \frac{1}{2} \quad \Rightarrow \quad b = 1$

Area bounded by curves and $x = 1$ is

$$\int_0^1 \left(\sqrt{a}x^{1/2} - \frac{x^2}{a}\right)dx = \frac{1}{2}\int_0^a \left(\sqrt{a}x^{1/2} - \frac{x^2}{a}\right)dx$$

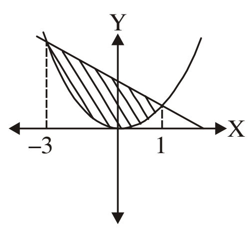
$\Rightarrow \quad \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$

$\Rightarrow \quad 4a\sqrt{a} - 2 = a^3$

$\Rightarrow \quad a^6 + 4a^3 + 4 = 16a^3$

$\Rightarrow \quad a^6 - 12a^3 + 4 = 0$

17. **(a)** Point of intersection of $y = x^2$ and $y = -2x + 3$ is obtained by $x^2 + 2x - 3 = 0$



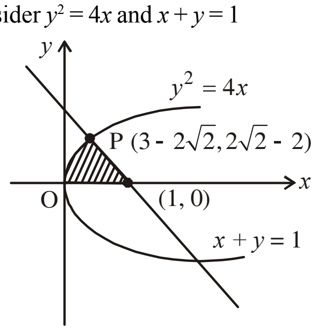
$\Rightarrow \quad x = -3, 1$

So, required area $= \int_{-3}^{1} (\text{line} - \text{parabola})dz$

$= \int_{-3}^{1} (3 - 2x - x^2)dx \quad = \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1}$

$= (3)4 - 2\left(\frac{1^2 - 3^2}{2}\right) - \left(\frac{1^3 + 3^3}{3}\right) = 12 + 8 - \frac{28}{3} = \frac{32}{3}$

18. **(b)** Consider $y^2 = 4x$ and $x + y = 1$



Substituting $x = 1 - y$ in the equation of parabola,
$y^2 = 4(1 - y) \Rightarrow y^2 + 4y - 4 = 0$

$\Rightarrow \quad (y + 2)^2 = 8 \Rightarrow y + 2 = \pm 2\sqrt{2}$

Hence, required area

$= \int_0^{3-2\sqrt{2}} 2\sqrt{x}\, dx + \frac{1}{2} \times (2\sqrt{2} - 2) \times (2\sqrt{2} - 2)$

$= \left[2 \times \frac{2}{3}x^{3/2}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}(8 + 4 - 8\sqrt{2})$

$= \frac{4}{3} \times (3 - 2\sqrt{2})\sqrt{3 - 2\sqrt{2}} + 6 - 4\sqrt{2}$

$= \frac{4}{3}(3 - 2\sqrt{2})(\sqrt{2} - 1) + 6 - 4\sqrt{2}$

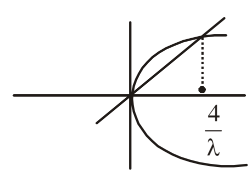
$[\because (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}]$

$= \frac{4}{3}(3\sqrt{2} - 3 - 4 + 2\sqrt{2}) + 6 - 4\sqrt{2}$

$= -\frac{10}{3} + \frac{8}{3}\sqrt{2} = a\sqrt{2} + b$

$\therefore \quad a = 8/3$ and $b = -10/3 \Rightarrow a - b = \frac{10}{3} + \frac{8}{3} = 6$

19. **(c)** Given parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$



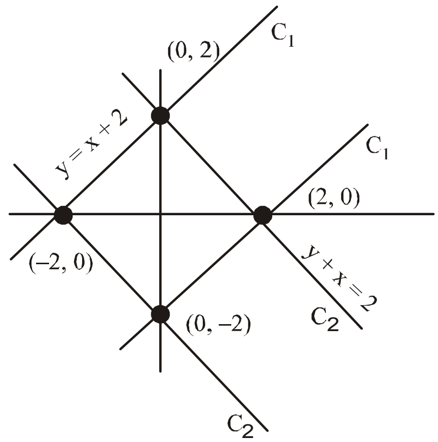
Putting $y = \lambda$ in $y^2 = 4\lambda x$, we get $x = 0, \frac{4}{\lambda}$

$\therefore \quad$ required area $= \int_0^{\frac{4}{\lambda}} (2\sqrt{\lambda x} - \lambda x)dx$

$= \left.\frac{2\sqrt{\lambda}.x^{3/2}}{3/2} - \frac{\lambda x^2}{2}\right|_0^{4/\lambda} = \frac{32}{3\lambda} - \frac{8}{\lambda}$

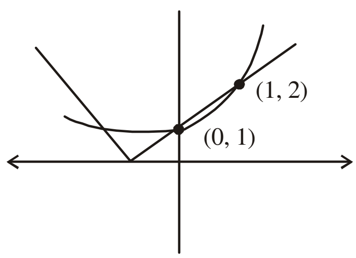
$= \frac{8}{3\lambda} = \frac{1}{9} \Rightarrow \lambda = 24$

20. **(a)** Let, $C_1 : |y - x| \le 2$
$C_2 : |y + x| \le 2$
By the diagram, region is square



Now, length of side $= \sqrt{2^2+2^2} = 2\sqrt{2}$

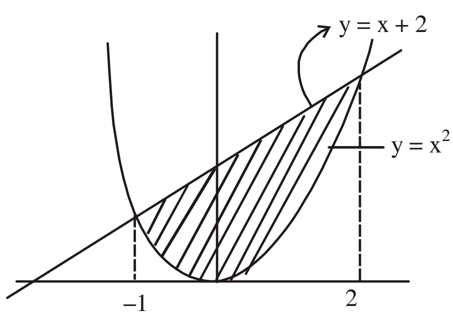
21. (d)



$$\text{Area} = \int_0^1 ((x+1)-2^x)dx \quad \left(\because \text{Area} = \int y dx\right)$$

$$= \left[\frac{x^2}{2}+x-\frac{2^x}{\ln 2}\right]_0^1 = \left(\frac{1}{2}+1-\frac{2}{\ln 2}\right)-\left(\frac{-1}{\ln 2}\right) = \frac{3}{2}-\frac{1}{\ln 2}$$

22. (b)

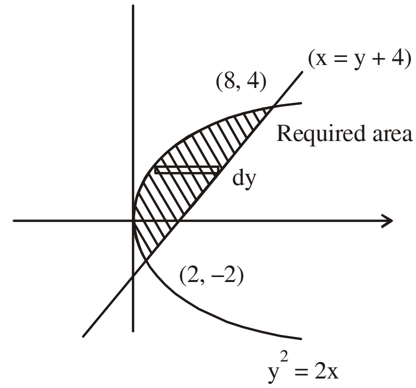


Required area is equal to the area under the curves $y \geq x^2$ and y d” x + 2

$$\therefore \text{ required area } \int_{-1}^{2} ((x+2)-x^2)dx$$

$$= \left(\frac{x^2}{2}+2x-\frac{x^3}{3}\right)_{-1}^{2} = \left(2+4-\frac{8}{3}\right)-\left(+\frac{1}{2}-2+\frac{1}{3}\right) = \frac{9}{2}$$

23. (d)



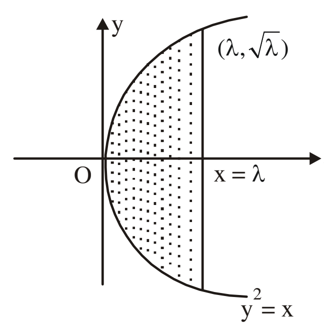
Given region, $A = \left\{(x,y) : \frac{y^2}{2} \leq x \leq y+4\right\}$

Hence, area $= \int_{-2}^{4} x\,dy = \int_{-2}^{4}\left(y+4-\frac{y^2}{2}\right)dy$

$$= \left[\frac{y^2}{2}+4y-\frac{y^3}{6}\right]_{-2}^{4} = \left(8+16-\frac{64}{6}\right)-\left(2-8+\frac{8}{6}\right)$$

$$= \left(24-\frac{32}{3}\right)-\left(-6+\frac{4}{3}\right) = \frac{40}{3}+\frac{14}{3} = \frac{54}{3} = 18$$

24. (d)



Area of the region $= 2\times\int_0^{\lambda} y dx = 2\int_0^{\lambda}\sqrt{x}\,dx$

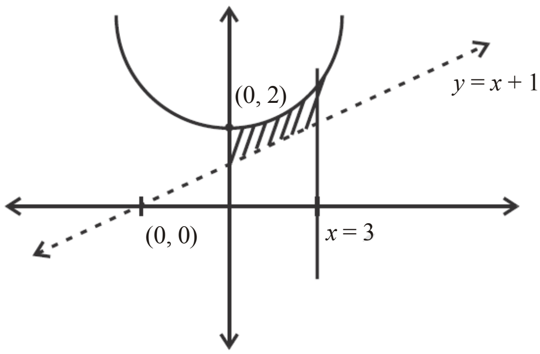
$$= 2\times\frac{2}{3}\pi^{\frac{3}{2}}$$

$$A(\lambda) = 2\times\frac{2}{3}(\lambda\times\sqrt{\lambda}) = \frac{4}{3}\lambda^{\frac{3}{2}}$$

Given, $\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{\frac{3}{2}}}{8} = \frac{2}{5}$

$$\lambda = \left(\frac{16}{5}\right)^{\frac{2}{3}} = 4.\left(\frac{4}{25}\right)^{\frac{1}{3}}$$

25. **(d)**

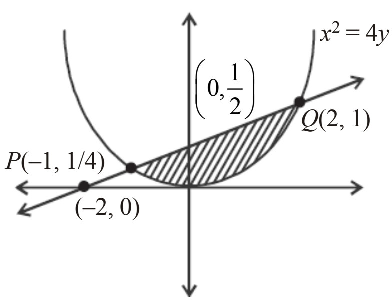


Area of the bounded region $\int_0^3 [(x^2+2)-(x+1)]dx$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3$

$= 9 - \frac{9}{2} + 3 = \frac{15}{2}$

26. **(b)**



Let points of intersection of the curve and the line be P and Q

$x^2 = 4\left(\frac{x+2}{4}\right)$

$x^2 - x - 2 = 0$

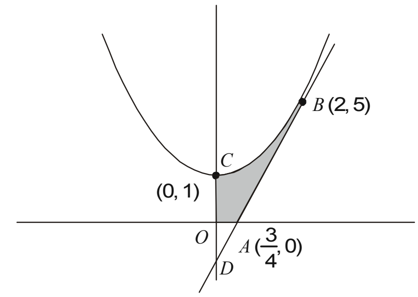
$x = 2, -1$

Point are (2, 1) and $\left(-1, \frac{1}{4}\right)$

$\text{Area} = \int_{-1}^{2}\left[\left(\frac{x+2}{4}\right) - \left(\frac{x^2}{4}\right)\right]dx = \left[\frac{x^2}{8} + \frac{1}{2}x - \frac{x^3}{12}\right]_{-1}^{2}$

$= \left(\frac{1}{2} + 1 - \frac{2}{3}\right) - \left(\frac{1}{8} - \frac{1}{2} + \frac{1}{12}\right) = \frac{9}{8}$

27. **(b)**



The equation of parabola $x^2 = y - 1$

The equation of tangent at (2, 5) to parabola is

$y - 5 = \left(\frac{dy}{dx}\right)_{(2,5)} (x-2)$

$y - 5 = 4(x-2)$

$4x - y = 3$

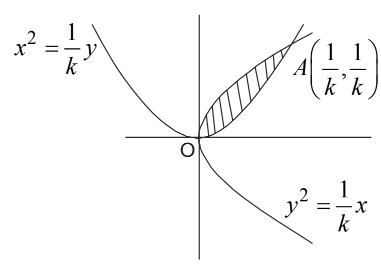
Then, the required area

$= \int_0^2 \{(x^2+1) - (4x-3)\}\, dx - \text{Area of } \Delta AOD$

$= \int_0^2 (x^2 - 4x + 4)\, dx - \frac{1}{2} \times \frac{3}{4} \times 3$

$= \left[\frac{(x-2)^3}{3}\right]_0^2 - \frac{9}{8} = \frac{37}{24}$

28. **(b)**



Two curves will intersect in the Ist quadrant at $A\left(\frac{1}{R}, \frac{1}{R}\right)$

$\because$ area of shaded region = 1.

$\therefore \int_0^{\frac{1}{k}} \left(\frac{\sqrt{x}}{\sqrt{k}} - kx^2\right) dx = 1$

$\Rightarrow \left(\frac{1}{\sqrt{k}} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^{\frac{1}{k}} - \left(k \cdot \frac{x^3}{3}\right)_0^{\frac{1}{k}} = 1$

$\Rightarrow \dfrac{2}{3\sqrt{k}}\cdot\dfrac{1}{k^{\frac{3}{2}}}-\dfrac{k}{3k^3}=1$

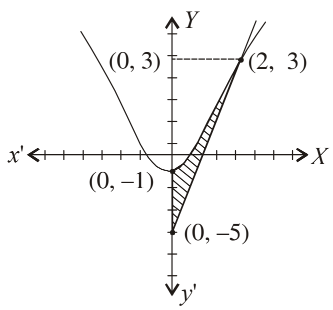
$\Rightarrow \dfrac{2}{3k^2}-\dfrac{1}{3k^2}=1$

$\Rightarrow 3k^2=1$

$\Rightarrow k=\pm\dfrac{1}{\sqrt{3}}$

$\therefore\ k=\dfrac{1}{\sqrt{3}}\ (\because k>0)$

29. **(a)**



$\because$ Curve is given as :

$y=x^2-1$

$\Rightarrow \dfrac{dy}{dx}=2x$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{(2,3)}=4$

$\therefore$ equation of tangent at (2, 3)

$(y-3)=4(x-2)$

$\Rightarrow y=4x-5$

but $x=0$

$\Rightarrow y=-5$

Here the curve cuts Y–axis

$\therefore$ required area

$=\dfrac{1}{4}\displaystyle\int_{-5}^{3}(y+5)dy-\int_{-1}^{3}\sqrt{y+1}\,dy$

$=\dfrac{1}{4}\left[\dfrac{y^2}{2}+5y\right]_{-5}^{3}\dfrac{-2}{3}\left[(y+1)^{3/2}\right]_{-1}^{3}$

$=\dfrac{1}{4}\left[\dfrac{9}{2}+15-\dfrac{25}{2}+25\right]$

$=-\dfrac{2}{3}\left[4^{3/2}-0\right]$

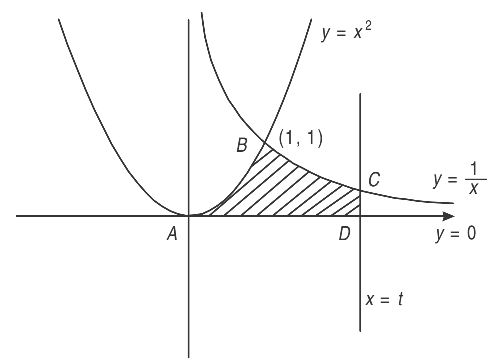
$=\dfrac{32}{4}-\dfrac{16}{3}=\dfrac{8}{3}$ sq-units.

30. **(b)** The intersection point of $y=x^2$ and $y=\dfrac{1}{x}$ is (1, 1)

Area bounded by the curves is the region ABCDA is given as:

$\text{Area}=\int_0^1 x^2\,dx+\int_1^t \dfrac{1}{x}\,dx$

$=\left[\dfrac{x^3}{3}\right]_0^1+\left[\ln(x)\right]_1^t=\dfrac{1}{3}+\ln(t)$

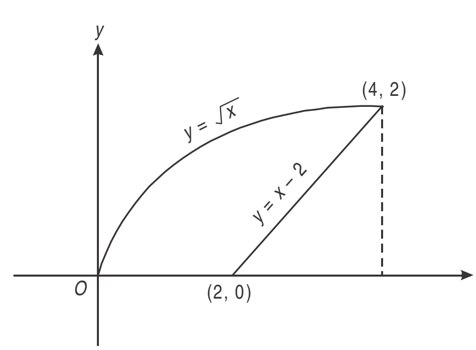


$\because$ area = 1

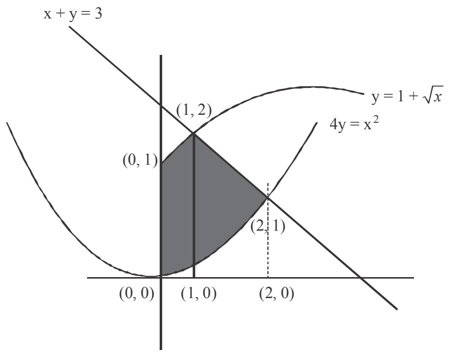
$\Rightarrow \dfrac{1}{3}+\ln(t)=1\quad\Rightarrow \ln(t)=\dfrac{2}{3}\quad\Rightarrow t=e^{\frac{2}{3}}$

31. **(b)** The intersection point of $y=x-2$ and $y=\sqrt{x}$ is (4, 2). The required area

$=\int_0^4\sqrt{x}dx-\dfrac{1}{2}\times2\times2=\dfrac{16}{3}-2=\dfrac{10}{3}$



32. **(a)**

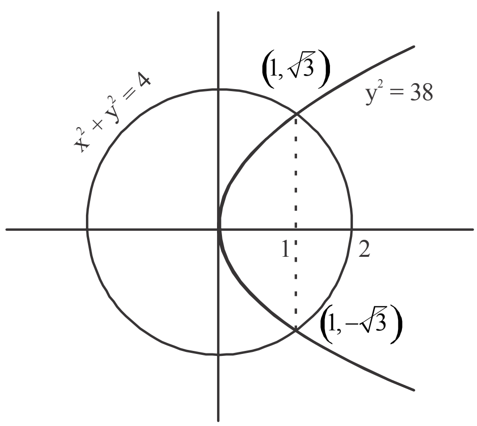


Area of shaded region

$$= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$$

$$= [x]_0^1 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + [3x]_1^2 - \left[\frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2 = \frac{5}{2}\text{sq.units}$$

33. **(d)**



From the equations we get;

$x^2 + 3x - 4 = 0$

$\Rightarrow \quad (x+4)(x-1) = 0 \Rightarrow x = -4, x = 1$

when $x = 1,\ y = \sqrt{3}$

$$\text{Area} = \int_0^1 \left(\int_0^1 \sqrt{3}\,.\sqrt{x}\,dx + \int_1^2 \sqrt{4-x^2}\,.\,dx\right) \times 2$$

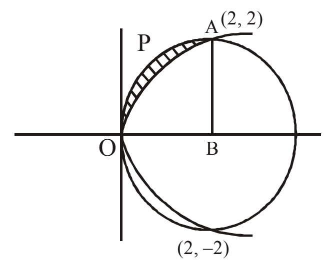
$$= \left(\sqrt{3}\left(\frac{x^{3/2}}{3/2}\right)_0^1 + \left(\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right)_1^2\right) \times 2$$

$$= \left\{\sqrt{3}\left(\frac{2}{3}\right) + \left\{2\,.\,\frac{\pi}{2} - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)\right\}\right\} \times 2$$

$$= \left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right) \times 2$$

$$= \left(\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}\right) \times 2 = \frac{1}{\sqrt{3}} + \frac{4\pi}{3}$$

34. **(d)**

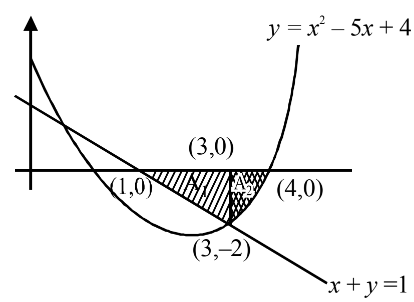


Points of intersection of the two curves are (0, 0), (2, 2) and (2, –2)

Area = Area (OPAB) – area under parabola (0 to 2)

$$= \frac{\pi \times (2)^2}{4} - \int_0^2 \sqrt{2}\sqrt{x}\ dx = \pi - \frac{8}{3}$$

35. **(a)**

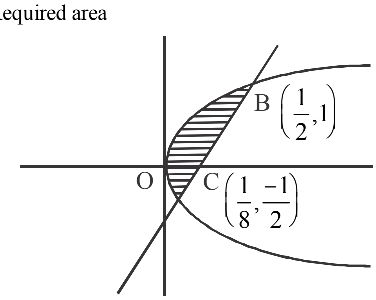


Required area $= A_1 + A_2$

$$= \frac{1}{2} \times 2 \times 2 + \left|\int_3^4 (x^2 - 5x + 4)\,dx\right|$$

$$= 2 + \frac{7}{6} = \frac{19}{6}\ \text{sq. units}$$

36. **(b)** Required area



$$= \int_{-1/2}^{1} \frac{y+1}{4}dy - \int_{-1/2}^{1} \frac{y^2}{2}dy$$

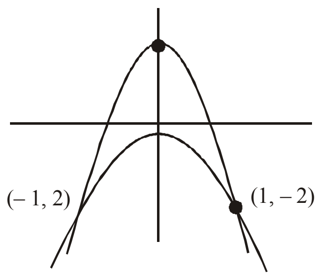
$$= \frac{1}{4}\left[\frac{y^2}{2}+y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$$

$$= \frac{1}{4}\left[\frac{3}{2}+\frac{3}{8}\right] - \frac{9}{48} = \frac{15}{32} - \frac{9}{48} = \frac{27}{96} = \frac{9}{32}$$

37. **(c)** Solving

$y + 2x^2 = 0$

$y + 3x^2 = 1$



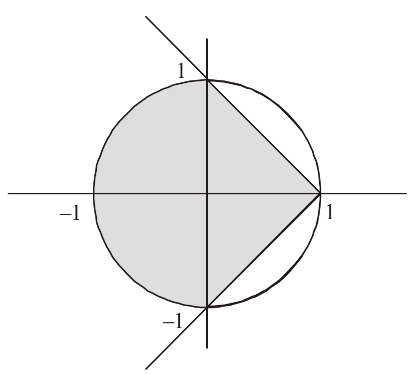
Point of intersection $(1, -2)$ and $(-1, -2)$

$$\text{Area} = 2\int_0^1 \left(\left(1-3x^2\right)-\left(-2x^2\right)\right)dx$$

$$2\int_0^1 \left(1-x^2\right)dx = 2\left(x-\frac{x^3}{3}\right)_0^1 = \frac{4}{3}$$

$= 15 - 6 = 9$ sq units

38. **(c)** Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.
Intersecting points are $x = 0, 1$



Area of shaded portion is the required area.

So, Required Area = Area of semi-circle + Area bounded by parabola

$$= \frac{\pi r^2}{2} + 2\int_0^1 \sqrt{1-x}\,dx = \frac{\pi}{2} + 2\int_0^1 \sqrt{1-x}\,dx$$

($\because$ radius of circle = 1)

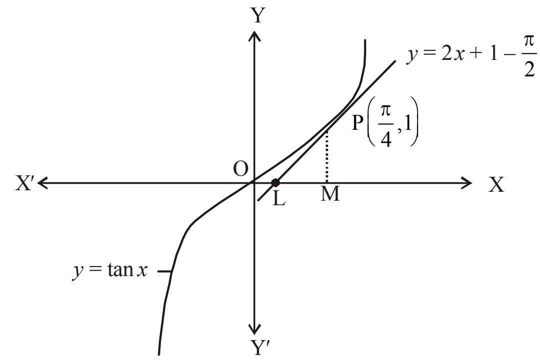
$$= \frac{\pi}{2} + 2\left[\frac{(1-x)^{3/2}}{-3/2}\right]_0^1 = \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3} \text{ sq. unit}$$

39. **(a)** The given curve is $y = \tan x$ ...(1)

when $x = \frac{\pi}{4}$, $y = 1$

Equation of tangent at P is

$$y - 1 = \left(\sec^2 \frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$



or $y = 2x + 1 - \frac{\pi}{2}$ ...(2)

Area of shaded region

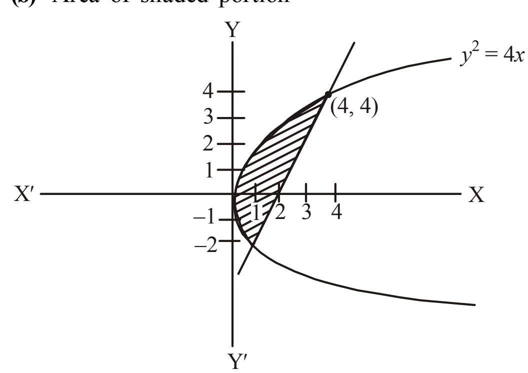
= area of OPMO – ar (ΔPLM)

$$= \int_0^{\frac{\pi}{4}} \tan x\,dx - \frac{1}{2}(\text{OM} - \text{OL})\text{PM}$$

$$= \left[\log \sec x\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\left\{\frac{\pi}{4} - \frac{\pi-2}{4}\right\} \times 1$$

$$= \frac{1}{2}\left[\log 2 - \frac{1}{2}\right] \text{ sq unit}$$

40. **(b)** Area of shaded portion



$$= \left|\int_2^4 \left(\frac{y+4}{2}\right)dy\right| - \left|\int_{-2}^4 \frac{y^2}{4}dy\right|$$

$$= \left|\frac{1}{2}\left[\frac{y^2}{2}+4y\right]_{-2}^{4}\right| - \left|\frac{1}{4}\left[\frac{y^3}{3}\right]_{-2}^{4}\right|$$

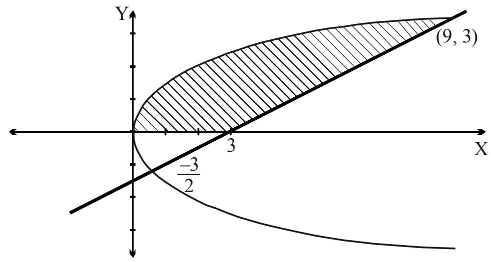
$$= \left|\frac{1}{2}[\{8+16\} - \{2-8\}]\right| - \left|\frac{1}{4}\left\{\frac{64}{3}+\frac{8}{3}\right\}\right| = 9$$

41. **(a)** Given curves are

$y = \sqrt{x}$ ...(1)

and $2y - x + 3 = 0$ ...(2)

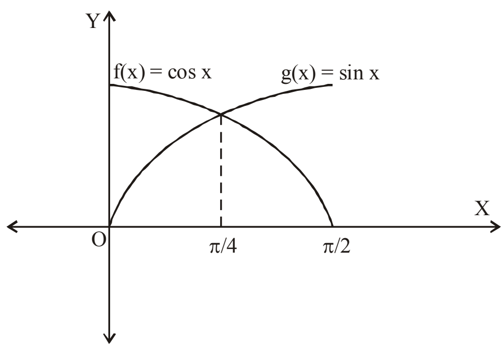
On solving both we get $y = -1, 3$



Required area $= \int_0^3 \left\{(2y+3) - y^2\right\} dy$

$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9.$

42. **(b)** $y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4} (\cos x - \sin x)\, dx$

$= 2\left[\sin x + \cos x\right]_0^{\pi/4}$

$= 2\left[\frac{2}{\sqrt{2}} - 1\right] = (2\sqrt{2} - 2)$ sq. units

43. **(d)** Required area $= \int_{y=1}^{4} \sqrt{\frac{y}{9}}\, dy$

$= \frac{1}{3}\int_{y=1}^{4} y^{1/2}\, dy = \frac{1}{3} \times \frac{2}{3}(y^{3/2})\Big|_1^4$

$= \frac{2}{9}[(4^{1/2})^3 - (1^{1/2})^3] = \frac{2}{9}[8-1]$

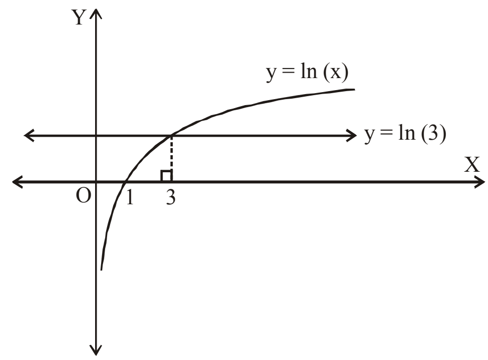
$= \frac{2}{9} \times 7 = \frac{14}{9}$ sq. units.

44. **(d)** To find the point of intersection of curves $y = \ln(x)$ and $y = \ln(3)$, put $\ln(x) = \ln(3)$

$\Rightarrow \ln(x) - \ln(3) = 0$

$\Rightarrow \ln(x) - \ln(3) = \ln(1)$

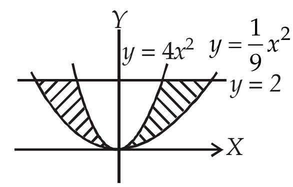
$\Rightarrow \frac{x}{3} = 1, \Rightarrow x = 3$



Required area $= \int_0^3 \ln(3)\, dx - \int_1^3 \ln(x)\, dx$

$= \left[x\ln(3)\right]_0^3 - \left[x\ln(x) - x\right]_1^3 = 2$

45. **(c)**



Required area $= 2\int_0^2 \left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right) dy$

$= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy = 2\left[\frac{2}{3} \times 3.y^{\frac{3}{2}} - \frac{1}{2} \times \frac{2}{3}.y^{\frac{3}{2}}\right]_0^2$

$= 2\left[2y^{\frac{3}{2}} - \frac{1}{3}y^{\frac{3}{2}}\right]_0^2 = 2 \times \left[\frac{5}{3}y^{\frac{3}{2}}\right]_0^2$

$= 2.\frac{5}{3}2\sqrt{2} = \frac{20\sqrt{2}}{3}$

46. **(b)** Intersecting points are $x = 1, 4$

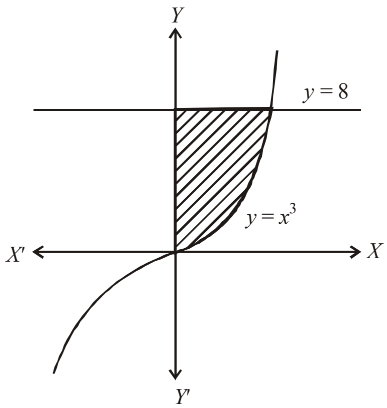
$\therefore$ Required area $= \int_1^4 \left[2\sqrt{x} - \left(\frac{2x+4}{3}\right)\right] dx$

$= \frac{2x^{3/2}}{3/2}\Big|_1^4 - \frac{2x^2}{3 \times 2}\Big|_1^4 - \frac{4}{3}x\Big|_1^4$

$= \frac{4}{3}\left(4^{3/2} - 1^{3/2}\right) - \frac{1}{3}(16-1) - \left[\frac{4}{3}(4) - \frac{4}{3}\right]$

$$= \frac{4}{3}(7) - 5 - 4 = \frac{28}{3} - 9 = \frac{28-27}{3} = \frac{1}{3}$$

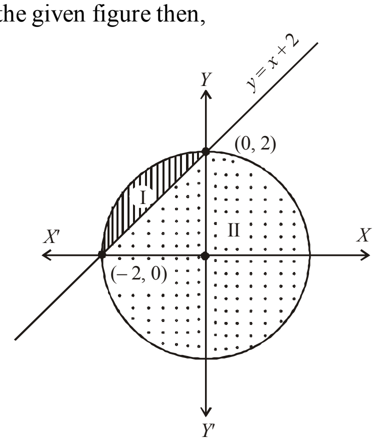
47. **(b)** Required Area $= \int_{y=0}^{8} y^{1/3} dy$



$$= \left.\frac{y^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right|_0^8 = \frac{3}{4}\left.\left(y^{4/3}\right)\right|_0^8$$

$$= \frac{3}{4}\left[(8)^{4/3} - 0\right] = \frac{3}{4}\left[2^4\right] = \frac{3}{4} \times 16 = 12 \text{ sq. unit.}$$

48. **(d)** Let I be the smaller portion and II be the greater portion of the given figure then,



Area of $I = \int_{-2}^{0} \left[\sqrt{4-x^2} - (x+2)\right] dx$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{0} - \left[\frac{x^2}{2} + 2x\right]_{-2}^{0}$$

$$= \left[2\sin^{-1}(-1)\right] - \left[-\frac{4}{2} + 4\right] = 2 \times \frac{\pi}{2} - 2 = \pi - 2$$

Now, area of II = Area of circle – area of I.

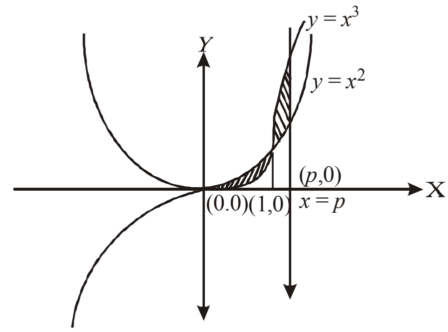
$$= 4\pi - (\pi - 2) = 3\pi + 2$$

Hence, required ratio $= \frac{\text{area of I}}{\text{area of II}} = \frac{\pi - 2}{3\pi + 2}$

49. **(d)** Given curves are $y = x^2$ and $y = x^3$

Also, $x = 0$ and $x = p, p > 1$

Now, intersecting point is $(1, 1)$



Required Area $= \int_0^1 \left(x^2 - x^3\right) dx + \int_1^p \left(x^3 - x^2\right) dx$

$$\frac{1}{6} = \left.\frac{x^3}{3} - \frac{x^4}{4}\right|_0^1 + \left.\frac{x^4}{4} - \frac{x^3}{3}\right|_1^p$$

$$\Rightarrow \frac{1}{6} = \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{p^4}{4} - \frac{p^3}{3} - \frac{1}{4} + \frac{1}{3}\right)$$

$$\Rightarrow \frac{1}{6} - \frac{1}{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{3} = \frac{3p^4 - 4p^3}{12}$$

$$\Rightarrow \frac{p^3(3p-4)}{12} = 0 \Rightarrow p^3(3p-4) = 0$$

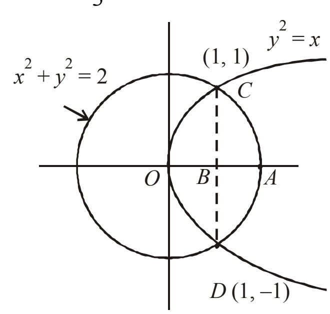
$$\Rightarrow p = 0 \text{ or } \frac{4}{3}$$

Since, it is given that $p > 1$

$\therefore$ p can not be zero.

Hence, $p = \frac{4}{3}$

50. **(b)**



Area of circle $= \pi\left(\sqrt{2}\right)^2 = 2\pi$

Area of $OCADO = 2\{\text{Area of } OCAO\}$

$= 2\ \{\text{area of } OCB + \text{area of } BCA\}$

$$= 2\int_0^1 y_p dx + 2\int_1^{\sqrt{2}} y_c dx$$

where $y_p = \sqrt{x}$ and $y_c = \sqrt{2-x^2}$

$$\therefore \text{ Required Area} = 2\int_0^1 \sqrt{x}\ dx + 2\int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$$

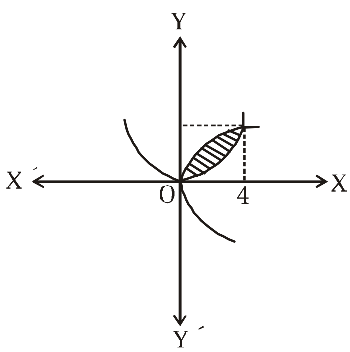
$$= 2\left[\frac{2}{3}.1-0\right]+2\left[\frac{x\sqrt{2-x^2}}{2}+\sin^{-1}\frac{x}{\sqrt{2}}\right]_1^{\sqrt{2}}$$

$$= \frac{4}{3}+2\left\{\frac{\pi}{2}-\frac{\pi}{4}-\frac{1}{2}\right\} = \frac{4}{3}+2\left\{\frac{\pi}{4}-\frac{1}{2}\right\} = \frac{3\pi+2}{6}$$

Bigger area $= 2\pi - \frac{3\pi+2}{6} = \frac{9\pi-2}{6}$

$\therefore$ Required Ratio $= \frac{9\pi-2}{3\pi+2}$ i.e., $9\pi-2 : 3\pi+2$

51. (b)



$$\text{Required area } = \int_0^4 \left(2\sqrt{x}-\frac{x^2}{4}\right)dx$$

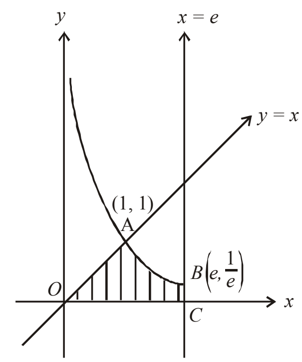
$$= \left[2\left(\frac{x^{3/2}}{3/2}\right)-\frac{x^3}{12}\right]_0^4$$

$$= \frac{4}{3}\times 8-\frac{64}{12} = \frac{32}{3}-16 = \frac{16}{3} \text{ sq. units}$$

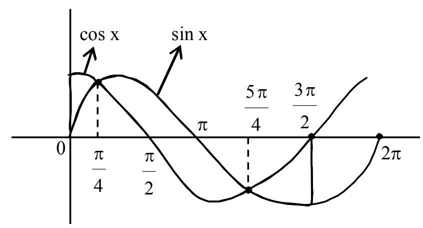
52. (b) Area of required region $AOCBO$

$$= \int_0^1 x dx + \int_1^e \frac{1}{x}dx = \left[\frac{x^2}{2}\right]_0^1 + \left[\log x\right]_1^e$$

$$= \frac{1}{2}+1 = \frac{3}{2} \text{ sq. units}$$



53. (d)



Area above x-axis = Area below x-axis

$\therefore$ Required area

$$= 2\left[\int_0^{\pi/4}(\cos x-\sin x)dx + \int_{\pi/4}^{\pi}\sin x dx - \int_{\pi/4}^{\pi/2}\cos x dx\right]$$

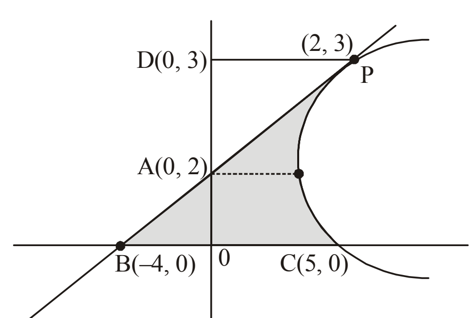
$$= 2\left[(\sin x+\cos x)_0^{\pi/4} + (-\cos x)_{\pi/4}^{\pi} - (\sin x)_{\pi/4}^{\pi/2}\right]$$

$$= 2\left[\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-(0+1)+\left(1+\frac{1}{\sqrt{2}}\right)-\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

$$= 2\left[\sqrt{2}-1+1+\frac{1}{\sqrt{2}}-1+\frac{1}{\sqrt{2}}\right]$$

$$= 2\left[\sqrt{2}+\sqrt{2}-1\right] = 4\sqrt{2}-2$$

54. (b)



For slope of tangents at (2, 3)

$(y-2)^2 = x-1$

$2(y-2)\frac{dy}{dx}=1 \Rightarrow \frac{dy}{dx}=\frac{1}{2(y-2)}$

$m=\left(\frac{dy}{dx}\right)_{(2,3)}=\frac{1}{2(3-2)}=\frac{1}{2}$

Equation of tangent

$y-3=\frac{1}{2}(x-2)$

$\Rightarrow x-2y+4=0$ ...(i)

The given parabola is $(y-2)^2=x-1$ ...(ii)

vertex (1, 2) and it meets x-axis at (5, 0)

Then required area = Ar ΔBOA + Ar (OCPD) – Ar (ΔAPD)

$=\frac{1}{2}\times 4\times 2+\int_0^3 x dy-\frac{1}{2}\times 2\times 1$

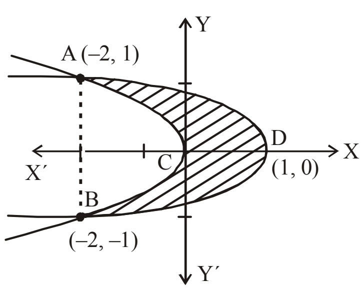
$=3+\int_0^3 (y-2)^2+1\, dy=3+\left[\frac{(y-2)^3}{3}+y\right]_0^3$

$=3+\left[\frac{1}{3}+3+\frac{8}{3}\right]=3+6=9$ sq. units

55. **(d)** Given $x+2y^2=0 \Rightarrow y^2=-\frac{x}{2}$

and $x+3y^2=1 \Rightarrow y^2=-\frac{1}{3}(x-1)$

On solving these two equations we get the points of intersection as (–2, 1), (–2, –1)



The required area is ACBDA, given by

$A=2\left\{\int_{-2}^{1}\frac{1}{\sqrt{3}}\sqrt{1-x}\,dx-\frac{1}{\sqrt{2}}\int_{-2}^{0}\sqrt{-x}\,dx\right\}$

$\Rightarrow 2\left\{\frac{1}{\sqrt{3}}\left[\frac{2}{3}(1-x)^{3/2}\right]_{-2}^{1}-\frac{1}{\sqrt{2}}\left[\frac{2}{3}(-x)^{3/2}\right]_{-2}^{0}\right\}$

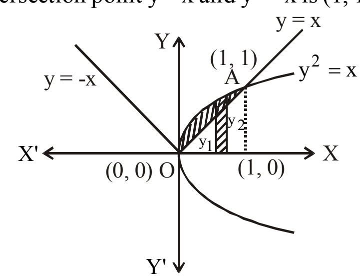
$\Rightarrow 2\left\{\left[-\frac{1}{\sqrt{3}}\times\frac{2}{3}\left(0-3^{3/2}\right)\right]-\left[\frac{-1}{\sqrt{2}}\times\frac{2}{3}\left(0-2^{3/2}\right)\right]\right\}$

$\Rightarrow 2\left\{\frac{2}{\cancel{3}\sqrt{\cancel{3}}}\times\cancel{3}\sqrt{\cancel{3}}-\frac{1}{\sqrt{\cancel{2}}}\times\frac{2}{3}\cdot 2\sqrt{\cancel{2}}\right\}$

$\Rightarrow 2\left\{2-\frac{4}{3}\right\}=2\left\{\frac{6-4}{3}\right\}=\frac{4}{3}$ sq. units

56. **(a)** It is clear from the figure, area lies between $y^2 = x$ and $y = x$

Intersection point $y = x$ and $y^2 = x$ is (1, 1)

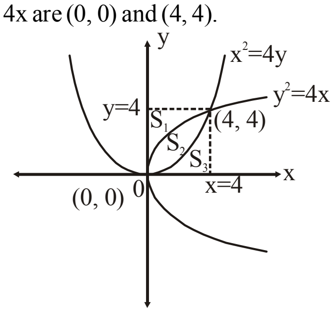


$\therefore$ Required area $=\int_0^1 (y_2-y_1)dx$

$=\int_0^1(\sqrt{x}-x)dx=\left[\frac{x^{3/2}}{3/2}-\frac{x^2}{2}\right]_0^1$

$=\frac{2}{3}\left[x^{3/2}\right]_0^1-\frac{1}{2}\left[x^2\right]_0^1=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$

57. **(d)** On solving, we get intersection points of $x^2=4y$ and $y^2 = 4x$ are (0, 0) and (4, 4).



By symmetry, we observe

$S_1=S_3=\int_0^4 y dx$

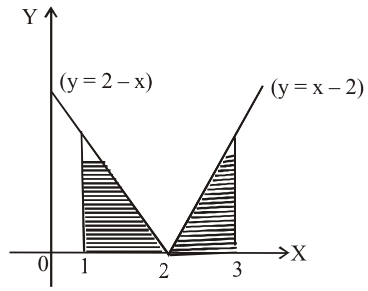
$=\int_0^4\frac{x^2}{4}dx=\left[\frac{x^3}{12}\right]_0^4=\frac{16}{3}$ sq. units

Also $S_2=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx=\left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12}\right]_0^4$

$=\frac{4}{3}\times 8-\frac{16}{3}=\frac{16}{3}$ sq. units

$\therefore S_1:S_2:S_3=1:1:1$

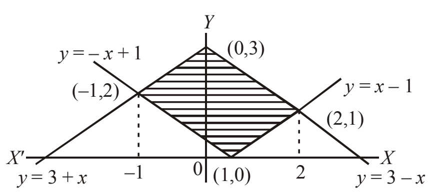
58. **(d)**



Required Area

$$A=2\int_2^3 (x-2)dx = 2\left[\frac{x^2}{2}-2x\right]_2^3 = 1$$

59. **(d)** Intersection point of $y = x - 1$ and $y = 3 - x$ is (2, 1) and eqns. $y = -x + 1$ and $y = 3 + x$ is (–1, 2)



$$A=\int_{-1}^{0}\{(3+x)-(-x+1)\}\,dx+$$

$$\int_0^1\{(3-x)-(-x+1)\}\,dx+\int_1^2\{(3-x)-(x-1)\}\,dx$$

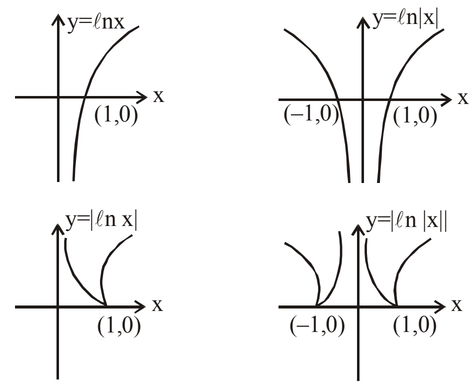
$$=\int_{-1}^{0}(2+2x)dx+\int_0^1 2dx+\int_1^2(4-2x)dx$$

$$=\left[2x+x^2\right]_{-1}^{0}+\left[2x\right]_0^1+\left[4x-x^2\right]_1^2$$

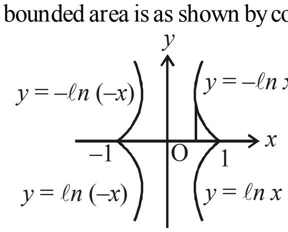
$$=0-(-2+1)+(2-0)+(8-4)-(4-1)$$

$$=1+2+4-3=4\ sq.\ \text{units}$$

60. **(a)** Separate graph of each curve



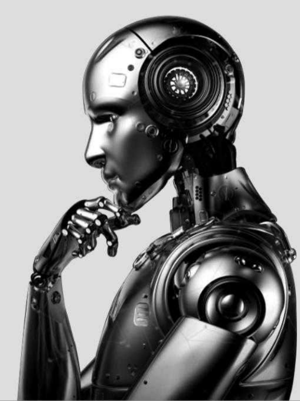
[Note: Graph of $y = |f(x)|$ can be obtained from the graph of the curve $y = f(x)$ by drawing the mirror image of the portion of the graph below x-axis, with respect to x-axis.
Hence the bounded area is as shown by combined all figure.



Required area $= 4\int_0^1 (-\ell n x)dx$

$= -4\left[x\ \ell n\ x - x\right]_0^1 = 4$ sq. units

24



Differential Equations

TOPIC 1 Ordinary Differential Equations, Order & Degree of Differential Equations, Formation of Differential Equations

1. The differential equation of the family of curves, $x^2=4b(y+b)$, $b\in R$, is: **[Jan. 8, 2020 (II)]**
   (a) $x(y')^2=x+2yy'$ (b) $x(y')^2=2yy'-x$
   (c) $xy''=y'$ (d) $x(y')^2=x-2yy'$

2. The differential equation representing the family of ellipse having foci either on the x-axis or on the y-axis centre at the origin and passing through the point (0, 3) is: **[Online April 16, 2018]**
   (a) $xyy'+y^2-9=0$ (b) $x+yy''=0$
   (c) $xyy''+x(y')^2-yy'=0$ (d) $xyy'-y^2+9=0$

3. If the differential equation representing the family of all circles touching x-axis at the origin is
   $\left(x^2-y^2\right)\dfrac{dy}{dx}=g(x)\,y$, then g(x) equals:
   [Online April 9, 2014]
   (a) $\dfrac{1}{2}x$ (b) $2x^2$
   (c) $2x$ (d) $\dfrac{1}{2}x^2$

4. **Statement-1:** The slope of the tangent at any point P on a parabola, whose axis is the axis of x and vertex is at the origin, is inversely proportional to the ordinate of the point P.
   Statement-2: The system of parabolas $y^2=4ax$ satisfies a differential equation of degree 1 and order 1.
   [Online April 9, 2013]
   (a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.
   (b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.
   (c) Statement-1 is true; Statement-2 is false.
   (d) Statement-1 is false; Statement-2 is true.

5. **Statement 1:** The degrees of the differential equations $\dfrac{dy}{dx}+y^2=x$ and $\dfrac{d^2y}{dx^2}+y=\sin x$ are equal.
   Statement 2: The degree of a differential equation, when it is a polynomial equation in derivatives, is the highest positive integral power of the highest order derivative involved in the differential equation, otherwise degree is not defined. **[Online May 12, 2012]**
   (a) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
   (b) Statement 1 is false, Statement 2 is true.
   (c) Statement 1 is true, Statement 2 is false.
   (d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

6. The differential equation which represents the family of curves $y=c_1e^{c_2x}$, where c_1, and c_2 are arbitrary constants, is **[2009]**
   (a) $y''=y'y$ (b) $yy''=y'$
   (c) $yy''=(y')^2$ (d) $y'=y^2$

7. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y=2$ is **[2009]**
   (a) $(x-2)y'^2=25-(y-2)^2$
   (b) $(y-2)y'^2=25-(y-2)^2$
   (c) $(y-2)^2y'^2=25-(y-2)^2$
   (d) $(x-2)^2y'^2=25-(y-2)^2$

8. The differential equation of all circles passing through the origin and having their centres on the x-axis is **[2007]**
   (a) $y^2=x^2+2xy\dfrac{dy}{dx}$ (b) $y^2=x^2-2xy\dfrac{dy}{dx}$
   (c) $x^2=y^2+xy\dfrac{dy}{dx}$ (d) $x^2=y^2+3xy\dfrac{dy}{dx}$

9. The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of **[2006]**
(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree

10. The differential equation representing the family of curves $y^2 = 2c(x+\sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows : **[2005]**
(a) order 1, degree 2 (b) order 1, degree 1
(c) order 1, degree 3 (d) order 2, degree 2

11. The differential equation for the family of circle $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is **[2004]**
(a) $(x^2+y^2)y' = 2xy$ (b) $2(x^2+y^2)y' = xy$
(c) $(x^2-y^2)y' = 2xy$ (d) $2(x^2-y^2)y' = xy$

12. The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively. **[2003]**
(a) 2, 3 (b) 2, 1
(c) 1, 2 (d) 3, 2.

13. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are **[2002]**
(a) $(1, \frac{2}{3})$ (b) (3, 1)
(c) (3, 3) (d) (1, 2)

TOPIC 2 General & Particular Solution of Differential Equation, Solution of Differential Equation by the Method of Separation of Variables, Solution of Homogeneous Differential Equations

14. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ is : **[Sep. 06, 2020 (I)]**
(where C is a constant of integration)
(a) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$
(b) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$
(c) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$
(d) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

15. If $y = \left(\frac{2}{\pi}x - 1\right)$ cosec x is the solution of the differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi}\text{cosec}\, x$, $0 < x < \frac{\pi}{2}$, then the function p(x) is equal to: **[Sep. 06, 2020 (II)]**
(a) cot x (b) cosec x
(c) sec x (d) tan x

16. If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is : **[Sep. 05, 2020 (I)]**
(a) 1 (b) – 1
(c) 0 (d) 2

17. The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is : **[Sep. 04, 2020 (II)]**
(where C is a constant of integration.)
(a) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$
(b) $x - \log_e(y+3x) = C$
(c) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$
(d) $x - 2\log_e(y+3x) = C$

18. Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and $\lim\limits_{t \to x}\frac{t^2f^2(x) - x^2f^2(t)}{t-x} = 0$.
If $f(x) = 1$, then x is equal to : **[Sep. 04, 2020 (II)]**
(a) $\frac{1}{e}$ (b) $2e$
(c) $\frac{1}{2e}$ (d) e

19. The solution curve of the differential equation,

$(1+e^{-x})(1+y^2)\frac{dy}{dx}=y^2$, which passes through the point $(0, 1)$, is : **[Sep. 03, 2020 (I)]**

(a) $y^2+1=y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$

(b) $y^2+1=y\left(\log_e\left(\frac{1+e^{x}}{2}\right)+2\right)$

(c) $y^2=1+y\log_e\left(\frac{1+e^{x}}{2}\right)$

(d) $y^2=1+y\log_e\left(\frac{1+e^{-x}}{2}\right)$

20. If $x^3dy+xy\,dx=x^2dy+2y\,dx;\ y(2)=e$ and $x>1$, then $y(4)$ is equal to : **[Sep. 03, 2020 (II)]**

(a) $\frac{3}{2}+\sqrt{e}$ (b) $\frac{3}{2}\sqrt{e}$

(c) $\frac{1}{2}+\sqrt{e}$ (d) $\frac{\sqrt{e}}{2}$

21. Let $y=y(x)$ be the solution of the differential equation,

$\frac{2+\sin x}{y+1}\cdot\frac{dy}{dx}=-\cos x,\ y>0,\ y(0)=1$. If $y(\pi)=a$ and $\frac{dy}{dx}$ at $x=\pi$ is b, then the ordered pair (a, b) is equal to : **[Sep. 02, 2020 (I)]**

(a) $\left(2,\frac{3}{2}\right)$ (b) $(1,-1)$

(c) $(1, 1)$ (d) $(2, 1)$

22. If a curve $y=f(x)$, passing through the point $(1, 2)$, is the solution of the differential equation,

$2x^2dy=(2xy+y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to : **[Sep. 02, 2020 (II)]**

(a) $\frac{1}{1+\log_e 2}$ (b) $\frac{1}{1-\log_e 2}$

(c) $1+\log_e 2$ (d) $\frac{-1}{1+\log_e 2}$

23. If $f2(x)=\tan^{-1}(\sec x+\tan x), -\frac{\pi}{2}<x<\frac{\pi}{2}$, and $f(0)=0$, then $f(1)$ is equal to: **[Jan. 9, 2020 (I)]**

(a) $\frac{\pi+1}{4}$ (b) $\frac{1}{4}$

(c) $\frac{\pi-1}{4}$ (d) $\frac{\pi+2}{4}$

24. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}; y(1)=1$; then a value of x satisfying $y(x)=e$ is: **[Jan. 9, 2020 (II)]**

(a) $\frac{1}{2}\sqrt{3}e$ (b) $\frac{e}{\sqrt{2}}$

(c) $\sqrt{2}e$ (d) $\sqrt{3}e$

25. Let $f(x)=(\sin(\tan^{-1}x)+\sin(\cot^{-1}x))^2-1$, $|x|>1$. If $\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$ and $y\left(\sqrt{3}\right)=\frac{\pi}{6}$, then $y\left(-\sqrt{3}\right)$ is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{6}$

(c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{3}$

26. Let $y=y(x)$ be a solution of the differential equation,

$\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0, |x|<1$.

If $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{\sqrt{3}}{2}$

27. If $y=y(x)$ is the solution of the differential equation, $e^y=e^x$ such that $y(0)=0$, then $y(1)$ is equal to: **[Jan. 7, 2020 (I)]**

(a) $1+\log_e 2$ (b) $2+\log_e 2$

(c) $2e$ (d) $\log_e 2$

28. The general solution of the differential equation (y^2-x^3) dx $-xydy=0$ $(x\neq 0)$ is : **[April 12, 2019 (II)]**

(a) $y^2-2x^2+cx^3=0$ (b) $y^2+2x^3+cx^2=0$

(c) $y^2+2x^2+cx^3=0$ (d) $y^2-2x^3+cx^2=0$

(where c is a constant of integration)

29. If $\cos x\frac{dy}{dx} - y\sin x = 6x, (0 < x < \frac{\pi}{2})$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to: **[April. 09, 2019 (II)]**

(a) $\frac{\pi^2}{2\sqrt{3}}$ (b) $-\frac{\pi^2}{2}$

(c) $-\frac{\pi^2}{2\sqrt{3}}$ (d) $-\frac{\pi^2}{4\sqrt{3}}$

30. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is: **[April. 08, 2019 (II)]**

(a) $x \log_e |y| = 2(x-1)$

(b) $x \log_e |y| = -2(x-1)$

(c) $x^2 \log_e |y| = -2(x-1)$

(d) $x \log_e |y| = x - 1$

31. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when $y(1) = 1$, is: **[Jan. 11, 2019 (II)]**

(a) $\log_e\left|\frac{2-x}{2-y}\right| = x - y$

(b) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

(c) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x + y - 2$

(d) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

32. If $\frac{dy}{dx} + \frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}, x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals: **[10 Jan 2019 I]**

(a) $\frac{1}{3} + e^6$ (b) $\frac{1}{3}$

(c) $-\frac{4}{3}$ (d) $\frac{1}{3} + e^3$

33. The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)\, dx + 2xy\, dy = 0$ which passes through (1, 1), is: **[Jan. 10, 2019 (II)]**

(a) a circle with centre on the x-axis.

(b) an ellipse with major axis along the y-axis.

(c) a circle with centre on the y-axis.

(d) a hyperbola with transverse axis along the x-axis.

34. Let $f : [0, 1] \to R$ be such that $f(xy) = f(x).f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is equal to: **[Jan. 09, 2019 (II)]**

(a) 3 (b) 4

(c) 2 (d) 5

35. The curve satisfying the differential equation, $(x^2 - y^2)\, dx + 2xydy = 0$ and passing through the point (1, 1) is **[Online April 15, 2018]**

(a) a circle of radius two (b) a circle of radius one

(c) a hyprbola (d) an ellipse

36. If $(2 + \sin x)\frac{dy}{dx} + (y+1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to : **[2017]**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$

(c) $-\frac{2}{3}$ (d) $-\frac{1}{3}$

37. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\, dx = x\, dy$, then $f\left(-\frac{1}{2}\right)$ is equal to : **[2016]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $-\frac{2}{5}$ (d) $-\frac{4}{5}$

38. If f(x) is a differentiable function in the interval ($(0, \infty)$ such that f(a) = 1 and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$, for each $x > 0$, then $f\left(\frac{3}{2}\right)$ is equal to : **[Online April 9, 2016]**

(a) $\frac{23}{18}$ (b) $\frac{13}{6}$

(c) $\frac{25}{9}$ (d) $\frac{31}{18}$

39. The solution of the differential equation $ydx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then f(a) is equal to : **[Online April 11, 2015]**

(a) 4 (b) 3

(c) 1 (d) 2

40. If y(x) is the solution of the differential equation

$(x+2)\frac{dy}{dx}=x^2+4x-9, x\neq -2$ and y(0) = 0, then y (–4) is equal to : **[Online April 10, 2015]**

(a) 0 (b) 2

(c) 1 (d) –1

41. Let the population of rabbits surviving at time t be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$. If $p(0)=100$, then $p(t)$ equals: **[2014]**

(a) $600-500\,e^{t/2}$ (b) $400-300\,e^{-t/2}$

(c) $400-300\,e^{t/2}$ (d) $300-200\,e^{-t/2}$

42. If the general solution of the differential equation $y'=\frac{y}{x}+\Phi\left(\frac{x}{y}\right)$, for some function Φ, is given by $y\ ln\,|cx|=x$, where c is an arbitrary constant, then $\Phi(2)$ is equal to: **[Online April 11, 2014]**

(a) 4 (b) $\frac{1}{4}$

(c) -4 (d) $-\frac{1}{4}$

43. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx}=100-12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[2013]**

(a) 2500 (b) 3000

(c) 3500 (d) 4500

44. If a curve passes through the point $\left(2,\frac{7}{2}\right)$ and has slope $\left(1-\frac{1}{x^2}\right)$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is – 2 is : **[Online April 23, 2013]**

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

45. Consider the differential equation :

$$\frac{dy}{dx}=\frac{y^3}{2\left(xy^2-x^2\right)}$$

[Online April 22, 2013]

Statement-1: The substitution $z=y^2$ transforms the above equation into a first order homogenous differential equation.

Statement-2: The solution of this differential equation is $y^2e^{-y^2/x}=C$.

(a) Both statements are false.

(b) Statement-1 is true and statement-2 is false.

(c) Statement-1 is false and statement-2 is true.

(d) Both statements are true.

46. The population $p\ (t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt}=0.5\ \text{p(t)}-450$. If $p\ (0)=850$, then the time at which the population becomes zero is : **[2012]**

(a) $2ln\,18$ (b) $ln\,9$

(c) $\frac{1}{2}ln\,18$ (d) $ln\,18$

47. Let $y\ (x)$ be a solution of $\frac{(2+\sin x)}{(1+y)}\frac{dy}{dx}=\cos x$. If $y\ (0)=2$, then $y\left(\frac{\pi}{2}\right)$ equals **[Online May 7, 2012]**

(a) $\frac{5}{2}$ (b) 2

(c) $\frac{7}{2}$ (d) 3

48. The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact is given by : **[2011RS]**

(a) $2y-3x=0$ (b) $y=\frac{6}{x}$

(c) $x^2+y^2=13$ (d) $\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=2$

49. Let I be the purchase value of an equipment and $V\ (t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt}=-k(T-t)$, where $k>0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is **[2011]**

(a) $I-\frac{kT^2}{2}$ (b) $I-\frac{k(T-t)^2}{2}$

(c) e^{-kT} (d) $T^2-\frac{1}{k}$

50. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to : **[2011]**

(a) 5 (b) 13
(c) -2 (d) 7

51. The solution of the differential equation $\frac{dy}{dy} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is **[2008]**

(a) $y = \ln x + x$ (b) $y = x \ln x + x^2$
(c) $y = xe^{(x-1)}$ (d) $y = x \ln x + x$

52. The normal to a curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a **[2007]**

(a) circle (b) hyperbola
(c) ellipse (d) parabola.

53. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is **[2005]**

(a) $y \log\left(\frac{x}{y}\right) = cx$ (b) $x \log\left(\frac{y}{x}\right) = cy$
(c) $\log\left(\frac{y}{x}\right) = cx$ (d) $\log\left(\frac{x}{y}\right) = cy$

54. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ **[2002]**

(a) $\frac{e^{-2x}}{4}$ (b) $\frac{e^{-2x}}{4} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-4x} + cx + d$

TOPIC 3 Linear Differential Equation of First Order Different Equation of the form: $\frac{d^2y}{dx^2} = F(x)$, Solution by Inspection Method

55. Let $y = y(x)$ be the solution of the differential equation

$$\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x,\ x \in \left(0, \frac{\pi}{2}\right).$$

If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to : **[Sep. 05, 2020 (II)]**

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$
(c) $\sqrt{2} - 2$ (d) $\frac{1}{\sqrt{2}} - 1$

56. Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x),\ x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to : **[Sep. 04, 2020 (I)]**

(a) $2 + \frac{\pi}{2}$ (b) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$
(c) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (d) $1 + \frac{\pi}{2}$

57. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation, $(x+1)dy = ((x+1)^2 + y - 3)dx$, $y(2) = 0$, then $y(3)$ is equal to _____. **[NA Jan. 09, 2020 (I)]**

58. Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is: **[Jan. 7, 2020 (II)]**

(a) $2 - e$ (b) $-e$
(c) 2 (d) $2 + e$

59. Consider the differential equation, $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of y is 1 when $x = 1$, then the value of x for which $y = 2$, is : **[April 12, 2019 (I)]**

(a) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (b) $\frac{3}{2} - \frac{1}{\sqrt{e}}$
(c) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (d) $\frac{3}{2} - \sqrt{e}$

60. If y = y(x) is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to : **[April 10, 2019 (I)]**

(a) $e - 2$ (b) $\frac{1}{2} - e$
(c) $2 + \frac{1}{e}$ (d) $\frac{1}{e} - 2$

61. Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y (0) = 1. Then : **[April 10, 2019 (II)]**

(a) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

(b) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

(c) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(d) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

62. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$ with $y(1) = 1$, is: **[April 09, 2019 (I)]**

(a) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (b) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

(c) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (d) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

63. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2 \frac{dy}{dx} + 2x(x^2+1)y = 1$ such that $y(0) = 0$. If $\sqrt{a}\, y(1) = \frac{\pi}{32}$, then the value of '$a$' is : **[April 08, 2019 (I)]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) $\frac{1}{16}$

64. Let y = y(x) be the solution of the differential equation, $x\frac{dy}{dx} + y = x \log_e x, (x > 1)$. If $2y(2) = \log_e 4 - 1$, then y(e) is equal to : **[Jan. 12, 2019 (I)]**

(a) $-\frac{e}{2}$ (b) $-\frac{e^2}{2}$

(c) $\frac{e}{4}$ (d) $\frac{e^2}{4}$

65. If a curve passes through the point (1, – 2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point :**[Jan. 12, 2019 (II)]**

(a) $(3, 0)$ (b) $(\sqrt{3}, 0)$

(c) $(-1, 2)$ (d) $(-\sqrt{2}, 1)$

66. If y(x) is the solution of the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, x > 0$, where $y(1) = \frac{1}{2}e^{-2}$, then : **[Jan. 11, 2019 (I)]**

(a) $y(\log_e 2) = \log_e 4$

(b) $y(\log_e 2) = \frac{\log_e 2}{4}$

(c) $y(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

(d) y (x) is decreasing in (0, 1)

67. Let f be a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}$, $(x > 0)$ and $f(1) \neq 4$. Then $\lim_{x \to 0^+} x f\left(\frac{1}{x}\right)$: **[Jan. 10, 2019 (II)]**

(a) exists and equals $\frac{4}{7}$. (b) exists and equals 4.

(c) does not exist. (d) exists and equals 0.

68. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to: **[Jan. 09, 2019 (I)]**

(a) $\frac{7}{64}$ (b) $\frac{1}{4}$

(c) $\frac{49}{16}$ (d) $\frac{13}{16}$

69. Let $y - y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : **[2018]**

(a) $\frac{-8}{9\sqrt{3}}\pi^2$ (b) $-\frac{8}{9}\pi^2$

(c) $-\frac{4}{9}\pi^2$ (d) $\frac{4}{9\sqrt{3}}\pi^2$

70. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1, & x \in [0, 1] \\ 0, & otherwise \end{cases}$

If $y(0) = 0$, then $y\left(\frac{3}{2}\right)$ is **[Online April 15, 2018]**

(a) $\frac{e^2 - 1}{2e^3}$ (b) $\frac{e^2 - 1}{e^3}$

(c) $\frac{1}{2e}$ (d) $\frac{e^2 + 1}{2e^4}$

71. The curve satisfying the differential equation, ydx–(x + 3y^2) dy = 0 and passing through the point (1, 1), also passes through the point : **[Online April 8, 2017]**

(a) $\left(\frac{1}{4},-\frac{1}{2}\right)$ (b) $\left(-\frac{1}{3},\frac{1}{3}\right)$

(c) $\left(\frac{1}{3},-\frac{1}{3}\right)$ (d) $\left(\frac{1}{4},\frac{1}{2}\right)$

72. The solution of the differential equaiton $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$, and y(0) = 1, is given by : **[Online April 10, 2016]**

(a) $y^2 = 1+\frac{x}{\sec x+\tan x}$ (b) $y=1+\frac{x}{\sec x+\tan x}$

(c) $y=1-\frac{x}{\sec x+\tan x}$ (d) $y^2=1-\frac{x}{\sec x+\tan x}$

73. Let y(x) be the solution of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x,(x\ge 1)$. Then y (e) is equal to: **[2015]**

(a) 2 (b) 2e

(c) e (d) 0

74. If $\frac{dy}{dx}+y\tan x=\sin 2x$ and $y(0)=1$, then $y(\pi)$ is equal to: **[Online April 19, 2014]**

(a) 1 (b) – 1

(c) – 5 (d) 5

75. The general solution of the differential equation, $\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$, is : **[Online April 12, 2014]**

(a) $y\sqrt{\tan x}=x+c$ (b) $y\sqrt{\cot x}=\tan x+c$

(c) $y\sqrt{\tan x}=\cot x+c$ (d) $y\sqrt{\cot x}=x+c$

76. The equation of the curve passing through the origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$ is **[Online April 25, 2013]**

(a) $(1+x^2)y=x^3$ (b) $3(1+x^2)y=2x^3$

(c) $(1+x^2)y=3x^3$ (d) $3(1+x^2)y=4x^3$

77. The integrating factor of the differential equation $\left(x^2-1\right)\frac{dy}{dx}+2xy=x$ is **[Online May 26, 2012]**

(a) $\frac{1}{x^2-1}$ (b) x^2-1

(c) $\frac{x^2-1}{x}$ (d) $\frac{x}{x^2-1}$

78. The general solution of the differential equation $\frac{dy}{dx}+\frac{2}{x}y=x^2$ is **[Online May 19, 2012]**

(a) $y=cx^{-3}-\frac{x^2}{4}$ (b) $y=cx^3-\frac{x^2}{4}$

(c) $y=cx^2+\frac{x^3}{5}$ (d) $y=cx^{-2}+\frac{x^3}{5}$

79. Consider the differential equation **[2011RS]**

$y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If $y(1)=1$, then x is given by :

(a) $4-\frac{2}{y}-\frac{e^{\frac{1}{y}}}{e}$ (b) $3-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$

(c) $1+\frac{1}{y}-\frac{e^{\frac{1}{y}}}{e}$ (d) $1-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$

80. Solution of the differential equation $\cos x\,dy=y(\sin x-y)\,dx,\ 0<x<\frac{\pi}{2}$ is **[2010]**

(a) $y\sec x=\tan x+c$

(b) $y\tan x=\sec x+c$

(c) $\tan x=(\sec x+c)y$

(d) $\sec x=(\tan x+c)y$

81. Solution of the differential equation $ydx+(x+x^2y)dy=0$ is **[2004]**

(a) $\log y=Cx$ (b) $-\frac{1}{xy}+\log y=C$

(c) $\frac{1}{xy}+\log y=C$ (d) $-\frac{1}{xy}=C$

82. The solution of the differential equation $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$, is **[2003]**

(a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$

(b) $(x-2)=ke^{2\tan^{-1}y}$

(c) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

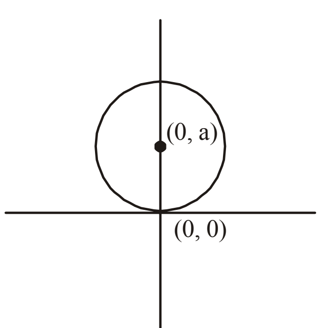
(d) $xe^{\tan^{-1}y}=\tan^{-1}y+k$

Hints & Solutions

1. **(a)** Since, $x^2 = 4b(y + b)$
$x^2 = 4by + 4b^2$
$2x = 4by'$
$\Rightarrow \quad b = \dfrac{x}{2y'}$
So, differential equation is
$$x^2 = \frac{2x}{y'}.y + \left(\frac{x}{y'}\right)^2$$
$x(y')^2 = 2yy' + x$

2. **(c)** $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
Since, it passes through (0, 3)
$\therefore \dfrac{0}{a^2} + \dfrac{9}{b^2} = 1$
$\Rightarrow b^2 = 9$
$\therefore$ eq. of ellipse becomes:
$$\frac{x^2}{a^2} + \frac{y^2}{9} = 1$$
differential w.r.t.x, we get;
$$\frac{2x}{a^2} + \frac{2y}{9}\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{y}{x}\left(\frac{dy}{dx}\right) = \frac{-9}{a^2}$$
Again differentiating w.r.t.x, we get;
$$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{x\frac{dy}{dx} - y}{x^2}\frac{dy}{dx} = 0$$
$\Rightarrow xyy'' + x(y')^2 - yy' = 0$

3. **(c)** Since family of all circles touching x-axis at the origin



$\therefore$ Eqn is $(x)^2 + (y - a)^2 = a^2$
where $(0, a)$ is the centre of circle.
$$\Rightarrow x^2 + y^2 + a^2 - 2ay = a^2$$
$$\Rightarrow x^2 + y^2 - 2ay = 0 \qquad \text{...(1)}$$
Differentiate both side w.r.t 'x', we get
$$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$$
$$\Rightarrow x + y\frac{dy}{dx} - a\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}} = a$$
Put value of 'a' in eqn (1), we get
$$x^2 + y^2 - 2y\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right] = 0$$
$$\Rightarrow (x^2 + y^2)\frac{dy}{dx} - 2y^2\frac{dy}{dx} - 2xy = 0$$
$$\Rightarrow (x^2 + y^2 - 2y^2)\frac{dy}{dx} = 2xy$$
$$\Rightarrow (x^2 - y^2)\frac{dy}{dx} = 2xy \equiv g(x)y$$
Hence, $g(x) = 2x$

4. **(b)** Statement -1 : $y^2 = \pm 4ax$
$$\Rightarrow \frac{dy}{dx} = \pm 2a.\frac{1}{y} \Rightarrow \frac{dy}{dx} \propto \frac{1}{y}$$
Statement -2 : $y^2 = 4ax \Rightarrow 2y\dfrac{dy}{dx} = 4a$
Thus both statements are true but statement-2 is not a correct explanation for statement-1.

5. **(d)** **Statement - 1**
Given differential equations are $\dfrac{dy}{dx} + y^2 = x$ and
$$\frac{d^2y}{dx^2} + y = \sin x$$
Their degrees are 1.
Both have equal degree.
Also, Statement - 2 is the correct explanation for Statement - 1.

6. **(c)** We have $y = c_1 e^{c_2 x}$

Differentiate it w.r. to x

$\Rightarrow\ y' = c_1 c_2 e^{c_2 x} = c_2 y$

$\Rightarrow\ \dfrac{y'}{y} = c_2$ Differentiate it w.r. to x

$\Rightarrow\ \dfrac{y''y-(y')^2}{y^2}=0 \Rightarrow\ y''y=(y')^2$

7. **(c)** Let the centre of the circle be $(h, 2)$

$\therefore$ Equation of circle is

$(x-h)^2+(y-2)^2=25$ ...(1)

Differentiating with respect to x, we get

$2(x-h)+2(y-2)\dfrac{dy}{dx}=0$

$\Rightarrow x-h=-(y-2)\dfrac{dy}{dx}$

Substituting in equation (1) we get

$(y-2)^2\left(\dfrac{dy}{dx}\right)^2+(y-2)^2=25$

$\Rightarrow (y-2)^2 (y')^2 = 25-(y-2)^2$

8. **(a)** General equation of circles passing through origin and having their centres on the x-axis is

$x^2+y^2+2gx=0$ ...(i)

On differentiating w.r.t x, we get

$2x+2y\,.\,\dfrac{dy}{dx}+2g=0 \Rightarrow g=-\left(x+y\dfrac{dy}{dx}\right)$

Putting in (i)

$x^2+y^2+2\left\{-\left(x+y\dfrac{dy}{dx}\right)\right\}.x=0$

$\Rightarrow\ x^2+y^2-2x^2-2x\dfrac{dy}{dx}.y=0$

$\Rightarrow\ y^2=x^2+2xy\dfrac{dy}{dx}$

9. **(d)** $Ax^2+By^2=1$ ...(i)

Differentiate w.r. to x

$Ax+By\dfrac{dy}{dx}=0$ ....(ii)

Again differentiate w.r. to x

$A+By\dfrac{d^2y}{dx^2}+B\left(\dfrac{dy}{dx}\right)^2=0$ ....(iii)

From (ii) and (iii)

$x\left\{-By\dfrac{d^2y}{dx^2}-B\left(\dfrac{dy}{dx}\right)^2\right\}+By\dfrac{dy}{dx}=0$

Dividing both sides by –B, we get

$xy\dfrac{d^2y}{dx^2}+x\left(\dfrac{dy}{dx}\right)^2-y\dfrac{dy}{dx}=0$

Therefore order 2 and degree 1.

10. **(c)** $y^2=2c(x+\sqrt{c})$ .... (i)

Differentite it w.r. to x

$2yy'=2c.1$ or $yy'=c$ .... (ii)

[On putting value of c from (ii) in (i)]

$\Rightarrow\ y^2=2yy'\,(x+\sqrt{yy'})$

On simplifying, we get

$(y-2xy')^2=4yy'^3$ .... (iii)

Hence equation (iii) is of order 1 and degree 3.

11. **(c)** $x^2+y^2-2ay=0$ ......(1)

Differentiate w.r. to x,

$2x+2y\dfrac{dy}{dx}-2a\dfrac{dy}{dx}=0 \Rightarrow a=\dfrac{x+yy'}{y'}$

Putting in (1) we get, $x^2+y^2-2\left(\dfrac{x+yy'}{y'}\right)y=0$

$\Rightarrow (x^2+y^2)y'-2xy-2y^2y'=0$

$\Rightarrow (x^2-y^2)y'=2xy$

12. **(c)** $y^2=4a(x-h)$,

Differentiating $2yy_1=4a \Rightarrow yy_1=2a$

Again differentiating, we get

$\Rightarrow y_1{}^2+yy_2=0$

Degree $=1$, order $=2$.

13. **(c)** $\left(1+3\dfrac{dy}{dx}\right)^2=\left(\dfrac{4d^3y}{dx^3}\right)^3$

$\Rightarrow \left(1+3\dfrac{dy}{dx}\right)^2=16\left(\dfrac{d^3y}{dx^3}\right)^3$

Order = 3, degree 3

14. **(a)** $\sqrt{1+x^2}\cdot\sqrt{1+y^2}=-xy\dfrac{dy}{dx}$

$\displaystyle\int\frac{\sqrt{1+x^2}}{x}dx=-\int\frac{y}{\sqrt{1+y^2}}dy$

Let $x=\tan\theta \Rightarrow dx=\sec^2\theta d\theta$

$\Rightarrow \displaystyle\int\frac{\sec^3\theta d\theta}{\tan\theta}=-\int\frac{2y}{2\sqrt{1+y^2}}dy$

$\Rightarrow \displaystyle\int\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cdot\cos^2\theta}d\theta=-\sqrt{1+y^2}$

$\Rightarrow \displaystyle\int(\tan\theta\cdot\sec\theta+\operatorname{cosec}\theta)d\theta=-\sqrt{1+y^2}$

$\Rightarrow \sec\theta+\log_e|\operatorname{cosec}\theta-\cot\theta|=-\sqrt{1+y^2}+C$

$\therefore \sqrt{1+x^2}+\log_e\left|\dfrac{\sqrt{1+x^2}-1}{x}\right|=-\sqrt{1+y^2}+C$

$$\Rightarrow \sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$$

15. **(a)** $\because y=\left(\frac{2}{\pi}x-1\right)\operatorname{cosec}x$

$$\frac{dy}{dx}=\frac{2}{\pi}\operatorname{cosec}x-\left(\frac{2}{\pi}x-1\right)\operatorname{cosec}x\cdot\cot x$$

$$=\operatorname{cosec}x\left[\frac{2}{\pi}-\left(\frac{2}{\pi}x-1\right)\cot x\right]$$

$$\Rightarrow \frac{dy}{dx}-\frac{2}{\pi}\operatorname{cosec}x=y\cot x \quad ...(i)$$

It is given that,

$$\Rightarrow \frac{dy}{dx}-\frac{2}{\pi}\operatorname{cosec}x=-yp(x) \quad ...(ii)$$

By comparision of (i) and (ii), we get

$p(x)=\cot x$

16. **(b)** $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}=-e^x$

$$\int\frac{dy}{2+y}=-\int\frac{e^x}{5+e^x}dx$$

$$\Rightarrow \log_e|2+y|\cdot\log_e|5+e^x|=\log_e C$$

$$\Rightarrow |(2+y)(5+e^x)|=C \qquad \because y(0)=1$$

$C=18.$

$$\therefore (2+y)\cdot(5+e^x)=18$$

When $x=\log_e 13$ then $(2+y)\cdot 18=18$

$$\Rightarrow 2+y=\pm 1$$

$$\therefore y=-1,-3$$

$$\therefore y(\ln 13)=-1$$

17. **(a)** Let $y+3x=t$

$$\Rightarrow \frac{dy}{dx}+3=\frac{dt}{dx}$$

Putting these value in given differential equation

$$\frac{dt}{dx}=\frac{t}{\log_e t}$$

$$\Rightarrow \int\frac{\log_e t}{t}dt=\int dx$$

$$\Rightarrow \frac{(\log_e t)^2}{2}=x-C$$

$$\Rightarrow x-\frac{1}{2}(\ln(y+3x))^2=C$$

18. **(a)** $\lim\limits_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$

$$\Rightarrow \lim_{t\to x}\frac{2tf^2(x)-2x^2f(t)\cdot f'(t)}{1}=0$$

Using L'Hospital's rule

$$\Rightarrow f(x)=xf'(x)$$

$$\int\frac{f'(x)}{f(x)}dx=\int\frac{1}{x}dx$$

$$\log_e f(x)=\log_e x+\log_e C$$

$$\Rightarrow f(x)=Cx, \qquad \because f(1)=e$$

$$\Rightarrow C=e;\ \text{so } f(x)=ex$$

When $f(x)=1=ex\Rightarrow x=\frac{1}{e}$

19. **(c)** $\int\left(\frac{y^2+1}{y^2}\right)dy=\int\frac{e^x dx}{e^x+1}$

$$\Rightarrow y-\frac{1}{y}=\log_e|e^x+1|+c$$

$\because$ Passes through (0, 1).

$\therefore c=-\log_e 2$

$$\Rightarrow y^2-1=y\log_e\left(\frac{e^x+1}{2}\right)$$

$$\Rightarrow y^2=1+y\log_e\left(\frac{e^x+1}{2}\right)$$

20. **(b)** $x^3dy+xy\,dyx=2y\,dx+x^2\,dy$

$$\Rightarrow (x^3-x^2)dy=(2-x)y\,dx$$

$$\Rightarrow \frac{dy}{y}=\frac{2-x}{x^2(x-1)}dx$$

$$\Rightarrow \int\frac{dy}{y}=\int\frac{2-x}{x^2(x-1)}dx \quad ...(i)$$

Let $\frac{2-x}{x^2(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$$\Rightarrow 2-x=A(x-1)+B(x-1)+Cx^2$$

Compare the coefficients of x, x^2 and constant term.

$C=1$, $B=-2$ and $A=-1$

$\therefore \int \frac{dy}{y} = \int \left\{\frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-1}\right\} dx$

$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln|x-1| + C$

$\therefore y(2) = e$

$\Rightarrow 1 = -\ln 2 + 1 + 0 + C$ $\quad [\because \log e = 1]$

$\Rightarrow C = \ln 2$

$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln|x-1| + \ln 2$

At $x = 4$,

$\Rightarrow \ln y(4) = -\ln 4 + \frac{1}{2} + \ln 3 + \ln 2$

$\Rightarrow \ln y(4) = \ln\left(\frac{3}{2}\right) + \frac{1}{2} = \ln\left(\frac{3}{2}e^{1/2}\right)$

$[\because \log m + \log n = \log(mn)]$

$\Rightarrow y(4) = \frac{3}{2}e^{1/2}$

21. (c) The given differential equation is

$\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y > 0$

$\Rightarrow \frac{dy}{y+1} = -\frac{\cos x}{2+\sin x}dx$

Integrate both sides,

$\int \frac{dy}{y+1} = \int \frac{(-\cos x)dx}{2+\sin x}$

$\ln|y+1| = -\ln|2+\sin x| + \ln C$

$\Rightarrow \ln|y+1| + \ln|2+\sin x| = \ln C$

$\Rightarrow \ln|(y+1)(2+\sin x)| = \ln C$

$\because y(0) = 1 \Rightarrow \ln 4 = \ln C \Rightarrow C = 4$

$\therefore (y+1)(2+\sin x) = 4$

$\Rightarrow y = \frac{4}{2+\sin x} - 1$

$\therefore y = \frac{2-\sin x}{2+\sin x} \Rightarrow y(\pi) = \frac{2-\sin\pi}{2+\sin\pi} = 1$

$\Rightarrow a = 1$

Now, $\frac{dy}{dx} = \frac{(2+\sin x)(-\cos x) - (2-\sin x)\cdot\cos x}{(2+\sin x)^2}$

$\left.\frac{dy}{dx}\right|_{x=\pi} = 1 \Rightarrow b = 1.$

Ordered pair $(a, b) = (1, 1)$.

22. (a) $\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$

It is homogeneous differential equation.

$\therefore$ Put $y = vx$

$\Rightarrow v + x\frac{dv}{dx} = v + \frac{v^2}{2} \Rightarrow \int 2\frac{dv}{v^2} = \int \frac{dx}{x}$

$\Rightarrow \frac{-2}{v} = \log_e x + c \Rightarrow \frac{-2x}{y} = \log_e x + c$

Put $x = 1, y = 2$, we get $c = -1$

$\Rightarrow \frac{-2x}{y} = \log_e x - 1$

Hence, put $x = \frac{1}{2} \Rightarrow y = \frac{1}{1+\log_e 2}$

23. (a) $f'(x) = \tan^{-1}(\sec x + \tan x)$

$= \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) = \frac{\pi}{4}+\frac{x}{2}$

Integrate both sides, we get

$\int (f'(x))\,dx = \int \left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + C$

$\because f(0) = 0$

$C = 0 \Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

So, $f(1) = \frac{\pi+1}{4}$

24. (d) The given differential equation,

$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Then, $v + x\frac{dv}{dx} = \frac{vx^2}{x^2 + v^2x^2} = \frac{v}{1+v^2}$

$\Rightarrow \quad \frac{1+v^2}{v^3}dv = -\frac{1}{x}dx$

$\Rightarrow \quad \int\left(\frac{1}{v^3} + \frac{1}{v}\right)dv = \int\frac{-1}{x}dx$

$\Rightarrow \quad \frac{-1}{2}\left(\frac{1}{v^2}\right) + \ln v = -\ln x + c$

$\Rightarrow \quad -\frac{x^2}{2y^2} = -\ln y + c \qquad \left[\because \ v = \frac{y}{x}\right]$

When $x = 1, y = 1$, then $-\frac{1}{2} = c$

$\Rightarrow \quad x^2 = y^2(1 + 2\ln y)$

At $y = e, x^2 = e^2(3)$

$\Rightarrow \quad x = \pm\sqrt{3}e$

So, $\quad x = \sqrt{3}e$

25. **(b)** $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1} f(x)\right)$

$2y = \sin^{-1} f(x) + C = \sin^{-1}(\sin(2\tan^{-1}x)) + C$

$\Rightarrow 2\left(\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + C$

$\frac{\pi}{3} = \frac{\pi}{3} + C \quad \therefore \quad C = 0$

for $x = -\sqrt{3}$, $2y = \sin^{-1}\left(\sin\left(\frac{-2\pi}{6}\right)\right) + 0$

$\Rightarrow 2y = \frac{-\pi}{3} \Rightarrow y = \frac{-\pi}{6}$

26. **(c)** The given differential eqn. is

$\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0 \Rightarrow \sin^{-1}y + \sin^{-1}x = c$

At $\quad x = \frac{1}{2}, y = \frac{\sqrt{3}}{2} \Rightarrow c = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}y = \cos^{-1}x$

Hence, $y\left(-\frac{1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

$= \sin\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2}}$

27. **(a)** Let $e^y = t$

$e^y\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \quad \frac{dt}{dx} - t = e^x \qquad \left[\because \ e^y\frac{dy}{dx} - e^y = e^x\right]$

I.F. $= e^{\int -1.dx} = e^{-x}$

$t(e^{-x}) = \int e^x.e^{-x}dx \Rightarrow e^{y-x} = x + c$

Put $x = 0, y = 0$, then we get $c = 1$

$e^{y-x} = x + 1$

$y = x + \log_e(x + 1)$

Put $x = 1 \quad \therefore \quad y = 1 + \log_e 2$

28. **(b)** Given differential equation can be written as,

$y^2dx - xydy = x^3dx$

$\Rightarrow \quad \frac{(ydx - xdy)y}{x^2} = xdx \Rightarrow -yd\left(\frac{y}{x}\right) = xdx$

$\Rightarrow \quad -\frac{y}{x}.d\left(\frac{y}{x}\right) = dx \Rightarrow -\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + c_1$

$\Rightarrow \quad 2x^3 + cx^2 + y^2 = 0 \qquad$ [Here, $c = 2c_1$]

29. **(c)** $\cos x dy - (\sin x)\, y\, dx = 6x\, dx$

$\Rightarrow \int d(y\cos x) = \int 6x\,dx \Rightarrow y\cos x = 3x^2 + C$ ...(1)

Given, $y\left(\frac{\pi}{3}\right) = 0$

Putting $x = \frac{\pi}{3}$ and $y = 0$ in eq. (1), we get

$(10)\times\left(\frac{1}{2}\right) = \frac{3\pi^2}{9} + C \Rightarrow C\frac{-\pi^2}{3}$

So, from (1) $y\cos x = 3x^2 - \frac{\pi^2}{3}$

Now, put $x = \frac{\pi}{6}$ in the above equation,

$y\frac{\sqrt{3}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3} \Rightarrow \frac{\sqrt{3}y}{2} = \frac{-3\pi^2}{12} \Rightarrow y = \frac{-\pi^2}{2\sqrt{3}}$

30. **(1)** Given $\frac{dy}{dx} = \frac{2y}{x^2}$

Integrating both sides, $\int\frac{dy}{y} = 2\int\frac{dx}{x^2}$

$\Rightarrow \ln|y| = -\frac{2}{x} + C$ ...(i)

Equation (i) passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, i.e., (1, 1)

$\therefore\ C = 2$

Now, $\ln|y| = -\frac{2}{x} + 2$

$x \ln|y| = -2(1-x) \Rightarrow x\ln|y| = 2(x-1)$

31. **(b)** The given differential equation

$\frac{dy}{dx} = (x-y)^2 \quad ...(1)$

Let $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

Now, from equation (1)

$\left(1 - \frac{dt}{dx}\right) = (t)^2$

$\Rightarrow 1 - t^2 = \frac{dt}{dx} \Rightarrow \int dx = \int \frac{dt}{1-t^2}$

$\Rightarrow -x = \frac{1}{2\times 1} ln\left|\frac{t-1}{t+1}\right| + c$

$\Rightarrow -x = \frac{1}{2} ln\left|\frac{x-y-1}{x-y+1}\right| + c$

$\because$ The given condition $y(1) = 1$

$-1 = \frac{1}{2}\ln\left|\frac{1-1-1}{1-1+1}\right| + c \Rightarrow c = -1$

Hence, $2(x-1) = -\ln\left|\frac{1-x+y}{1-y+x}\right|$

32. **(a)** Given, $\frac{dy}{dx} + \frac{3}{\cos^2 x} y = \frac{1}{\cos^2 x}$

$\frac{dy}{dx} = \sec^2 x(1-3y)$

$\Rightarrow \int \frac{dy}{(1-3y)} = \int \sec^2 x\, dx$

$\Rightarrow -\frac{1}{3}\ln|1-3y| = \tan x + C \quad ...(i)$

$\because \quad y\left(\frac{\pi}{4}\right) = \frac{4}{3} \quad$ (Given)

$\Rightarrow -\frac{1}{3}\ln|1-4| = \tan\frac{\pi}{4} + C$

$\Rightarrow -\frac{1}{3}\ln 3 = C + 1 \Rightarrow C = -1 - \frac{1}{3}\ln 3$

$\therefore$ in eq. (i), we get

$\frac{-1}{3}\ln|1-3y| = \tan x - 1 - \frac{1}{3}\ln 3$

Put, $x = -\frac{\pi}{4}$

$\Rightarrow -\frac{1}{3}\ln|1-3y| = \tan\left(-\frac{\pi}{4}\right) - 1 - \frac{1}{3}\ln 3$

$= -1 - 1 - \frac{1}{3}\ln 3$

$\Rightarrow \ln|1-3y| = 6 + \ln 3$

$\Rightarrow \ln\left|\frac{1}{3} - y\right| = 6 \Rightarrow \left|\frac{1}{3} - y\right| = e^6 \Rightarrow y = \frac{1}{3} \pm e^6$

33. **(a)** $(x^2 - y^2)dx + 2xy\, dy = 0$

$y^2dx - 2xydy = x^2dx$

$2xydy - y^2dx = -x^2dx$

$d(xy^2) = -x^2\, dx$

$\frac{xd(y^2) - y^2 d(x)}{x^2} = -dx$

$d\left(\frac{y^2}{x}\right) = -dx$

$\int d\left(\frac{y^2}{x}\right) = -\int dx$

$\frac{y^2}{x} = -x + C \quad ...(1)$

Since, the above curve passes through the point (1, 1)

Then, $\frac{1^2}{1} = -1 + C \Rightarrow C = 2$

Now, the curve (1) becomes

$y^2 = -x^2 + 2x$

$\Rightarrow y^2 = -(x-1)^2 + 1$

$(x-1)^2 + y^2 = 1$

The above equation represents a circle with centre (1, 0) and centre lies on x-axis.

34. **(a)** $f(xy) = f(x).f(y) \quad ...(1)$

Put $x = y = 0$ in (1) to get $f(0) = 1$

Put $x = y = 1$ in (1) to get $f(1) = 0$ or $f(1) = 1$

$f(1) = 0$ is rejected else $y = 1$ in (1) gives $f(x) = 0$ imply $f(0) = 0$.

Hence, $f(0) = 1$ and $f(1) = 1$

By first principle derivative formula,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} f(x)\left(\frac{f\left(1+\frac{h}{x}\right) - f(1)}{h}\right)$$

$$\Rightarrow \quad f'(x) = \frac{f(x)}{x} f'(1)$$

$$\Rightarrow \quad \frac{f'(x)}{f(x)} = \frac{k}{x} \Rightarrow \ln f(x) = k \ln x + c$$

$f(1) = 1 \Rightarrow \ln 1 = k \ln 1 + c \Rightarrow c = 0$

$\Rightarrow \quad \ln f(x) = k \ln x \Rightarrow f(x) = x^k$ but $f(0) = 1$

$\Rightarrow \quad k = 0$

$\therefore \quad f(x) = 1$

$$\frac{dy}{dx} = f(x) = 1 \Rightarrow y = x + c, y(0) = 1 \Rightarrow c = 1$$

$\Rightarrow \quad y = x + 1$

$$\therefore \quad y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$$

35. (b) $(x^2 - y^2)\,dx + 2xydy = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2} \quad \Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$$

$$\Rightarrow \quad \frac{2vdv}{v^2 + 1} = -\frac{dx}{x}$$

After integrating, we get

$ln\,|\,v^2 + 1| = -\,ln\,|\,x\,| + lnc$

$$\frac{y^2}{x^2} + 1 = \frac{c}{x}$$

As curve passes through the point (1, 1), so $1 + 1 = c$

$\Rightarrow c = 2$

$x^2 + y^2 - 2x = 0$, which is a circle of radius one.

36. (b) We have $(2 + \sin x)\dfrac{dy}{dx} + (y+1)\cos x = 0$

$$\Rightarrow \frac{d}{dx}(2 + \sin x)(y+1) = 0$$

On integrating, we get

$(2 + \sin x)(y + 1) = C$

At $x = 0, y = 1$ we have

$(2 + \sin 0)(1 + 1) = C$

$\Rightarrow \quad C = 4$

$$\Rightarrow \quad y + 1 = \frac{4}{2 + \sin x}$$

$$y = \frac{4}{2 + \sin x} - 1$$

$$\text{Now } y\left(\frac{\pi}{2}\right) = \frac{4}{2 + \sin\frac{\pi}{2}} - 1 = \frac{4}{3} - 1 = \frac{1}{3}$$

37. (b) $y(1 + xy)dx = xdy$

$$\frac{xdy - ydx}{y^2} = xdx$$

$$\Rightarrow \int -d\left(\frac{x}{y}\right) = \int xdx$$

$$-\frac{x}{y} = \frac{x^2}{2} + C \text{ as } y(1) = -1 \Rightarrow C = \frac{1}{2}$$

$$\text{Hence, } y = \frac{-2x}{x^2 + 1} \Rightarrow f\left(\frac{-1}{2}\right) = \frac{4}{5}$$

38. (d) Let $L = \lim\limits_{t \to x} \dfrac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Applying L.H. rule

$$L = \lim_{t \to x} \frac{2t\,f(x) - x^2 f'(t)}{1} = 1$$

$2x\,f(x) - x^2 f'(x) = 1$

solving above differential equation, we get

$$f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$$

Put $x = \dfrac{3}{2}$

$$f\left(\frac{3}{2}\right) = \frac{2}{3} \times \left(\frac{3}{2}\right)^2 + \frac{1}{3} \times \frac{2}{3} = \frac{3}{2} + \frac{2}{9} = \frac{27 + 4}{18} = \frac{31}{18}$$

39. (b) Given differential equation is

$ydx - (x + 2y^2)\,dy = 0$

$\Rightarrow ydx - xdy - 2y^2dy = 0$

$$\Rightarrow \frac{ydx - xdy}{y^2} = 2dy$$

$$\Rightarrow d\left(\frac{x}{y}\right) = 2dy$$

Integrate both the side

$$\Rightarrow \frac{x}{y} = 2y + c$$

using $f(-1) = 1$, we get

$c = 1$

$\Rightarrow \frac{x}{y} = 2y + 1$

put y = 1, we get f(a) = 3

40. **(a)** $(x+2)\frac{dy}{dx} = x^2 + 4x - 9 \quad x \neq -2$

$\frac{dy}{dx} = \frac{x^2 + 4x - 9}{x+2}$

$dy = \frac{x^2 + 4x - 9}{x+2} dx$

$\int dy = \int \frac{x^2 + 4x - 9}{x+2} dx$

$y = \int \left(x + 2 - \frac{13}{x+2} \right) dx$

$y = \int (x+2)\, dx - 13 \int \frac{1}{x+2} dx$

$y = \frac{x^2}{2} + 2x - 13 \log|x+2| + c$

Given that y = (0) = 0

$0 = -13 \log 2 + c$

$y = \frac{x^2}{2} + 2x - 13 \log|x+2| + 13 \log 2$

$y(-4) = 8 - 8 - 13 \log 2 + 13 \log 2 = 0$

41. **(c)** Given differential equation is

$\frac{dp(t)}{dt} = \frac{1}{2} p(t) - 200$

By separating the variable, we get

$dp(t) = \left[\frac{1}{2} p(t) - 200 \right] dt$

$\Rightarrow \quad \frac{dp(t)}{\frac{1}{2} p(t) - 200} = dt$

Integrate on both the sides,

$\int \frac{d(p(t))}{\frac{1}{2} p(t) - 200} = \int dt$

Let $\frac{1}{2} p(t) - 200 = s \Rightarrow \frac{dp(t)}{2} = ds$

So, $\int \frac{d\ p(t)}{\left(\frac{1}{2} p(t) - 200 \right)} = \int dt$

$\Rightarrow \quad \int \frac{2ds}{s} = \int dt \quad \Rightarrow \ 2 \log s = t + c$

$\Rightarrow 2 \log \left(\frac{p(t)}{2} - 200 \right) = t + c$

$\Rightarrow \quad \frac{p(t)}{2} - 200 = e^{\frac{t}{2}} k$

Using given condition $p(t) = 400 - 300\, e^{t/2}$

42. **(d)** Given $\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{y}{x}\right)$ ...(1)

Let $\left(\frac{y}{x}\right) = v$ so that $y = xv$

or $\frac{dy}{dx} = x\frac{dv}{dx} + v$ ...(2)

from (1) & (2), $x\frac{dv}{dx} + \cancel{v} = \cancel{v} + \phi\left(\frac{1}{v}\right)$

or, $\frac{dv}{\phi\left(\frac{1}{v}\right)} = \frac{dx}{x}$

Integrating both sides, we get

$\int \frac{dx}{x} = \int \frac{dv}{\phi\left(\frac{1}{v}\right)} \Rightarrow \ln x + c = \int \frac{dv}{\phi\left(\frac{1}{v}\right)}$

(where c being constant of integration)

But, given $y = \frac{x}{\ln|cx|}$ is the general solution

so that $\frac{x}{y} = \frac{1}{v} = \ln|cx| = \int \frac{dv}{\phi\left(\frac{1}{v}\right)}$

Differentiating w.r.t v both sides, we get

$\phi\left(\frac{1}{v}\right) = \frac{-1}{v^2} \Rightarrow \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$

when $\frac{x}{y} = 2$ i.e. $\phi(2) = -\left(\frac{y}{x}\right)^2 = -\left(\frac{1}{2}\right)^2 = \left(\frac{-1}{4}\right)$

43. **(c)** Given, Rate of change is $\frac{dP}{dx} = 100 - 12\sqrt{x}$

$\Rightarrow \quad dP = (100 - 12\sqrt{x})\, dx$

By integrating

$\int dP = \int (100 - 12\sqrt{x})\, dx$

$P = 100x - 8x^{3/2} + C$

Given when $x = 0$ then $P = 2000$

$\Rightarrow \quad C = 2000$

Now when $x = 25$ then

$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000 = 4500 - 1000$

$\Rightarrow \quad P = 3500$

44. **(a)** Slope $= \frac{dy}{dx} = 1 - \frac{1}{x^2}$

$\Rightarrow \int dy = \int \left(1 - \frac{1}{x^2}\right) dx$

$\Rightarrow y = x + \frac{1}{x} + C$, which is the equation of the curve

since curve passes through the point $\left(2, \frac{7}{2}\right)$

$\therefore \frac{7}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 1$

$\therefore y = x + \frac{1}{x} + 1$

when $x = -2$, then $y = -2 + \frac{1}{-2} + 1 = \frac{-3}{2}$

45. **(d)** Given differential equation is

$\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}$

By substituting $z = y^2$, we get diff. eqn. as

$\frac{dz}{dx} = \frac{2z^2}{2(xz - x^2)} = \frac{z^2}{xz - x^2}$

Now, $\frac{dx}{dz} = \frac{x}{z} - \frac{x^2}{z^2} = \frac{x}{z}\left[1 - \frac{x}{z}\right] \approx F\left(\frac{x}{z}\right)$

Hence, statement-1 is true.

Now, $y^2 e^{-y^2/x} = C$ satisfies the given diff. equation

$\therefore$ It is the solution of given diff. equation.

Thus, statement-2 is also true.

46. **(a)** Given differential equation is

$\frac{dp(t)}{dt} = 0.5\,p(t) - 450$

$\Rightarrow \frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450$

$\Rightarrow \frac{dp(t)}{dt} = \frac{p(t) - 900}{2}$

$\Rightarrow 2\frac{dp(t)}{dt} = -[900 - p(t)]$

$\Rightarrow 2\frac{dp(t)}{900 - p(t)} = -dt$

Integrate both the side, we get

$-2\int \frac{dp(t)}{900 - p(t)} = \int dt$

Let $900 - p(t) = u$

$\Rightarrow -dp(t) = du$

$2\int \frac{du}{u} = \int dt \Rightarrow 2\ln u = t + c$ ...(i)

$\Rightarrow 2\ln[900 - p(t)] = t + c$

Given $t = 0, p(0) = 850$

$2\ln(50) = c$

Putting in (i)

$\Rightarrow 2\left[\ln\left(\frac{900 - p(t)}{50}\right)\right] = t$

$\Rightarrow 900 - p(t) = 50e^{\frac{t}{2}}$

$\Rightarrow p(t) = 900 - 50e^{\frac{t}{2}}$

let $p(t_1) = 0$

$0 = 900 - 50e^{\frac{t_1}{2}} \quad \therefore t_1 = 2\ln 18$

47. **(c)** Given differential equation is

$\frac{(2 + \sin x)}{(1 + y)} \cdot \frac{dy}{dx} = \cos x$

which can be rewritten as

$\frac{dy}{1 + y} = \frac{\cos x}{2 + \sin x} dx$

Integrate both the sides, we get

$\int \frac{dy}{1 + y} = \int \frac{\cos x\, dx}{2 + \sin x}$

$\Rightarrow \log(1 + y) = \log(2 + \sin x) + \log C$

$\Rightarrow 1 + y = C(2 + \sin x)$

Given $y(0) = 2$

$\Rightarrow 1 + 2 = C[2 + \sin 0] \Rightarrow C = \frac{3}{2}$

Now, $y\left(\frac{\pi}{2}\right)$ can be found as

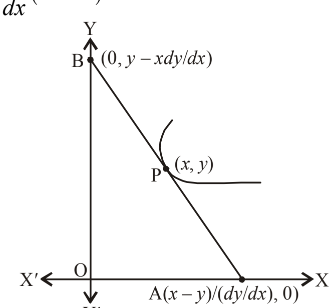
$1 + y = \frac{3}{2}\left(2 + \sin\frac{\pi}{2}\right) \Rightarrow 1 + y = \frac{9}{2}$

$\Rightarrow y = \frac{7}{2}$

Hence, $y\left(\frac{\pi}{2}\right) = \frac{7}{2}$

48. **(b)** Equation of tangent at P

$Y - y = \frac{dy}{dx}(X - x)$



X-intercept $= x - \frac{y}{dy/dx}$

Y-intercept $= y - \frac{xdy}{dx}$

Since P is mid-point of A and B

$x - \frac{y}{dy} = 2x$ and $y - \frac{xdy}{dx} = 2y$

$\Rightarrow \frac{-y}{\frac{dy}{dx}} = x$ and $\frac{-xdy}{dx} = y$

$\Rightarrow \frac{dx}{x} = -\frac{dy}{y}$

$\ell ny = -\ell nx + \ell nc$

$y = \frac{c}{x}$

Since the above line passes through the point (2, 3).

$\therefore \quad c = 6$

Hence $y = \frac{6}{x}$ is the required equation.

49. (a) $\frac{dV(t)}{dt} = -k(T-t)$

$\Rightarrow \int dV(t) = -k\int (T-t)dt$

$V(t) = \frac{k(T-t)^2}{2} + c$

at $t = 0, V(t) = I$

$I = \frac{kT^2}{2} + c$

$\Rightarrow c = I - \frac{kT^2}{2}$

$\Rightarrow V(t) = I + \frac{k}{2}(t^2 - 2tT)$

$V(T) = I + \frac{k}{2}(T^2 - 2T^2) \ = I - \frac{k}{2}T^2$

50. (d) $\frac{dy}{dx} = y + 3 \Rightarrow \int \frac{dy}{y+3} = \int dx$

$\Rightarrow \ell n|y+3| = x + c$

Given $y(0) = 2, \therefore \ell n 5 = c$

$\Rightarrow \ell n|y+3| = x + \ell n 5$

Put $x = \ell n\, 2$, then $\ell n\, |y + 3| = \ell n\, 2 + \ell n 5$

$\Rightarrow \ell n\,|y+3| = \ell n\, 10$

$\therefore \ y + 3 = \pm 10 \ \Rightarrow y = 7, -13$

51. (d) $\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x}$

It is homogeneous differential eqn.

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

we get

$v + x\frac{dv}{dx} = 1 + v \Rightarrow \int \frac{dx}{x} = \int dv$

$\Rightarrow v = \ln x + c \Rightarrow y = x\ln x + cx$

As $y(1) = 1$

$\therefore \ c = 1$ So solution is $y = x \ln x + x$

52. (b) Equation of normal at $P(x, y)$ is

$Y - y = -\frac{dx}{dy}(X - x)$

Coordinate of G at X axis is $(X, 0)$ (let)

$\therefore \ 0 - y = -\frac{dx}{dy}(X - x)$

$\Rightarrow \ y\frac{dy}{dx} = X - x$

$\Rightarrow \ X = x + y\frac{dy}{dx}$

$\therefore$ Co-ordinate of $G\left(x + y\frac{dy}{dx}, 0\right)$

Given distance of G from origin = twice of the abscissa of P.

$\because$ distance cannot be –ve, therefore abscissa x should be +ve

$\therefore \ x + y\frac{dy}{dx} = 2x \ \Rightarrow \ y\frac{dy}{dx} = x$

$\Rightarrow \ ydy = xdx$

On Integrating, we have $\frac{y^2}{2} = \frac{x^2}{2} + c_1$

$\Rightarrow \ x^2 - y^2 = -2c_1$

$\therefore$ the curve is a hyperbola

53. (c) $\frac{xdy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$

Put $y = vx$

$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow \ v + \frac{xdv}{dx} = v(\log v + 1)$

$\frac{xdv}{dx} = v \log v \ \Rightarrow \int \frac{dv}{v \log v} = \int \frac{dx}{x}$

Put $\log v = z$

$\frac{1}{v}dv = dz \ \Rightarrow \int \frac{dz}{z} = \int \frac{dx}{x}$

$\ln z = \ln x + \ln c$

$x = cx$ or $\log v = cx$ or $\log\left(\frac{y}{x}\right) = cx.$

54. (b) $\frac{d^2y}{dx^2} = e^{-2x}$; on integration $\frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$;

Again integrate we get $y = \frac{e^{-2x}}{4} + cx + d$

55. (c) $\frac{dy}{dx} + 2y\tan x = 2\sin x$

I.F. $= e^{\int 2\tan x\, dx} = \sec^2 x$

The solution of the differential equation is

$y \times \text{I.F.} = \int \text{I.F.} \times 2\sin x\, dx + C$

$\Rightarrow y \cdot \sec^2 x = \int 2\sin x \cdot \sec^2 x\, dx + C$

$\Rightarrow y\sec^2 x = 2\sec x + C$ ...(i)

When $x = \frac{\pi}{3}$, $y = 0$; then $C = -4$

$\therefore$ From (i), $y\sec^2 x = 2\sec x - 4$

$\Rightarrow y = \frac{2\sec x - 4}{\sec^2 x} \Rightarrow y\left(\frac{\pi}{4}\right) = \sqrt{2} - 2$

56. **(a)** $\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x)$

I.F. $= e^{-\int \frac{1}{x} dx} = \frac{1}{x}$

$\therefore \int d\left(\frac{y}{x}\right) = \int (x\cos x + \sin x)dx$

$\Rightarrow \frac{y}{x} = x\sin x + C$ $\quad \because y(\pi) = \pi \Rightarrow C = 1$

$y = x^2 \sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

$y' = 2x\sin x + x^2 \cos x + 1$

$y'' = 2\sin x - x^2 \sin x \Rightarrow y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4}$

$\therefore y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} + \frac{\pi^2}{4} + \frac{\pi}{2} = 2 + \frac{\pi}{2}$

57. **(c)** $(x+1)dy = ((x+1)^2 + (y-3))dx = 0$

$\Rightarrow \frac{dy}{dx} = (1+x) + \left(\frac{y-3}{1+x}\right)$

$\frac{dy}{dx} - \frac{1}{(1+x)}y = (1+x) - \frac{3}{(1+x)}$

I.F. $= e^{-\int \frac{1}{(1+x)} dx} = \frac{1}{(1+x)}$

$\therefore \frac{d}{dx}\left(\frac{y}{1+x}\right) = 1 - \frac{3}{(1+x)^2}$

$y = (1+x)\left[x + \frac{3}{(1+x)} + C\right]$

$\because$ At $x = 2, y = 0$

$\therefore$ $0 = 3(2 + 1 + C) \Rightarrow C = -3$

Then, $y = (1+x)\left[x + \frac{3}{1+x} - 3\right]$

Now, at $x = 3$, $y = (1+3)\left[3 + \frac{3}{1+3} - 3\right] = 3$

58. **(a)** The given differential equation is $\frac{dx}{dy} + x = y^2$

Comparing with $\frac{dx}{dy} + Px = Q$, where $P = 1$, $Q = y^2$

Now, I.F. $= e^{\int 1.dy} = e^y$

$x.e^y = \int (y^2)e^y.dy = y^2.e^y - \int 2y.e^y.dy$

$= y^2e^y - 2(y.e^y - e^y) + C$

$\Rightarrow$ $x.e^y = y^2e^y - 2ye^y + 2e^y + C$

$\Rightarrow$ $x = y^2 - 2y + 2 + C.\,e^{-y}$ ...(i)

As $y(0) = 1$, satisfying the given differential eqn,

$\therefore$ put $x = 0, y = 1$ in eqn. (i)

$0 = 1 - 2 + 2 + \frac{C}{e}$

$C = -e$

$y = 0, x = 0 - 0 + 2 + (-e)(e^{-0})$

$x = 2 - e$

59. **(b)** Consider the differential equation,

$y^2dx + \left(x - \frac{1}{y}\right)dy = 0$

$\Rightarrow \frac{dx}{dy} + \left(\frac{1}{y^2}\right)x = \frac{1}{y^3}$

I.F. $= e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

$\therefore x.e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \frac{1}{y^3}dy + c$

Put $-\frac{1}{y} = u \Rightarrow \frac{1}{y^2}dy = du$

$\Rightarrow$ $x.e^{-\frac{1}{y}} = -\int ue^u du + c = -ue^u + e^u + c$

$\Rightarrow$ $x.e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(\frac{1}{y} + 1\right) + c$

At $y = 1, x = 1$

$1 = 2 + ce \Rightarrow c = -\frac{1}{e} \Rightarrow x = \left(1 + \frac{1}{y}\right) - \frac{1}{e} e^{\frac{1}{y}}$

On putting $y = 2$, we get $x = \frac{3}{2} - \frac{1}{\sqrt{e}}$

60. **(a)** $\frac{dy}{dx} + y\sec^2 x = \sec^2 \times \tan x$

Given equation is linear differential equation.

$\text{IF} = e^{\int \sec^2 x dx} = e^{\tan x}$

$\Rightarrow y.e^{\tan x} = \int e^{\tan x} \sec^2 \times \tan x dx$

Put $\tan x = u = \sec^2 x\, dx = du$

$ye^{\tan x} = \int e^u u\, du \Rightarrow ye^{\tan x} = ue^u - e^u + c$

$\Rightarrow ye^{\tan x} = (\tan x - 1)\, e^{\tan x} + c$

$\Rightarrow y = (\tan x - 1) + c.\, e^{-\tan x}$

$\therefore\ y(0) = 0$ (given) $\Rightarrow 0 = -1 + c \Rightarrow c = 1$

Hence, solution of differential equation,

$y\left(-\frac{\pi}{4}\right) = -1 - 1 + e = -2 + e$

61. **(d)** Given differential equation is,

$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$

Here, $P = \tan x$, $Q = 2x + x^2 \tan x$

$\text{I.F.} = e^{\int \tan x dx} = e^{\ln|\sec x|} = |\sec x|$

$\therefore\ y(\sec x) = \int (2x + x^2 \tan x)\sec x dx$

$= \int x^2 \tan x \sec x dx + \int 2x \sec x dx = x^2 \sec x + c$

Given $y(0) = 1 \Rightarrow c = 1$

$\therefore\ y = x^2 + \cos x$ ...(i)

Now put $x = \frac{\pi}{4}$ and $x = \frac{-\pi}{4}$ in equation (i),

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$ and $y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$\Rightarrow y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = 0$

$\frac{dy}{dx} = 2x - \sin x$

$\therefore y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$ and $y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$

$\Rightarrow y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

62. **(c)** $\frac{dy}{dx} + \frac{2}{x}y = x$ $y(1) = 1$ (given)

Since, the above differential equation is the linear differential equation, then $I.F = e^{\int \frac{2}{x} dx} = x^2$

Now, the solution of the linear differential equation

$y \times x^2 = \int x^3 dx$

$\Rightarrow yx^2 = \frac{x^4}{4} + C$

$\because y(1) = 1$

$\therefore 1 \times 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$

$\therefore$ Solution becomes.

$y = \frac{x^2}{4} + \frac{3}{4x^2}$

63. **(d)** $(1 + x^2)^2 \frac{dy}{dx} + 2x(1 + x^2)y = 1$

$\Rightarrow\ \frac{dy}{dx} + \left(\frac{2x}{1 + x^2}\right)y = \frac{1}{(1 + x^2)^2}$

Since, the above differential equation is a linear differential equation

$\therefore$ I.F. $= e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$

Then, the solution of the differential equation

$\Rightarrow\ y(1 + x^2) = \int \frac{dx}{1 + x^2} + c$

$\Rightarrow\ y(1 + x^2) = \tan^{-1} x + c$ ...(1)

If $x = 0$ then $y = 0$ (given)

$\Rightarrow\ 0 = 0 + c$

$\Rightarrow\ c = 0$

Then, equation (1) becomes,

$\Rightarrow\ y(1 + x^2) = \tan^{-1} x$

Now put $x = 1$ in above equation, then

$2y = \frac{\pi}{4}$

$\Rightarrow\ 2\left(\frac{\pi}{32\sqrt{a}}\right) = \frac{\pi}{4}$ $\left[\sqrt{a}\, y(1) = \frac{\pi}{32}\right]$

$\Rightarrow \quad \sqrt{a}=\frac{1}{4}$

$\Rightarrow \quad a=\frac{1}{16}$

64. **(c)** Consider the differential equation,

$\frac{dy}{dx}+\frac{y}{x}=\log_e x$

$\because \quad IF=e^{\int\frac{1}{x}dx}=x$

$\therefore \quad yx=\int x\ln x\,dx$

$\Rightarrow \quad xy=\ln x\cdot\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}dx$

$\Rightarrow \quad xy=\frac{x^2}{2}\cdot\ln x-\frac{x^2}{4}+c$

Given, $2y(2)=\log_e 4-1$.

$\therefore \quad 2y=2\ln 2-1+c$

$\Rightarrow \quad \ln 4-1=\ln 4-1+c$

i.e. $c=0$

$\Rightarrow \quad xy=\frac{x^2}{2}\ln x-\frac{x^2}{4}$

$\Rightarrow \quad y=\frac{x}{2}\ln x-\frac{x}{4}$

$\Rightarrow \quad y(e)=\frac{e}{2}-\frac{e}{4}=\frac{e}{4}$

65. **(b)** $\because$ Slope of the tangent $=\frac{x^2-2y}{x}$

$\because \quad \frac{dy}{dx}=\frac{x^2-2y}{x}$

$\frac{dy}{dx}+\frac{2}{x}y=x$

I.F. $=e^{\int\frac{2}{x}dx}=e^{2\ln x}=x^2$

Solution of equation

$y\cdot x^2=\int x\cdot x^2dx$

$x^2y=\frac{x^4}{4}+C$

$\therefore$ curve passes through point (1, –2)

$(1)^2(-2)=\frac{1^4}{4}+C$

$\Rightarrow \quad C=\frac{-9}{4}$

Then, equation of curve

$y=\frac{x^2}{4}-\frac{9}{4x^2}$

Since, above curve satisfies the point.

Hence, the curve passes through $\left(\sqrt{3},0\right)$.

66. **(c)** Given differential equation is,

$\frac{dy}{dx}+\left(2+\frac{1}{x}\right)y=e^{-2x}, x>0$

$\text{IF}=e^{\int\left(2+\frac{1}{x}\right)dx}=e^{2x+\ln x}=xe^{2x}$

Complete solution is given by

$y(x)\cdot x\,e^{2x}=\int x\,e^{2x}\cdot e^{-2x}dx+c$

$=\int x\,dx+c$

$y(x)\cdot e^{2x}\cdot x=\frac{x^2}{2}+c$

Given, $y(1)=\frac{1}{2}e^{-2}$

$\therefore \quad \frac{1}{2}e^{-2}\cdot e^{2}\cdot 1=\frac{1}{2}+c\Rightarrow c=0$

$\therefore \quad y(x)=\frac{x^2}{2}\cdot\frac{e^{-2x}}{x}$

$y(x)=\frac{x}{2}\cdot e^{-2x}$

Differentiate both sides with respect to x,

$y'(x)=\frac{e^{-2x}}{2}(1-2x)<0\ \forall\ x\in\left(\frac{1}{2},1\right)$

Hence, $y(x)$ is decreasing in $\left(\frac{1}{2},1\right)$

67. **(b)** Let $y=f(x)$

$\frac{dy}{dx}+\left(\frac{3}{4x}\right)y=7$

I.F. $=e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\ln x}=x^{\left(\frac{3}{4}\right)}$

Solution of differential equation

$y\cdot x^{\frac{3}{4}}=\int 7\cdot x^{\frac{3}{4}}dx+C$

$y\cdot x^{\frac{3}{4}}=7\cdot\frac{x^{\frac{7}{4}}}{\left(\frac{7}{4}\right)}+C=4x^{\frac{7}{4}}+C$

$y = 4x + Cx^{-\frac{3}{4}}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{4}{x} + Cx^{\frac{3}{4}}$

$\Rightarrow \lim_{x\to 0^+} x \cdot f\left(\frac{1}{x}\right) = \lim_{x\to 0^+}\left(4 + Cx^{\frac{7}{4}}\right) = 4$

68. **(c)** Since, $x\frac{dy}{dx} + 2y = x^2$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x$

I.F. $= e^{\int\frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$.

Solution of differential equation is:

$y \cdot x^2 = \int x \cdot x^2 dx$

$y \cdot x^2 = \frac{x^4}{4} + C$ ...(1)

$\because \quad y(1) = 1$

$\therefore \quad C = \frac{3}{4}$

Then, from equation (1)

$y \cdot x^2 = \frac{x^4}{4} + \frac{3}{4}$

$\therefore \quad y = \frac{x^2}{4} + \frac{3}{4x^2}$

$\therefore \quad y\left(\frac{1}{2}\right) = \frac{1}{16} + 3 = \frac{49}{16}$

69. **(b)** Consider the given differential equation the

$\sin x dy + y\cos x dx = 4x dx$

$\Rightarrow d(y.\sin x) = 4x dx$

Integrate both sides

$\Rightarrow y.\sin x = 2x^2 + C$ ...(1)

$\Rightarrow y(x) = \frac{2x^2}{\sin x} + c$ ...(2)

$\because$ eq. (2) passes through $\left(\frac{\pi}{2}, 0\right)$

$\Rightarrow 0 = \frac{\pi^2}{2} + C$

$\Rightarrow C = -\frac{\pi^2}{2}$

Now, put the value of C in (1)

Then, $y\sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution

$\therefore y\left(\frac{\pi}{6}\right) = \left(2.\frac{\pi^2}{36} - \frac{\pi^2}{2}\right)2 = -\frac{8\pi^2}{9}$

70. **(a)** When $x \in [0, 1]$, then $\frac{dy}{dx} + 2y = 1 \Rightarrow y = \frac{1}{2} + C_1 e^{-2x}$

$\because y(0) = 0 \Rightarrow y(x) = \frac{1}{2} - \frac{1}{2}e^{-2x}$

Here, $y(1) = \frac{1}{2} - \frac{1}{2}\ e^{-2} = \frac{e^2 - 1}{2e^2}$

When $x \notin [0, 1]$, then $\frac{dy}{dx} + 2y = 0 \Rightarrow y = c_2\, e^{-2x}$

$\because y(1) = \frac{e^2 - 1}{2} \Rightarrow \frac{e^2 - 1}{2} = c^2 e^{-2} \Rightarrow C_2 = \frac{e^2 - 1}{2}$

$\therefore y(x)\left(\frac{e^2 - 1}{2}\right)e^{-2x} \Rightarrow y\left(\frac{3}{2}\right) = \frac{e^2 - 1}{2e^3}$

71. **(b)** $y\,dx - x\,dy - 3y^2\,dy = 0$

$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 3y \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$

if $= e^{-\int\frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$

$\therefore$ solution is $\frac{x}{y} = \int 3y\,.\,\frac{1}{y}\,dy$

$\Rightarrow \frac{x}{y} = 3y + c$

which passes through (1, 1)

$\therefore \quad 1 = 3 + c \Rightarrow c = -2$

$\therefore$ solution becomes

$\Rightarrow x = 3y^2 - 2y$

which also passes through $\left(-\frac{1}{3}, \frac{1}{3}\right)$

72. **(d)** $\frac{dy}{dx} + \frac{y}{2}\ \sec x = \frac{\tan x}{2y}$

$2y\frac{dy}{dx} + y^2 \sec x = \tan x$

Put $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\sec x = \tan x$

$I.f = e^{\int \sec x dx} = e^{\ln(\sec x + \tan x)} = \sec x + \tan x$

$$\frac{dt}{dx}(\sec x + \tan x) + t \sec x (\sec x + \tan x)$$
$$= \tan x(\sec x + \tan x)$$
$$\int d(t(\sec x + \tan x)) = \int \tan x(\sec x + \tan x)\,dx$$
$$t(\sec x + \tan x) = \sec x + \tan x - x$$
$$t = 1 - \frac{x}{\sec x + \tan x} \Rightarrow y^2 = 1 - \frac{x}{\sec x + \tan x}$$

73. (a) Given, $\frac{dy}{dx} + \left(\frac{1}{x \log x}\right)y = 2$

$$\text{I.F.} = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$$
$$y.\log x = \int 2\log x\,dx + c$$
$$y\log x = 2[x\log x - x] + c$$
Put $x = 1$, $y.0 = -2 + c$
$c = 2$
Put $x = e$
$y \log e = 2e(\log e - 1) + c$
$y(e) = c = 2$

74. (c) Let $\frac{dy}{dx} + y\tan x = \sin 2x$

$$\text{I.F} = e^{\int \tan x\,dx} = e^{-\log\cos x} = \sec x$$
Required solution is
$$y(\sec x) = \int \sin 2x \sec x\,dx + c$$
$$y(\sec x) = \int \frac{2\sin x\cos x}{\cos x}dx + c$$
$$y(\sec x) = 2\int \sin x\,dx + c$$
$y(\sec x) = -2\cos x + c$ ...(1)
Given $y(0) = 1$
$\therefore$ put $x = 0$ and $y = 1$, we get
$1(\sec 0) = -2\cos 0 + c$
$\Rightarrow c = 1 + 2 \Rightarrow c = 3$
$\therefore$ from eqn (1), we have
$y\sec x = -2\cos x + 3$ ...(2)
To find $y(\pi)$, put $x = \pi$ in eqn (2), we get
$y(\sec\pi) = -2\cos\pi + 3$
$y = -2(-1)(-1) + 3(-1) = -2 - 3 = -5$

75. (d) Given, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$

or, $\frac{dy}{dx} = \frac{y}{\sin 2x} + \sqrt{\tan x}$

or, $\frac{dy}{dx} - y\,\text{cosec}\,2x = \sqrt{\tan x}$ ...(1)

Now, integrating factor (I.F) $= e^{\int -\text{cosec}\,2x}$

or, I.F $= e^{-\frac{1}{2}\log|\tan x|} = e^{\log\left(\sqrt{\tan x}\right)^{-1}}$

$= \frac{1}{\sqrt{\tan x}} = \sqrt{\cot x}$

Now, general solution of eq. (1) is written as

$y(\text{I. F.}) = \int Q(\text{I.F.})\,dx + c$

$\therefore\ y\sqrt{\cot x} = \int \sqrt{\tan x}.\sqrt{\cot x}\,dx + c$

$\therefore\ y\sqrt{\cot x} = \int 1.dx + c$

$\boxed{\therefore\ y\sqrt{\cot x} = x + c}$

76. (d) Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$
$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$$
This is linear diff. equation
$$\text{I.F} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$$
$\therefore$ Solution is
$$y(1+x^2) = \int \frac{4x^2}{1+x^2} \times 1 + x^2 + C$$
$$\Rightarrow y(1+x^2) = \frac{4x^3}{3} + C$$
$\Rightarrow$ Required curve is
$3y(1+x^2) = 4x^3$ ($\because$ C = 0)

77. (b) Given differential equation is $(x^2 - 1)\frac{dy}{dx} + 2xy = x$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{x^2-1}.y = \frac{x}{x^2-1}$$
This is in linear form.

Integrating factor $= e^{\int \frac{2x}{x^2-1}dx} = e^{\int \frac{dt}{t}}$ where $t = x^2 - 1$

$= e^{\log t} = x^2 - 1$

Hence, required integrating factor $= x^2 - 1$.

78. (d) Given differential equation is

$$\frac{dy}{dx} + \frac{2}{x}.y = x^2$$
This is of the linear form.
$$\therefore P = \frac{2}{x}, Q = x^2$$
$$\text{I.F} = e^{\int \frac{2}{x}dx} = e^{\log x^2} = x^2$$

Solution is

$$y.x^2 = \int x^2.x^2 dx + c = \frac{x^5}{5} + c$$

$$y = \frac{x^3}{5} + cx^{-2}$$

79. **(c)** $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

It is linear differential eqn.

$$\text{I.F.} = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

So $x.e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$

Let $\frac{-1}{y} = t$

$\Rightarrow \frac{1}{y^2} dy = dt$

$\Rightarrow I = -\int te^t dt = e^t - te^t = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$

$\Rightarrow xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$

$\Rightarrow x = 1 + \frac{1}{y} + c.e^{1/y}$

Given $y(1) = 1$

$\therefore c = -\frac{1}{e}$

$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$

80. **(d)** $\cos x\, dy = y(\sin x - y)\, dx$

$$\frac{dy}{dx} = y \tan x - y^2 \sec x$$

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x \qquad \text{...(i)}$$

Let $\frac{1}{y} = t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

Putting in (i)

$$-\frac{dt}{dx} - t\tan x = -\sec x$$

$$\Rightarrow \frac{dt}{dx} + (\tan x)t = \sec x$$

$$\text{I.F.} = e^{\int \tan x\, dx} = e^{\log|\sec x|} = \sec x$$

Solution : $t \sec x = \int \sec x \sec x\, dx$

$\Rightarrow \frac{1}{y}\sec x = \tan x + c$

81. **(b)** $ydx + (x + x^2 y)dy = 0$

$$\Rightarrow \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2,$$

It is Bernoulli form. Divide by x^2

$$x^{-2}\frac{dx}{dy} + x^{-1}\left(\frac{1}{y}\right) = -1.$$

put $x^{-1} = t, -x^{-2}\frac{dx}{dy} = \frac{dt}{dy}$ we get,

$$-\frac{dt}{dy} + t\left(\frac{1}{y}\right) = -1 \Rightarrow \frac{dt}{dy} - \left(\frac{1}{y}\right)t = 1$$

It is linear differential eqn. in t.

$$\text{I.F} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = y^{-1}$$

$\therefore$ Solution is $t(y^{-1}) = \int (y^{-1})dy + C$

$$\Rightarrow \frac{1}{x}.\frac{1}{y} = \log y + C \Rightarrow \log y - \frac{1}{xy} = C$$

82. **(c)** $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1} y}}{(1+y^2)}$$

It is form of linear differential equation.

$$I.F = e^{\int \frac{1}{(1+y^2)} dy} = e^{\tan^{-1} y}$$

$$x(e^{\tan^{-1} y}) = \int \frac{e^{\tan^{-1} y}}{1+y^2} e^{\tan^{-1} y} dy$$

$$x(e^{\tan^{-1} y}) = \frac{e^{2\tan^{-1} y}}{2} + C \qquad \left[\because \int e^{2x} dx = \frac{e^{2x}}{2}\right]$$

$$\therefore 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$$

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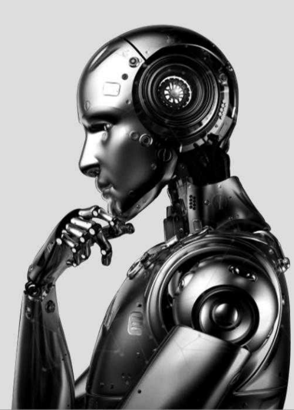


Vector Algebra

TOPIC 1 Algebra of Vectors, Section Formula, Linear Dependence & Independence of Vectors, Position Vector of a Point, Modulus of a Vector, Collinearity of Three points, Coplanarity of Three Vectors & Four Points, Vector Inequality

1. Let $a,b,c \in \mathbf{R}$ be such that $a^2+b^2+c^2=1$. If $a\cos\theta - b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is : **[Sep. 03, 2020 (II)]**
(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{9}$ (d) 0

2. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda:1\ (\lambda>0)$. If O is the region and $\overrightarrow{OB}\cdot\overrightarrow{OP}-3|\overrightarrow{OA}\times\overrightarrow{OP}|^2=6$, then λ is equal to ___________. **[NA Sep. 02, 2020 (II)]**

3. If the vectors, $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$, $\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and $\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$ $(a \in \mathrm{R})$ are coplanar and $3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$, then the value of λ is _____. **[NA Jan. 9, 2020 (I)]**

4. Let $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ has the magnitude 12 then one such vector is : **[April 12, 2019 (I)]**
(a) $4\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$ (b) $4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$
(c) $4\left(2\hat{i}+2\hat{j}-\hat{k}\right)$ (d) $4\left(-2\hat{i}-2\hat{j}+\hat{k}\right)$

5. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to : **[April 12, 2019 (I)]**
(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$

6. Let $\alpha \in \mathrm{R}$ and the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=\alpha\hat{i}-2\hat{j}+3\hat{k}$. Then the set $S=(\alpha:\vec{a},\vec{b}$ and $\vec{c}$ are coplanar) **[April 12, 2019 (II)]**
(a) is singleton
(b) is empty
(c) contains exactly two positive numbers
(d) contains exactly two numbers only one of which is positive

7. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and $\theta\in(0,\pi)$ with $\hat{k}$, then a value of θ is: **[April 09, 2019 (II)]**
(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{5\pi}{12}$ (d) $\frac{2\pi}{3}$

8. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$, are co-planar, is : **[Jan. 12, 2019 (I)]**
(a) –1 (b) 0 (c) 1 (d) 2

9. Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}, \vec{b}=\hat{i}+\lambda\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+\left(\lambda^2-1\right)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a}\times\vec{c}$ is : **[Jan. 11, 2019 (I)]**
(a) $-10\hat{i}-5\hat{j}$ (b) $-14\hat{i}-5\hat{j}$
(c) $-14\hat{i}+5\hat{j}$ (d) $-10\hat{i}+5\hat{j}$

10. Let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is : **[Jan. 11, 2019 (II)]**

(a) 4 (b) 3 (c) 2 (d) 1

11. Let $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is: **[Jan. 10, 2019 (II)]**

(a) -4 (b) -3 (c) 4 (d) 3

12. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}-24$, then $|\vec{u}|^2$ is equal to : **[2018]**

(a) 315 (b) 256 (c) 84 (d) 336

13. Let ABC be a triangle whose circumcentre is at P. If the position vectors A, B, C and P are $\vec{a},\vec{b},\vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is : **[Online April 10, 2016]**

(a) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (b) $\vec{a}+\vec{b}+\vec{c}$

(c) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (d) $\vec{0}$

14. If the vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is **[2013]**

(a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{45}$

15. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then the value of α for which the vectors $\vec{u}=(\alpha-2)\vec{a}+\vec{b}$ and $\vec{v}=(2+3\alpha)\vec{a}-3\vec{b}$ are collinear is : **[Online April 23, 2013]**

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $-\frac{3}{2}$ (d) $-\frac{2}{3}$

16. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=r\hat{i}+\hat{j}+(2r-1)\hat{k}$ are three vectors such that $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$, then r is equal to **[Online May 19, 2012]**

(a) 1 (b) -1 (c) 0 (d) 2

17. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is : **[2011RS]**

(a) $\vec{a}$ (b) $\vec{c}$ (c) $\vec{0}$ (d) $\vec{a}+\vec{c}$

18. If the $p\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+q\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+r\hat{k}$ $(p\neq q\neq r\neq 1)$ vector are coplanar, then the value of $pqr-(p+q+r)$ is **[2011RS]**

(a) 2 (b) 0 (c) -1 (d) -2

19. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β? **[2008]**

(a) $\alpha=2,\ \beta=2$ (b) $\alpha=1,\ \beta=2$

(c) $\alpha=2,\ \beta=1$ (d) $\alpha=1,\ \beta=1$

20. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overline{AB},\overline{BC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overline{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is **[2006]**

(a) $\frac{AB^2+AC^2}{(AB)^2(AC)^2}$ (b) $\frac{(AB)(AC)}{AB+AC}$

(c) $\frac{1}{AB}+\frac{1}{AC}$ (d) $\frac{1}{AD}$

21. If C is the mid point of AB and P is any point outside AB, then **[2005]**

(a) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$

(b) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$

(c) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$

(d) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{0}$

22. Let a, b and c be distinct non- negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is **[2005]**

(a) the Geometric Mean of a and b

(b) the Arithmetic Mean of a and b

(c) equal to zero

(d) the Harmonic Mean of a and b

23. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c},\ \lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non coplanar for **[2004]**
(a) no value of λ
(b) all except one value of λ
(c) all except two values of λ
(d) all values of λ

24. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals **[2004]**
(a) 0 (b) $\lambda\vec{b}$ (c) $\lambda\vec{c}$ (d) $\lambda\vec{a}$

25. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a **[2003]**
(a) parallelogram but not a rhombus
(b) square
(c) rhombus
(d) rectangle.

26. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $(1,a,a^2)$, $(1,b,b^2)$ and $(1,c,c^2)$ are non- coplanar, then the product abc equals **[2003]**
(a) 0 (b) 2 (c) –1 (d) 1

27. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ & $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **[2003]**
(a) $\sqrt{288}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

TOPIC 2 Scalar or Dot Product of two Vectors, Projection of a Vector Along any other Vector, Component of a Vector

28. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is ______. **[NA Sep. 06, 2020 (I)]**

29. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}|=|\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is ______. **[NA Sep. 06, 2020 (II)]**

30. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}|=2, |\vec{b}|=4$ and $|\vec{c}|=4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a}+\vec{b}-\vec{c}|$ is ______. **[NA Sep. 05, 2020 (II)]**

31. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$. Then $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$ is equal to ________. **[NA Sep. 02, 2020 (I)]**

32. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is ______. **[NA Jan. 9, 2020 (I)]**

33. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k}, \vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $\vec{w}=2\hat{i}+\hat{j}+\hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be: **[Jan. 8, 2020 (I)]**
(a) $\frac{7}{6\sqrt{6}}$ (b) $\frac{7}{6\sqrt{3}}$ (c) $\frac{5}{7}$ (d) $\frac{5}{3\sqrt{3}}$

34. A vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ $(\alpha,\beta\in\boldsymbol{R})$ lies in the plane of the vectors, $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then: **[Jan. 7, 2020 (I)]**
(a) $\vec{a}\cdot\hat{i}+3=0$ (b) $\vec{a}\cdot\hat{i}+1=0$
(c) $\vec{a}\cdot\hat{k}+2=0$ (d) $\vec{a}\cdot\hat{k}+4=0$

35. Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}, \vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$ Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is: **[Jan. 10, 2019 (I)]**
(a) $(1, 3, 1)$ (b) $\left(-\frac{1}{2}, 4, 0\right)$
(c) $\left(\frac{1}{2}, 4, -2\right)$ (d) $(1, 5, 1)$

36. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$.
If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to: **[Jan. 09, 2019 (II)]**
(a) $\sqrt{32}$ (b) 6 (c) $\sqrt{22}$ (d) 4

37. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}, -\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+q\hat{j}-4\hat{k}$, then the point (p, q) lies on a line : **[Online April 9, 2016]**

(a) making an obtuse angle with the positive direction of x–axis
(b) parallel to x–axis
(c) parallel to y–axis
(d) making an acute angle with the positive direction of x–axis

38. In a parallelogram ABD, $|\overrightarrow{AB}|=a, |\overrightarrow{AD}|=b$ and $|\overrightarrow{AC}|=c$, then $\overrightarrow{DA}.\overrightarrow{AB}$ has the value : **[Online April 11, 2015]**

(a) $\frac{1}{2}(a^2+b^2+c^2)$ (b) $\frac{1}{2}(a^2-b^2+c^2)$
(c) $\frac{1}{2}(a^2+b^2-c^2)$ (d) $\frac{1}{3}(b^2+c^2-a^2)$

39. If $\hat{x}, \hat{y}$ and $\hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of
$|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2$ **[Online April 12, 2014]**

(a) $\frac{3}{2}$ (b) 3 (c) $3\sqrt{3}$ (d) 6

40. If $|\vec{a}|=2, |\vec{b}|=3$ and $|2\vec{a}-\vec{b}|=5$, then $|2\vec{a}+\vec{b}|$ equals: **[Online April 9, 2014]**

(a) 17 (b) 7 (c) 5 (d) 1

41. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors satisfying $\hat{a}-\sqrt{3}\hat{b}+\hat{c}=\vec{0}$, then the angle between the vectors $\hat{a}$ and $\hat{c}$ is : **[Online April 22, 2013]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

42. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector of the type $\vec{b}+\lambda\vec{c}$ for some scalar λ, whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is : **[Online April 9, 2013]**

(a) $2\hat{i}+\hat{j}+5\hat{k}$ (b) $2\hat{i}+3\hat{j}-3\hat{k}$
(c) $2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+3\hat{j}+3\hat{k}$

43. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB}=\vec{q}, \overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[2012]**

(a) $\vec{r}=3\vec{q}-\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (b) $\vec{r}=-\vec{q}+\frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$
(c) $\vec{r}=\vec{q}-\frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (d) $\vec{r}=-3\vec{q}-\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

44. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[2012]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

45. If $a+b+c=0$, $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is **[Online May 19, 2012]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

46. A unit vector which is perpendicular to the vector $2\hat{i}-\hat{j}+2\hat{k}$ and is coplanar with the vectors $\hat{i}+\hat{j}-\hat{k}$ and $2\hat{i}+2\hat{j}-\hat{k}$ is **[Online May 12, 2012]**

(a) $\frac{2\hat{j}+\hat{k}}{\sqrt{5}}$ (b) $\frac{3\hat{i}+2\hat{j}-2\hat{k}}{\sqrt{17}}$
(c) $\frac{3\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{17}}$ (d) $\frac{2\hat{i}+2\hat{j}-2\hat{k}}{3}$

47. $ABCD$ is parallelogram. The position vectors of A and C are respectively, $3\hat{i}+3\hat{j}+5\hat{k}$ and $\hat{i}-5\hat{j}-5\hat{k}$. If M is the midpoint of the diagonal DB, then the magnitude of the projection of $\overrightarrow{OM}$ on $\overrightarrow{OC}$, where O is the origin, is **[Online May 7, 2012]**

(a) $7\sqrt{51}$ (b) $\frac{7}{\sqrt{50}}$ (c) $7\sqrt{50}$ (d) $\frac{7}{\sqrt{51}}$

48. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu)=$ **[2010]**

(a) $(2,-3)$ (b) $(-2,3)$
(c) $(3,-2)$ (d) $(-3,2)$

49. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **[2008]**

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

50. The values of a, for which points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right angled triangle with $C=\frac{\pi}{2}$ are **[2006]**

(a) 2 and 1 (b) – 2 and – 1
(c) – 2 and 1 (d) 2 and – 1

51. Let $\bar{u},\bar{v},\bar{w}$ be such that $|\bar{u}|=1, |\bar{v}|=2, |\bar{w}|=3$. If the projection $\bar{v}$ along $\bar{u}$ is equal to that of $\bar{w}$ along $\bar{u}$ and $\bar{v}$, $\bar{w}$ are perpendicular to each other then $|\bar{u}-\bar{v}+\bar{w}|$ equals **[2004]**

(a) 14 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 2

52. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$, $|\vec{b}|=2, |\vec{c}|=3$, then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to **[2003]**

(a) 1 (b) 0 (c) –7 (d) 7

53. If $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ thus what will be the value of $|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$ **[2002]**

(a) 25 (b) 50 (c) –25 (d) –50

54. If sdaa $\vec{a},\vec{b},\vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is **[2002]**

(a) 60° (b) 30° (c) 45° (d) 90°

TOPIC 3 Vector or Cross Product of two vectors, Area of a Parallelogram & Triangle, Scalar & Vector Tripple Product

55. If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a}=\hat{i}+\hat{j}+n\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c}=\hat{i}+n\hat{j}+3\hat{k}\,(n\geq 0)$, is 158 cu.units, then: **[Sep. 05, 2020 (I)]**

(a) $\vec{a}\cdot\vec{c}=17$ (b) $\vec{b}\cdot\vec{c}=10$
(c) $n=7$ (d) $n=9$

56. Let x_0 be the point of local maxima of $f(x)=\vec{a}\cdot(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x=x_0$ is : **[Sep. 04, 2020 (I)]**

(a) – 4 (b) – 30 (c) 14 (d) – 22

57. If $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$, then the value of $|\hat{i}\times(\vec{a}\times\hat{i})|^2+|\hat{j}\times(\vec{a}\times\hat{j})|^2+|\hat{k}\times(\vec{a}\times\hat{k})|^2$ is equal to ________________. **[NA Sep. 04, 2020 (II)]**

58. Let $\vec{b}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}, |\vec{b}|=5$, $\vec{b}\cdot\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{b}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to _____. **[NA Jan. 9, 2020 (II)]**

59. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}\cdot\vec{a}=0$, then $\vec{c}\cdot\vec{b}$ is equal to: **[Jan. 8, 2020 (II)]**

(a) $-\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) –1

60. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. if $\lambda=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda,\vec{d})$ is equal to: **[Jan. 7, 2020 (II)]**

(a) $\left(\frac{3}{2},3\vec{a}\times\vec{c}\right)$ (b) $\left(-\frac{3}{2},3\vec{c}\times\vec{b}\right)$

(c) $\left(\frac{3}{2},3\vec{b}\times\vec{c}\right)$ (d) $\left(-\frac{3}{2},3\vec{a}\times\vec{b}\right)$

61. Let $\vec{\alpha}=3\hat{i}+\hat{j}$ and $\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$. If $\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1\times\vec{\beta}_2$ is equal to: **[April 09, 2019 (I)]**

(a) $-3\hat{i}+9\hat{j}+5\hat{k}$ (b) $3\hat{i}-9\hat{j}-5\hat{k}$

(c) $\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$ (d) $\frac{1}{2}(3\hat{i}-9\hat{j}+5\hat{k})$

62. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is : **[April 08, 2019 (I)]**

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{6}$ (c) $3\sqrt{6}$ (d) $\sqrt{\frac{3}{2}}$

63. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if : **[April 08, 2019 (II)]**

(a) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$ (b) $r \ge 5\sqrt{\frac{3}{2}}$

(c) $0 < r \le \sqrt{\frac{3}{2}}$ (d) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$

64. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to : **[Jan. 12, 2019 (II)]**

(a) 30° (b) 90° (c) 60° (d) 45°

65. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to: **[Jan 09, 2019]**

(a) $\frac{19}{2}$ (b) 9 (c) 8 (d) $\frac{17}{2}$

66. If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i} + 7\hat{j} + 8\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is **[Online April 15, 2018]**

(a) $\frac{1}{2}(4\hat{i} + 8\hat{j} + 11\hat{k})$ (b) $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

(c) $\frac{1}{4}(8\hat{i} + 14\hat{j} + 9\hat{k})$ (d) $\frac{1}{3}(6\hat{i} + 11\hat{j} + 15\hat{k})$

67. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Then $|\vec{b}|$ equals? **[Online April 16, 2018]**

(a) $\sqrt{\frac{11}{3}}$ (b) $\frac{\sqrt{11}}{3}$ (c) $\frac{11}{\sqrt{3}}$ (d) $\frac{11}{3}$

68. If $\vec{a}, \vec{b}$, and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$, then $|\vec{a} \times \vec{c}|$ is equal to **[Online April 15, 2018]**

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{15}}{4}$ (c) $\frac{15}{16}$ (d) $\frac{\sqrt{15}}{16}$

69. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to : **[2017]**

(a) $\frac{1}{8}$ (b) $\frac{25}{8}$ (c) 2 (d) 5

70. If the vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\vec{b_1}$, parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{b_2}$, perpendicular to $\vec{a}$, then $\vec{b_1} \times \vec{b_2}$ is equal to : **[Online April 9, 2017]**

(a) $-3\hat{i} + 3\hat{j} - 9\hat{k}$ (b) $6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

(c) $-6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k}$ (d) $3\hat{i} - 3\hat{j} + 9\hat{k}$

71. The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is : **[Online April 8, 2017]**

(a) 26 (b) 65 (c) 20 (d) 52

72. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[2016]**

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

73. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is : **[2015]**

(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$

74. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$, then $2|\vec{c}|$ is equal to : **[Online April 10, 2015]**

(a) $\sqrt{55}$ (b) $\sqrt{37}$ (c) $\sqrt{51}$ (d) $\sqrt{43}$

75. If $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = \lambda [\vec{a}\,\vec{b}\,\vec{c}]^2$ then λ is equal to **[2014]**

(a) 0 (b) 1 (c) 2 (d) 3

76. If $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$, then the magnitude of the projection of $\vec{x} \times \vec{y}$ on $\vec{z}$ is: **[Online April 19, 2014]**

(a) 12 (b) 15 (c) 14 (d) 13

77. If $|\vec{c}|^2 = 60$ and $\vec{c}\times(\hat{i}+2\hat{j}+5\hat{k}) = \vec{0}$, then a value of $\vec{c}.(-7\hat{i}+2\hat{j}+3\hat{k})$ is: **[Online April 11, 2014]**

(a) $4\sqrt{2}$ (b) 12 (c) 24 (d) $12\sqrt{2}$

78. Let $\vec{a} = 2\hat{i}+\hat{j}-2\hat{k}, \vec{b} = \hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\bullet\vec{c} = |\vec{c}|, |\vec{c}-\vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 30°, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ equals: **[Online April 25, 2013]**

(a) $\frac{1}{2}$ (b) $\frac{3\sqrt{3}}{2}$ (c) 3 (d) $\frac{3}{2}$

79. The vector $(\hat{i}\times\vec{a}.\vec{b})\hat{i}+(\hat{j}\times\vec{a}.\vec{b})\hat{j}+(\hat{k}\times\vec{a}.\vec{b})\hat{k}$ is equal to : **[Online April 9, 2013]**

(a) $\vec{b}\times\vec{a}$ (b) $\vec{a}$ (c) $\vec{a}\times\vec{b}$ (d) $\vec{b}$

80. **Statement 1:** The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ lie in the same plane if and only if $\vec{a}.\left(\vec{b}\times\vec{c}\right) = 0$

Statement 2: The vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if $\vec{u}.\vec{v} = 0$ where $\vec{u}\times\vec{v}$ is a vector perpendicular to the plane of $\vec{u}$ and $\vec{v}$. **[Online May 26, 2012]**

(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1.

81. If $\vec{u} = \hat{j}+4\hat{k}$, $\vec{v} = \hat{i}+3\hat{k}$ and $\vec{w} = \cos\theta\hat{i}+\sin\theta\hat{j}$ are vectors in 3-dimensional space, then the maximum possible value of $|\vec{u}\times\vec{v}.\vec{w}|$ is **[Online May 12, 2012]**

(a) $\sqrt{3}$ (b) 5 (c) $\sqrt{14}$ (d) 7

82. **Statement 1:** If the points (1, 2, 2), (2, 1, 2) and (2, 2, z) and (1, 1, 1) are coplanar, then $z = 2$.
Statement 2: If the 4 points P, Q, R and S are coplanar, then the volume of the tetrahedron $PQRS$ is 0. **[Online May 12, 2012]**

(a) Statement 1 is false,, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

83. If $\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$, $\vec{b} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c} = \lambda\hat{i}+\hat{j}+(2\lambda-1)\hat{k}$ are coplanar vectors, then λ is equal to **[Online May 7, 2012]**

(a) 0 (b) -1 (c) 2 (d) 1

84. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to **[2011]**

(a) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (b) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

(c) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (d) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

85. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i}+\hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$, then the value of $(2\vec{a}-\vec{b})\left[(\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b})\right]$ is **[2011]**

(a) –3 (b) 5 (c) 3 (d) –5

86. Let $\vec{a} = \hat{j}-\hat{k}$ and $\vec{c} = \hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is **[2010]**

(a) $2\hat{i}-\hat{j}+2\hat{k}$ (b) $\hat{i}-\hat{j}-2\hat{k}$

(c) $\hat{i}+\hat{j}-2\hat{k}$ (d) $-\hat{i}+\hat{j}-2\hat{k}$

87. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality
$[3\vec{u}\; p\vec{v}\; p\vec{\omega}]-[p\vec{v}\;\vec{\omega}\; q\vec{u}]-[2\vec{\omega}\; q\vec{v}\; q\vec{u}] = 0$
holds for : **[2009]**

(a) exactly two values of (p, q)
(b) more than two but not all values of (p, q)
(c) all values of (p, q)
(d) exactly one value of (p, q)

88. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{i}-\hat{j}+2\hat{k}$ and $\vec{c} = x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals **[2007]**

(a) -4 (b) -2 (c) 0 (d) 1.

89. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times3\hat{v}$ is a unit vector for **[2007]**

(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ

90. If $(\bar{a}\times\bar{b})\times\bar{c} = \bar{a}\times(\bar{b}\times\bar{c})$ where $\bar{a}, \bar{b}$ and $\bar{c}$ are any three vectors such that $\bar{a}.\bar{b}\neq0$, $\bar{b}.\bar{c}\neq0$ then $\bar{a}$ and $\bar{c}$ are **[2006]**

(a) inclined at an angle of $\frac{\pi}{3}$ between them
(b) inclined at an angle of $\frac{\pi}{6}$ between them
(c) perpendicular
(d) parallel

91. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on **[2005]**

(a) only y (b) only x
(c) both x and y (d) neither x nor y

92. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then **[2005]**

$[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}] = [\vec{a}\quad \vec{b}+\vec{c}\quad \vec{b}]$ for

(a) exactly one value of λ
(b) no value of λ
(c) exactly three values of λ
(d) exactly two values of λ

93. For any vector $\vec{a}$, the value of

$(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$ is equal to **[2005]**

(a) $3\vec{a}^2$ (b) $\vec{a}^2$ (c) $2\vec{a}^2$ (d) $4\vec{a}^2$

94. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be non-zero vectors such that $(\bar{a}\times\bar{b})\times\bar{c} = \frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$. If θ is the acute angle between the vectors $\bar{b}$ and $\bar{c}$, then $\sin\theta$ equals **[2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

95. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non- coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals **[2003]**

(a) $3\vec{u}.\vec{v}\times\vec{w}$ (b) 0
(c) $\vec{u}.(\vec{v}\times\vec{w})$ (d) $\vec{u}.\vec{w}\times\vec{v}$.

96. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1) B(2, 1, 3) and C(-1, 1, 2). Then the angle between the faces OAB and ABC will be **[2003]**

(a) 90^o (b) $\cos^{-1}\left(\frac{19}{35}\right)$
(c) $\cos^{-1}\left(\frac{17}{31}\right)$ (d) 30^o

97. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$, then $|\vec{w}.\hat{n}|$ is equal to **[2003]**

(a) 3 (b) 0 (c) 1 (d) 2.

98. If $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c} =$ **[2002]**

(a) abc (b) –1 (c) 0 (d) 2

99. $\vec{a} = 3\hat{i} - 5\hat{j}$ and $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|$ **[2002]**

(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$
(c) $34:39:45$ (d) $39:35:34$

100. If the vectors $\vec{c}$, $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\hat{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handed system then $\vec{c}$ is: **[2002]**

(a) $z\hat{i} - x\hat{k}$ (b) $\vec{0}$
(c) $y\hat{j}$ (d) $-z\hat{i} + x\hat{k}$

101. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}] = 4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] =$ **[2002]**

(a) 16 (b) 64 (c) 4 (d) 8

102. If $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a}\times\vec{b})^2$ is equal to **[2002]**

(a) 48 (b) 16
(c) $\vec{a}$ (d) None of these

TOPIC 4 Scalar Product of Four Vectors, Reciprocal System of Vector, Application of Vectors in Mechanics

103. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is **[2007]**

(a) $\tan^{-1}\frac{bc}{a(c-a)}$ (b) $\tan^{-1}\frac{bc}{a}$
(c) $\tan^{-1}\frac{b}{ac}$ (d) 45°.

104. A body weighing 13 kg is suspended by two strings 5m and 12m long, their other ends being fastened to the extremities of a rod 13m long. If the rod be so held that the body hangs immediately below the middle point, then tensions in the strings are **[2007]**

(a) 5 kg and 12 kg (b) 5 kg and 13 kg
(c) 12 kg and 13 kg (d) 5 kg and 5 kg

105. The resultant of two forces Pn and 3n is a force of 7n. If the direction of 3n force were reversed, the resultant would be $\sqrt{19}\,n$. The value of P is **[2007]**

(a) 3 n (b) 4 n (c) 5 n (d) 6 n.

106. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10 m/s^2$, then the height above the point P from where the body began to fall is **[2006]**

(a) 720m (b) 900m (c) 320m (d) 680m

107. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is **[2006]**

(a) 90° (b) 120° (c) 45° (d) 60°

108. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is: **[2005]**

(a) $2:1$ (b) $3:\sqrt{2}$ (c) $3:2$ (d) $3:2\sqrt{2}$

109. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance **[2005]**

(a) $\dfrac{2H}{A-B}$ (b) $\dfrac{H}{A+B}$

(c) $\dfrac{H}{2(A+B)}$ (d) $\dfrac{H}{A-B}$

110. A particle is projected from a point O with velocity u at an angle of 60° with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by **[2005]**

(a) $\dfrac{u}{3}$ (b) $\dfrac{u}{2}$ (c) $\dfrac{2u}{3}$ (d) $\dfrac{u}{\sqrt{3}}$

111. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to **[2004]**

(a) 1 (b) $4u^2/g^2$

(c) $u^2/2g$ (d) u^2/g

112. A velocity $\dfrac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is **[2004]**

(a) $\dfrac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s (b) $\dfrac{1}{4}(\sqrt{3}-1)$ m/s

(c) $\dfrac{1}{4}$ m/s (d) $\dfrac{1}{8}$ m/s

113. A paticle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5km/hr. If $AB = 12$ km and $BC = 5$ km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively **[2004]**

(a) $\dfrac{13}{9}$ km/h and $\dfrac{17}{9}$ km/h

(b) $\dfrac{13}{4}$ km/h and $\dfrac{17}{4}$ km/h

(c) $\dfrac{17}{9}$ km/h and $\dfrac{13}{9}$ km/h

(d) $\dfrac{17}{4}$ km/h and $\dfrac{13}{4}$ km/h

114. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC are in equilibrium. Then $\vec{P}:\vec{Q}:\vec{R}$ is **[2004]**

(a) $\operatorname{cosec}\dfrac{A}{2}:\operatorname{cosec}\dfrac{B}{2}:\operatorname{cosec}\dfrac{C}{2}$

(b) $\sin\dfrac{A}{2}:\sin\dfrac{B}{2}:\sin\dfrac{C}{2}$

(c) $\sec\dfrac{A}{2}:\sec\dfrac{B}{2}:\sec\dfrac{C}{2}$

(d) $\cos\dfrac{A}{2}:\cos\dfrac{B}{2}:\cos\dfrac{C}{2}$

115. In a right angle ΔABC, $\angle A = 90°$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is **[2004]**

(a) 9 (b) 4 (c) 5 (d) 3

116. With two forces acting at point, the maximum affect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are **[2004]**

(a) $\left(2+\dfrac{1}{2}\sqrt{3}\right)N$ and $\left(2-\dfrac{1}{2}\sqrt{3}\right)N$

(b) $\left(2+\sqrt{3}\right)N$ and $\left(2-\sqrt{3}\right)N$

(c) $\left(2+\dfrac{1}{2}\sqrt{2}\right)N$ and $\left(2-\dfrac{1}{2}\sqrt{2}\right)N$

(d) $\left(2+\sqrt{2}\right)N$ and $\left(2-\sqrt{2}\right)N$

117. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by **[2004]**

(a) 15 (b) 30 (c) 25 (d) 40

118. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in **[2003]**

(a) H.P (b) A.G.P (c) A.P (d) G.P.

119. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time **[2003]**

(a) $\frac{u\cos\alpha}{f}$ (b) $\frac{u\sin\alpha}{f}$ (c) $\frac{f\cos\alpha}{u}$ (d) $u\sin\alpha$

120. Two stones are projected from the top of a cliff h metres high , with the same speed u, so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals **[2003]**

(a) $u\sqrt{\frac{2}{gh}}$ (b) $\sqrt{\frac{2u}{gh}}$ (c) $2g\sqrt{\frac{u}{h}}$ (d) $2h\sqrt{\frac{u}{g}}$

121. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by **[2003]**

(a) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ (b) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$

(c) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$ (d) $\sqrt{2s(f+r)}$

122. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed,then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is **[2003]**

(a) $2:3:1$ (b) $3:1:1$ (c) $2:3:2$ (d) $1:2:3$.

123. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle the moment of the couple thus formed is $\vec{H}$. If instead , the force $\vec{P}$ are turned through an angle α, then the moment of couple becomes **[2003]**

(a) $\vec{H}\sin\alpha-\vec{G}\cos\alpha$ (b) $\vec{G}\sin\alpha-\vec{H}\cos\alpha$

(c) $\vec{H}\sin\alpha+\vec{G}\cos\alpha$ (d) $\vec{G}\sin\alpha+\vec{H}\cos\alpha$.

124. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}-3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is **[2003]**

(a) 50 units (b) 20 units

(c) 30 units (d) 40 units.

125. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are **[2002]**

(a) $T=w\cos\theta \quad R=w\tan\theta$

(b) $T=2w\cos\theta \quad R=w$

(c) $T=w \quad R=w\sin\theta$

(d) $T=w\sin\theta \quad R=w\cot\theta$

126. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are **[2002]**

(a) 13, 5 (b) 12, 6 (c) 14, 4 (d) 11, 7

Hints & Solutions

1. **(a)** $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right) = k$

$$a=\frac{k}{\cos\theta},\ b=\frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)},\ c=\frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

$$ab+bc+ca = k^2\frac{\left[\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right]}{\cos\left(\theta+\frac{4\pi}{3}\right)\cdot\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$=k^2\left[\frac{\cos\theta+2\cos(\theta+\pi)\cdot\cos\left(\frac{\pi}{3}\right)}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{\cos\theta-2\cos\theta\cdot\frac{1}{2}}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)}\right]=0$$

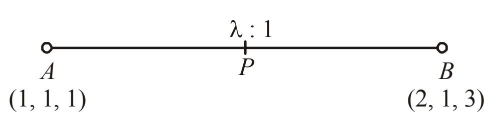
$$\cos\phi=\frac{(a\hat{i}+b\hat{j}+c\hat{k})\cdot(b\hat{i}+c\hat{j}+a\hat{k})}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{b^2+c^2+a^2}}$$
$$=ab+bc+ca=0$$

$$\phi=\frac{\pi}{2}$$

2. **(0.8)**

Let position vector of A and B be $\vec{a}$ and $\vec{b}$ respectively.

$\therefore$ Position vector of P is $\overrightarrow{OP}=\dfrac{\lambda\vec{b}+\vec{a}}{\lambda+1}$



Given $\overrightarrow{OB}\cdot\overrightarrow{OP}-3|\overrightarrow{OA}\times\overrightarrow{OP}|^2=6$

$$\Rightarrow \vec{b}\cdot\left(\frac{\lambda\vec{b}+\vec{a}}{\lambda+1}\right)-3\left|\vec{a}\times\frac{\lambda\vec{b}+\vec{a}}{\lambda+1}\right|^2=6$$

$$\Rightarrow\frac{\vec{a}\cdot\vec{b}+\lambda|\vec{b}|^2}{\lambda+1}-\frac{3\lambda^2}{(\lambda+1)^2}|\vec{a}\times\vec{b}|^2=6$$

$(\because \vec{a}\times\vec{b}=2\hat{i}-\hat{j}-\hat{k}$ and $\vec{a}\cdot\vec{b}=6)$

$$\Rightarrow\frac{6+14\lambda}{\lambda+1}-\frac{18\lambda^2}{(\lambda+1)^2}=6$$

$$\Rightarrow 6+\frac{8\lambda}{\lambda+1}-\frac{18\lambda^2}{(\lambda+1)^2}=6$$

Let $\dfrac{\lambda}{\lambda+1}=t$

$\Rightarrow 18t^2-8t=0\Rightarrow 2t(9t-4)=0$

$\Rightarrow t=0,\ \dfrac{4}{9}$

$\therefore \dfrac{\lambda}{\lambda+1}=\dfrac{4}{9}\Rightarrow\lambda=\dfrac{4}{5}=0.8.$

3. **(1.0)** $\begin{vmatrix}a+1 & a & a\\ a & a+1 & a\\ a & a & a+1\end{vmatrix}=0$

$\Rightarrow\ 3a+1=0\ \Rightarrow\ a=-\dfrac{1}{3}$

The given vectors

$$\vec{p}=\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}-\frac{1}{3}\hat{k}=\frac{1}{3}(2\hat{i}-\hat{j}-\hat{k})$$

$$\vec{q}=\frac{1}{3}(-\hat{i}+2\hat{j}-\hat{k})$$

$$\vec{r}=\frac{1}{3}(-\hat{i}-\hat{j}+2\hat{k})$$

Now, $\vec{p}.\vec{q}=\dfrac{1}{9}(-2-2+1)=-\dfrac{1}{3}$

$$\vec{r}\times\vec{q}=\frac{1}{9}\begin{vmatrix}i & j & k\\ -1 & 2 & -1\\ -1 & -1 & 2\end{vmatrix}$$

$$=\frac{1}{9}(i(4-1)-j(-2-1)+k(1+2))$$

$$=\frac{1}{9}(3i+3j+3k)=\frac{i+j+k}{3}$$

$|\vec{r}\times\vec{q}|=\dfrac{1}{3}\sqrt{3}\quad\Rightarrow\quad|\vec{r}\times\vec{q}|^2=\dfrac{1}{3}$

$3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$

$\Rightarrow\ 3.\dfrac{1}{9}-\lambda.\dfrac{1}{3}=0\ \Rightarrow\ \lambda=1$

4. **(b)** Let vector be $\lambda[(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})]$

Given, $a=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

$\therefore$ $\vec{a}+\vec{b}=4\hat{i}+4\hat{j}$ and $\vec{a}-\vec{b}=2\hat{i}+4\hat{k}$

$\therefore$ vector $=\lambda[(4\hat{i}+4\hat{j})\times(2\hat{i}+4\hat{k})]$

$=\lambda[16\hat{i}-16\hat{j}-8\hat{k}]=8\lambda[2\hat{i}-2\hat{j}-\hat{k}]$

Given that magnitude of the vector is 12.

$\therefore$ $12=8|\lambda|\sqrt{4+4+1}\Rightarrow|\lambda|=\dfrac{1}{2}$

$\therefore$ required vector is $\pm4\ (2\hat{i}-2\hat{j}-\hat{k})$

5. **(b)** Volume of the parallelepiped is,

$$V=\left\|\begin{matrix}1 & \lambda & 1\\ 0 & 1 & \lambda\\ \lambda & 0 & 1\end{matrix}\right\|=|1(1)+\lambda(\lambda^2)+1(-\lambda)|$$

$=|\lambda^3-\lambda+1|$

Let $f(x)=x^3-x+1$

On differentiating, $f'(x)=3x^2-1$

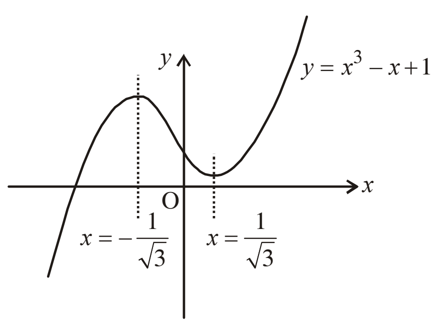
Now, $f'(x)=0$

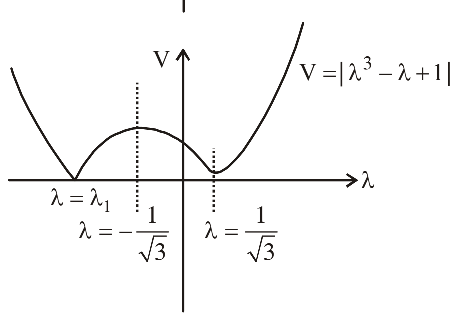
$\Rightarrow x=\pm\dfrac{1}{\sqrt{3}}$

and $f''(x)=6x$

Since, $f''\left(\dfrac{1}{\sqrt{3}}\right)>0$

$\therefore$ $x=\dfrac{1}{\sqrt{3}}$ is point of local minima.





For $\lambda=\lambda_1$, volume of parallelopiped is zero.

$\therefore$ vectors are coplanar.

6. **(b)** Let, three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar,

then $[\vec{a},\vec{b},\vec{c}]=0$

$$\Rightarrow\begin{vmatrix}\alpha & 1 & 3\\ 2 & 1 & -\alpha\\ \alpha & -2 & 3\end{vmatrix}=0\Rightarrow\alpha^2+6=0$$

$\because$ no real value of 'α' exist.

$\therefore$ set S is an empty set.

7. **(d)** Let $\cos\alpha$, $\cos\beta$, $\cos\gamma$ be direction cosines of a.

$\therefore$ $\cos\alpha=\cos\dfrac{\pi}{3},\ \cos\beta=\cos\dfrac{\pi}{4}$ and $\cos\gamma=\cos\theta$

$\Rightarrow\cos^2\dfrac{\pi}{3}+\cos^2\dfrac{\pi}{4}+\cos^2\theta=1\Rightarrow\cos^2\theta=\dfrac{1}{4}$

$\Rightarrow\cos\theta=\pm\dfrac{1}{2}\Rightarrow\theta=\dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

8. **(a)** $\because$ Three vectors $(\mu\hat{i}+\hat{j}+\hat{k}),(\hat{i}+\mu\hat{j}+\hat{k}$ and $(\hat{i}+\hat{j}+\mu\hat{k})$ are copalnar.

$$\therefore\begin{vmatrix}\mu & 1 & 1\\ 1 & \mu & 1\\ 1 & 1 & \mu\end{vmatrix}=0$$

$\Rightarrow$ $\mu(\mu^2-1)+1-\mu+1-\mu=0$

$\Rightarrow$ $(1-\mu)[2-\mu(\mu+1)]=0$

$\Rightarrow$ $(1-\mu)[\mu^2+\mu-2]=0$

$\Rightarrow$ $\mu=1,-2$

Therefore, sum of all real values $=1-2=-1$

9. **(d)** $\because$ $\overline{a},\overline{b}$ and $\overline{c}$ are coplanar

$$\therefore\begin{vmatrix}1 & 2 & 4\\ 1 & \lambda & 4\\ 2 & 4 & (\lambda^2-1)\end{vmatrix}=0$$

$\Rightarrow$ $\lambda^3-\lambda-16+2(8-\lambda^2+1)+4(4-2\lambda)=0$

$\Rightarrow$ $\lambda^3-2\lambda^2-9\lambda+18=0$

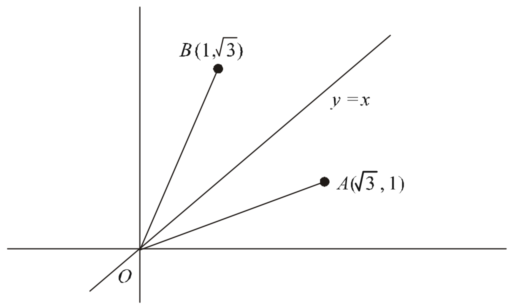
i.e., $(\lambda-2)(\lambda-3)(\lambda+3)=0$

For $\lambda=2$, $\vec{c}=2\hat{i}+4\hat{j}+3\hat{k}$

$$\therefore\ \vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 4\\ 2 & 4 & 3\end{vmatrix}=-10\hat{i}+5\hat{j}$$

For $\lambda=3$ or -3, $\overline{c}=2\overline{a}\Rightarrow\overline{a}\times\overline{c}=0$ (Rejected)

10. **(d)** Since, the angle bisector of acute angle between OA and OB would be $y = x$



Since, the distance of C from bisector $= \frac{3}{\sqrt{2}}$

$$\Rightarrow \left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}} = 2\beta = \pm 3 + 1$$

$\beta = 2$ or $\beta = -1$

Hence, the sum of all possible value of $\beta = 2 + (-1) = 1$

11. **(a)** Let $\vec{\alpha}$ and $\vec{\beta}$ are collinear for same k

i.e., $\vec{\alpha} = k\vec{\beta}$

$(\lambda - 2)\ \vec{a} + \vec{b} = k((4\lambda - 2)\ \vec{a} + 3\vec{b})$

$(\lambda - 2)\ \vec{a} + \vec{b} = k(4\lambda - 2)\ \vec{a} + 3k\vec{b}$

$(\lambda - 2 - k(4\lambda - 2))\ \vec{a} + \vec{b}\ (1 - 3k) = 0$

But $\vec{a}$ and $\vec{b}$ are non-collinear, then

$\lambda - 2 - k(4\lambda - 2) = 0,\ 1 - 3k = 0$

$\Rightarrow\ k = \frac{1}{3}$ and $\lambda - 2 - \frac{1}{3}(4\lambda - 2) = 0$

$3\lambda - 6 - 4\lambda + 2 = 0$

$\lambda = -4$

12. **(d)** $\because \vec{u}, \vec{a}\ \&\ \vec{b}$ are coplanar

$\therefore \vec{u} = \lambda(\vec{a} \times \vec{b}) \times \vec{a} \quad = \lambda\{\vec{a}^2.\vec{b} - (\vec{a}.\vec{b})\vec{a}\}$

$= \lambda\{-4\hat{i} + 8\hat{j} + 16\hat{k}\} = \lambda'\{-\hat{i} + 2\hat{j} + 4\hat{k}\}.$

Also, $\vec{u}.\vec{b} = 24 \ \Rightarrow \lambda' = 4$

$\therefore \quad \vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow \quad |\vec{u}|^2 = 336$

13. **(c)** Position vector of centriod $\vec{G} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$

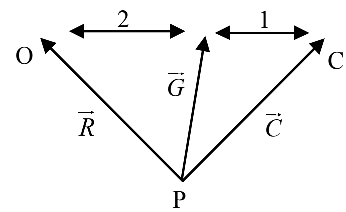
Position vector of circum centre $\vec{C} = \frac{\vec{a} + \vec{b} + \vec{c}}{4}$

$$\vec{G} = \frac{2\vec{C} + \vec{r}}{3}$$

$3\vec{G} = 2\vec{C} + \vec{r}$

$$\vec{r} = 3\vec{G} - 2\vec{C} = (\vec{a} + \vec{b} + \vec{c}) - 2\left(\frac{\vec{a} + \vec{b} + \vec{c}}{4}\right)$$

$$= \frac{\vec{a} + \vec{b} + \vec{c}}{2}$$

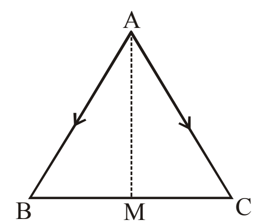


14. **(c)** We have,

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0 \Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

Let M be mid-point of BC

Now, $\overrightarrow{BM} = \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2} \quad \left(\because \quad \overrightarrow{BM} = \frac{\overrightarrow{BC}}{2}\right)$



Also, we have

$\overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA} = 0$

$\Rightarrow \quad \overrightarrow{AB} + \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2} = \overrightarrow{AM}$

$\Rightarrow \quad 1\,\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow \quad \left|\overrightarrow{AM}\right| = \sqrt{33}$

15. **(b)** Since, $\vec{u}$ and $\vec{v}$ are collinear, therefore $k\vec{u} + \vec{v} = 0$

$\Rightarrow [k(\alpha - 2) + 2 + 3\alpha]\ \vec{a}\ + (k - 3)\ \vec{b}\ = 0$ ...(i)

Since $\vec{a}$ and $\vec{b}$ are non-collinear, then for some constant m and n,

$m\vec{a} + n\vec{b} = 0 \ \Rightarrow\ m = 0, n = 0$

Hence from equation (i)

$k - 3 = 0 \ \Rightarrow\ k = 3$

And $k(\alpha - 2) + 2 + 3\alpha = 0$

$\Rightarrow 3(\alpha - 2) + 2 + 3\alpha = 0 \ \Rightarrow\ \alpha = \frac{2}{3}$

16. **(c)** Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, and $\vec{c} = r\hat{i} + \hat{j} + (2r-1)\hat{k}$

Since, $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$ therefore, $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

$$\therefore \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ r & 1 & 2r-1 \end{vmatrix} = 0$$

$\Rightarrow 1\,(6r - 3 + 1) + 2\,(4r - 2 + r) + 3\,(2 - 3r) = 0$

$\Rightarrow 6r - 2 + 10r - 4 + 6 - 9r = 0$

$\Rightarrow r = 0$

17. **(c)** As per question

$\vec{a} + 3\vec{b} = \lambda\vec{c}$ ....(i)

$\vec{b} + 2\vec{c} = \mu\vec{a}$ ....(ii)

On solving equations (i) and (ii)

$(1+3\mu)\vec{a} - (\lambda+6)\vec{c} = 0$

As $\vec{a}$ and $\vec{c}$ are non collinear,

$\therefore$ $1 + 3\mu = 0$ and $\lambda + 6 = 0$

From (i), $\vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$

18. **(d)** The given vectors are coplanar then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$\Rightarrow$ $p(qr-1) + 1(1-r) + 1(1-q) = 0$

$\Rightarrow$ $pqr - p + 1 - r + 1 - q = 0$

$\Rightarrow$ $pqr - (p+q+r) = -2$

19. **(d)** $\because$ $\vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$

$\therefore$ $\vec{a} = \vec{b} + \lambda\vec{c}$

$\Rightarrow$ $\alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \hat{i} + \hat{j} + \lambda(\hat{j} + \hat{k})$

$\Rightarrow$ $\alpha = 1, 2 = 1 + \lambda, \beta = \lambda$

$\Rightarrow$ $\alpha = 1, \beta = 1$

ALTERNATE SOLUTION

$\because$ $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$.

$\therefore$ $\vec{a} = \lambda(\hat{b} + \hat{c})$

$\Rightarrow \alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \dfrac{\lambda(\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{2}}$

$\Rightarrow \alpha = \dfrac{\lambda}{\sqrt{2}}, \lambda = \sqrt{2}, \beta = \dfrac{\lambda}{\sqrt{2}}$

$\Rightarrow \alpha = \beta = 1$

20. **(d)** If we consider unit vectors $\hat{i}$ and $\hat{j}$ in the direction AB and AC respectively and its magnitude $\dfrac{1}{AB}$ and $\dfrac{1}{AC}$ respectively, then as per quesiton, forces along AB and AC respectively are

$\left(\dfrac{1}{AB}\right)\hat{i}$ and $\left(\dfrac{1}{AC}\right)\hat{j}$

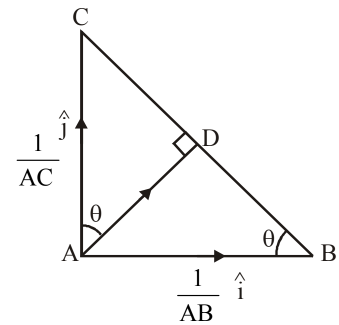
$\therefore$ Their resultant along $AD = \left(\dfrac{1}{AB}\right)i + \left(\dfrac{1}{AC}\right)j$

$\therefore$ Magnitude of resultant is

$$= \sqrt{\left(\frac{1}{AB}\right)^2 + \left(\frac{1}{AC}\right)^2} = \sqrt{\frac{AC^2 + AB^2}{AB^2 + AC^2}}$$

$[\because AC^2 + AB^2 = BC^2]$

$$= \frac{BC}{AB.AC}$$



$\because$ $\Delta ABC \sim \Delta DBA$

$\Rightarrow \dfrac{BC}{AB} = \dfrac{AC}{AD} \Rightarrow \dfrac{BC}{AB \times AC} = \dfrac{1}{AD}$

$\therefore$ The required magnitude of resultant becomes $\dfrac{1}{AD}$.

21. **(a)** $\overrightarrow{PA} + \overrightarrow{AP} = 0$ and $\overrightarrow{PC} + \overrightarrow{CP} = 0$

$\Rightarrow$ $\overrightarrow{PA} + \overrightarrow{AC} + \overrightarrow{CP} = 0$ .... (i)

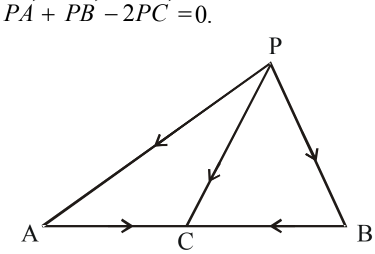
Similarly, $\overrightarrow{PB} + \overrightarrow{BC} + \overrightarrow{CP} = 0$ .... (ii)

Adding eqn. (i) and (ii), we get

$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{AC} + \overrightarrow{BC} + 2\overrightarrow{CP} = 0.$

Since $\overrightarrow{AC} = -\overrightarrow{BC}$ & $\overrightarrow{CP} = -\overrightarrow{PC}$

$\Rightarrow$ $\overrightarrow{PA} + \overrightarrow{PB} - 2\overrightarrow{PC} = 0.$



22. **(a)** Vector $a\vec{i} + a\vec{j} + c\vec{k}$, $\vec{i} + \vec{k}$ and $c\vec{i} + c\vec{j} + b\vec{k}$ are coplanar

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

$\therefore c$ is G.M. of a and b.

23. **(c)** If vectors $\vec{a}+2\vec{b}+3\vec{c}, \lambda\vec{b}+4\vec{c},$ and $(2\lambda-1)\vec{c}$ are coplanar then $\begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & 2\lambda-1\end{vmatrix}=0$

$\Rightarrow \lambda(2\lambda-1)=0 \Rightarrow \lambda=0$ or $\frac{1}{2}$

$\therefore$ Forces are noncoplanar for all λ, except $\lambda=0, \frac{1}{2}$

24. **(c)** Given that $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$

Let $\vec{a}+2\vec{b}=t\vec{c}$ and $\vec{b}+3\vec{c}=s\,\vec{a}$, where t and s are scalars

$\therefore\ \vec{a}+2\vec{b}+6\vec{c}=t\vec{c}+6\vec{c}$

$=(t+6)\vec{c}$ [using $\vec{a}+2\vec{b}=t\vec{c}$]

$=\lambda\vec{c}$, where $\lambda=t+6$

25. **(none)** Given that

$A=(7,-4,7), B=(1,-6,10), C=(-1,-3,4)$ and $D=(5,-1,5)$

$\therefore\ AB=\sqrt{(7-1)^2+(-4+6)^2+(7-10)^2}$

$=\sqrt{36+4+9}=7$

Similarly, $BC=7, CD=\sqrt{41},\ DA=\sqrt{17}$

$\therefore$ None of the options is satisfied.

26. **(c)** Given $\begin{vmatrix}a & a^2 & 1+a^3\\ b & b^2 & 1+b^3\\ c & c^2 & 1+c^3\end{vmatrix}=0$

$\Rightarrow \begin{vmatrix}a & a^2 & 1\\ b & b^2 & 1\\ c & c^2 & 1\end{vmatrix}+\begin{vmatrix}a & a^2 & a^3\\ b & b^2 & b^3\\ c & c^2 & c^3\end{vmatrix}=0$

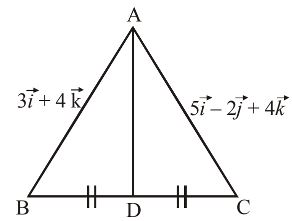
$\Rightarrow (1+abc)\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=0$

Given that $(1,a,a^2),(1,b,b^2)$ and $(1,c,c^2)$ are non-coplanar

$\therefore\ \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\neq 0$ (*given condition*)

$\therefore\ 1+abc=0 \Rightarrow abc=-1$

27. **(d)**



Given that AD is median of ΔABC.

$\therefore\ \overrightarrow{AD}=\frac{(3+5)i+(0-2)j+(4+4)k}{2}=4i-j+4k$

$\left|\overrightarrow{AD}\right|=\sqrt{16+16+1}=\sqrt{33}$

28. **(4)**

Let angle between $\vec{a}$ and $\vec{b}$ be θ.

$|\vec{a}+\vec{b}|=\sqrt{1+1+2\cos\theta}=2\left|\cos\frac{\theta}{2}\right|$ $[\because |a|=|b|=1]$

Similarly, $|\vec{a}-\vec{b}|=2\left|\sin\frac{\theta}{2}\right|$

So, $\sqrt{3}\,|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|=2\left[\sqrt{3}\left|\cos\frac{\theta}{2}\right|+\left|\sin\frac{\theta}{2}\right|\right]$

$\because$ Maximum value of $(a\cos\theta+b\sin\theta)=\sqrt{a^2+b^2}$

$\therefore$ Maximum value $=2\sqrt{(\sqrt{3})^2+(1)^2}=4.$

29. **(1.00)**

$\because |\vec{x}+\vec{y}|=|\vec{x}|$

Squaring both sides we get

$|\vec{x}|^2+2\vec{x}.\vec{y}+|\vec{y}|^2=|\vec{x}|^2$

$\Rightarrow 2\vec{x}.\vec{y}+\vec{y}\cdot\vec{y}=0$ ...(i)

Also $2\vec{x}+\lambda\vec{y}$ and $\vec{y}$ are perpendicular

$\therefore 2\vec{x}\cdot\vec{y}+\lambda\vec{y}\cdot\vec{y}=0$ ...(ii)

Comparing (i) and (ii), $\lambda=1$

30. **(6.00)**

$\because$ Projection of $\vec{b}$ on $\vec{a}$ = Projection of $\vec{c}$ on $\vec{a}$

$\therefore \vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$

Given, $\vec{b}\cdot\vec{c}=0$

$\because |\vec{a}+\vec{b}-\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{c}-2\vec{a}\cdot\vec{c}$
$=4+16+16=36.$

$\Rightarrow |\vec{a}+\vec{b}-\vec{c}|^2=6$

31. **(2)**

$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=-2$

Now, $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$

$=2|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2+4(\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c})=2$

32. **(8)** Let $P(1,-1,3), Q(2,-4,11), R(-1,2,3)$ and $S(3,-2,10)$

Then, $\overrightarrow{PQ}=\hat{i}-3\hat{j}+8\hat{k}$

Projection of $\overrightarrow{PQ}$ on $\overrightarrow{RS}$

$$=\frac{\overrightarrow{PQ}.\overrightarrow{RS}}{|\overrightarrow{RS}|}=\frac{4+12+56}{\sqrt{(4)^2+(4)^2+(7)^2}}=8$$

33. **(b)** It is given that $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k},\ \ \vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $w=2\hat{i}+\hat{j}+\hat{k}$

Volume of parallelopiped $=[\vec{u}\ .\ \vec{v}\ .\ \vec{w}]$

$$\Rightarrow\quad \pm1=\begin{vmatrix}1&1&\lambda\\1&1&3\\2&1&1\end{vmatrix}\Rightarrow-\lambda+3=\pm1\Rightarrow\lambda=2 \text{ or } \lambda=4$$

For $\lambda=2$

$$\cos\theta=\frac{2+1+2}{\sqrt{6}\sqrt{6}}=\frac{5}{6}$$

For $\lambda=4$

$$\cos\theta=\frac{2+1+4}{\sqrt{6}\sqrt{18}}=\frac{7}{6\sqrt{3}}$$

34. **(c)** Angle bisector between $\vec{b}$ and $\vec{c}$ can be

$\vec{a}=\lambda(\hat{b}+\hat{c})$ or $\vec{a}=\mu(\hat{b}-\hat{c})$

$$\text{If } \vec{a}=\lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}+\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$$

$$=\frac{\lambda}{3\sqrt{2}}[3\hat{i}+3\hat{j}+\hat{i}-\hat{j}+4\hat{k}]$$

$$=\frac{\lambda}{3\sqrt{2}}[4\hat{i}+2\hat{j}+4\hat{k}]$$

Compare with $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{2\lambda}{3\sqrt{2}}=2\quad\Rightarrow\quad\lambda=3\sqrt{2}$$

$\vec{a}=4\hat{i}+2\hat{j}+4\hat{k}$

Not satisfy any option

$$\text{Now consider } \vec{a}=\mu\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}-\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$$

$$\vec{a}=\frac{\mu}{3\sqrt{2}}(3\hat{i}+3\hat{j}-\hat{i}+\hat{j}-4\hat{k})$$

$$=\frac{\mu}{3\sqrt{2}}(2\hat{i}+4\hat{j}-4\hat{k})$$

Compare with $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{4\mu}{3\sqrt{2}}=2\quad\Rightarrow\quad\mu=\frac{3\sqrt{2}}{2}$$

$\vec{a}=\hat{i}+2\hat{j}-2\hat{k}$

$\therefore\quad \vec{a}\,.\,\vec{k}+2=\left(\hat{i}+2\hat{j}-2\hat{k}\right)\cdot\hat{k}+2$

$=-2+2=0$

35. **(b)** $\because\ \vec{b}=2\vec{a}$

$\therefore\quad 4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$\therefore\quad 3-\lambda_2=2\lambda_1$ ...(1)

$\because\ \vec{a}$ is perpendicular to $\vec{c}$

$\therefore\quad \vec{a}\cdot\vec{c}=0$

$\Rightarrow\quad 6+6\lambda_1+3(\lambda_3-1)=0$

$\Rightarrow\quad 2+2\lambda_1+\lambda_3-1=0$

$\Rightarrow\quad \lambda_3=-2\lambda_1-1$ ...(2)

Since $\left(\frac{-1}{2},4,0\right)$ satisfies equation (1) and (2). Hence, one of possible value of

$$\lambda_1=-\frac{1}{2},\ \lambda_2=4 \text{ and } \lambda_3=0$$

36. **(b)** Projection of $\vec{b}$ on $\vec{a}$ = $\dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\dfrac{b_1+b_2+2}{4}$

According to question $\dfrac{b_1+b_2+2}{2}=\sqrt{1+1+2}=2$

$\Rightarrow\quad b_1+b_2=2$ ...(1)

Since, $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$.

Hence, $\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}=0$

$\Rightarrow\quad 8+5b_1+b_2+2=0$ ...(2)

From (1) and (2),

$b_1=-3,\ b_2=5$

$\Rightarrow\quad \vec{b}=-3\cdot\hat{i}+5\hat{j}+\sqrt{2}\hat{k}$

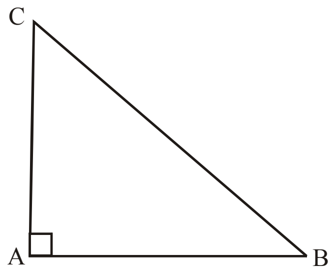
$|\vec{b}|=\sqrt{9+25+2}=6$

37. **(d)** $\overline{AB}=-4\hat{i}+2\hat{j}+(p+1)\hat{k}$

$\overline{AC}=2\hat{i}+(q-1)\hat{j}-3\hat{k}$

$\overline{AB}\perp\overline{AC}$

$\Rightarrow \quad \overline{AB}.\overline{AC} = 0$



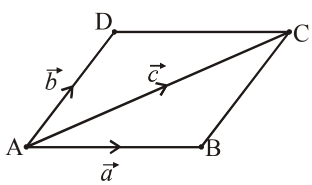
$-8 + 2(q-1) - 3(p+1) = 0$
$3p - 2q + 13 = 0$
(p, q) lies on $3x - 2y + 13 = 0$

slope $= \dfrac{3}{2}$

$\therefore$ Acute angle with x-axis

38. **(c)** Let $|\overrightarrow{AB}| = a$, $|\overrightarrow{AD}| = b$ and $|\overrightarrow{AC}| = c$

We have $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$



On squaring both the side, we get

$|\overrightarrow{AB}|^2 + |\overrightarrow{AD}|^2 + 2\,\overrightarrow{AB}.\overrightarrow{AD} = |\overrightarrow{AC}|^2$

$\Rightarrow a^2 + b^2 + 2\,\overrightarrow{AB}.(-\overrightarrow{DA}) = c^2$

$\Rightarrow 2\,\overrightarrow{AB}.\overrightarrow{DA} = a^2 + b^2 - c^2$

$\Rightarrow \overrightarrow{DA}.\overrightarrow{AB} = \dfrac{1}{2}(a^2 + b^2 - c^2)$

39. **(b)** $(\hat{x} + \hat{y} + \hat{z})^2 \geq 0$

$\Rightarrow \quad 3 + 2\Sigma\,\hat{x}.\hat{y} \geq 0$

$\Rightarrow \quad 2\Sigma\,\hat{x}.\hat{y} \geq -3$

Now, $|\hat{x} + \hat{y}|^2 + |\hat{y} + \hat{z}|^2 + |\hat{z} + \hat{x}|^2$

$= 6 + 2\Sigma\,\hat{x}.\hat{y} \geq 6 + (-3)$

$\Rightarrow \quad |\hat{x} + \hat{y}|^2 + |\hat{y} + \hat{z}|^2 + |\hat{z} + \hat{x}|^2 \geq 3$

40. **(c)** Given $|2\vec{a} - \vec{b}| = 5$

$\sqrt{(2|\vec{a}|)^2 + |\vec{b}|^2 - 2\times|2\vec{a}||\vec{b}|\cos\theta} = 5$

Putting values of $|\vec{a}|$ and $|\vec{b}|$, we get

$(2\times 2)^2 + (3)^2 - 24\cos\theta = 25$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \dfrac{\pi}{2}$

$|2\vec{a} + \vec{b}| = \sqrt{16 + 9 + 24\cos\theta} = \sqrt{25} = 5$

41. **(b)** Let angle between $\hat{a}$ and $\hat{c}$ be θ.

Now, $\hat{a} - \sqrt{3}\,\hat{b} + \hat{c} = \vec{0}$

$\Rightarrow (\hat{a} + \hat{c}) = \sqrt{3}\,\hat{b}$

$\Rightarrow (\hat{a} + \hat{c}).(\hat{a} + \hat{c}) = 3(\hat{b}.\hat{b})$

$\Rightarrow \hat{a}.\hat{a} + \hat{a}.\hat{c} + \hat{c}.\hat{a} + \hat{c}.\hat{c} = 3\times 1$

$\Rightarrow 1 + 2\cos\theta + 1 = 3$

$\Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

42. **(b)** Let $\vec{d} = \vec{b} + \lambda\,\vec{c}$

$\therefore \vec{d} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

$= (1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}$

If θ be the angle between $\vec{d}$ and $\vec{a}$, then projection of $\vec{d}$ or $(\vec{b} + \lambda\,\vec{c})$ on $\vec{a}$

$= |\vec{d}|\cos\theta = |\vec{d}|\left(\dfrac{\vec{d}.\vec{a}}{|\vec{d}||\vec{a}|}\right) = \dfrac{\vec{d}.\vec{a}}{|\vec{a}|}$

$= \dfrac{2(\lambda+1) - (\lambda+2) - (2\lambda+1)}{\sqrt{4+1+1}} = \dfrac{-\lambda-1}{\sqrt{6}}$

But projection of $\vec{d}$ on $\vec{a} = \sqrt{\dfrac{2}{3}}$

$\therefore \; -\dfrac{\lambda+1}{\sqrt{6}} = \sqrt{\dfrac{2}{3}} \Rightarrow \dfrac{\lambda^2 + 2\lambda + 1}{6} = \dfrac{2}{3}$

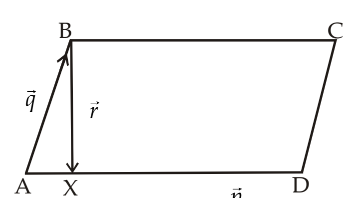
$\Rightarrow \lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda^2 + 3\lambda - \lambda - 3 = 0$

$\Rightarrow \lambda(\lambda+3) - 1(\lambda+3) = 0, \Rightarrow \lambda = 1, -3$

when $\lambda = 1$, then $\vec{b} + \lambda\,\vec{c} = 2\hat{i} + 3\hat{j} - 3\hat{k}$

when $\lambda = -3$, then $\vec{b} + \lambda\,\vec{c} = -2\hat{i} - \hat{j} + 5\hat{k}$

43. **(b)** Let $ABCD$ be a parallelogram such that $\overline{AB} = \vec{q}$, $\overline{AD} = \vec{p}$ and $\angle BAD$ be an acute angle.
We have



$\overline{AX} = \left(\dfrac{\vec{p}.\vec{q}}{|\vec{p}|}\right)\left(\dfrac{\vec{p}}{|\vec{p}|}\right) = \dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

From triangle law

Let $\vec{r} = \overline{BX} = \overline{BA} + \overline{AX} = -\vec{q} + \dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

44. **(c)** Given that $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ and $|\hat{a}| = |\hat{b}| = 1$

Since $\vec{c}$ and $\vec{d}$ are perpendicular to each other

$\therefore \quad \vec{c}\cdot\vec{d} = 0$

$\Rightarrow \quad (\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0$

$\Rightarrow \quad 5 + 6\hat{a}\cdot\hat{b} - 8 = 0$

$\Rightarrow \quad \hat{a}\cdot\hat{b} = \dfrac{1}{2} \Rightarrow \sin\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

45. **(a)** Let $a + b + c = 0 \Rightarrow (a + b) = -c$

$\Rightarrow \quad (a + b)^2 = c^2$

$\Rightarrow \quad a^2 + b^2 + 2a.b = c^2$

$\Rightarrow \quad 9 + 25 + 2.3.5\cos\theta = 49$

$\left(\because \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5 \text{ and } \left|\vec{c}\right| = 7\right)$

$\therefore \; \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

46. **(d)** Let $x\hat{i} + y\hat{j} + z\hat{k}$ be the required unit vector.

Since $\hat{a}$ is perpendicular to $(2\hat{i} - \hat{j} + 2\hat{k})$.

$\therefore \quad 2x - y + 2z = 0$ .... (i)

Since vector $x\hat{i} + y\hat{j} + z\hat{k}$ is coplanar with the vector $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} + 2\hat{j} - \hat{k}$.

$\therefore \quad x\hat{i} + y\hat{j} + z\hat{k} = p\,(\hat{i} + \hat{j} - \hat{k}) + q\,(2\hat{i} + 2\hat{j} - \hat{k})$,

where p and q are some scalars.

$\Rightarrow \quad x\hat{i} + y\hat{j} + z\hat{k} = (p + 2q)\hat{i} + (p + 2q)\hat{j} - (p + q)\hat{k}$

$\Rightarrow \quad x = p + 2q,\; y = p + 2q,\; z = -p - q$

Now from equation (i),

$2p + 4q - p - 2q - 2p - 2q = 0$

$\Rightarrow \quad -p = 0 \Rightarrow p = 0$

$\therefore \quad x = 2q,\; y = 2q,\; z = -q$

Since vector $x\hat{i} + y\hat{j} + z\hat{k}$ is a unit vector, therefore

$|x\hat{i} + y\hat{j} + z\hat{k}| = 1$

$\Rightarrow \quad \sqrt{x^2 + y^2 + z^2} = 1$

$\Rightarrow \quad x^2 + y^2 + z^2 = 1$

$\Rightarrow \quad 4q^2 + 4q^2 + q^2 = 1$

$\Rightarrow \quad 9q^2 = 1 \Rightarrow q = \pm\dfrac{1}{3}$

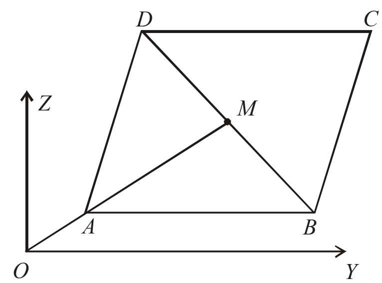
When $q = \dfrac{1}{3}$, then $x = \dfrac{2}{3}, y = \dfrac{2}{3}, z = -\dfrac{1}{3}$

When $q = -\dfrac{1}{3}$, then $x = -\dfrac{2}{3}, y = -\dfrac{2}{3}, z = \dfrac{1}{3}$

Here required unit vector is $\dfrac{2}{3}\hat{i} + \dfrac{2}{3}\hat{j} - \dfrac{1}{3}\hat{k}$

or $-\dfrac{2}{3}\hat{i} - \dfrac{2}{3}\hat{j} + \dfrac{1}{3}\hat{k}$.

47. **(d)**



In a parallelogram, diagonals bisect each other. So, mid point of DB is also the mid-point of AC.

Mid-point of $M = 2\hat{i} - \hat{j}$

Direction ratio of $OC = (1, -5, -5)$

Direction ratio of $OM = (2, -1, 0)$

Angle θ between OM and OC is given by

$$\cos\theta = \frac{(1\times 2) + (-5)(-1) + (-5)(0)}{\sqrt{2^2 + (-1)^2}\sqrt{(1)^2 + (-5)^2 + (-5)^2}}$$

$$= \frac{2+5}{\sqrt{5}\sqrt{51}} = \frac{7}{\sqrt{5}\sqrt{51}}$$

Projection of $\overrightarrow{OM}$ on $\overrightarrow{OC}$ is given by

$|OM|.\cos\theta = \sqrt{5} \times \dfrac{7}{\sqrt{5}\times\sqrt{51}} = \dfrac{7}{\sqrt{51}}$

48. **(d)** Given that, $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually orthogonal

$\therefore \; \vec{a}.\vec{b} = 0,\; \vec{b}.\vec{c} = 0,\; \vec{c}.\vec{a} = 0$

$\Rightarrow 2\lambda + 4 + \mu = 0$ ...(i)

$\lambda - 1 + 2\mu = 0$ ...(ii)

On solving (i) and (ii), we get $\lambda = -3, \mu = 2$

49. **(d)** Clearly $\vec{a} = -\dfrac{8}{7}\vec{c}$

$\Rightarrow \vec{a} \,||\, \vec{c}$ and are opposite in direction

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

50. **(a)** $\overrightarrow{CA} = (2 - a)\hat{i} + 2\hat{j}$; $\overrightarrow{CB} = (1 - a)\hat{i} - 6\hat{k}$

$\left[\because \overrightarrow{CA} \perp \overrightarrow{CB}\right]$

$\therefore \quad \overrightarrow{CA}\cdot\overrightarrow{CB} = 0 \Rightarrow (2 - a)(1 - a) = 0$

$\Rightarrow \quad a = 2, 1$

51. **(c)** Projection of $\vec{v}$ along $\vec{u}$ $= \dfrac{\vec{v}.\vec{u}}{|\vec{u}|} = \vec{v}.\vec{u}$

projection of $\vec{w}$ along $\vec{u}$ $= \dfrac{\vec{w}.\vec{u}}{|\vec{u}|} = \vec{w}.\vec{u}$

Given $\vec{v}.\vec{u} = \vec{w}.\vec{u}$ ....(1)

Also , $\vec{v}.\vec{w}=0$ $[\because \ \vec{v}\perp\vec{w}]$ ....(2)

Now $|\vec{u}-\vec{v}+\vec{w}|^2$

$=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2-2\vec{u}.\vec{v}-2\vec{v}.\vec{w}+2\vec{u}.\vec{w}$

$=1+4+9+0$ [From (1) and (2)] $=14$

$\therefore |\vec{u}-\vec{v}+\vec{w}|=\sqrt{14}$

52. **(c)** Given that $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$1+4+9+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\dfrac{-1-4-9}{2}=-7$

53. **(a)** Given that, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2|+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow 25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-25\cdot$

$\therefore |\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|=25\cdot$

54. **(a)** Given that $\vec{a}+\vec{b}+\vec{c}=0\Rightarrow \vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow |\vec{b}+\vec{c}|^2=|\vec{a}|^2=5^2+3^2+2\vec{b}\cdot\vec{c}=7^2$

$\Rightarrow 2|\vec{b}||\vec{c}|\cos\theta=49-34=15$;

$\Rightarrow 2\times 5\times 3\cos\theta=15$;

$\Rightarrow \cos\theta=1/2; \Rightarrow \theta=\dfrac{\pi}{3}=60°$

55. **(b)** We know that the volume of parallelopiped

$=[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix}=158$$

$\Rightarrow (12+n^2)-1(6+n)+n(2n-4)=158$

$\Rightarrow 3n^2-5n-152=0$

$\Rightarrow 3n^2-24n+19n-152=0$

$\Rightarrow 3n(n-8)+19(n-8)=0$

$\Rightarrow n=8$ or $n=\dfrac{-19}{3}$

$\Rightarrow n=8$ $\quad(\because n\geq 0)$

$\therefore \vec{a}=\hat{i}+\hat{j}+8\hat{k}, \vec{b}=2\hat{i}+4\hat{j}-8\hat{k}$ and

$\vec{c}=\hat{i}+8\hat{j}+3\hat{k}$

$\vec{a}\cdot\vec{c}=1+8+24=33$

$\vec{b}\cdot\vec{c}=2+32-24=10$

56. **(d)** It is given that

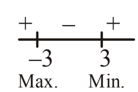
$$f(x)=\vec{a}\cdot(\vec{b}\times\vec{c})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}=x^3-27x+26$$

$\Rightarrow f(x)=x^3-27x+26$

$\Rightarrow f'(x)=3x^2-27$

For critical point $f'(x)=0$

$\Rightarrow 3x^2-27=0\Rightarrow x=-3, 3$



The local maxima of $f(x)$ is, $x_0=-3$.

Then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$

$=-2x-2x-3-14-2x-x+7x+4+3x=3x-13$

So, value at $x=x_0, =\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=3x-13$

$=3\times(-3)-13=-22.$

57. **(18)**

$\hat{i}\times(\vec{a}\times\hat{i})=(\hat{i}\cdot\hat{i})\vec{a}-(\hat{i}\cdot\vec{a})\hat{i}=\hat{j}+2\hat{k}$

Similarly, $\hat{j}\times(\vec{a}\times\hat{j})=2\hat{i}+2\hat{k}$,

$\hat{k}\times(\vec{a}\times\hat{k})=2\hat{i}+\hat{j}$

$\therefore |\hat{j}+2\hat{k}|^2+|2\hat{i}+2\hat{k}|^2+|2\hat{i}+\hat{j}|^2=5+8+5=18.$

58. **(30)** $\vec{b}.\vec{c}=10 \Rightarrow |\vec{b}|\ |\vec{c}|\cos\left(\dfrac{\pi}{3}\right)=10$

$\Rightarrow \quad 5.|\vec{c}|.\dfrac{1}{2}=10 \Rightarrow |\vec{c}|=4$

Since, is perpendicular to the vector $\vec{b}\times\vec{c}$, then

$\vec{a}.(\vec{b}\times\vec{c})=0$

Now, $|\vec{a}\times(\vec{b}\times\vec{c})|=|\vec{a}||\vec{b}\times\vec{c}|\sin\left(\dfrac{\pi}{2}\right)$

$=\sqrt{3}\times|\vec{b}||\vec{c}|\sin\dfrac{\pi}{3}\times 1$

Hence, $|\vec{a}\times(\vec{b}\times\vec{c})|=30.$

59. **(c)** $\vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\vec{b}\times\vec{a})$

$\Rightarrow \quad -(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}$

$\Rightarrow \quad -4\vec{c} = 6(\hat{i}-\hat{j}+\hat{k}) - 4(\hat{i}-2\hat{j}-\hat{k})$

$\Rightarrow \quad -4\vec{c} = 2\hat{i}-2\hat{j}+2\hat{k}$

$\Rightarrow \quad \vec{c} = \frac{1}{2}(\hat{i}+\hat{j}+\hat{k})$

$\Rightarrow \quad \vec{b}.\vec{c} = -\frac{1}{2}$

60. (d) $\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = 0$

$3+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = \frac{-3}{2} \Rightarrow \lambda = \frac{-3}{2}$

$\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times(-\vec{a}-\vec{b})+(-\vec{a}-\vec{b})\times\vec{a} \quad [\because \vec{c} = -\vec{a}-\vec{b}]$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}$

$\vec{d} = 3(\vec{a}\times\vec{b})$

61. (c) $\vec{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2}$ ...(1)

Since, $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$.

$\therefore \overrightarrow{\beta_2}.\vec{\alpha} = 0$

Since, $\overrightarrow{\beta_1}$ is parallel to $\vec{a}$.

then $\overrightarrow{\beta_1} = \lambda\vec{\alpha}$ (say)

$\vec{a}.\vec{\beta} = \vec{a}.\overrightarrow{\beta_1} - \vec{\alpha}.\overrightarrow{\beta_2}$

$\Rightarrow 5 = \lambda\alpha^2 \Rightarrow 5 = \lambda\times 10 \qquad \left(\because |\vec{\alpha}| = \sqrt{10}\right).$

$\Rightarrow \lambda = \frac{1}{2} \qquad \therefore \overrightarrow{B_1} = \frac{\vec{\alpha}}{2}$

$\therefore \vec{\beta}_1 = \frac{\vec{\alpha}}{2}$

Cross product with $\overrightarrow{B_1}$ in equation (1)

$\Rightarrow \vec{\beta}\times\overrightarrow{B_1} = -\overrightarrow{B_2}\times\overrightarrow{B_1}$

$\Rightarrow \vec{\beta}\times\overrightarrow{B_1} = \overrightarrow{B_1}\times\overrightarrow{B_2} \Rightarrow \vec{\beta}_1\times\vec{\beta}_2 = \frac{\vec{\beta}\times\vec{\alpha}}{2}$

$$\Rightarrow \overrightarrow{B_1}\times\overrightarrow{B2} = \frac{1}{2}\begin{vmatrix} \hat{i} & j & k \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$$

$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right] \qquad = \frac{1}{2}\left[-3\hat{i}+9\hat{j}+5\hat{k}\right]$

62. (d) Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$

$\therefore$ vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a}\times\vec{b}$

$$\therefore \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}-2\hat{j}+\hat{k}$$

Now, projection of vector $\vec{c} = 2\hat{i}+3\hat{j}+\hat{k}$ on $\vec{a}\times\vec{b}$ is

$= \left|\frac{\vec{c}.(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}\right| = \left|\frac{2-6+1}{\sqrt{6}}\right| = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$

63. (b) Given, $\vec{a} = 3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+\hat{k}$

$$\text{Now, } \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$$

$= (2+x)\hat{i}+(x-3)\hat{j}-5\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{(2+x)^2+(x-3)^2+(-5)^2} = r$

$\Rightarrow r = \sqrt{4+x^2+4x+x^2+9-6x+25}$

$= \sqrt{2x^2-2x+38} = \sqrt{2\left(x^2-x+\frac{1}{4}\right)+38-\frac{1}{2}}$

$= \sqrt{2\left(x-\frac{1}{2}\right)^2+\frac{75}{2}} \Rightarrow r \ge \sqrt{\frac{75}{2}} \Rightarrow r \ge 5\sqrt{\frac{3}{2}}$

64. (a) Since, $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors

$\therefore \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Then, $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}\vec{b}$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{2}\vec{b}$

$\therefore \quad \vec{a}\cdot\vec{c} = \frac{1}{2}$ and $\vec{a}\cdot\vec{b} = 0$

$|\vec{a}||\vec{c}|\cos\beta = \frac{1}{2}$ and $|\vec{a}||\vec{b}|\cos\alpha = 0$

$\Rightarrow \quad \beta = 60°$ and $\alpha = 90°$

Hence, $|\alpha-\beta| = |90°-60°| = 30°$

65. (a) $\because |\vec{a}\times\vec{c}|^2 = |\vec{a}|^2|\vec{c}|^2 - (\vec{a}.\vec{c})^2$

$\Rightarrow \quad |-\vec{b}|^2 = 2|\vec{c}|^2 - 16 \quad \Rightarrow 3 = 2|\vec{c}|^2 - 16$

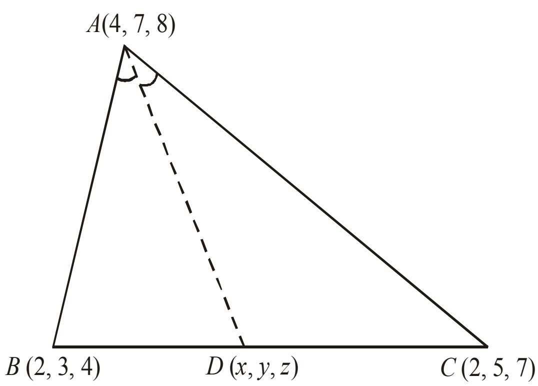
$\Rightarrow \quad |\vec{c}|^2 = \frac{19}{2}$

66. **(b)** Suppose angular bisector of A meets BC at D (x, y, z)
Using angular bisector theorem,

$$\frac{AB}{AC}=\frac{BD}{DC}$$

$$\frac{BD}{DC}=\frac{\sqrt{(4-2)^2+(7-3)^2+(8-4)^2}}{\sqrt{(4-2)^2+(7-5)^2+(8-7)^2}}$$

$$=\frac{\sqrt{2^2+4^2+4^2}}{\sqrt{2^2+2^2+1^2}}=\frac{6}{3}=2$$



So, $D(x,y,z)\equiv\left(\frac{(2)(2)+(1)(2)}{2+1},\frac{(2)(5)+(1)(3)}{2+1},\frac{(2)(7)+(1)(4)}{2+1}\right)$

$$D(x,y,z)\equiv\left(\frac{6}{3},\frac{13}{3},\frac{18}{3}\right)$$

Therefore, position vector of point $P=\frac{1}{3}(6i+13j+18k)$

67. **(a)** $\because\ \vec{a}=\hat{i}+\hat{j}+\hat{k}\Rightarrow|\vec{a}|=\sqrt{3}$

& $\vec{c}=\hat{j}-\hat{k}\Rightarrow|\vec{c}|\sqrt{2}$

Now, $\vec{a}\times\vec{b}=\vec{c}$ (Given)

$\Rightarrow|\vec{a}||\vec{b}|\sin\theta=|\vec{c}|$

$\Rightarrow|\vec{a}||\vec{b}|\sin\theta=\sqrt{2}$ ...[i]

Also $\vec{a}\cdot\vec{b}=3$

$\Rightarrow|\vec{a}||\vec{b}|\cos\theta=3$ ....[ii]

Dividing [i] by [ii], we get

$\tan\theta=\frac{\sqrt{2}}{3}\ \therefore\ \sin\theta=\frac{\sqrt{2}}{\sqrt{11}}$

Substituting value of $\sin\theta$ in [i] we get

$$\sqrt{3}|\vec{b}|\frac{\sqrt{2}}{\sqrt{11}}=\sqrt{2}$$

$$|\vec{b}|=\frac{\sqrt{11}}{\sqrt{3}}$$

68. **(b)** $\because\ \vec{a}+2\vec{b}+2\vec{c}=\vec{0}$ [Given]

$\Rightarrow\vec{a}+2\vec{c}=-2\vec{b}\Rightarrow(\vec{a}+2\vec{c})\cdot(\vec{a}+2\vec{c})=(-2\vec{b})(-2\vec{b})$

$\Rightarrow\vec{a}\cdot\vec{a}+4\vec{c}\cdot\vec{c}+4\vec{a}\cdot\vec{c}=4\vec{b}\cdot\vec{b}\Rightarrow1+4+4\vec{a}\cdot\vec{c}=4$

$\Rightarrow\vec{a}\cdot\vec{c}=\frac{-1}{4}$

$\because\ |\vec{a}\cdot\vec{c}|^2+|\vec{a}\times\vec{c}|^2=1$ ($\vec{a}$ is unit vector)

$\Rightarrow\ \frac{1}{16}+|\vec{a}\times\vec{c}|^2=1$

$\Rightarrow\ |\vec{a}\times\vec{c}|^2=\frac{15}{16}\ \Rightarrow\ |\vec{a}\times\vec{c}|=\frac{\sqrt{15}}{4}$

69. **(c)** Given : $\vec{a}=2\hat{i}+\hat{j}-2\hat{k},\ \vec{b}=\hat{i}+\hat{j}$

$\Rightarrow\ |\vec{a}|=3$

$\therefore\ \vec{a}\times\vec{b}=2\hat{i}-2\hat{j}+\hat{k}$

$|\vec{a}\times\vec{b}|=\sqrt{2^2+2^2+1^2}=3$

We have $(\vec{a}\times\vec{b})\times\vec{c}=|\vec{a}\times\vec{b}||\vec{c}|\sin 30\ n$

$\Rightarrow\ |(\vec{a}\times\vec{b})\times\vec{c}|=3|\vec{c}|.\frac{1}{2}\ \Rightarrow3=3|\vec{c}|.\frac{1}{2}$

$\therefore\ |\vec{c}|=2$

Now $|\vec{c}-\vec{a}|=3$

On squaring, we get

$\Rightarrow\ |\vec{c}|^2+|\vec{a}|^2-2\vec{a}.\vec{c}=9\Rightarrow4+9-2\,\vec{a}.\vec{c}=9$

$\Rightarrow\ \vec{a}.\vec{c}=2[\because\vec{c}.\vec{a}=\vec{a}.\vec{c}]$

70. **(b)** $\overrightarrow{b_1}=\frac{(\vec{b_1}\cdot\vec{a})\hat{a}}{1}=\left\{\frac{(3\hat{j}+4\hat{k}).(\hat{i}+\hat{j})}{\sqrt{2}}\right\}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$$=\frac{3(\hat{i}+\hat{j})}{\sqrt{2}\times\sqrt{2}}=\frac{3(\hat{i}+\hat{j})}{2}$$

$\vec{b}_1+\vec{b}_2=\vec{b}$

$\Rightarrow\ \vec{b}_2=\vec{b}-\overrightarrow{b_1}=(3\hat{j}+4\hat{k})-\frac{3}{2}(\hat{i}+\hat{j})$

$\Rightarrow\ \boxed{\overrightarrow{b_2}=-\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+4\hat{k}}$

& $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\\frac{3}{2}&\frac{3}{2}&0\\-\frac{3}{2}&\frac{3}{2}&4\end{vmatrix}$

$\Rightarrow\ \overrightarrow{b_1}\times\overrightarrow{b_2}=\hat{i}(6)-\hat{j}(6)+\hat{k}\left(-\frac{9}{4}+\frac{9}{4}\right)$

$\Rightarrow\ 6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$

71. **(b)** Let; $d_1 = 8\hat{i} - 6\hat{j} + 0\hat{k}$ & $d_2 = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$\therefore \quad |d_1 \times d_2| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix} = \left|72\hat{i} - (-96)\hat{j} + 50\hat{k}\right|$

$\Rightarrow \quad |d_1 \times d_2| = \sqrt{16900} = 130$

$\therefore$ Area of parallelogram $= \frac{1}{2}|d_1 \times d_2| = \frac{1}{2} \times 130 = 65$

72. **(b)** $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$

On comparing both sides

$\vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2} \Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$

[$\because \vec{a}$ and $\vec{b}$ are unit vectors]

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\theta = \frac{5\pi}{6}$

73. **(c)** $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \quad -\vec{c} \times (\vec{a} \times \vec{b}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \quad -(\vec{c}.\vec{b})\vec{a} + (\vec{c}.\vec{a})\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \quad -|\vec{b}||\vec{c}|\cos\theta\vec{a} + (\vec{c}.\vec{a})\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\because \quad \vec{a}, \vec{b}, \vec{c}$ are non collinear, the above equation is possible only when

$-\cos\theta = \frac{1}{3}$ and $\vec{c}.\vec{a} = 0$

$\Rightarrow \quad \cos\theta = -\frac{1}{3}$

$\Rightarrow \quad \sin\theta = \frac{2\sqrt{2}}{3}$; $\theta \in$ II quad

74. **(a)** $|\vec{a} + \vec{b}| = \sqrt{3}$

angle between $\vec{a}$ and $\vec{b}$ is 60°.

$\vec{a} \times \vec{b}$ is $\perp^r$ to plane containing $\vec{a}$ and $\vec{b}$

$\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$

$\vec{c} = \sqrt{|\vec{a}|^2 + 4|\vec{b}|^2 + 2.2|\vec{a}|^2 \cos 60° \vec{n}_1} + 3|\vec{a}||\vec{b}|\sin 60° \vec{n}_2$

$+3|\vec{a}||\vec{b}|\sin 60°.\vec{n}_2$

$\vec{n}_1 \perp^r \vec{n}_2$

$|\vec{c}|^2 = (1 + 4 + 2) + 9 \times \frac{3}{4} \Rightarrow |\vec{c}|^2 = 7 + 27/4 = 55/4$

$2|\vec{c}| = \sqrt{55}$

75. **(b)** L.H.S $= (\vec{a} \times \vec{b}).[(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})]$

$= (\vec{a} \times \vec{b}).[(\vec{b} \times \vec{c}.\vec{a})\vec{c} - (\vec{b} \times \vec{c}.\vec{c})\vec{a}]$

$= (\vec{a} \times \vec{b}).[[\vec{b}\,\vec{c}\,\vec{a}]\vec{c}] \quad [\because \vec{b} \times \vec{c}.\vec{c} = 0]$

$= [\vec{a}\ \vec{b}\,\vec{c}].(\vec{a} \times \vec{b}.\vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}]^2$

$= [\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$

So $\lambda = 1$

76. **(c)** Let $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$

Now, $\vec{x} \times \vec{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -1 \\ 1 & 4 & -3 \end{vmatrix} = 22\hat{i} + 8\hat{j} + 18\hat{k}$

Projection of $\vec{x} \times \vec{y}$ on $\vec{z} = \frac{(\vec{x} + \vec{y}).(\vec{z})}{|\vec{z}|}$

$= \frac{22(3) + 8(-4) + 18(-12)}{\sqrt{9 + 16 + 144}} = \frac{-182}{13} = -14$

Now, magnitude of projection = 14.

77. **(d)** Let, $\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

Given, $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 1 & 2 & 5 \end{vmatrix} = \vec{0}$

$\Rightarrow (5b - 2c)\hat{i} - (5a - c)\hat{j} + (2a - b)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Comparing both sides, we get

$5b - 2c = 0$; $5a - c = 0$; $2a - b = 0$

or $5b = 2c$; $5a = c$; $2a = b$

Also given $|\vec{c}|^2 = 60 \Rightarrow a^2 + b^2 + c^2 = 60$

Putting the value of b and c in above eqn., we get

$a^2 + (2a)^2 + (5a)^2 = 60$

$\Rightarrow a^2 + 4a^2 + 25a^2 = 60 \Rightarrow 30a^2 = 60$

$a^2 = 2$

$a = \pm\sqrt{2}$; $b = 2\sqrt{2}$; $c = 5\sqrt{2}$

Now, $\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore \vec{c} = \sqrt{2}\hat{i} + 2\sqrt{2}\hat{j} + 5\sqrt{2}\hat{k}$

Value of $\vec{c}.(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is

$(\sqrt{2}\hat{i} + 2\sqrt{2}\hat{j} + 5\sqrt{2}\hat{k}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$

$= -7\sqrt{2} + 4\sqrt{2} + 15\sqrt{2} = 12\sqrt{2}$

78. **(d)** $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

and $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a}\times\vec{b}|=\sqrt{4+4+1}=3$

Now, $|\vec{c}-\vec{a}|=2\sqrt{2} \Rightarrow |\vec{c}-\vec{a}|^2=8$

$\Rightarrow |\vec{c}-\vec{a}|.(\vec{c}-\vec{a})=8$

$\Rightarrow |\vec{c}|^2+|\vec{a}|^2-2\vec{c}.\vec{a}=8$

$\Rightarrow |\vec{c}|^2+9-2|\vec{c}|=8$

$\Rightarrow (|\vec{c}|-1)^2=0 \Rightarrow |\vec{c}|=1$

$\therefore |(\vec{a}\times\vec{b})\times\vec{c}|=|\vec{a}\times\vec{b}||\vec{c}|\sin 30°=3\times1\times\frac{1}{2}=\frac{3}{2}$

79. **(c)** $(\hat{i}\times\vec{a}\cdot\vec{b})\hat{i}+(\hat{j}\times\vec{a}\cdot\vec{b})\hat{j}+(\hat{k}\times\vec{a}\cdot\vec{b})\hat{k}$

$=(\hat{i}\cdot\vec{a}\times\vec{b})\hat{i}+(\hat{j}\cdot\vec{a}\times\vec{b})\hat{j}+(\hat{k}\cdot\vec{a}\times\vec{b})\hat{k}$

$(\because \vec{a}\times\vec{b}\cdot\vec{c}=\vec{a}\cdot\vec{b}\times\vec{c})$

$=(\vec{a}\times\vec{b})\hat{i}+(\vec{a}\times\vec{b})\hat{j}+(\vec{a}\times\vec{b})\hat{k}=\vec{a}\times\vec{b}$

80. **(c)** Statement - 1

The vectors $\vec{a},\vec{b}$ and $\vec{c}$ lie in the same plane.

$\Rightarrow \vec{a},\vec{b}$ and $\vec{c}$ are coplanar.

We know, the necessary and sufficient conditions for three vectors to be coplanar is that $[\vec{a}\vec{b}\vec{c}]=0$

i.e. $\vec{a}\cdot(\vec{b}\times\vec{c})=0$

Hence, statement-1 is true.

81. **(b)** Let $\vec{u}=\hat{j}+4\hat{k}$, $\vec{v}=\hat{i}-3\hat{k}$ and

$\vec{w}=\cos\theta\hat{i}+\sin\theta\hat{j}$

Now, $\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 4\\ 1 & 0 & -3\end{vmatrix}=\hat{i}(-3)-\hat{j}(-4)+\hat{k}(-1)$

$=-3\hat{i}+4\hat{j}-\hat{k}$

Now, $(\vec{u}\times\vec{v})\cdot\vec{w}=(-3\hat{i}+4\hat{j}-\hat{k}).(\cos\theta\hat{i}+\sin\theta\hat{j})$

$=-3\cos\theta+4\sin\theta$

Now, maximum possible value of

$|-3\cos\theta+4\sin\theta|=\sqrt{(-3)^2+(4)^2}=\sqrt{25}=5$

82. **(a)** **Statement - 1**

Points (1, 2, 2), (2, 1, 2), (2, 2, z) and (1, 1, 1) are coplanar then $z=2$ which is false.

$\because \begin{vmatrix}1 & -1 & 0\\ 1 & 0 & z-2\\ 0 & -1 & -1\end{vmatrix}=0$

$\Rightarrow 1(z-2)+1(-1)=0\Rightarrow z=3$

Statement - 2 is the true statement.

83. **(a)** Since $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and

$\vec{c}=\lambda\hat{i}+\hat{j}+(2\lambda-1)\hat{k}$ are coplanar

therefore $[\vec{a}\,\vec{b}\,\vec{c}]=0$

i.e., $\begin{vmatrix}1 & 2 & \lambda\\ -2 & 3 & 1\\ 3 & -1 & 2\lambda-1\end{vmatrix}=0$

$\Rightarrow 1(6\lambda-2)-2(-4\lambda-1)+\lambda(-7)=0$

$\Rightarrow (6\lambda-2)+8\lambda+2+2+2\lambda-9\lambda=0$

$\Rightarrow 7\lambda=0\Rightarrow\lambda=0$

84. **(c)** Given that $\vec{a}.\vec{b}\neq 0$, $\vec{a}.\vec{d}=0$

Now, $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{d})\vec{b}-(\vec{a}.\vec{b})\vec{d}$

$\Rightarrow (\vec{a}.\vec{b})\vec{d}=-(\vec{a}.\vec{c})\vec{b}+(\vec{a}.\vec{b})\vec{c}$

$\vec{d}=\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

85. **(d)** $(2\vec{a}-\vec{b}).\left((\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b})\right)$

$=(2\vec{a}-\vec{b})\cdot((\vec{a}\times\vec{b})\times\vec{a}+2(\vec{a}\times\vec{b})\times\vec{b})$

$=(2\vec{a}-\vec{b})((\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}+2(\vec{a}.\vec{b})\vec{b}-2(\vec{b}.\vec{b})\vec{a})$

$=(2\vec{a}-\vec{b})(\vec{b}-0+0-2\vec{a})$

From given values we get

$\vec{a}.\vec{b}=0$ and $\vec{b}.\vec{b}=1$

$=-4\vec{a}.\vec{a}-\vec{b}.\vec{b}=-5$

86. **(d)** Given that

$\vec{c}=\vec{b}\times\vec{a} \Rightarrow \vec{b}.\vec{c}=\vec{b}.(\vec{b}\times\vec{a}) \Rightarrow \vec{b}.\vec{c}=0$

$\Rightarrow (b_1\hat{i}+b_2\hat{j}+b_3\hat{k}).(\hat{i}-\hat{j}-\hat{k})=0,$

where $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$b_1-b_2-b_3=0$ ...(i)

and $\vec{a}.\vec{b}=3\Rightarrow(\hat{j}-\hat{k}).(b_1\hat{i}+b_2\hat{j}+b_3\hat{k})=3$

$\Rightarrow b_2-b_3=3$

From equation (i)

$b_1=b_2+b_3=(3+b_3)+b_3=3+2b_3$

$\vec{b}=(3+2b_3)\hat{i}+(3+b_3)\hat{j}+b_3\hat{k}$

From the option given, it is clear that b_3 equal to either 2 or -2.

If $b_3=2$ then $\vec{b}=7\hat{i}+5\hat{j}+2\hat{k}$ which is not possible

If $b_3=-2$, then $\vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

87. **(d)** $\because \vec{u},\vec{v},\vec{w}$ are non coplanar vectors

$\therefore [\vec{u},\vec{v},\vec{w}]\neq 0$

Now, $[3\vec{u},p\vec{v},p\vec{w}]-[p\vec{v},p\vec{w},q\vec{u}]$

$-[2\vec{w},q\vec{v},q\vec{u}]=0$

$\Rightarrow\ 3p^2[\vec{u},\vec{v},\vec{w}]-pq[\vec{v},\vec{w},\vec{u}]$
$-2q^2[\vec{w},\vec{v},\vec{u}]=0$

$\Rightarrow\ 3p^2[\vec{u},\vec{v},\vec{w}]-pq[\vec{u},\vec{v},\vec{w}]+2q^2[\vec{u},\vec{v},\vec{w}]$

$\Rightarrow\ (3p^2-pq+2q^2)\ [\vec{u},\vec{v},\vec{w}]=0$

$\Rightarrow\ 3p^2-pq+2q^2=0$
$(\because[\vec{u},\vec{v},\vec{w}]=0)$

$\Rightarrow\ 2p^2+p^2-pq+\frac{q^2}{4}+\frac{7q^2}{4}=0$

$\Rightarrow\ 2p^2+\left(p-\frac{q}{2}\right)^2+\frac{7}{4}q^2=0$

$\Rightarrow\ p=0, q=0, p=q/2$

This is possible only when $p=0, q=0$

$\therefore$ There is exactly one value of (p, q).

88. **(b)** Given $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and

$\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$

Given that $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=0$

i.e. $\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ x & (x-2) & -1\end{vmatrix}=0$

$\Rightarrow\ 1[1-2(x-2)]-1[-1-2x]+1[x-2+x]=0$
$\Rightarrow\ 1-2x+4+1+2x+2x-2=0$
$\Rightarrow\ 2x=-4 \Rightarrow\ x=-2$

89. **(b)** Given that $|2\hat{u}\times 3\hat{v}|=1$ and θ is acute angle between $\hat{u}$ and $\hat{v}$, $|\hat{u}|=1, |\hat{v}|=1$

$\Rightarrow\ |2\hat{u}\times 3\hat{v}|=6|\hat{u}||\hat{v}||\sin\theta|=1$

$\Rightarrow\ 6|\sin\theta|=1\ \Rightarrow\ \sin\theta=\frac{1}{6}$

Hence, there is exactly one value of θ for which $2\hat{u}\times 3\hat{v}$ is a unit vector.

90. **(d)** $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{a}.\vec{b}\neq 0$, $\vec{b}.\vec{c}\neq 0$

$\Rightarrow\ (\vec{a}.\vec{c}).\vec{b}-(\vec{b}.\vec{c})\vec{a}=(\vec{a}.\vec{c}).\vec{b}-(\vec{a}.\vec{b}).\vec{c}$

$\Rightarrow (\vec{a}.\vec{b}).\vec{c}=(\vec{b}.\vec{c})\vec{a}\ \Rightarrow\ \vec{a}\,||\,\vec{c}$.

91. **(d)** Given that

$\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and

$\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=\vec{a}.(\vec{b}\times\vec{c})=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$

$=1[1+x-y-x+x^2]-[x^2-y]$

$=1-y+x^2-x^2+y=1$

Hence $[\vec{a}\ \vec{b}\ \vec{c}]$ is independent of x and y both.

92. **(b)** Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

Given that

$\left[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}\right]=\left[\vec{a}\ \vec{b}+\vec{c}\ \vec{b}\right]$

$\Rightarrow\begin{vmatrix}\lambda(a_1+b_1) & \lambda(a_2+b_2) & \lambda(a_3+b_3)\\ \lambda^2 b_1 & \lambda^2 b_2 & \lambda^2 b_3\\ \lambda c_1 & \lambda c_2 & \lambda c_3\end{vmatrix}$

$=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$\Rightarrow\lambda^4\begin{vmatrix}a_1+b_1 & a_2+b_2 & a_3+b_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$

$=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$R_1\to R_1-R_2$ in 1st det.

and $R_2\to R_2-R_3$ in 2nd det.

$\Rightarrow\lambda^4\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=\begin{vmatrix}a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$\Rightarrow\lambda^4=-1$

Hence λ has no real values.

93. **(c)** Let $\vec{a}=x\vec{i}+y\vec{j}+z\vec{k}$

$\vec{a}\times\vec{i}=z\vec{j}-y\vec{k}\ \Rightarrow\left|\vec{a}\times\vec{i}\right|^2=y^2+z^2$

Similarly, $\left|\vec{a}\times\vec{j}\right|^2=x^2+z^2$ and $\left|\vec{a}\times\vec{k}\right|^2=x^2+y^2$

Adding all above equation

$\Rightarrow\ \left|\vec{a}\times\vec{i}\right|^2+\left|\vec{a}\times\vec{j}\right|^2+\left|\vec{a}\times\vec{k}\right|^2$

$=2(x^2+y^2+z^2)\ =2|\vec{a}|^2$

94. **(a)** Given that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Clearly $\vec{a}$ and $\vec{b}$ are non collinear

$\Rightarrow(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Comparing both side.

$\therefore\ \vec{a}.\vec{c}=0$ and $-\vec{b}.\vec{c}=\dfrac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \cos\theta=\dfrac{-1}{3}$

$\therefore\ \sin\ \theta=\sqrt{1-\dfrac{1}{9}}=\dfrac{2\sqrt{2}}{3}$

[θ is acute angle between $\vec{b}$ and $\vec{c}$]

95. **(c)** $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w})\quad [\because \vec{v}\times\vec{v}=0]$

$=\vec{u}.(\vec{u}\times\vec{v})-\vec{u}.(\vec{u}\times\vec{w})+\vec{u}.(\vec{v}\times\vec{w})+\vec{v}.(\vec{u}\times\vec{v})$

$-\vec{v}.(\vec{u}\times\vec{w})+\vec{v}.(\vec{v}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})+\vec{w}.(\vec{u}\times\vec{w})\ -\vec{w}.(\vec{v}\times\vec{w})$

We know that $[\vec{a},\vec{a},\vec{b}]=0$

$=\vec{u}.(\vec{v}\times\vec{w})-\vec{v}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$

$=[\vec{u}\vec{v}\vec{w})]+[\vec{v}\vec{w}\vec{u})]-[\vec{w}\vec{u}\vec{v}]=\vec{u}.(\vec{v}\times\vec{w})$

96. **(b)** Normal vector of the face OAB

$$=\overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&1\\2&1&3\end{vmatrix}=5\hat{i}-\hat{j}-3\hat{k}$$

Normal vector of the face ABC

$$=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&2\\-2&-1&1\end{vmatrix}=\hat{i}-5\hat{j}-3\hat{k}$$

Angle between the faces = angle between their normals

$$\cos\theta=\left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right|=\frac{19}{35}\text{ or }\theta=\cos^{-1}\left(\frac{19}{35}\right)$$

97. **(a)** Given that $\vec{u}.\hat{n}=0$ and $\vec{v}.\vec{n}=0$

$\Rightarrow\ \vec{n}$ is perpendicular both $\vec{u}$ *and* $\vec{v}$,

$\therefore\quad \hat{n}=\dfrac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|}$

$$\hat{n}=\frac{\begin{vmatrix}i&j&k\\1&1&0\\1&-1&0\end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-k$$

$|\vec{\omega}.\hat{n}|=|(i+2j+3k).(-k)|=|-3|=3$

98. **(c)** Let $\vec{a}+\vec{b}+\vec{c}=\vec{r}$. Then $\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{r}$

$\Rightarrow 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{a}\times\vec{r}$

$\Rightarrow \vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{a}\times\vec{r}\Rightarrow\vec{a}\times\vec{r}=\vec{0}$

$[\because\ \vec{a}\times\vec{b}=\vec{c}\times\vec{a}\,]$

Similarly $\vec{b}\times\vec{r}=\vec{0}$ & $\vec{c}\times\vec{r}=\vec{0}$

Above three conditions can be hold if and only if $\vec{r}=\vec{0}$

99. **(b)** We have $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-5&0\\6&3&0\end{vmatrix}=39\hat{k}=\vec{c}$

Also $|\vec{a}|=\sqrt{34},|\vec{b}|=\sqrt{45},|\vec{c}|=39$;

$\therefore\quad |\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39$.

100. **(a)** Given that $\vec{a},\vec{c},\vec{b}$ form a right handed system,

$$\therefore\quad \vec{c}=\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&0\\x&y&z\end{vmatrix}=z\hat{i}-x\hat{k}$$

101. **(a)** $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$

$=(\vec{a}\times\vec{b}).\ \left\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\right\}$

$\because\ \vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

$=(\vec{a}\times\vec{b})\cdot\left\{(\vec{m}\cdot\vec{a})\vec{c}-(\vec{m}\cdot\vec{c})\vec{a}\right\}$ (where $\vec{m}=\vec{b}\times\vec{c}$)

$=\{(\vec{a}\times\vec{b})\cdot\vec{c}\}\cdot\{(\vec{a}\cdot(\vec{b}\times\vec{c})\}=[\vec{a}\ \vec{b}\ \vec{c}]^2=4^2=16$.

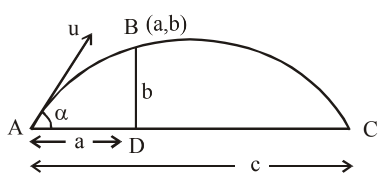
102. **(b)** Since, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\dfrac{\pi}{6}=4\times2\times\dfrac{\sqrt{3}}{2}=4\sqrt{3}$.

We know that, $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

$\Rightarrow\ (\vec{a}\times\vec{b})^2+48=16\times4$

$\Rightarrow\ (\vec{a}\times\vec{b})^2=16$

103. **(a)** Let B be the top of the wall whose coordinates will be (a, b). Range $(R)=c$



B lies on the trajectory

$\therefore\quad y=x\tan\alpha-\dfrac{1}{2}\mathrm{g}\dfrac{x^2}{u^2\cos^2\alpha}$

$\Rightarrow\quad b=a\tan\alpha-\dfrac{1}{2}\mathrm{g}\dfrac{a^2}{u^2\cos^2\alpha}$

$\Rightarrow\quad b=a\tan\alpha\left[1-\dfrac{ga}{2u^2\cos^2\alpha\tan\alpha}\right]$

$=a\tan\alpha\left[1-\dfrac{a}{\dfrac{2u^2}{g}\cos^2\alpha.\dfrac{\sin\alpha}{\cos\alpha}}\right]$

$=a\tan\alpha\left[1-\dfrac{a}{\dfrac{u^2.2\sin\alpha\cos\alpha}{g}}\right]$

$$= a\tan\alpha\left[1-\frac{a}{\frac{u^2\sin 2\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1-\frac{a}{R}\right] \qquad \left(\because\ R=\frac{u^2\sin^2\alpha}{g}\right)$$

$$\Rightarrow\quad b = a\tan\alpha\left[1-\frac{a}{c}\right]$$

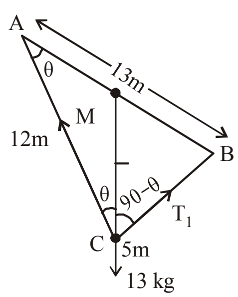
$$\Rightarrow\quad b = a\tan\alpha\,.\left(\frac{c-a}{c}\right)$$

$$\Rightarrow\quad \tan\alpha = \frac{bc}{a(c-a)}$$

The angle of projection,

$$\alpha = \tan^{-1}\frac{bc}{a(c-a)}$$

104. (a)



In ΔABC

$\because\ 13^2 = 5^2 + 12^2 \Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow \angle ACB = 90°$

M is mid point of the hypotenuse AB, therefore $MA = MB = MC$

$\Rightarrow \angle A = \angle ACM = \theta$

Applying Lami's theorem at C, we get

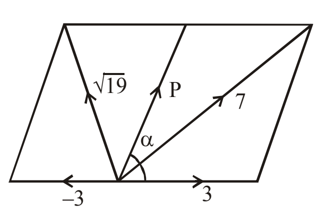
$$\frac{T_1}{\sin(180-\theta)} = \frac{T_2}{\sin(90+\theta)} = \frac{13kg}{\sin 90°}$$

$\Rightarrow\ T_1 = 13\sin\theta$ and $T_2 = 13\cos\theta$

$\Rightarrow\ T_1 = 13\times\frac{5}{13}$ and $T_2 = 13\times\frac{12}{13}$

$\Rightarrow\ T_1 = 5$ kg and $T_2 = 12$ kg

105. (c) Given that : Force $P = Pn$, $Q = 3n$, resultant $R = 7n$ & $P' = Pn$, $Q' = (-3)n$, $R' = \sqrt{19}\,n$



We know that $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$\Rightarrow\ (7)^2 = P^2 + (3)^2 + 2\times P\times 3\cos\alpha$

$\Rightarrow\ 49 = P^2 + 9 + 6P\cos\alpha$

$\Rightarrow\ 40 = P^2 + 6P\cos\alpha$ .....(i)

and $\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2P\times -3\cos\alpha$

$\Rightarrow\ 19 = P^2 + 9 - 6P\cos\alpha$

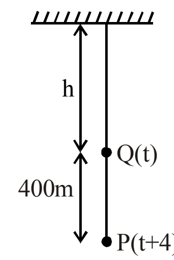
$\Rightarrow\ 10 = P^2 - 6P\cos\alpha$ .....(ii)

Adding (i) and (ii) $50 = 2P^2$

$\Rightarrow\ P^2 = 25 \Rightarrow P = 5n.$

106. (a) We know that $h = \frac{1}{2}gt^2$

and $h + 400 = \frac{1}{2}g(t+4)^2$



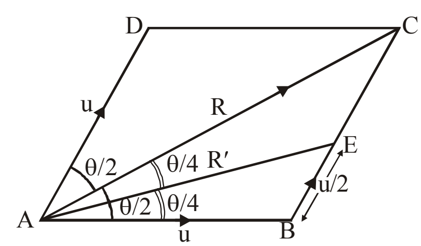
Subtracting, we get $400 = 8g + 4gt$

$\Rightarrow\ t = 8$ sec

$\therefore\ h = \frac{1}{2}\times 10\times 64 = 320m$

$\therefore$ Required height $= 320 + 400 = 720\ m$

107. (b) Let two velocities u and u at an angle θ to each other the resultant is given by



$R^2 = u^2 + u^2 + 2u^2\cos\theta = 2u^2(1+\cos\theta)$

$\Rightarrow\ R^2 = 4u^2\cos^2\theta/2$ or $R = 2u\cos\frac{\theta}{2}$

Now in second case, the new resultant AE (i.e., R') bisects $\angle CAB$, therefore using angle bisector theorem in ΔABC, we get

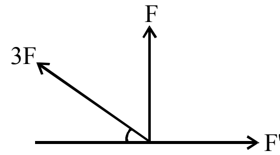
$$\frac{AB}{AC} = \frac{BE}{EC} \Rightarrow \frac{u}{R} = \frac{u/2}{u/2} \Rightarrow R = u$$

$$\Rightarrow\ 2u\cos\frac{\theta}{2} = u$$

$$\Rightarrow\quad \cos\frac{\theta}{2} = \frac{1}{2} = \cos 60° \Rightarrow \frac{\theta}{2} = 60°$$

or $\quad\theta = 120°$

108. (d) According to question $F' = 3F\cos\theta$ and $F = 3F\sin\theta$



$\Rightarrow F' = 2\sqrt{2}\,F$

$\Rightarrow F : F' :: 3 : 2\sqrt{2}$.

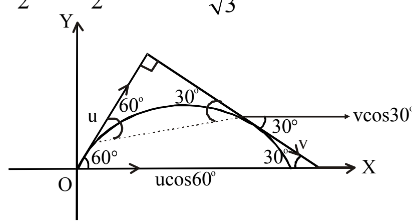
109. (b) Let A and B be displaced by a distance x then Change in moment of $(A+B)$ = applied moments

$\Rightarrow (A+B)\times x = H \Rightarrow x = \dfrac{H}{A+B}$

110. (d) As per question $u\cos 60° = v\cos 30°$

(as horizontal component of velocity remains the same)

$\Rightarrow u\cdot\dfrac{1}{2} = v\cdot\dfrac{\sqrt{3}}{2}$ or $v = \dfrac{1}{\sqrt{3}}u$



111. (b) For same horizontal range the angles of projection must be α and $\dfrac{\pi}{2}-\alpha$

$\therefore\quad t_1 = \dfrac{2u\sin\alpha}{g}$ ....(i)

$t_2 = \dfrac{2u\sin\left(\dfrac{\pi}{2}-\alpha\right)}{g} = \dfrac{2u\cos\alpha}{g}$ ....(ii)

Squaring and adding eqn. (i) and (ii),

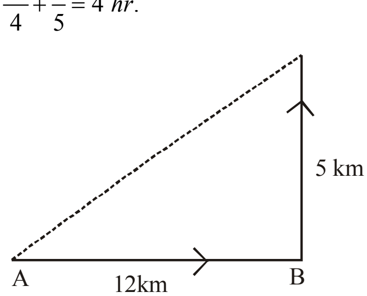
$\therefore\quad t_1^2 + t_2^2 = \dfrac{4u^2}{g^2}$

112. (a) Given $v = \dfrac{1}{4}$ m/s, component along OB

$= \dfrac{v\sin 30°}{\sin(45°+30°)} = \dfrac{\frac{1}{4}\times\frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \dfrac{\sqrt{6}-\sqrt{2}}{8}$

113. (d) Time taken by the particle in complete journey

$T = \dfrac{12}{4} + \dfrac{5}{5} = 4\ hr.$



$\therefore$ Average speed $= \dfrac{12+5}{4} = \dfrac{17}{4}$

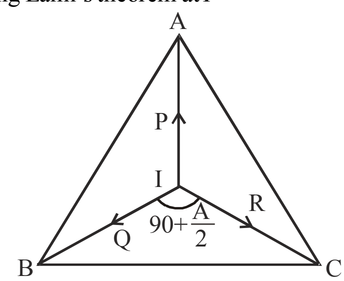
Average velocity $= \sqrt{\dfrac{12^2+5^2}{4}} = \dfrac{13}{4}$

114. (d) Let I is incentre of ΔABC.

$\therefore$ IA, IB, IC are bisectors of the angles A, B and C.

Now $\angle BIC = 180 - \dfrac{B}{2} - \dfrac{C}{2} = 90° + \dfrac{A}{2}$ *etc.*

Applying Lami's theorem at I

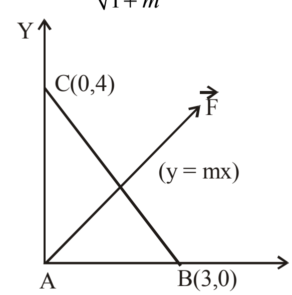


$$\frac{P}{\sin(90°+\frac{A}{2})} = \frac{Q}{\sin\left(90°+\frac{B}{2}\right)} = \frac{R}{\sin\left(90°+\frac{C}{2}\right)}$$

$\Rightarrow\quad P : Q : R = \cos\dfrac{A}{2} : \cos\dfrac{B}{2} : \cos\dfrac{C}{2}$

115. (c) Since, the moment about A is zero, hence $\vec{F}$ passes through A. Taking A as origin. Let the line of action of force $\vec{F}$ be $y = mx$. (see figure)

Moment about $B = \dfrac{3m}{\sqrt{1+m^2}}|\vec{F}| = 9$ ....(1)



Moment about $C = \dfrac{4}{\sqrt{1+m^2}}|\vec{F}| = 16$....(2)

Dividing (1) by (2), we get

$m = \dfrac{3}{4} \Rightarrow |\vec{F}| = 5N.$

116. (c) Let forces be P and Q. then $P+Q=4$ ....(1)

and $P^2+Q^2=3^2$ ....(2)

Solving eqns. (1) and (2), we get the forces

$\left(2+\dfrac{\sqrt{2}}{2}\right)N$ and $\left(2-\dfrac{\sqrt{2}}{2}\right)N$

117. (d) Resultant of forces

$\vec{F} = 4\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} + \hat{j} - \hat{k} = 7\hat{i} + 2\hat{j} - 4\hat{k}$

Displacement

$\vec{d} = 5\hat{i} + 4\hat{j} + \hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k}) \;\; = 4\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore$ Work done $= \vec{F}.\vec{d} = 28 + 4 + 8 = 40$

118. (a) Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

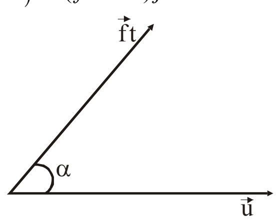
We have $R_1 = \dfrac{u^2}{g(1+\sin\beta)}$ and $R_2 = \dfrac{u^2}{g(1-\sin\beta)}$

Adding above equations

$\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2g}{u^2}$ or $\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2}{R}$ $\quad \left[\because R = \dfrac{u^2}{g}\right]$

$\therefore R_1, R, R_2$ are in H.P.

119. (a) Let the two velocities be $\vec{v}_1 = u\hat{i}$ and $\vec{v}_2 = (ft\cos\alpha)\hat{i} + (ft\sin\alpha)\hat{j}$



$\therefore$ Relative velocity of second with respect to first

$\vec{v} = \vec{v}_2 - \vec{v}_1 = (ft\cos\alpha - u)\hat{i} + ft\sin\alpha\;\hat{j}$

$\Rightarrow \;|\vec{v}|^2 = (ft\cos\alpha - u)^2 + (ft\sin\alpha)^2$

$= f^2t^2 + u^2 - 2uft\cos\alpha$

For $|\vec{v}|$ to be min and max. we should have

$\dfrac{d|v|^2}{dt} = 0 \Rightarrow 2f^2t - 2uf\cos\alpha = 0$

$\Rightarrow \;t = \dfrac{u\cos\alpha}{f}$

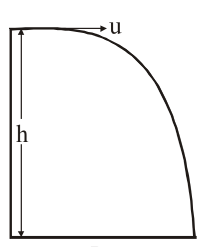
Also $\dfrac{d^2|v|^2}{dt^2} = 2f^2 = +ve$

$\therefore |v|^2$ and hence $|v|$ is least at the time $\dfrac{u\cos\alpha}{f}$

120. (a) Given that the stone projected horizontally. For horizontal motion,

Distance = speed × time $\Rightarrow R = ut$

and for vertical motion

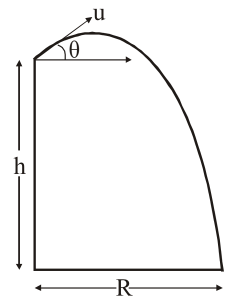


$h = 0 \times t + \dfrac{1}{2}gt^2$

$\Rightarrow \;t = \sqrt{\dfrac{2h}{g}}$

$\therefore$ We get $R = u\sqrt{\dfrac{2h}{g}}$ ...(1)

When the stone projected at an angle θ, for horizontal and vertical motions, we have



$R = u\cos\theta \times t$ ....(2)

and $\;h = -u\sin\theta \times t + \dfrac{1}{2}gt^2$ ....(3)

From eqns. (1) and (2) we get

$u\sqrt{\dfrac{2h}{g}} = u\cos\theta \times t$

$\Rightarrow \;t = \dfrac{1}{\cos\theta}\sqrt{\dfrac{2h}{g}}$

Putting the value of t in eq (3) we get

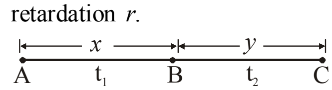
$h = -\dfrac{u\sin\theta}{\cos\theta}\sqrt{\dfrac{2h}{g}} + \dfrac{1}{2}g\left[\dfrac{2h}{g\cos^2\theta}\right]$

$h = -u\sqrt{\dfrac{2h}{g}}\tan\theta + h\sec^2\theta$

$h = -u\sqrt{\dfrac{2h}{g}}\tan\theta + h\tan^2\theta + h$

$\tan^2\theta - u\sqrt{\dfrac{2}{hg}}\tan\theta = 0; \therefore \;\tan\theta = u\sqrt{\dfrac{2}{hg}}$

121. (a) Let the body travels from A to B with constant acceleration t and from B to C with constant retardation r.



If $AB = x$, $BC = y$, time taken from A to $B = t_1$ and time taken from B to $C = t_2$, then $s = x + y$ and $t = t_1 + t_2$

For the motion from A to B

$v^2 = u^2 + 2fs \Rightarrow v^2 = 2fx \;(\because u = 0)$

$\Rightarrow \;x = \dfrac{v^2}{2f}$ ....(1)

and $\;v = u + ft \;\;\Rightarrow v = ft_1$

$\Rightarrow \;t_1 = \dfrac{v}{f}$ ...(2)

For the motion from B to C

$v^2 = u^2 + 2fs$

$\Rightarrow\ 0 = v^2 - 2ry \Rightarrow y = \frac{v^2}{2r}$ ...(3)

and $v = u + ft \Rightarrow 0 = v - rt_2$

$\Rightarrow\ t_2 = \frac{v}{r}$

Adding equations (1) and (3), we get

$$x + y = \frac{v^2}{2}\left[\frac{1}{f} + \frac{1}{r}\right] = s$$

Adding equations (2) and (4), we get

$$t_1 + t_2 = v\left[\frac{1}{f} + \frac{1}{r}\right] = t$$

$$\therefore \quad \frac{t^2}{2s} = \frac{v^2\left[\frac{1}{f} + \frac{1}{r}\right]^2}{2 \times \frac{v^2}{2}\left(\frac{1}{f} + \frac{1}{r}\right)} = \frac{1}{f} + \frac{1}{r}$$

$$\Rightarrow\ t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$$

122. (c) $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ .... (1)

When $\vec{Q}$ and $\vec{R}$ are doubled

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ .... (2)

When $\vec{Q}$ is reversed and $\vec{R}$ is doubled

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ .... (3)

Adding (1) and (3), $5R^2 = 2P^2 + 2Q^2$

$\Rightarrow 2P^2 + 2Q^2 - 5R^2 = 0$ .... (4)

Applying $(3)\times 2 + (2)$, $12R^2 = 3P^2 + 6Q^2$

$\Rightarrow 3P^2 + 6Q^2 - 12R^2 = 0$ ....(5)

From (4) and (5)

$$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$$

$$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6} \text{ or } P^2 : Q^2 : R^2 = 2:3:2$$

123. (c) We know that $\vec{G} = \vec{r} \times \vec{p}$; $|\vec{G}| = |\vec{r}||\vec{p}|\sin\theta$

$|\vec{H}| = |\vec{r}||\vec{p}|\cos\theta$ $\quad \left[\because \sin(90^o + \theta) = \cos\theta\right]$

$G = |\vec{r}||\vec{p}|\sin\theta$ .....(1)

$H = |\vec{r}||\vec{p}|\cos\theta$ .....(2)

$x = |\vec{r}||\vec{p}|\sin(\theta + \alpha)$ .....(3)

From (1), (2) & (3), $x = \vec{G}\cos\alpha + \vec{H}\sin\alpha$.

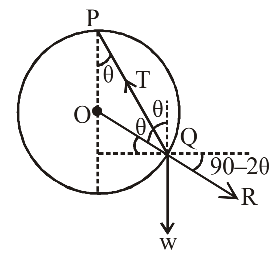
124. (d) $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7i + 2j - 4k$

$\vec{d}$ = Positon Vector of $\vec{B}$ – Position Vector of $\vec{A}$

$= 4i + 2j - 2k$

$W = \vec{F}.\vec{d} = 28 + 4 + 8 = 40$ unit

125. (b) From figure $\angle TQW = 180 - \theta$; $\angle RQW = 2\theta$; $\angle RQT = 180 - \theta$



Applying Lami's theorem at Q.

$$\frac{T}{\sin 2\theta} = \frac{R}{\sin(180-\theta)} = \frac{W}{\sin(180-\theta)}$$

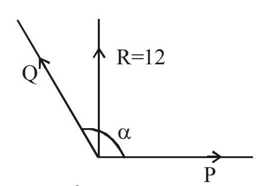
$\Rightarrow\ R = W$ and $T = 2W\cos\theta$

126. (a) Given that $P + Q = 18$ ....(1)

We know that

$P^2 + Q^2 + 2PQ\cos\alpha = 144$ ......(2)

$$\tan 90^o = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$



$\Rightarrow\ P + Q\cos\alpha = 0$ ......(3)

From (2) and (3),

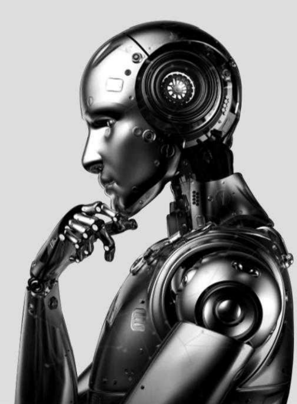
$Q^2 - P^2 = 144 \Rightarrow (Q-P)(Q+P) = 144$

$$\therefore\quad Q - P = \frac{144}{18} = 8$$

From (1), On solving, we get $Q = 13$, $P = 5$

26

Three Dimensional Geometry



TOPIC 1 **Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Condition of Parallelism & Perpendicularity of two Lines, Projection of a Point on a Line, Projection of a Line Segment Joining two Points**

1. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to: **[April 10, 2019 (I)]**
   (a) 1 (b) 2
   (c) –1 (d) –2

2. The vertices B and C of a “ABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A (1, –1, 2), is: **[April 09, 2019 (II)]**
   (a) $5\sqrt{17}$ (b) $2\sqrt{34}$
   (c) 6 (d) $\sqrt{34}$

3. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then distance of R from the origin is : **[April 08, 2019 (II)]**
   (a) $2\sqrt{14}$ (b) $2\sqrt{21}$
   (c) 6 (d) $\sqrt{53}$

4. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1, 1, 2) and O(0, 0, 0). The angle between the faces OPQ and PQR is: **[Jan. 12, 2019 (I)]**
   (a) $\cos^{-1}\left(\frac{17}{31}\right)$ (b) $\cos^{-1}\left(\frac{19}{35}\right)$
   (c) $\cos^{-1}\left(\frac{9}{35}\right)$ (d) $\cos^{-1}\left(\frac{7}{31}\right)$

5. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is: **[2018]**
   (a) $\frac{2}{3}$ (b) $\frac{1}{3}$
   (c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$

6. An angle between the lines whose direction cosines are given by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is **[Online April 15, 2018]**
   (a) $\cos^{-1}\left(\frac{1}{8}\right)$ (b) $\cos^{-1}\left(\frac{1}{6}\right)$
   (c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{1}{4}\right)$

7. ABC is triangle in a plane with vertices A (2, 3, 5), B (–1, 3, 2) and C (λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is : **[Online April 10, 2016]**
   (a) 1130 (b) 1348
   (c) 1077 (d) 676

8. The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2+m^2+n^2$ is **[2014]**
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

9. Let A (2, 3, 5), B (– 1, 3, 2) and C (λ, 5, μ) be the vertices of a ΔABC. If the median through A is equally inclined to the coordinate axes, then: **[Online April 11, 2014]**
   (a) $5\lambda - 8\mu = 0$ (b) $8\lambda - 5\mu = 0$
   (c) $10\lambda - 7\mu = 0$ (d) $7\lambda - 10\mu = 0$

10. A line in the 3-dimensional space makes an angle θ $\left(0 < \theta \leq \frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of θ is the interval: **[Online April 9, 2014]**
    (a) $\left(0, \frac{\pi}{4}\right]$ (b) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

(c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$

11. Let ABC be a triangle with vertices at points A (2, 3, 5), B (–1, 3, 2) and C (λ, 5, μ) in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ) is equal to : **[Online April 25, 2013]**

(a) (10, 7) (b) (7, 5)
(c) (7, 10) (d) (5, 7)

12. If the projections of a line segment on the x, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the line segment is : **[Online April 23, 2013]**

(a) 12 (b) 7
(c) 9 (d) 6

13. The acute angle between two lines such that the direction cosines l, m, n, of each of them satisfy the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$ is : **[Online April 22, 2013]**

(a) 15° (b) 30°
(c) 60° (d) 45°

14. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals **[2010]**

(a) 45° (b) 60°
(c) 75° (d) 30°

15. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[2009]**

(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

(c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) 6, – 3, 2

16. If a line makes an angle of $\pi/4$ with the positive directions of each of x- axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is **[2007]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

17. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals **[2004]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

TOPIC 2 Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Condition for Coplanarity of two Lines Perpendicular Distance of a Point from a Line, Shortest Distance between two Skew Lines, Distance Between two Parallel Lines.

18. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is: **[Sep. 06, 2020 (II)]**

(a) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$ (b) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

(c) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$ (d) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

19. If (a, b, c) is the image of the point (1, 2, –3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then $a + b + c$ is equals to: **[Sep. 05, 2020 (I)]**

(a) 2 (b) – 1
(c) 3 (d) 1

20. The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$ **[Sep. 03, 2020 (I)]**

(a) do not intersect for any values of l and m
(b) intersect for all values of l and m
(c) intersect when $l = 2$ and $m = \frac{1}{2}$
(d) intersect when $l = 1$ and $m = 2$

21. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is: **[Jan. 08, 2020 (I)]**

(a) $2\sqrt{30}$ (b) $\frac{7}{2}\sqrt{30}$
(c) $3\sqrt{30}$ (d) 3

22. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is , then α is equal to _________. **[NA Jan. 07, 2020 (II)]**

23. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are : **[April 10, 2019 (II)]**

(a) $(1, 0, 2)$ (b) $(2, 0, 1)$
(c) $(-1, 0, 4)$ (d) $(4, 0, -1)$

24. The length of the perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is : **[April 08, 2019 (I)]**
(a) greater than 3 but less than 4
(b) less than 2
(c) greater than 2 but less than 3
(d) greater than 4

25. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy- plane has coordinates : **[Jan. 11, 2019 (II)]**
(a) $(2,-4,-7)$ (b) $(2, 4, 7)$
(c) $(2,-4, 7)$ (d) $(-2, 4, 7)$

26. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then: **[Jan. 09, 2019 (II)]**
(a) $ab' + bc' + 1 = 0$
(b) $cc' + a + a' = 0$
(c) $bb' + cc' + 1 = 0$
(d) $aa' + c + c' = 0$

27. If the angle between the lines,
$\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{P}=\frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then P is equal to **[Online April 16, 2018]**
(a) $-\frac{7}{4}$ (b) $\frac{2}{7}$
(c) $-\frac{4}{7}$ (d) $\frac{7}{2}$

28. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is : **[Online April 10, 2016]**
(a) 2 (b) 4
(c) 3 (d) 1

29. The shortest distance between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ lies in the interval : **[Online April 9, 2016]**
(a) $(3,4]$ (b) $(2,3]$
(c) $[1,2)$ (d) $[0,1)$

30. Equation of the line of the shortest distance between the lines $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$ and $\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}$ is: **[Online April 19, 2014]**
(a) $\frac{x}{1}=\frac{y}{-1}=\frac{z}{-2}$ (b) $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{-2}$
(c) $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{1}$ (d) $\frac{x}{-2}=\frac{y}{1}=\frac{z}{2}$

31. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have **[2013]**
(a) any value (b) exactly one value
(c) exactly two values (d) exactly three values

32. If two lines L_1 and L_2 in space, are defined by
$L_1=\left\{x=\sqrt{\lambda}y+\left(\sqrt{\lambda}-1\right),\ z=\left(\sqrt{\lambda}-1\right)y+\sqrt{\lambda}\right\}$ and
$L_2=\left\{x=\sqrt{\mu}y+\left(1-\sqrt{\mu}\right),\ z=\left(1-\sqrt{\mu}\right)y+\sqrt{\mu}\right\}$
then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that : **[Online April 23, 2013]**
(a) $\sqrt{\lambda}+\sqrt{\mu}=1$ (b) $\lambda \neq \mu$
(c) $\lambda+\mu=0$ (d) $\lambda=\mu$

33. If the lines $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z+1}{3}$ and $\frac{x+2}{2}=\frac{y-k}{3}=\frac{z}{4}$ are coplanar, then the value of k is : **[Online April 9, 2013]**
(a) $\frac{11}{2}$ (b) $-\frac{11}{2}$
(c) $\frac{9}{2}$ (d) $-\frac{9}{2}$

34. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to: **[2012]**
(a) -1 (b) $\frac{2}{9}$
(c) $\frac{9}{2}$ (d) 0

35. The distance of the point $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line that passes through the point $2\hat{i}+3\hat{j}-4\hat{k}$ and is parallel to the vector $6\hat{i}+3\hat{j}-4\hat{k}$ is **[Online May 26, 2012]**

(a) 9 (b) 8
(c) 7 (d) 10

36. **Statement 1:** The shortest distance between the lines $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ and $\frac{x-1}{4}=\frac{y-1}{-2}=\frac{z-1}{4}$ is $\sqrt{2}$.

Statement 2: The shortest distance between two parallel lines is the perpendicular distance from any point on one of the lines to the other line. **[Online May 19, 2012]**

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1

37. The coordinates of the foot of perpendicular from the point (1, 0, 0) to the line

$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ are **[Online May 12, 2012]**

(a) $(2,-3,8)$ (b) $(1,-1,-10)$
(c) $(5,-8,-4)$ (d) $(3,-4,-2)$

38. The length of the perpendicular drawn from the point $(3,-1,11)$ to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is : **[2011RS]**

(a) $\sqrt{29}$ (b) $\sqrt{33}$
(c) $\sqrt{53}$ (d) $\sqrt{66}$

39. **Statement-1:** The point $A(1, 0, 7))$ is the mirror image of the point $B(1, 6, 3)$ in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement-2: The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

40. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is **[2010]**

(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$
(c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

41. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to **[2008]**

(a) –5 (b) 5
(c) 2 (d) –2

42. If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is **[2005]**

(a) $(-1,2)$ (b) $(-1,-2)$
(c) $(1,-2)$ (d) $\left(1,-\frac{1}{2}\right)$

43. The angle between the lines 2x = 3y = – z and 6x = – y = – 4z is **[2005]**

(a) 0° (b) 90°
(c) 45° (d) 30°

44. If the straight lines **[2004]**

$x=1+s, y=-3-\lambda s, z=1+\lambda s$

and $x=\frac{t}{2}, y=1+t, z=2-t$, with parameters s and t respectively, are co-planar, then λ equals.

(a) 0 (b) –1
(c) $-\frac{1}{2}$ (d) –2

45. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x=y+a=z$ and $x+a=2y=2z$. The co-ordinates of each of the points of intersection are given by **[2004]**

(a) $(2a,3a,3a),(2a,a,a)$ (b) $(3a,2a,3a),(a,a,a)$
(c) $(3a,2a,3a),(a,a,2a)$ (d) $(3a,3a,3a),(a,a,a)$

46. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ **[2003]**

and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(a) k = 3 or –2 (b) k = 0 or –1
(c) k = 1 or –1 (d) k = 0 or –3.

47. The two lines $x=ay+b, z=cy+d$ and $x=a'y+b', z=c'y+d'$ will be perpendicular, if and only if **[2003]**

(a) $aa'+cc'+1=0$
(b) $aa'+bb'+cc'+1=0$
(c) $aa'+bb'+cc'=0$
(d) $(a+a')(b+b')+(c+c')=0$.

TOPIC 3 Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Plane Containing two Lines, Angle Between two Planes, Angle Between a Plane and a Line, Distance Between two Parallel Planes, Position of Point and Line wrt a Plane, Projection of a Line on a Plane

48. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x+y+z+1=0,\ 2x-y+z+3=0$ is : **[Sep. 06, 2020 (I)]**

(a) 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

49. If for some $\alpha\in\mathbf{R}$, the lines $L_1:\frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $L_2:\frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point : **[Sep. 05, 2020 (II)]**

(a) $(10, 2, 2)$ (b) $(2, -10, -2)$

(c) $(10, -2, -2)$ (d) $(-2, 10, 2)$

50. If the equation of a plane P, passing through the intersection of the planes, $x+4y-z+7=0$ and $3x+y+5z=8$ is $ax+by+6z=15$ for some $a, b\in\mathbf{R}$, then the distance of the point $(3, 2, -1)$ from the plane P is ___________. **[Sep. 04, 2020 (I)]**

51. The distance of the point $(1, -2, 3)$ from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is : **[NA Sep. 04, 2020 (II)]**

(a) $\frac{7}{5}$ (b) 1

(c) $\frac{1}{7}$ (d) 7

52. The foot of the perpendicular drawn from the point $(4, 2, 3)$ to the line joining the points $(1, -2, 3)$ and $(1, 1, 0)$ lies on the plane : **[Sep. 03, 2020 (I)]**

(a) $2x+y-z=1$ (b) $x-y-2z=1$

(c) $x-2y+z=1$ (d) $x+2y-z=1$

53. The plane which bisects the line joining the points $(4, -2, 3)$ and $(2, 4, -1)$ at right angles also passes through the point: **[Sep. 03, 2020 (II)]**

(a) $(4, 0, 1)$ (b) $(0, -1, 1)$

(c) $(4, 0, -1)$ (d) $(0, 1, -1)$

54. Let a plane P contain two lines

$\vec{r}=\hat{i}+\lambda(\hat{i}+\hat{j}),\ \lambda\in\mathbf{R}$ and $\vec{r}=-\hat{j}+\mu(\hat{j}-\hat{k}),\ \mu\in\mathbf{R}.$

If $Q(\alpha,\beta,\gamma)$ is the foot of the perpendicular drawn from the point $M(1, 0, 1)$ to P, then $3(\alpha+\beta+\gamma)$ equals ____________. **[NA Sep. 03, 2020 (II)]**

55. The plane passing through the points $(1, 2, 1)$, $(2, 1, 2)$ and parallel to the line, $2x=3y$, $z=1$ also through the point : **[Sep. 02, 2020 (I)]**

(a) $(0, 6, -2)$ (b) $(-2, 0, 1)$

(c) $(0, -6, 2)$ (d) $(2, 0, -1)$

56. A plane passing through the point $(3, 1, 1)$ contains two lines whose direction ratios are $1, -2, 2$ and $2, 3, -1$ respectively. If this plane also passes through the point $(\alpha, -3, 5)$, then α is equal to : **[Sep. 02, 2020 (II)]**

(a) 5 (b) -10

(c) 10 (d) -5

57. If for some α and β in $\mathbf{R}$, the intersection of the following three planes

$x+4y-2z=1$

$x+7y-5z=\beta$

$x+5y+\alpha z=5$

is a line in R^3, then $\alpha+\beta$ is equal to: **[Jan. 9, 2020 (I)]**

(a) 0 (b) 10

(c) 2 (d) -10

58. If the distance between the plane, $23x-10y-2z+48=0$ and the plane containing the lines

$\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$

and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda\in R)$

is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______. **[NA Jan. 9, 2020 (II)]**

59. The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane? **[Jan. 8, 2020 (II)]**

(a) $(1, 1, 1)$ (b) $(1, -1, 1)$
(c) $(-1, -1, 1)$ (d) $(-1, -1, -1)$

60. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is: **[Jan. 7, 2020 (I)]**

(a) (6, 5, 2) (b) (6, 5, –2)
(c) (4, 3, 2) (d) (3, 4, –2)

61. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to: **[April 12, 2019 (I)]**

(a) 14 (b) $\sqrt{14}$
(c) $2\sqrt{7}$ (d) $2\sqrt{14}$

62. A plane which bisects the angle between the two given planes $2x-y+2z-4=0$ and $x+2y+2z-2=0$, passes through the point : **[April 12, 2019 (II)]**

(a) (1, –4, 1) (b) (1, 4, –1)
(c) (2, 4, 1) (d) (2, –4, 1)

63. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$ is : **[April 12, 2019 (II)]**

(a) 3 (b) $\frac{1}{3}$
(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

64. If Q (0, –1, –3) is the image of the point P in the plane $3x-y+4z=2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is : **[April 10, 2019 (I)]**

(a) $2\sqrt{13}$ (b) $\frac{\sqrt{91}}{4}$
(c) $\frac{\sqrt{91}}{2}$ (d) $\frac{\sqrt{65}}{2}$

65. If the plane $2x-y+2z+3=0$ has the distances $\frac{1}{2}$ and $\frac{2}{3}$ units from the planes $4x-2y+4z+\lambda=0$ and $2x-y+2z+\mu=0$, respectively, then the maximum value of $\lambda+\mu$ is equal to : **[April 10, 2019 (II)]**

(a) 9 (b) 15
(c) 5 (d) 13

66. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x+2y+3z=15$ at a point P, then the distance of P from the origin is: **[April 09 2019I]**

(a) $\sqrt{5}/2$ (b) $2\sqrt{5}$
(c) 9/2 (d) 7/2

67. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y-z+5=0$, also passes through the point: **[April 09 2019I]**

(a) $(-\sqrt{2}, 1, -4)$ (b) $(\sqrt{2}, -1, 4)$
(c) $(-\sqrt{2}, -1, -4)$ (d) $(\sqrt{2}, 1, 4)$

68. Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to: **[April 09, 2019 (II)]**

(a) $17/\sqrt{5}$ (b) $63\sqrt{5}$
(c) $205\sqrt{5}$ (d) $11/\sqrt{5}$

69. The equation of a plane containing the line of intersection of the planes $2x-y-4=0$ and $y+2z-4=0$ and passing through the point (1, 1, 0) is : **[April 08 2019 I]**

(a) $x-3y-2z=-2$ (b) $2x-z=2$
(c) $x-y-z=0$ (d) $x+3y+z=4$

70. The vector equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$ is : **[April 08, 2019 (II)]**

(a) $\vec{r}\times(\hat{i}-\hat{k})+2=0$ (b) $\vec{r}.(\hat{i}-\hat{k})-2=0$
(c) $\vec{r}\times(\hat{i}+\hat{k})+2=0$ (d) $\vec{r}.(\hat{i}-\hat{k})+2=0$

71. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is : **[Jan. 12, 2019 (I)]**

(a) $11\sqrt{6}$ (b) $11/\sqrt{6}$
(c) 11 (d) $6\sqrt{11}$

72. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kx=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is **[Jan. 12, 2019 (II)]**

(a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{\frac{3}{5}}$

(c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$

73. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point- $(-1, -1, 1)$. Then S is equal to : **[Jan. 12, 2019 (II)]**

(a) $\{\sqrt{3}\}$ (b) $\{\sqrt{3}, -\sqrt{3}\}$

(c) $\{1, -1\}$ (d) $\{3, -3\}$

74. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points? **[Jan. 11, 2019 (I)]**

(a) $(2, 2, 0)$ (b) $(-2, 2, 2)$

(c) $(0, -2, 2)$ (d) $(2, 0, -2)$

75. The direction ratios of normal to the plane through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are : **[Jan. 11, 2019 (I)]**

(a) $2, -1, 1$ (b) $2, \sqrt{2}, -\sqrt{2}$

(c) $\sqrt{2}, 1, -1$ (d) $2\sqrt{3}, 1, -1$

76. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to : **[Jan. 11, 2019 (II)]**

(a) 12 (b) 7

(c) 5 (d) 17

77. The plane which bisects the line segment joining the points $(-3, -3, 4)$ and $(3, 7, 6)$ at right angles, passes through which one of the following points? **[Jan. 10, 2019 (II)]**

(a) $(-2, 3, 5)$ (b) $(4, -1, 7)$

(c) $(2, 1, 3)$ (d) $(4, 1, -2)$

78. On which of the following lines lies the point of inter-section of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$? **[Jan. 10, 2019 (II)]**

(a) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

(b) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(c) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(d) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

79. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$ **[Jan 09 2019I]**

(a) is inconsistent when $a = 4$

(b) has a unique solution for $|a| = \sqrt{3}$

(c) has infinitely many solutions for $a = 4$

(d) is inconsistent when $|a| = \sqrt{3}$

80. The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is: **[Jan 09 2019I]**

(a) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

(b) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(c) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(d) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

81. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point: **[Jan 09 2019I]**

(a) $(-3, 0, -1)$ (b) $(-3, 1, 1)$

(c) $(3, 3, -1)$ (d) $(3, 2, 1)$

82. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is: **[Jan. 09, 2019 (II)]**

(a) $x - 2y + z = 0$

(b) $3x + 2y - 3z = 0$

(c) $x + 2y - 2z = 0$

(d) $5x + 2y - 4z = 0$

83. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is : **[2018]**

(a) $\frac{1}{3\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{4\sqrt{2}}$

84. The sum of the intercepts on the coordinate axes of the plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, -1, 2)$ and $(1, 1, 1)$ is **[Online April 16, 2018]**

(a) 12 (b) -8
(c) -4 (d) 4

85. A variable plane passes through a fixed point $(3, 2, 1)$ and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to $yz-plane$ through A, a second plane is drawn parallel $zx-plane$ through B and a third plane is drawn parallel to $xy-plane$ through C. Then the locus of the point of intersection of these three planes, is **[Online April 15, 2018]**

(a) $x+y+z=6$ (b) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

(c) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (d) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$

86. An angle between the plane, $x+y+z=5$ and the line of intersection of the planes, $3x+4y+z-1=0$ and $5x+8y+2z+14=0$, is **[Online April 15, 2018]**

(a) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (b) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

(c) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (d) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

87. A plane bisects the line segment joining the points $(1, 2, 3)$ and $(-3, 4, 5)$ at right angles. Then this plane also passes through the point. **[Online April 15, 2018]**

(a) $(-3, 2, 1)$ (b) $(3, 2, 1)$
(c) $(1, 2, -3)$ (d) $(-1, 2, 3)$

88. If the image of the point P(1, –2, 3) in the plane, $2x+3y-4z+22=0$ measured parallel to line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to : **[2017]**

(a) $6\sqrt{5}$ (b) $3\sqrt{5}$

(c) $2\sqrt{42}$ (d) $\sqrt{42}$

89. The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$, having normal perpendicular to both the lines **[2017]**

$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is :

(a) $\frac{10}{\sqrt{74}}$ (b) $\frac{20}{\sqrt{74}}$

(c) $\frac{10}{\sqrt{83}}$ (d) $\frac{5}{\sqrt{83}}$

90. If $x=a, y=b, z=c$ is a solution of the system of linear equations **[Online April 9, 2017]**

$x+8y+7z=0$
$9x+2y+3z=0$
$x+y+z=0$

such that the point (a, b, c) lies on the plane $x+2y+z=6$, then $2a+b+c$ equals :

(a) –1 (b) 0
(c) 1 (d) 2

91. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is : **[Online April 9, 2017]**

(a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$

(b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$

(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$

(d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$

92. If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x-4y+3z=2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is : **[Online April 9, 2017]**

(a) 2 (b) 1
(c) 0 (d) 3

93. The coordinates of the foot of the perpendicular from the point $(1, -2, 1)$ on the plane containing the lines,

$\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and

$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$, is : **[Online April 8, 2017]**

(a) $(2, -4, 2)$ (b) $(-1, 2, -1)$
(c) $(0, 0, 0)$ (d) $(1, 1, 1)$

94. The line of intersection of the planes

$\vec{r}.(3\hat{i}-\hat{j}+\hat{k})=1$ and

$\vec{r}.(\hat{i}+4\hat{j}-2\hat{k})=2$, is : **[Online April 8, 2017]**

(a) $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$

(b) $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$

(c) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

(d) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$

95. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to : **[2016]**

(a) 5 (b) 2
(c) 26 (d) 18

96. The distance of the point (1, –5, 9) from the plane $x-y+z=5$ measured along the line $x=y=z$ is : **[2016]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{20}{3}$
(c) $3\sqrt{10}$ (d) $10\sqrt{3}$

97. The distance of the point (1, –2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes $x-y+2z=3$ and $2x-2y+z+12=0$, is : **[Online April 9, 2016]**

(a) 2 (b) $\sqrt{2}$
(c) $2\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

98. The equation of the plane containing the line $2x-5y+z=3$; $x+y+4z=5$, and parallel to the plane, $x+3y+6z=1$, is: **[2015]**

(a) $x+3y+6z=7$ (b) $2x+6y+12z=-13$
(c) $2x+6y+12z=13$ (d) $x+3y+6z=-7$

99. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=16$, is **[2015]**

(a) $3\sqrt{21}$ (b) 13
(c) $2\sqrt{14}$ (d) 8

100. The shortest distance between the z–axis and the line $x+y+2z-3=0=2x+3y+4z-4$, is **[Online April 11, 2015]**

(a) 1 (b) 2
(c) 4 (d) 3

101. A plane containing the point (3, 2, 0) and the line $\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ also contains the point : **[Online April 11, 2015]**

(a) (0, 3, 1) (b) (0, 7, –10)
(c) (0, –3, 1) (d) 0, 7, 10

102. If the points (1, 1, λ) and (–3, 0, 1) are equidistant from the plane, $3x+4y-12z+13=0$, then λ satisfies the equation : **[Online April 10, 2015]**

(a) $3x^2+10x-13=0$ (b) $3x^2-10x+21=0$
(c) $3x^2-10x+7=0$ (d) $3x^2+10x-7=0$

103. If the shortest distance between the lines $\frac{x-1}{\alpha}=\frac{y+1}{-1}=\frac{z}{1}, (\alpha \neq -1)$ and $x+y+z+1=0=2x-y+z+3$ is $\frac{1}{\sqrt{3}}$, then a value α is : **[Online April 10, 2015]**

(a) $-\frac{16}{19}$ (b) $-\frac{19}{16}$
(c) $\frac{32}{19}$ (d) $\frac{19}{32}$

104. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line: **[2014]**

(a) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$
(b) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$
(c) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$
(d) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

105. If the angle between the line $2(x+1)=y=z+4$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is $\frac{\pi}{6}$, then the value of λ is: **[Online April 19, 2014]**

(a) $\frac{135}{7}$ (b) $\frac{45}{11}$
(c) $\frac{45}{7}$ (d) $\frac{135}{11}$

106. If the distance between planes, $4x-2y-4z+1=0$ and $4x-2y-4z+d=0$ is 7, then d is: **[Online April 12, 2014]**

(a) 41 or –42 (b) 42 or –43
(c) –41 or 43 (d) –42 or 44

107. A symmetrical form of the line of intersection of the planes $x=ay+b$ and $z=cy+d$ is **[Online April 12, 2014]**

(a) $\frac{x-b}{a}=\frac{y-1}{1}=\frac{z-d}{c}$
(b) $\frac{x-b-a}{a}=\frac{y-1}{1}=\frac{z-d-c}{c}$
(c) $\frac{x-a}{b}=\frac{y-0}{1}=\frac{z-c}{d}$
(d) $\frac{x-b-a}{b}=\frac{y-1}{0}=\frac{z-d-c}{d}$

108. The plane containing the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$ passes through the point: **[Online April 11, 2014]**

(a) $(1,-2,5)$ (b) $(1,0,5)$
(c) $(0,3,-5)$ (d) $(-1,-3,0)$

109. Equation of the plane which passes through the point of intersection of lines

$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and

$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

and has the largest distance from the origin is: **[Online April 9, 2014]**

(a) $7x+2y+4z=54$ (b) $3x+4y+5z=49$
(c) $4x+3y+5z=50$ (d) $5x+4y+3z=57$

110. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **[2013]**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$
(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

111. The equation of a plane through the line of intersection of the planes $x+2y=3$, $y-2z+1=0$, and perpendicular to the first plane is : **[Online April 25, 2013]**

(a) $2x-y-10z=9$ (b) $2x-y+7z=11$
(c) $2x-y+10z=11$ (d) $2x-y-9z=10$

112. Let Q be the foot of perpendicular from the origin to the plane $4x-3y+z+13=0$ and R be a point $(-1,-6)$ on the plane. Then length QR is : **[Online April 22, 2013]**

(a) $\sqrt{14}$ (b) $\sqrt{\frac{19}{2}}$
(c) $3\sqrt{\frac{7}{2}}$ (d) $\frac{3}{\sqrt{2}}$

113. A vector $\vec{n}$ is inclined to x-axis at 45°, to y-axis at 60° and at an acute angle to z-axis. If $\vec{n}$ is a normal to a plane passing through the point $(\sqrt{2},-1,1)$ then the equation of the plane is : **[Online April 9, 2013]**

(a) $4\sqrt{2}x+7y+z-2$ (b) $2x+y+2z=2\sqrt{2}+1$
(c) $3\sqrt{2}x-4y-3z=7$ (d) $\sqrt{2}x-y-z=2$

114. A equation of a plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is : **[2012]**

(a) $x-2y+2z-3=0$ (b) $x-2y+2z+1=0$
(c) $x-2y+2z-1=0$ (d) $x-2y+2z+5=0$

115. The equation of a plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point $(0,7,-7)$ is **[Online May 26, 2012]**

(a) $x+y+z=0$
(b) $x+2y+z=21$
(c) $3x-2y+5z+35=0$
(d) $3x+2y+5z+21=0$

116. Consider the following planes

$P: x+y-2z+7=0$
$Q: x+y+2z+2=0$
$R: 3x+3y-6z-11=0$ **[Online May 26, 2012]**

(a) P and R are perpendicular
(b) Q and R are perpendicular
(c) P and Q are parallel
(d) P and R are parallel

117. If the three planes $x=5$, $2x-5ay+3z-2=0$ and $3bx+y-3z=0$ contain a common line, then(a, b) is equal to **[Online May 19, 2012]**

(a) $\left(\frac{8}{15},-\frac{1}{5}\right)$ (b) $\left(\frac{1}{5},-\frac{8}{15}\right)$
(c) $\left(-\frac{8}{15},\frac{1}{5}\right)$ (d) $\left(-\frac{1}{5},\frac{8}{15}\right)$

118. A line with positive direction cosines passes through the point $P(2,-1,2)$ and makes equal angles with the coordinate axes. If the line meets the plane $2x+y+z=9$ at point Q, then the length PQ equals **[Online May 7, 2012]**

(a) $\sqrt{2}$ (b) 2
(c) $\sqrt{3}$ (d) 1

119. The values of a for which the two points $(1, a, 1)$ and $(-3, 0, a)$ lie on the opposite sides of the plane $3x+4y-12z+13=0$, satisfy **[Online May 7, 2012]**

(a) $0<a<\frac{1}{3}$ (b) $-1<a<0$
(c) $a<-1$ or $a<\frac{1}{3}$ (d) $a=0$

120. The distance of the point $(1,-5,9)$ from the plane $x-y+z=5$ measured along a straight $x=y=z$ is **[2011RS]**

(a) $10\sqrt{3}$ (b) $5\sqrt{3}$
(c) $3\sqrt{10}$ (d) $3\sqrt{5}$

121. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals **[2011]**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

122. **Statement -1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x-y+z=5$.
Statement -2: The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4). **[2010]**
(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

123. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$. Then (α, β) equals **[2009]**
(a) $(-6, 7)$ (b) $(5, -15)$
(c) $(-5, 5)$ (d) $(6, -17)$

124. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then **[2008]**
(a) $a=2, b=8$ (b) $a=4, b=6$
(c) $a=6, b=4$ (d) $a=8, b=2$

125. Let L be the line of intersection of the planes $2x+3y+z=1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then cos α equals **[2007]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$.

126. The image of the point (–1, 3, 4) in the plane $x-2y=0$ is **[2006]**

(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) $(15, 11, 4)$

(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) None of these

127. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(i-j+4k)$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ is **[2005]**

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$

(c) $\frac{3}{10}$ (d) $\frac{10}{3}$

128. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}\,z+4=0$ is such that $\sin\theta=\frac{1}{3}$ then the value of λ is **[2005]**

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

129. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **[2004]**

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{3}{2}$

130. Two system of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a', b', c' from the origin then **[2003]**

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(b) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(d) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$.

131. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x+y=3$ are **[2002]**

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$

(c) $1, 1, 2$ (d) $\sqrt{2}, 1, 1$

132. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is **[2002]**

(a) $x-y+z=1$ (b) $x+y+z=5$
(c) $x+2y-z=1$ (d) $2x-y+z=5$

TOPIC 4 Sphere and Miscellaneous Problems on Sphere

133. If (2, 3, 5) is one end of a diameter of the sphere $x^2+y^2+z^2-6x-12y-2z+20=0$, then the cooordinates of the other end of the diameter are **[2007]**

(a) (4, 3, 5) (b) (4, 3, –3)
(c) (4, 9, –3) (d) (4, –3, 3).

134. The plane $x+2y-z=4$ cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius **[2005]**

(a) 3 (b) 1
(c) 2 (d) $\sqrt{2}$

135. If the plane 2ax – 3ay + 4az + 6 = 0 passes through the midpoint of the line joining the centres of the spheres

$x^2+y^2+z^2+6x-8y-2z=13$ and

$x^2+y^2+z^2-10x+4y-2z=8$ then a equals **[2005]**

(a) – 1 (b) 1
(c) – 2 (d) 2

136. The intersection of the spheres

$x^2+y^2+z^2+7x-2y-z=13$ and

$x^2+y^2+z^2-3x+3y+4z=8$ is the same as the intersection of one of the sphere and the plane **[2004]**

(a) $2x-y-z=1$ (b) $x-2y-z=1$
(c) $x-y-2z=1$ (d) $x-y-z=1$

137. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is **[2003]**

(a) 4 (b) 1
(c) 2 (d) 3

138. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is **[2003]**

(a) 39 (b) 26
(c) $11\frac{4}{13}$ (d) 13.

Hints & Solutions

1. **(c)** Given, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}=p$ (let) and point P $(\beta, 0, \beta)$

Any point on line A $= (p, 1, -p-1)$

Now, DR of AP a” $< p-\beta, 1-0, -p-1-\beta >$

Which is perpendicular to line.

$\therefore (p-\beta)\,1+0.1-1\,(-p-1-\beta)=0$

$\Rightarrow p-\beta+p+1+\beta=0 \Rightarrow p=\frac{-1}{2}$

$\therefore$ Point $A\left(\frac{-1}{2}, 1-\frac{1}{2}\right)$

Given that distance AP $=\sqrt{\frac{3}{2}} \Rightarrow AP^2=\frac{3}{2}$

$$\Rightarrow\left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2=\frac{3}{2} \text{ or } 2\left(\beta+\frac{1}{2}\right)^2=\frac{1}{2}$$

$$\Rightarrow\left(\beta+\frac{1}{2}\right)^2=\frac{1}{4} \Rightarrow \beta=0,-1,(\beta\neq 0)$$

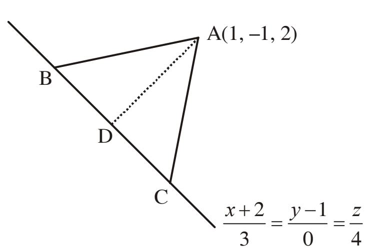
$\therefore \beta=-1$

2. **(d)** Let a point D on BC $=(3\lambda-2, 1, 4\lambda)$

$\overrightarrow{AD}=(3\lambda-3)\hat{i}+2\hat{j}+(4\lambda-2)\hat{k}$

$\because \overrightarrow{AD}\perp\overrightarrow{BC}$, $\therefore \overrightarrow{AD}.\overrightarrow{BC}=0$

$$\Rightarrow (3\lambda-3)+3+2(0)+(4\lambda-2)4=0 \Rightarrow \lambda=\frac{17}{25}$$



Hence, $D=\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$$\left|\overrightarrow{AD}\right|=\sqrt{\left(\frac{1}{25}-1\right)^2+(2)^2+\left(\frac{68}{25}-2\right)^2}$$

$$=\sqrt{\frac{(24)^2+4(25)^2+(18)^2}{25}}=\sqrt{\frac{3400}{25}}=\frac{2\sqrt{34}}{5}$$

Area of triangle $=\frac{1}{2}\times\left|\overrightarrow{BC}\right|\times\left|\overrightarrow{AD}\right|$

$$=\frac{1}{2}\times 5\times\frac{2\sqrt{34}}{5}=\sqrt{34} \qquad [\mapsto BC=5]$$

3. **(a)** Here, P, Q, R are collinear

$\therefore \overrightarrow{PR}=\lambda\overrightarrow{PQ}$

$2\hat{i}+(y+3)\hat{j}+(z-4)\hat{k}=\lambda[6\hat{i}+3\hat{j}+6\hat{k}]$

$\Rightarrow 6\lambda=2,\ y+3=3\lambda,\ z-4=6\lambda$

$\Rightarrow \lambda=\frac{1}{3}, y=-2, z=6$

$\therefore$ Point $R\,(4,-2,6)$

Now, OR $=\sqrt{(4)^2+(-2)^2+(6)^2}=\sqrt{56}=2\sqrt{14}$

4. **(b)** Let $\vec{v}_1$ and $\vec{v}_2$ be the vectors perpendicular to the plane OPQ and PQR respectively.

$$\vec{v}_1=\overrightarrow{PQ}\times\overrightarrow{OQ}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 2 & 1 & 3\end{vmatrix}$$

$=5\hat{i}-\hat{j}-3\hat{k}$

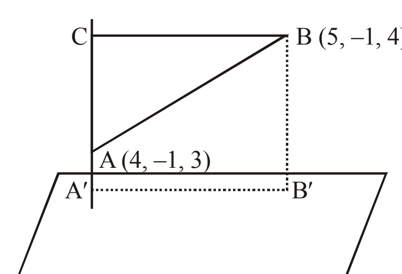
$$\vec{v}_2=\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ -2 & -1 & 1\end{vmatrix}$$

$=\hat{i}-5\hat{j}-3\hat{k}$

$$\because \quad \cos\theta=\frac{\vec{v}_1\cdot\vec{v}_2}{|\vec{v}_1||\vec{v}_2|}=\frac{5+5+9}{25+1+9}=\frac{19}{35}$$

$$\therefore \quad \theta=\cos^{-1}\left(\frac{19}{35}\right)$$

5. **(c)**



$$AC = \overrightarrow{AB}.\hat{AC} = \left(\hat{i}+\hat{k}\right).\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{2-\frac{4}{3}} = \sqrt{\frac{2}{3}}$

$\therefore$ Length of projection $= \sqrt{\frac{2}{3}}$

6. **(b)** Given

$l + 3m + 5n = 0$ (1)

and $5lm - 2mn + 6nl = 0$ (2)

From eq. (1) we have

$l = -3m - 5n$

Put the value of l in eq. (2), we get;

$5(-3m-5n)m - 2mn + 6n(-3m-5n) = 0$

$\Rightarrow 15m^2 + 45mn + 30n^2 = 0$

$\Rightarrow m^2 + 3mn + 2n^2 = 0$

$\Rightarrow m^2 + 2mn + mn + 2n^2 = 0$

$\Rightarrow (m+n)(m+2n) = 0$

$\therefore m = -n$ or $m = -2n$

For $m = -n, l = -2n$

And for $m = -2n, l = n$

$\therefore (l, m, n) = (-2n, -n, n)$ Or $(l, m, n) = (n, -2n, n)$

$\Rightarrow (l, m, n) = (-2, -1, 1)$ Or $(l, m, n) = (1, -2, 1)$

Therefore, angle between the lines is given as:

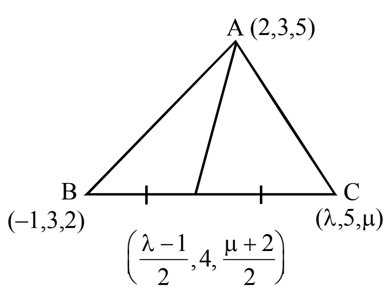
$$\cos(\theta) = \frac{(-2)(1) + (-1).(-2) + (1)(1)}{\sqrt{6}.\sqrt{6}}$$

$$\Rightarrow \cos(\theta) = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

7. **(b)** DR's of AD are $\frac{\lambda-1}{2} - 2, 4-3, \frac{\mu+2}{2} - 5$

i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

$\because$ This median is making equal angles with coordinate axes, therefore,



$$\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$$

$\Rightarrow \lambda = 7$ & $\mu = 10$

$\therefore \lambda^3 + \mu^3 = 5 = 1348$

8. **(c)** Given, $l + m + n = 0$ and $l^2 = m^2 + n^2$

Now, $(-m-n)^2 = m^2 + n^2$

$\Rightarrow mn = 0 \Rightarrow m = 0$ or $n = 0$

If $m = 0$ then $l = -n$

We know $l^2 + m^2 + n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

i.e. $(l_1, m_1, n_1) = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

If $n = 0$ then $l = -m$

$l^2 + m^2 + n^2 = 1 \Rightarrow 2m^2 = 1$

$\Rightarrow m = \pm\frac{1}{\sqrt{2}}$

Let $m = \frac{1}{\sqrt{2}}$

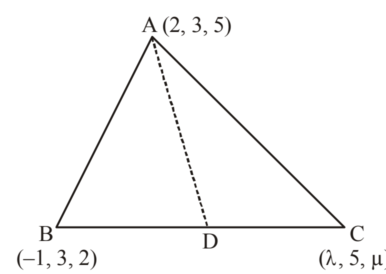
$\Rightarrow l = -\frac{1}{\sqrt{2}}$ and $n = 0$

$(l_2, m_2, n_2) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

9. **(c)** If D be the mid-point of BC, then

$$D = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$$



Direction ratios of AD are $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

Since median AD is equally inclined with coordinate axes, therefore direction ratios of AD will be equal, i.e,

$$\frac{\left(\frac{\lambda-5}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2} = \frac{1}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}$$

$$= \frac{\left(\frac{\mu-8}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}$$

$\Rightarrow \left(\frac{\lambda-5}{2}\right)^2 = 1 = \left(\frac{\mu-8}{2}\right)^2$

$\Rightarrow$ λ = 7, 3 and μ = 10, 6

If λ = 7 and μ = 10

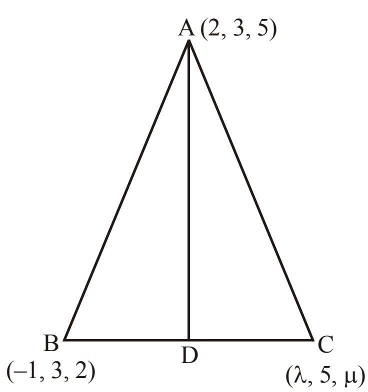
Then $\frac{\lambda}{\mu} = \frac{7}{10} \Rightarrow 10\lambda - 7\mu = 0$

10. **(c)** It makes θ with x and y-axes.
$l = \cos\theta$, $m = \cos\theta$, $n = \cos(\pi - 2\theta)$
we have $l^2 + m^2 + n^2 = 1$
$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2(\pi - 2\theta) = 1$
$\Rightarrow 2\cos^2\theta + (-\cos 2\theta)^2 = 1$
$\Rightarrow 2\cos^2\theta - 1 + \cos^2 2\theta = 0$
$\Rightarrow \cos 2\theta - [1 + \cos 2\theta] = 0$
$\Rightarrow \cos 2\theta = 0$ or $\cos 2\theta = -1$
$\Rightarrow 2\theta = \pi/2$ or $2\theta = \pi$

$\Rightarrow \theta = \pi/4$ or $\theta = \frac{\pi}{2}$

$\Rightarrow \theta = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

11. **(c)** Since AD is the median



$\therefore$ $D = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

Now, dR's of AD is

$a = \left(\frac{\lambda-1}{2} - 2\right) = \frac{\lambda-5}{2}$

$b = 4 - 3 = 1$, $c = \frac{\mu+2}{2} - 5 = \frac{\mu-8}{2}$

Also, a, b, c are dR's
$\therefore$ $a = kl, b = km, c = kn$ where $l = m = n$
and $l^2 + m^2 + n^2 = 1$

$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$

Now, $a = 1, b = 1$ and $c = 1$

$\Rightarrow \lambda = 7$ and $\mu = 10$

12. **(b)** Length of the line segment

$= \sqrt{(2)^2 + (3)^2 + (6)^2} = 7$

13. **(c)** Let l_1, m_1, n_1 and l_2, m_2, n_2 be the d.c of line 1 and 2 respectively, then as given

$l_1 + m_1 + n_1 = 0$
and $l_2 + m_2 + n_2 = 0$
and $l_1^2 + m_1^2 - n_1^2 = 0$ and
$l_2^2 + m_2^2 - n_2^2 = 0$
($\because$ $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$)
Angle between lines, θ is
$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ ...(1)
As given $l^2 + m^2 = n^2$ and $l + m = -n$
$\Rightarrow (-n)^2 - 2lm = n^2 \Rightarrow 2lm = 0$ or $lm = 0$
So $l_1 m_1 = 0, l_2 m_2 = 0$
If $l_1 = 0, m_1 \neq 0$ then $l_1 m_2 = 0$
If $m_1 = 0, l_1 \neq 0$ then $l_2 m_1 = 0$
If $l_2 = 0, m_2 \neq 0$ then $l_2 m_1 = 0$
If $m_2 = 0, l_2 \neq 0$ then $l_1 m_2 = 0$
Also $l_1 l_2 = 0$ and $m_1 m_2 = 0$
$l^2 + m^2 - n^2 = l^2 + m^2 + n^2 - 2n^2 = 0$

$\Rightarrow 1 - 2n^2 = 0 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore n_1 = \pm\frac{1}{\sqrt{2}}, n_2 = \pm\frac{1}{\sqrt{2}}$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$ (acute angle)

14. **(b)** As per question, direction cosines of the line :

$\ell = \cos 45° = \frac{1}{\sqrt{2}}$, $m = \cos 120° = \frac{-1}{2}$, $n = \cos\theta$

where θ is the angle, which line makes with positive z-axis.

We know that, $\ell^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{2}$ (θ being acute)

$\Rightarrow \theta = \frac{\pi}{3}$

15. **(b)** Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are 6, – 3, 2 then

$x_2 - x_1, = 6$; $y_2 - y_1 = -3$; $z_2 - z_1 = 2$

So that direction ratios of $\overrightarrow{PQ}$ are 6, – 3, 2

$\therefore$ Direction cosines of $\overrightarrow{PQ}$ are

$$\frac{6}{\sqrt{6^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+2^2}},$$

$$\frac{2}{\sqrt{6^2+(-3)^2+2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

16. **(b)** Let the line makes an angle θ with the positive direction of z-axis. Given that lines makes angle $\frac{\pi}{4}$ with x-axis and y-axis.

$\therefore\ l = \cos\frac{\pi}{4}, m = \cos\frac{\pi}{4}, n = \cos\theta$

We know that, $l^2 + m^2 + n^2 = 1$

$\therefore\ \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1$

$\Rightarrow\ \frac{1}{2} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow\ \cos^2\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

Hence, angle with positive direction of the z-axis is $\frac{\pi}{2}$.

17. **(c)** As per question the direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$

$\therefore\ \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

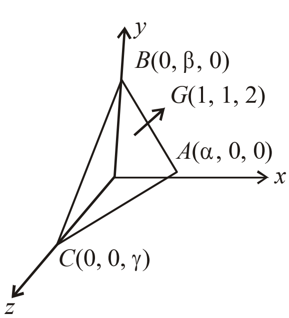
$\because\ 2\cos^2\theta = 1 - \cos^2\theta$

$\Rightarrow\ 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)

$\Rightarrow\ 2\cos^2\theta = 3 - 3\cos^2\theta$

$\therefore\ \cos^2\theta = \frac{3}{5}$

18. **(c)**



$\therefore\ \alpha = 3, \beta = 3$ and $\gamma = 6$ as G is centroid.

$\therefore$ The equation of plane is

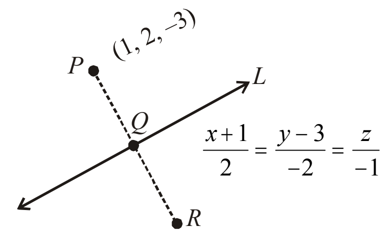
$$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$$

$$\Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{6}=1 \Rightarrow 2x+2y+z=6$$

$\therefore$ The required line is, $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

19. **(a)** $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}=\lambda$

Any point on line $= Q(2\lambda-1, -2\lambda+3, -\lambda)$



$\therefore$ D.r. of $PQ = [2\lambda-2, -2\lambda+1, -\lambda+3]$

D.r. of given line $= [2, -2, -1]$

$\because$ PQ is perpendicular to line L

$\therefore\ 2(2\lambda-2)-2(-2\lambda+1)-1(-\lambda+3)=0$

$\Rightarrow 4\lambda-4+4\lambda-2+\lambda-3=0$

$\Rightarrow 9\lambda-9=0 \Rightarrow \lambda=1$

$\because$ Q is mid point of $PR = Q = (1, 1, -1)$

$\therefore$ Coordinate of image $R = (1, 0, 1) = (a, b, c)$

$\therefore\ a+b+c=2.$

20. **(a)** $L_1 \equiv \vec{r} = (\hat{i}-\hat{j})+\ell(2\hat{i}+\hat{k})$

$L_2 \equiv \vec{r} = (2\hat{i}-\hat{j})+m(\hat{i}+\hat{j}-\hat{k})$

Equating coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$ of L_1 and L_2

$2\ell+1=m+2$ ...(i)

$-1=-1+m \Rightarrow m=0$ ...(ii)

$\ell=-m$ ...(iii)

$\Rightarrow m=\ell=0$ which is not satisfy eqn. (i) hence lines do not intersect for any value of ℓ and m.

21. **(c)** $\overrightarrow{AB}=6\hat{i}+15\hat{j}+3\hat{k}$

$\vec{p}=\hat{i}+4\hat{j}+22\hat{k}$

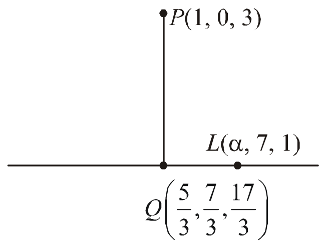
$\vec{q}=\hat{i}+\hat{j}+7\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix} i & j & k \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix}=6\hat{i}+15\hat{j}-3\hat{k}$$

Shortest distance between the lines is

$$=\frac{|\overrightarrow{AB}.(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}=\frac{|36+225+9|}{\sqrt{36+225+9}}=3\sqrt{30}$$

22. **(4)** Since, PQ is perpendicular to L

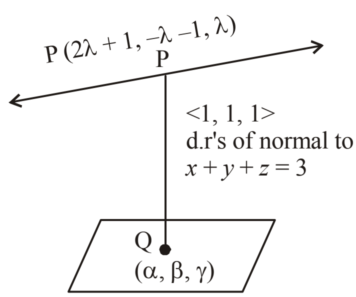


$$\therefore \quad \left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right)+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$$

$$\Rightarrow \quad \frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0$$

$$\Rightarrow \quad \frac{2\alpha}{3}=\frac{24}{9} \quad \Rightarrow \quad \alpha=4$$

23. **(b)**



Let co-ordinates of Q be (α, β, γ), then

$\alpha+\beta+\gamma=3$...(i)

$\alpha-\beta+\gamma=3$...(ii)

$\Rightarrow \alpha+\gamma=3$ and $\beta=0$

Equating direction ratio's of PQ, we get

$$\frac{\alpha-2\lambda-1}{1}=\frac{\lambda+1}{1}=\frac{\gamma-\lambda}{1}$$

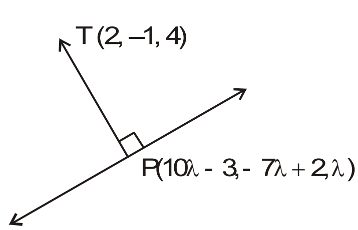
$\Rightarrow \alpha=3\lambda+2, \gamma=2\lambda+1$

Substituting the values of α and γ in equation (i), we get

$\Rightarrow 5\lambda+3=3 \Rightarrow \lambda=0$

Hence, point is Q (2, 0, 1)

24. **(a)** Let P be the foot of perpendicular from point $T(2, -1, 4)$ on the given line. So P can be assumed as $P(10\lambda-3, -7\lambda+2, \lambda)$



DR's of $TP \propto$ to $10\lambda-5, -7\lambda+3, \lambda-4$

$\because$ TP and given line are perpendicular, so

$10(10\lambda-5)-7(-7\lambda+3)+1(\lambda-4)=0$

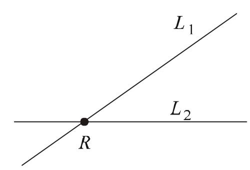
$$\Rightarrow \quad \lambda=\frac{1}{2}$$

$$\Rightarrow TP=\sqrt{(10\lambda-5)^2+(-7\lambda+3)^2+(\lambda-4)^2}$$

$$=\sqrt{0+\frac{1}{4}+\frac{49}{4}}=\sqrt{12.5}=3.54$$

Hence, the length of perpendicular is greater than 3 but less than 4.

25. **(a)** Let the coordinate of P with respect to line



$$\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}=\lambda$$

$$\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}=\mu$$

$L_1=(\lambda+3, 3\lambda-1, -\lambda+6)$

and coordinate of P w.r.t.

line $L_2=(7\mu-5, -6\mu+2, 4\mu+3)$

$\therefore \quad \lambda-7\mu=-8, 3\lambda+6\mu=3, \lambda+4\mu=3$

From above equation : $\lambda=-1, \mu=1$

$\therefore$ Coordinate of point of intersection $R=(2, -4, 7)$.

Image of R w.r.t. xy plane $=(2, -4, -7)$.

26. **(d)** First line is: $x=ay+b, z=cy+d$

$$\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$$

and another line is: $x=a'z+b', y=c'z+d'$

$$\Rightarrow \quad \frac{x-b'}{a'}=\frac{y-d'}{c'}=\frac{z}{1}$$

$\because$ Both lines are perpendicular to each other

$\therefore \quad aa'+c'+c=0$

27. **(d)** Let θ be the angle between the two lines

Here direction cosines of $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ are 2, 2, 1

Also second line can be written as:

$$\frac{x-5}{2}=\frac{y-2}{\frac{P}{7}}=\frac{z-3}{4}$$

$\therefore$ its direction cosines are $2, \frac{P}{7}, 4$

Also, $\cos\theta = \frac{2}{3}$ (Given)

$\because \quad \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

$\Rightarrow \frac{2}{3} = \left|\frac{(2\times 2)+\left(2\times\frac{P}{7}\right)+(1\times 4)}{\sqrt{2^2+2^2+1^2}\sqrt{2^2+\frac{P^2}{49}+4^2}}\right|$

$= \frac{4+\frac{2P}{7}+4}{3\times\sqrt{2^2+\frac{P^2}{49}+4^2}}$

$\Rightarrow \left(4+\frac{P}{7}\right)^2 = 20+\frac{P^2}{49} \Rightarrow 16+\frac{8P}{7}+\frac{P^2}{49} = 20+\frac{P^2}{49}$

$\Rightarrow \frac{8P}{7} = 4 \Rightarrow P = \frac{7}{2}$

28. **(c)** Lines are coplanar

$$\begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2(4-\lambda^4)+4(\lambda^2-2)=0$

$\Rightarrow 4-\lambda^4+2\lambda^2-4=0 \Rightarrow \lambda^2(\lambda^2-2)=0$

$\Rightarrow \lambda = 0, \sqrt{2}, -\sqrt{2}$

29. **(b)** Shortest distance between two lines

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

$\therefore$ The shortest distance between given lines are

$$\left|\frac{\begin{vmatrix} -2 & 4 & 5 \\ 2 & 2 & 1 \\ -1 & 8 & 4 \end{vmatrix}}{\sqrt{(8-8)^2+(-1-8)^2+(16+2)^2}}\right|$$

$= \left|\frac{0-36+90}{\sqrt{405}}\right| = \frac{54}{20.1} = 2.68$

30. **(b)** Let equation of the required line be

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ ...(i)

Given two lines

$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ ...(ii)

and $\frac{x-1}{0} = \frac{y+1}{0} = \frac{z}{1}$ ...(iii)

Since the line (i) is perpendicular to both the lines (ii) and (iii), therefore

$a - b + c = 0$ ...(iv)

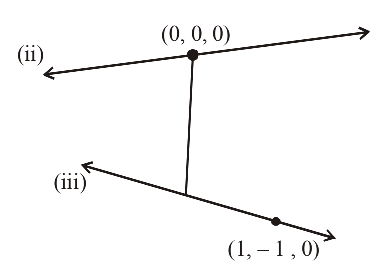
$-2b + c = 0$ ...(v)

From (iv) and (v) $c = 2b$ and $a + b = 0$, which are not satisfy by options (c) and (d). Hence options (c) and (d) are rejected.

Thus point (x_1, y_1, z_1) on the required line will be either $(0, 0, 0)$ or $(1, -1, 0)$.

Now foot of the perpendicular from point $(0, 0, 0)$ to the line (iii)

$= (1, -2r - 1, r)$



The direction ratios of the line joining the points $(0, 0, 0)$ and $(1, -2r - 1, r)$ are $1, -2r-1, r$

Since sum of the x and y-coordinate of direction ratio of the required line is 0.

$\therefore$ $1 - 2r - 1 = 0, \Rightarrow r = 0$

Hence direction ratio are 1, – 1, 0

But the z-direction ratio of the required line is twice the y-direction ratio of the required line

i.e. $0 = 2\,(-1)$, which is not true.

Hence the shortest line does not pass through the point (0, 0, 0). Therefore option (a) is also rejected.

31. **(c)** Given lines will be coplanar

$$\text{If}\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \;\; -1(1+2k) - (1+k^2) + 1(2-k) = 0$

$\Rightarrow \;\; k = 0, -3$

32. **(d)** For L_1,

$$x = \sqrt{\lambda}\,y + (\sqrt{\lambda} - 1) \;\Rightarrow\; y = \frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} \qquad \text{...(i)}$$

$$z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda} \;\Rightarrow\; y = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1} \qquad \text{...(ii)}$$

From (i) and (ii)

$$\frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} = \frac{y - 0}{1} = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1} \qquad \text{...(A)}$$

The equation (A) is the equation of line L_1.

Similarly equation of line L_2 is

$$\frac{x - (1 - \sqrt{\mu})}{\sqrt{\mu}} = \frac{y - 0}{1} = \frac{z - \sqrt{\mu}}{1 - \sqrt{\mu}} \qquad \text{...(B)}$$

Since $L_1 \perp L_2$, therefore

$\sqrt{\lambda}\,\sqrt{\mu} + 1 \times 1 + (\sqrt{\lambda} - 1)\,(1 - \sqrt{\mu}) = 0$

$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0 \;\Rightarrow\; \sqrt{\lambda} = -\sqrt{\mu}$

$\Rightarrow \lambda = \mu$

33. **(a)** Two given planes are coplanar, if

$$\begin{vmatrix} -2-(-1) & k-1 & 0-(-1) \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & k-1 & 1 \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$\Rightarrow (-1)\,(4-9) - (k-1)\,(8-6) + 6 - 2 = 0$

$\Rightarrow k = \dfrac{11}{2}$

34. **(c)** Given lines are $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$

and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1}$

$\therefore \;\; a_1 = \hat{i} - \hat{j} + \hat{k}\,,\; \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$a_2 = 3\hat{i} + k\hat{j}\,,\; \vec{b}_2 = \hat{i} + 2\hat{j} + \hat{k}$

Given lies are intersect if

$$\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1||\vec{b}_2|} = 0$$

$\Rightarrow \;\; (\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

$$\Rightarrow \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \;\; 2\,(3-8) - (k+1)\,(2-4) - 1\,(4-3) = 0$

$\Rightarrow \;\; 2\,(-5) - (k+1)\,(-2) - 1\,(1) = 0$

$\Rightarrow \;\; -10 + 2k + 2 - 1 = 0 \;\Rightarrow\; k = \dfrac{9}{2}$

35. **(c)** Point is (–1, 2, 6)

Line passes through the point (2, 3, –4) parallel to vector whose direction ratios is 6, 3, – 4.

Equation is $\dfrac{x-2}{6} = \dfrac{y-3}{3} = \dfrac{z+4}{-4} = \lambda$

Any point on this line is given by $x = 6\lambda + 2$, $y = 3\lambda + 3$, $z = -4\lambda - 4$

Now, d.Rs of line passing through (–1, 2, 6) and $\perp$ to this line is

$\{(x + 1), (y - 2), (z - 6)\}$

So, $6\,(x + 1) + 3\,(y - 2) - 4\,(z - 6) = 0$

$\Rightarrow 6x + 3y - 4z + 24 = 0$

Now, $6\,(6\lambda + 2) + 3\,(3\lambda + 3) + 4\,(4\lambda + 4) + 24 = 0$

$\Rightarrow 61\lambda + 61 = 0 \Rightarrow \lambda = -1$

So, $x = -4$, $y = 0$, $z = 0$

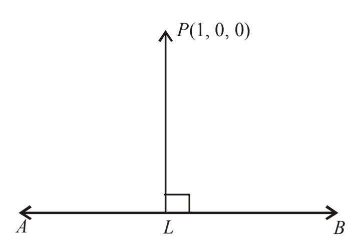
Now, distance between (–1, 2, 6) and (– 4, 0, 0) is $\sqrt{9+4+36} = \sqrt{49} = 7$

36. **(c)** On solving we will get shortest distance $\neq \sqrt{2}$

37. **(d)** Let the equation of AB is

$$\frac{x-1}{2} = \frac{y-(-1)}{-3} = \frac{z-(-10)}{8} = k$$

Let L be the foot of the perpendicular drawn from $P(1, 0, 0)$.



$\therefore \;\; L = (2k+1, -3k-1, 8k-10)$.

Now, direction ratio of $PL = (2k, -3k-1, 8k-10)$ and direction ratio of $AB = (2, -3, 8)$

Since, PL is perpendicular to AB

$\therefore \quad 2(2k)-3(-3k-1)+8(8k-10)=0$

Now, $k=\dfrac{2(1-1)+(-3)(0+1)+8(0+10)}{(2)^2+(-3)^2+(8)^2}$

$=\dfrac{0-3+80}{4+9+64}=\dfrac{77}{77}=1$

$\therefore$ Required co-ordinate $=L=(2+1,-3-1,8-10)$
$=(3,-4,-2)$.

38. **(c)** Any point on line $\dfrac{x}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}=\alpha$ is

$(2\alpha,3\alpha+2,4\alpha+3)$

$\Rightarrow$ Direction ratio of the $\perp$ line is
$2\alpha-3, 3\alpha+3, 4\alpha-8$. and
Direction ratio of the given line are 2, 3, 4

$\Rightarrow \quad 2(2\alpha-3)+3(3\alpha+3)+4(4\alpha-8)=0$

$\Rightarrow \quad 29\alpha-29=0$

$\Rightarrow \quad \alpha=1$

$\Rightarrow$ Foot of $\perp$ is $(2, 5, 7)$

$\Rightarrow$ Length $\perp$ is $\sqrt{1^2+6^2+4^2}=\sqrt{53}$

39. **(a)** The direction ratio of the line segment AB is 0, 6, –4 and the direction ratio of the given line is 1, 2, 3.
Clearly $1\times 0+2\times 6+3\times(-4)=0$
So, the given line is perpendicular to line AB.
Also, the mid point of A and B is (1, 3, 5) which satisfy the given line.
So, the image of B in the given line is A statement-1 and 2 both true but 2 is not correct explanation. of 1.

40. **(c)** Slope of line L $=-\dfrac{b}{5}$

Slope of line K $=-\dfrac{3}{c}$

Line L is parallel to line k.

$\Rightarrow \dfrac{b}{5}=\dfrac{3}{c} \Rightarrow bc=15$

(13, 32) is a point on L.

$\therefore \dfrac{13}{5}+\dfrac{32}{b}=1 \Rightarrow \dfrac{32}{b}=-\dfrac{8}{5}$

$\Rightarrow b=-20 \Rightarrow c=-\dfrac{3}{4}$

Equation of K :

$y-4x=3 \Rightarrow 4x-y+3=0$

Distance between L and K $=\dfrac{|52-32+3|}{\sqrt{17}}=\dfrac{23}{\sqrt{17}}$

41. **(a)** When the two lines intersect then shortest distance between them is zero i.e.

$\dfrac{(\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}=0$

$\Rightarrow \quad (\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2=0$

where $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}_1=k\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}_2=2\hat{i}+3\hat{j}+\hat{k}$, $\hat{b}_2=3\hat{i}+k\hat{j}+2\hat{k}$

$$\Rightarrow \begin{vmatrix}1 & 1 & -2\\ k & 2 & 3\\ 3 & k & 2\end{vmatrix}=0$$

$\Rightarrow \quad 1(4-3k)-1(2k-9)-2(k^2-6)=0$

$\Rightarrow \quad -2k^2-5k+25=0 \Rightarrow k=-5$ or $\dfrac{5}{2}$

$\because$ k is an integer, therefore $k=-5$

42. **(c)** a, b, c are in H.P. $\Rightarrow \dfrac{1}{a},\dfrac{1}{b},\dfrac{1}{c}$ are in A.P.

$\Rightarrow \dfrac{2}{b}=\dfrac{1}{a}+\dfrac{1}{c} \Rightarrow \dfrac{1}{a}-\dfrac{2}{b}+\dfrac{1}{c}=0$

$\therefore \quad \dfrac{x}{a}+\dfrac{y}{a}+\dfrac{1}{c}=0$ passes through $(1,-2)$

43. **(b)** The given lines are $2x=3y=-z$

or $\quad \dfrac{x}{3}=\dfrac{y}{2}=\dfrac{z}{-6}$ [Dividing by 6]

and $6x=-y=-4z$

or $\quad \dfrac{x}{2}=\dfrac{y}{-12}=\dfrac{z}{-3}$ [Dividing by 12]

$\therefore$ Angle between two lines is

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta=\frac{3.2+2.(-12)+(-6).(-3)}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}$$

$$=\frac{6-24+18}{\sqrt{49}\sqrt{157}}=0 \Rightarrow \theta=90^\circ$$

44. **(d)** The given lines are

$x-1=\dfrac{y+3}{-\lambda}=\dfrac{z-1}{\lambda}=s$ ....(1)

and $2x=y-1=\dfrac{z-2}{-1}=t$ ....(2)

The lines are coplanar, if

$$\begin{vmatrix}0-1 & 1-(-3) & 2-1\\ 1 & -\lambda & \lambda\\ \frac{1}{2} & 1 & -1\end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix}-1 & 4 & 1\\ 1 & -\lambda & \lambda\\ \frac{1}{2} & 1 & -1\end{vmatrix}=0$$

Apply $c_2 \to c_2 + c_3$; $\begin{vmatrix} -1 & 5 & 1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix} = 0$

$\Rightarrow -5\left(-1-\frac{\lambda}{2}\right) = 0 \Rightarrow \lambda = -2$

45. **(b)** Let a point on the line $x = y + a = z = \lambda$ is $(\lambda, \lambda - a, \lambda)$ and a point on the line

$x + a = 2y = 2z = \mu$ is $\left(\mu - a, \frac{\mu}{2}, \frac{\mu}{2}\right)$, then Direction ratio of the line joining these points are

$\lambda - \mu + a,\ \lambda - a - \frac{\mu}{2},\ \lambda - \frac{\mu}{2}$

If it respresents the required line whose $d \cdot r.$ be 2, 1, 2, then

$$\frac{\lambda - \mu + a}{2} = \frac{\lambda - a - \frac{\mu}{2}}{1} = \frac{\lambda - \frac{\mu}{2}}{2}$$

on solving we get $\lambda = 3a, \mu = 2a$

$\therefore$ The required points of intersection are

$(3a, 3a\text{-}a, 3a)$ and $\left(2a - a, \frac{2a}{2}, \frac{2a}{2}\right)$

or $(3a, 2a, 3a)$ and (a, a, a)

46. **(d)** Two planes are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix} = 0$$

$\Rightarrow\ 1[2 + 2k - (k+2)(1-k)] = 0$

$\Rightarrow\ 2 + 2k - (-k^2 - k + 2) = 0$

$k^2 + 3k = 0 \Rightarrow k(k+3) = 0$

or $k = 0$ or -3

47. **(a)** $\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}; \frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}.$

For perpendicularity of lines,

$aa' + 1 + cc' = 0$

48. **(b)** For line of intersection of planes $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$:

$$\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\hat{i} + \hat{j} - 3\hat{k}$$

Put $y = 0$, we get $x = -2$ and $z = 1$

$L_2 : \overline{r} = (-2\hat{i} + \hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$ and

$L_1 : \overline{r} = (\hat{i} - \hat{j}) + \mu(-\hat{j} + \hat{k})$ (Given)

Now, $\overline{b}_1 \times \overline{b}_2 = -2[\hat{i} + \hat{j} + \hat{k}]$ and $\overline{a}_2 - \overline{a}_1 = -3\hat{i} + \hat{j} + \hat{k}$

$\therefore$ Shortest distance $= \frac{1}{\sqrt{3}}$

49. **(b)** Since, lince are coplanar

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(-1 - 5 + \alpha) - 3(2 - \alpha) + 2(10 - 2\alpha + \alpha) = 0$

$\therefore \alpha = -4$

$\therefore$ Equation of $L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

$\therefore$ Point $(2, -10, -2)$ lies on line L_2.

50. **(3.00)**

Equation of plane P is

$(x + 4y - z + 7) + \lambda(3x + y + 5z - 8) = 0$

$\Rightarrow x(1 + 3\lambda) + y(4 + \lambda) + z(-1 + 5\lambda) + (7 - 8\lambda) = 0$

$\Rightarrow \frac{1+3\lambda}{a} = \frac{4+\lambda}{b} = \frac{5\lambda - 1}{6} = \frac{7 - 8\lambda}{-15}$

From last two ratios, $\lambda = -1$

$\Rightarrow \frac{-2}{a} = \frac{3}{b} = -1$

$\therefore a = 2, b = -3$

$\therefore$ Equation of plane is, $2x - 3y + 6z - 15 = 0$

Distance $= \frac{|6 - 6 - 6 - 15|}{7} = \frac{21}{7} = 3.$

51. **(b)** Equation of line through point $P(1, -2, 3)$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

So, any point on line $= Q(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

Since, this point lies on plane $x - y + 2 = 5$

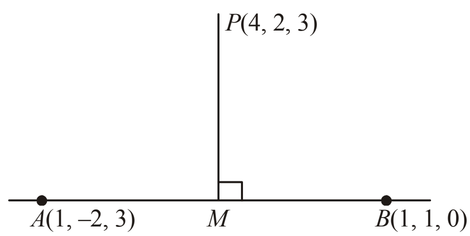
$\therefore 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow \lambda = \frac{1}{7}$

$\therefore$ Point of intersection line and plane, $Q = \left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$

$\therefore$ Required distance PQ

$$= \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(-\frac{11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2} = 1$$

52. **(a)** Equation of line through points $(1, -2, 3)$ and $(1, 1, 0)$ is



$$\frac{x-1}{0} = \frac{y-1}{-3} = \frac{z-0}{3-0} \; (=\lambda \text{ say})$$

$\therefore M(1, -\lambda+1, \lambda)$

Direction ratios of PM $= [-3, -\lambda-1, \lambda-3]$

$\because PM \perp AB$

$\therefore (-3)\cdot 0 + (-1-\lambda)(-1) + (\lambda-3)\cdot 1 = 0$

$\therefore \lambda = 1$

$\therefore$ Foot of perpendicular $= (1, 0, 1)$

This point satisfy the plane $2x + y - z = 1$.

53. **(c)** Direction ratios of normal to the plane are $<1, -3, 2>$.
Plane passes through $(3, 1, 1)$.
Equation of plane is,

$1(x-3) - 3(y-1) + 2(z-1) = 0$

$\Rightarrow x - 3y + 2z - 2 = 0$

54. **(5)**

Normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$

Direction ratios of normal to the plane $= <-1, 1, 1>$

Equation of plane

$-1(x-1) + 1(y-0) + 1(z-0) = 0$

$\Rightarrow x - y - z - 1 = 0$

If (x, y, z) is foot of perpendicular of $M(1, 0, 1)$ on the plane then

$$\Rightarrow \frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-0-1-1)}{3}$$

$\therefore x = \frac{4}{3}, y = -\frac{1}{3}, z = \frac{2}{3}$

$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$

$\therefore 3(\alpha + \beta + \gamma) = 3 \times \frac{5}{3} = 5.$

55. **(b)** Let plane passes through $(2, 1, 2)$ be

$a(x-2) + b(y-1) + (z-2) = 0$

It also passes through $(1, 2, 1)$

$\therefore -a + b - c = 0 \Rightarrow a - b + c = 0$

The given line is

$\frac{x}{3} = \frac{y}{2} = \frac{z-1}{0}$ is parallel to plane

$\therefore 3a + 2b + c(0) = 0$

$$\Rightarrow \frac{a}{0-2} = \frac{b}{3-0} = \frac{c}{2+3}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{2+3}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{-5}$$

$\therefore$ plane is $2x - 4 - 3y + 3 - 5z + 10 = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

The plane satisfies the point $(-2, 0, 1)$.

56. **(a)** $\because$ Plane contains two lines

$$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix}$$

$= \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$

$= -4\hat{i} + 5\hat{j} + 7\hat{k}$

So, equation of plane is

$-4(x-3) + 5(y-1) + 7(z-1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$

This also passes through $(\alpha, -3, 5)$

So, $-4\alpha - 15 + 35 = 0$

$\Rightarrow -4\alpha = -20 \Rightarrow \alpha = 5.$

57. **(b)** $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$

$\Rightarrow \quad (7\alpha+25)-(4\alpha+10)+(-20+14)=0$

$\Rightarrow \quad 3\alpha+9=0 \Rightarrow \alpha=-3$

Also, $D_z=0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix}=0$

$\Rightarrow 1(35-5\beta)-(15)+1(4\beta-7)=0 \Rightarrow \beta=13$

Hence, $\alpha+\beta=-3+13=10$

58. **(3)** Since, the line $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ contains the point $(-1, 3, -1)$ and line $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}$ contains the point $(-3, -2, 1)$.

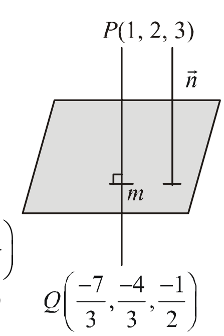
Then, the distance between the plane $23x-10y-2z+48=0$ and the plane containing the lines = perpendicular distance of plane $23x-10y-2z+48=0$ either from $(-1, 3, -1)$ or $(-3, -2, 1)$.

$$=\left|\frac{23(-1)-10(3)-2(-1)}{\sqrt{(23)^2+(10)^2+(-2)^2}}\right|=\frac{3}{\sqrt{633}}$$

It is given that distance between the planes

$$=\frac{k}{\sqrt{633}} \Rightarrow \frac{k}{\sqrt{633}}=\frac{3}{\sqrt{633}} \Rightarrow k=3$$

59. **(b)** $\vec{n}=\frac{-7}{3}-1, \frac{-4}{3}-2, \frac{-1}{3}-3$



$\vec{n}=\frac{10}{3}, \frac{10}{3}, \frac{10}{3}$

$D.r$ of normal to the plane $(1, 1, 1)$

Midpoint of P and Q is $\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

$\therefore$ Equation of required plane Q

$\vec{r}.\vec{n}=\vec{a}.\vec{n}$

$\vec{r}.(\hat{i}+\hat{j}+\hat{k})=\frac{-2}{3}+\frac{1}{3}+\frac{4}{3}$

$\therefore$ Equation of plane is $x+y+z=1$

60. **(b)** Equation of plane is $x+y-2z=3$

$\Rightarrow \quad \frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\frac{-2(2+1-12-3)}{6}$

$\Rightarrow \quad (x, y, z)=(6, 5, -2)$

61. **(d)** Let points $P(3\lambda+2, 2\lambda-1, -\lambda+1)$ and $Q(3\mu+2, 2\mu-1, -\mu+1)$

$\because P$ lies on $2x+3y-z+13=0$

$\therefore \quad 6\lambda+4+6\lambda-3+\lambda-1+13=0$

$\Rightarrow \quad 13\lambda=-13 \Rightarrow \lambda=-1$

Hence, $P(-1, -3, 2)$

Similarly, Q lies on $3x+y+4z=16$

$\therefore \quad 9\mu+6+2\mu-1-4\mu+4=16$

$\Rightarrow \quad 7\mu=7 \Rightarrow \mu=1$

Hence, Q is $(5, 1, 0)$

Now, $PQ=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

62. **(d)** The equations of angle bisectors are,

$$\frac{x+2y+2z-2}{3}=\pm\frac{2x-y+2z-4}{3}$$

$\Rightarrow \quad x-3y-2=0$

or $\quad 3x+y+4z-6=0$

$(2, -4, 1)$ lies on the second plane.

63. **(c)** The equation of plane containing two given lines is,

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}=0$$

On expanding, we get $x-y-z=0$

Now, the length of perpendicular from $(2, 1, 4)$ to this plane

$$=\left|\frac{2-1-4}{\sqrt{1^2+1^2+1^2}}\right|=\sqrt{3}$$

64. **(c)** Image of Q $(0, -1, -3)$ in plane is,

$$\frac{(x-0)}{3}=\frac{(y+1)}{-1}=\frac{z+3}{+4}=\frac{-2(1-12-2)}{9+1+16}=1$$

$\Rightarrow x=3, y=-2, z=1$

$\Rightarrow$ P $(3, -2, 1)$, Q $(0, -1, -3)$, R $(3, -1, -2)$

$\therefore$ Area of ΔPQR is

$$\frac{1}{2}|\vec{Q}P\times\vec{Q}R|=\frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 4 \\ 3 & 0 & 1 \end{matrix}\right\|$$

$$=\frac{1}{2}|\{\hat{i}(-1)-\hat{j}(3-12)+\hat{k}(3)\}|$$

$$=\frac{1}{2}\sqrt{(1+81+9)}=\frac{\sqrt{91}}{2}$$

65. **(d)** Let, $P_1: 2x-y+2z+3=0$

$P_2: 2x-y+2z+\frac{\lambda}{2}=0$

$P_3: 2x-y+2z+\mu=0$

Given, distance between P_1 and P_2 is $\frac{1}{3}$

$$\frac{1}{3}=\frac{\left|3-\frac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3-\frac{\lambda}{2}\right|=1 \Rightarrow \lambda_{max}=8$$

And distance between P_1 and P_3 is $\frac{2}{3}$

$$\frac{2}{3}=\frac{|\mu-3|}{\sqrt{9}} \Rightarrow \mu_{max}=5$$

$\Rightarrow (\lambda+\mu)_{max}=13$

66. **(c)** Let point on line be P $(2k+1, 3k-1, 4k+2)$

Since, point P lies on the plane $x+2y+3z=15$

$\therefore 2k+1+6k-2+12k+6=15$

$$\Rightarrow k=\frac{1}{2}$$

$$\therefore P \equiv \left(2, \frac{1}{2}, 4\right)$$

Then the distance of the point P from the origin is

$$OP=\sqrt{4+\frac{1}{4}+16}=\frac{9}{2}$$

67. **(d)** Let the required plane passing through the points $(0, -1, 0)$ and $(0, 0, 1)$ be $\frac{x}{\lambda}+\frac{y}{-1}+\frac{z}{1}=1$ and the given plane is $y-z+5=0$

$$\therefore \cos\frac{\pi}{4}=\frac{-1-1}{\sqrt{\left(\frac{1}{\lambda^2}+1+1\right)}\sqrt{2}}$$

$$\Rightarrow \lambda^2=\frac{1}{2} \Rightarrow \frac{1}{\lambda}=\pm\sqrt{2}$$

Then, the equation of plane is

$\pm\sqrt{2}x-y+z=1$

Then the point $(\sqrt{2}, 1, 4)$ satisfies the equation of plane

68. **(d)** Let the plane be

$P \equiv (2x+3y+z+5)+\lambda(x+y+z-6)=0$

$\because$ above plane is perpendicular to xy plane.

$$\therefore \left((2+\lambda)\hat{i}+(3+\lambda)\hat{j}+(1+\lambda)\hat{k}\right).\hat{k}=0 \Rightarrow \lambda=-1$$

Hence, the equation of the plane is,

$P \equiv x+2y+11=0$

Distance of the plane P from $(0, 0, 256)$

$$\left|\frac{0+0+11}{\sqrt{5}}\right|=\frac{11}{\sqrt{5}}$$

69. **(c)** Let the equation of required plane be;

$(2x-y-4)+\lambda(y+2z-4)=0$

$\because$ This plane passes through the point $(1, 1, 0)$ then $(2-1-4)+\lambda(1+0-4)=0$

$\Rightarrow \lambda=-1$

Then, equation of required plane is,

$(2x-y-4)-(y+2z-4)=0$

$\Rightarrow \quad 2x-2y-2z=0 \Rightarrow x-y-z=0$

70. **(d)** Equation of the plane passing through the line of intersection of $x+y+z=1$ and $2x+3y+4z=5$ is

$(2x+3y+4z-5)+\lambda(x+y+z-1)=0$

$\Rightarrow (2+\lambda)x+(3+\lambda)y+(4+\lambda)z+(-5-\lambda)=0 \quad$ ...(i)

$\because$ plane (i) is perpendicular to the plane $x-y+z=0$

$\therefore (2+\lambda)(1)+(3+\lambda)(-1)+(4+\lambda)(1)=0$

$2+\lambda-3-\lambda+4+\lambda=0 \Rightarrow \lambda=-3$

Hence, equation of required plane is

$-x+z-2=0$ or $x-z+2=0$

$\Rightarrow \bar{r}.(\hat{i}-\hat{k})+2=0$

71. **(b)** $\because$ plane containing both lines.

$$\therefore \quad \text{D.R. of plane}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 5 & 7\\ 1 & 4 & 7\end{vmatrix}=7\hat{i}-14\hat{j}+7\hat{k}$$

Now, equation of plane is,

$7(x-1)-14(y-4)+7(z+4)=0$

$\Rightarrow \quad x-1-2y+8+z+4=0$

$\Rightarrow \quad x-2y+z+11=0$

Hence, distance from $(0, 0, 0)$ to the plane,

$$=\frac{11}{\sqrt{1+4+1}}=\frac{11}{\sqrt{6}}$$

72. **(a)** Let angle between line and plane is θ, then

$$\sin\theta=\left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|$$

$$=\left|\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}-K\hat{k})}{\sqrt{9}\cdot\sqrt{1+4+K^2}}\right|$$

$$=\left|\frac{2-2\_2K}{3\sqrt{5+K^2}}\right|=\frac{2|K|}{3\sqrt{4+K^2}}$$

Since, $\cos\theta=\frac{2\sqrt{2}}{3} \Rightarrow \sin\theta=\frac{1}{3}$

Then, $\frac{2|K|}{3\sqrt{5+K^2}}=\frac{1}{3} \Rightarrow 4K^2=5+K^2$

$$3K^2=5 \Rightarrow K=\pm\sqrt{\frac{5}{3}}$$

73. **(b)** Let $A(-\lambda^2, 1, 1), B(1, -\lambda^2, 1), C(1, 1, -\lambda^2), D(-1, -1, 1)$ lie on same plane, then

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & 1-\lambda^2 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$\Rightarrow \ (\lambda^2+1)((1-\lambda^2)^2-4)=0$

$\Rightarrow \ (3-\lambda^2)(\lambda^2+1)=0 \Rightarrow \lambda^2=3$

$\lambda = \pm\sqrt{3}$

Hence, $S = \{-\sqrt{3}, \sqrt{3}\}$

74. **(d)** Let normal to the required plane is $\vec{n}$

$\Rightarrow$ $\vec{n}$ is perpendicular to both vector $2\hat{i}-\hat{j}+3\hat{k}$ and $2\hat{i}+3\hat{j}-3\hat{k}$.

$$\Rightarrow \ \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -8\hat{i}+8\hat{j}+8\hat{k}$$

$\Rightarrow$ Equation of the required plane is

$\Rightarrow \ (x-3)(-8)+(y+2)\times 8+(z-1)\times 8=0$

$\Rightarrow \ (x-3)(-1)+(y+2)\times 1+(z-1)\times 1=0$

$\Rightarrow \ x-3-y-2-z+1=0$

$\because \ x-y-z=4$ passes through $(2, 0, -2)$

$\therefore$ plane contains $(2, 0, -2)$.

75. **(b, c)** Let the d.r's of the normal be $\langle a, b, c\rangle$

Equation of the plane is

$a(x-0)+b(y+1)+c(z-0)=0$

$\because$ It passes through $(0, 0, 1)$

$\therefore \ b+c=0$

Also $\dfrac{0\cdot a+b-c}{\sqrt{a^2+b^2+c^2\cdot\sqrt{2}}} = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \ b-c=\sqrt{a^2+b^2+c^2}$

And $b+c=0$

$\Rightarrow \ b=\pm\dfrac{1}{\sqrt{2}}a.$

$\therefore$ The d.r's are $\sqrt{2}, 1, -1$ or $2, \sqrt{2}, -\sqrt{2}$

76. **(b)** Let the normal to the required plane is $\vec{n}$, then

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 20\hat{i}+8\hat{j}-12$$

$\therefore$ Equation of the plane

$(x-3)\times 20+(y-4)\times 8+(z-2)\times(-12)=0$

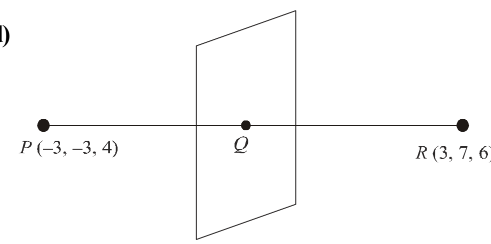
$5x-15+2y-8-3z+6=0$

$5x+2y-3z-17=0 \ \ldots(1)$

Since, equation of plane (1) passes through $(2, \alpha, \beta)$, then

$10+2\alpha-3\beta-17=0 \Rightarrow 2\alpha-3\beta=7$

77. **(d)**



$Q\,(0, 2, 5)$

Since, direction ratios of normal to the plane

is $\vec{n}=6\hat{i}+10\hat{j}+2\hat{k}$

Then, equation of the plane is

$(x-0)6+(y-2)10+(z-5)2=0$

$3x+5y-10+z-5=0$

$3x+5y+z=15 \qquad \ldots(1)$

Since, plane (1) satisfies the point $(4, 1, -2)$

Hence, required point is $(4, 1, -2)$

78. **(c)** Let any point on the line $\dfrac{x-4}{2}=\dfrac{y-5}{2}=\dfrac{z-3}{1}$ be $A(2\lambda+4, 2\lambda+5, \lambda+3)$ which lies on the plane $x+y+z=2$

$\Rightarrow \ 2\lambda+4+2\lambda+5+\lambda+3=2$

$\Rightarrow \ 5\lambda=-10 \Rightarrow \lambda=-2$

Then, the point of intersection is $(0, 1, 1)$

which lies on the line $\dfrac{x-1}{1}=\dfrac{y-3}{2}=\dfrac{z+4}{-5}$

79. **(d)** Since the system of linear equations are

$x+y+z=2 \ \ldots(1)$

$2x+3y+2z=5 \ \ldots(2)$

$2x+3y+(a^2-1)z=a+1 \ \ldots(3)$

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$$

(Applying $R_3 \to R_3 - R_2$)

$$\Rightarrow \ \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix}$$

$=a^2-3$

When, $\Delta=0 \Rightarrow a^2-3=0 \Rightarrow |a|=\sqrt{3}$

If $a^2 = 3$, then plane represented by eqn (2) and eqn (3) are parallel.

Hence, the given system of equation is inconsistent.

80. **(c)** Let any point on the intersecting line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda \text{ (say)}$$

is $(-3\lambda - 1, 2\lambda + 3, -\lambda + 2)$

Since, the above point lies on a line which passes through the point $(-4, 3, 1)$

Then, direction ratio of the required line

$= <-3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1>$

or $<-3\lambda + 3, 2\lambda, -\lambda + 1>$

Since, line is parallel to the plane

$x + 2y - z - 5 = 0$

Then, perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$

Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda+1)(-1) = 0$

$\Rightarrow \quad \lambda = -1$

Now direction ratio of the required line $= <6, -2, 2>$ or $<3, -1, 1>$

Hence required equation of the line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

81. **(d)** Since, equation of plane through intersection of planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(2x + 3y - z + 4) + \lambda(x + y + z - 1) = 0$

$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ ...(1)

But, the above plane is parallel to y-axis then

$(2 + \lambda) \times 0 + (3 + \lambda) \times 1 + (-1 + \lambda) \times 0 = 0$

$\Rightarrow \lambda = -3$

Hence, the equation of required plane is

$-x - 4z + 7 = 0$

$\Rightarrow x + 4z - 7 = 0$

Therefore, (3, 2, 1) the passes through the point.

82. **(a)** Let the direction ratios of the plane containing lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is $<a, b, c>$

$\therefore \quad 3a + 4b + 2c = 0$

$\quad 4a + 2b + 3c = 0$

$$\therefore \quad \frac{a}{12-4} = \frac{b}{8-9} = \frac{c}{6-16}$$

$$\frac{a}{8} = \frac{b}{-1} = \frac{c}{-10}$$

$\therefore \quad$ Direction ratio of plane $= <-8, 1, 10>$.

Let the direction ratio of required plane is $<l, m, n>$

Then $-8l + m + 10n = 0$ ...(1)

and $2l + 3m + 4n = 0$ ...(2)

From (1) and (2),

$$\frac{l}{-26} = \frac{m}{52} = \frac{n}{-26}$$

$\therefore \quad$ D.R.s are $<1, -2, 1>$

$\therefore \quad$ Equation of plane: $x - 2y + z = 0$

83. **(a)** Equation of plane passing through the line of intersection of first two planes is:

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

or $x(\lambda + 2) - y(2 + \lambda) + z(\lambda + 3) + (\lambda - 2) = 0$ ...(i)

is having infinite number of solution with $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$, then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 5$

Now put $\lambda = 5$ in (i), we get

$7x - 7y + 8z + 3 = 0$

Now perpendicular distance from (0, 0, 0) to the place containing L_1 and $L_2 = \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

84. **(c)** Equation of plane passing through three given points is:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 1+2 & -1+2 & 2-2 \\ 1+2 & 1+2 & 1-2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 3 & 1 & 0 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow -x + 3y + 6z - 8 = 0$

$$\Rightarrow \frac{x}{8} - \frac{3y}{8} - \frac{6z}{8} + \frac{8}{8} = 0$$

$$\Rightarrow \frac{x}{8} - \frac{y}{\frac{8}{3}} - \frac{z}{\frac{8}{6}} = -1$$

$$\Rightarrow \frac{x}{-8} + \frac{y}{\frac{8}{3}} + \frac{z}{\frac{8}{6}} = 1$$

$\therefore$ Sum of intercepts $= -8+\frac{8}{3}+\frac{8}{6}=-4$

85. **(c)** If a, b, c are the intercepts of the variable plane on the x, y, z axes respectively, then the equation of the plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

And the point of intersection of the planes parallel to the xy, yz and zx planes is (a, b, c).

As the point (3, 2, 1) lies on the variable plane, so

$$\frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$$

Therefore, the required locus is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

86. **(d)** Normal to $3x+4y+z=1$ is $3\hat{i}+4\hat{j}+\hat{k}$.

Normal to $5x+8y+2z=-14$ is $5\hat{i}+8\hat{j}+2\hat{k}$

The line of intersection of the planes is perpendicular to both normals, so, direction ratios of the intersection line are directly proportional to the cross product of the normal vectors.

Therefore the direction ratios of the line is $-\hat{j}+4\hat{k}$

Hence the angle between the plane $x+y+z+5=0$ and the intersection line is $\sin^{-1}\left(\frac{-1+4}{\sqrt{17}\sqrt{3}}\right)=\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

87. **(a)** Since the plane bisects the line joining the points (1, 2, 3) and (–3, 4, 5) then the plane passes through the midpoint of the line which is :

$$\left(\frac{1-3}{2},\frac{2+4}{2},\frac{5+3}{2}\right)\equiv\left(\frac{-2}{2},\frac{6}{2},\frac{8}{2}\right)\equiv(-1,3,4).$$

As plane cuts the line segment at right angle, so the direction cosines of the normal of the plane are $(-3-1, 4-2, 5-3)=(-4, 2, 2)$

So the equation of the plane is : $-4x+2y+2z=\lambda$

As plane passes through $(-1, 3, 4)$ so

$-4(-1)+2(3)+2(4)=\lambda \Rightarrow \lambda=18$

Therefore, equation of plane is : $-4x+2y+2z=18$

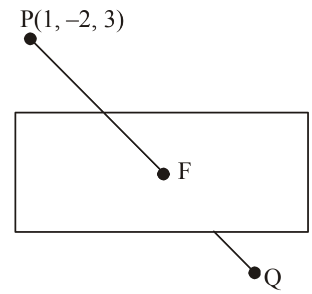
Now, only $(-3, 2, 1)$ satisfies the given plane as

$-4(-3)+2(2)+2(1)=18$

88. **(c)** Equation of line PQ is

$$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}$$

Let F be $(\lambda+1, 4\lambda-2, 5\lambda+3)$



Since F lies on the plane

$\therefore\ 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$

$2\lambda+2+12\lambda-6-20\lambda-12+22=0$

$\Rightarrow\ -6\lambda+6=0 \Rightarrow \lambda=1$

$\therefore$ F is (2, 2, 8)

$PQ=2\,PF=2\sqrt{1^2+4^2+5^2}=2\sqrt{42}$

89. **(c)** Let the plane be

$a(x-1)+b(y+1)+c(z+1)=0$

Normal vector

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 2 & -1 & -1\end{vmatrix}=5\hat{i}+7\hat{j}+3\hat{k}$$

So plane is $5(x-1)+7(y+1)+3(z+1)=0$

$\Rightarrow\ 5x+7y+3z+5=0$

Distance of point $(1, 3, -7)$ from the plane is

$$\frac{5+21-21+5}{\sqrt{25+49+9}}=\frac{10}{\sqrt{83}}$$

90. **(c)** $x+8y+7z=0$

$9x+2y+3z=0$

$x+y+z=0$

$x=\lambda \quad y=6\lambda \quad z=-7\lambda$

$x=\lambda \quad y=6\lambda \quad z=-7\lambda$

$$\text{Now,}\quad \begin{array}{c|c} \lambda+12\lambda-7\lambda=6 & \therefore\ 2\lambda+6\lambda-7\lambda \\ 6\lambda=6 & =\lambda \\ \lambda=1 & =1 \end{array}$$

91. **(a)** Suppose centroid be (h, k, ℓ)

$\therefore$ x – intp $=3h$, y – intp $=3k$, z – intp $=3\ell$

Equation $\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3\ell}=1$

$\therefore$ Distance from (0, 0, 0)

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9\ell^2}}}\right|=3$$

$\Rightarrow \boxed{\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1}$

92. **(c)** Point $(3, -2, -\lambda)$ on p line $2x-4y+3z-2=0$
$=6+8-3\lambda-2=0=3\lambda=12$

$\boxed{\lambda=4}$

Now,

$$\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+4}{-2}=k_1 \quad ...(i)$$

$$\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}=k_2 \quad ...(ii)$$

Point on equation (i) P $(k_1+3, -k_1-2, -2k_1-4)$
Point on equation (ii) Q $(12k_2+1, 9k_2, 4k_2)$
$k_1+3=12k_2+1 \mid -k_1-2=9k_2 \mid -2k_1-4=4k_2$
$k_2=0$
$k_1=-2$
p (1, 0, 0) lie on equation of a line 1
gives shortest distance = 0

93. **(c)** $\vec{n}=\vec{n_1}\times\vec{n_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & 7 & 8\\ 3 & 5 & 7\end{vmatrix}=(9,-18,9)=(1,-2,1)$

$\therefore$ Equation of plane is
$1(x+1)-2(y-1)+(z-3)=0$
$\Rightarrow x-2y+z=0$
foot to z

$$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-1}{1}=-\frac{[1+4+1]}{6}$$

$\boxed{x=0, y=0, z=0}$

94. **(c)** $\vec{n}=\vec{n_1}\times\vec{n_2}$

$$\Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ 1 & 4 & -2\end{vmatrix}=\hat{i}(-2)-\hat{j}(-7)+\hat{k}(13)$$

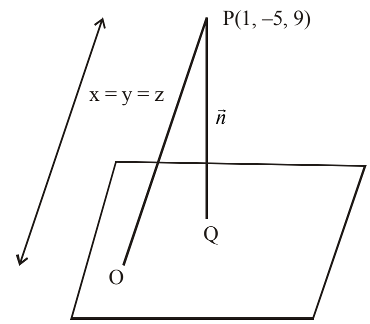
$\Rightarrow \vec{n}=-2\hat{i}+7\hat{j}+13\hat{k}$

Now,
$3x-y+z=1$
$x+4y-2z=2$
but $z=0$ & solving the given
$x=6/13$ & $y=5/13$
$\therefore$ required equation of a line is

$$\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}$$

95. **(b)** Line lies in the plane $\Rightarrow (3, -2, -4)$ lie in the plane
$\Rightarrow 3\ell-2m+4=9$ or $3\ell-2m=5$ ....(1)
Also, $\ell, m, -1$ are dr's of line perpendicular to plane and 2, -1, 3 are dr's of line lying in the plane
$\Rightarrow 2\ell-m-3=0$ or $2\ell-m=3$ ....(2)
Solving (1) and (2) we get $\ell=1$ and $m=-1$
$\Rightarrow \ell^2+m^2=2$.

96. **(d)**



Eq^n of PO : $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$
$\Rightarrow x=\lambda+1;\ y=\lambda-5; z=\lambda+9.$
Putting these in eq^n of plane :
$\lambda+1-\lambda+5+\lambda+9=5$
$\Rightarrow \lambda=-10$
$\Rightarrow$ O is $(-9, -15, -1)$
$\Rightarrow$ distance OP $=10\sqrt{3}$

97. **(c)** Let equation of plane be
$a(x-1)+b(y-2)+c(z-2)=0$ .....(1)
(1) is perpendicular to given planes then
$a-b+2c=0$
$2a-2b+c=0$
Solving above equation $c=0$ and $a=b$
equation of plane (1) can be
$x+y-3=0$
distance from $(1, -2, 4)$ will be

$$D=\frac{|1-2-3|}{\sqrt{1+1}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$$

98. **(a)** Equation of the plane containing the lines
$2x-5y+z=3$ and $x+y+4z=5$ is
$2x-5y+z-3+\lambda(x+y+4z-5)=0$
$\Rightarrow (2+\lambda)x+(-5+\lambda)y+(1+4\lambda)z+(-3-5\lambda)=0$ ...(i)
Since the plane (i) parallel to the given plane
$x+3y+6z=1$

$$\therefore \frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$$

$$\Rightarrow \lambda=-\frac{11}{2}$$

Hence equation of the required plane is

$$\left(2-\frac{11}{2}\right)x+\left(-5-\frac{11}{2}\right)y+\left(1-\frac{44}{2}\right)z+\left(-3+\frac{55}{2}\right)=0$$

$\Rightarrow (4-11)x+(-10-11)y+(2-44)z+(-6+55)=0$

$\Rightarrow -7x-21y-42z+49=0$

$\Rightarrow x+3y+6z-7=0$

$\Rightarrow x+3y+6z=7$

99. (b) General point on given line $\equiv P(3r+2, 4r-1, 12r+2)$

Point P must satisfy equation of plane

$(3r+2)-(4r-1)+(12r+2)=16$

$11r+5=16$

$r=1$

$P(3\times 1+2, 4\times 1-1, 12\times 1+2)=P(5, 3, 14)$

distance between P and (1, 0, 2)

$D=\sqrt{(5-1)^2+3^2+(14-2)^2}=13$

100. (b) The equation of any plane passing through given line is

$(x+y+2z-3)+\lambda(2x+3y+4z-4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(2+4\lambda)z-(3+4\lambda)=0$

If this plane is parallel to z–axis then normal to the plane will be perpendicular to z–axis.

$\therefore (1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1)=0$

$\lambda=-\frac{1}{2}$

Thus, Required plane is

$(x+y+2z-3)-\frac{1}{2}(2x+3y+4z-4)=0 \Rightarrow y+2=0$

$\therefore S.D=\frac{2}{\sqrt{(1)^2}}=2$

101. (c) Equation of the plane containing the given line

$\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ is

$A(x-1)+B(y-2)+C(z-3)=0$ ...(i)

where $A+5B+4C=0$ ...(ii)

Since the point (3, 2, 0) contains in the plane (i), therefore

$2A+0.B-3C=0$ ...(iii)

From equations (ii) and (iii),

$\frac{A}{-15-0}=\frac{B}{6+3}=\frac{C}{0-10}=k$ (let)

$\Rightarrow A=-15k, B=9k$ and $C=-10k$

Putting the value of A, B and C in equation (i), we get

$-15(x-1)+9(y-2)-10(z-3)=0$...(iv)

Now the coordinates of the point (0, –3, 1) satisfy the equation of the plane (iv) as

$-15(0-1)+9(-3-2)-10(1-3)$

$=15-45+20=0$

Hence the point (0, –3, 1) contains in the plane.

102. (c) $|3+4-12\lambda+13|=|-9+0-12+13|$

$\Rightarrow |-12\lambda+20|=|8| \Rightarrow |3\lambda-5|=2$

$\Rightarrow 9\lambda^2+25-30\lambda=4 \Rightarrow 9\lambda^2-30\lambda+21=0$

$\Rightarrow 3\lambda^2-10\lambda+7=0$

103. (c) Plane passing through $x+y+z+1=0$ and $2x-y+z+3=0$ is $x+y+z+1+\lambda(2x-y+z+3)=0$

$\Rightarrow (2\lambda+1)x+(1-\lambda)y+(1+\lambda)z+3\lambda+1=0$

Parallel to the given line if

$\alpha(2\lambda+1)-1(1-\lambda)+1(1+\lambda)=0$

$\Rightarrow \alpha=\frac{-2\lambda}{2\lambda+1}$ ...(i)

Also, $\left|\frac{2\lambda+1-(1-\lambda)+0+3\lambda+1}{\sqrt{(2\lambda+1)^2+(1-\lambda)^2+(1+\lambda)^2}}\right|=\frac{1}{\sqrt{3}}$

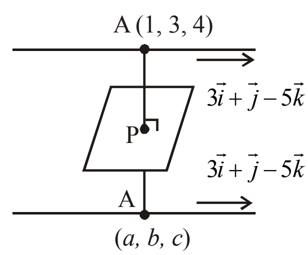
$\Rightarrow \lambda=0, \frac{-32}{102}$; $\alpha=0$ or $\alpha=\frac{32}{19}$

104. (c) $\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda$(let)

$\Rightarrow a=2\lambda+1$

$b=3-\lambda$

$c=4+\lambda$



$P=\left(\frac{a+1}{2},\frac{b+3}{2},\frac{c+4}{2}\right)=\left(\lambda+1,\frac{6-\lambda}{2},\frac{\lambda+8}{2}\right)$

$\therefore \quad 2(\lambda+1)-\frac{6-\lambda}{2}+\frac{\lambda+8}{2}+3=0$

$3\lambda+6=0 \Rightarrow \lambda=-2$

$a=-3, b=5, c=2$

Required line is $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

105. (c) Given equation of line can be written as

$\frac{x+1}{1}=\frac{y}{2}=\frac{z+4}{2}$

Eqn of plane is $2x-y+\sqrt{\lambda}z+4=0$

Since, angle between the line and the plane is $\frac{\pi}{6}$ therefore

$\sin\frac{\pi}{6}=\frac{2(1)+2(-1)+2(\sqrt{\lambda})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$

$$\frac{1}{2}=\frac{2-2+2\sqrt{\lambda}}{\sqrt{9}\sqrt{5+\lambda}}$$

$$\Rightarrow \frac{\sqrt{\lambda}}{\sqrt{5+\lambda}}=\frac{3}{4}\Rightarrow\frac{\lambda}{5+\lambda}=\frac{9}{16}$$

$$\Rightarrow 7\lambda = 45 \Rightarrow \lambda=\frac{45}{7}$$

106. **(c)** Given planes are

$4x-2y-4z+1=0$

and $4x-2y-4z+d=0$

They are parallel.

Distance between them is $\pm 7=\dfrac{d-1}{\sqrt{16+4+16}}$

$\Rightarrow \dfrac{d-1}{6}=\pm 7 \Rightarrow d = 42 + 1$

or $-42 + 1$ i.e. $d = -41$ or 43.

107. **(b)** Given two planes :

$x - ay - b = 0$ and $cy - z + d = 0$

Let, l, m, n be the direction ratio of the required line. Since the required line is perpendicular to normal of both the plane, therefore $l - am = 0$ and $cm - n = 0$

$\Rightarrow l - am + 0.n = 0$ and $0.l + cm - n = 0$

$\therefore \dfrac{l}{a-0}=\dfrac{m}{0+1}=\dfrac{n}{c-0}$

Hence, $d.R$ of the required line are a, 1, c.

Hence, options (c) and (d) are rejected.

Now, the point $(a + b, 1, c + d)$ satisfy the equation of the two given planes.

$\therefore$ Option (b) is correct.

108. **(b)** Equation of the plane containing the line

$\dfrac{x-1}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$ is

$a(x - 1) + b(y - 2) + c(z - 3) = 0$ .... (i)

where $a.1 + b.2 + c.3 = 0$

i.e., $a + 2b + 3c = 0$ .... (ii)

Since the plane (i) parallel to the line

$\dfrac{x}{1}=\dfrac{y}{1}=\dfrac{z}{4}$

$\therefore$ $a.1 + b.1 + c.4 = 0$

i.e., $a + b + 4c = 0$ ...... (iii)

From (ii) and (iii),

$\dfrac{a}{8-3}=\dfrac{b}{3-4}=\dfrac{c}{1-2}=k$ (let)

$\therefore$ $a = 5k$, $b = -k$, $c = -k$

On putting the value of a, b and c in equation (i),

$5(x - 1) - (y - 2) - (z - 3) = 0$

$\Rightarrow 5x - y - z = 0$ .... (iv)

when $x = 1$, $y = 0$ and $z = 5$; then

L.H.S. of equation (iv) $= 5x - y - 2$

$= 5 \times 1 - 0 - 5 = 0$

= R.H.S. of equation (iv)

Hence coordinates of the point (1, 0, 5) satisfy the equation plane represented by equations (iv),

Therefore the plane passes through the point (1,0,5)

109. **(c)** Given equation of lines are

$\dfrac{x-1}{3}=\dfrac{y-2}{1}=\dfrac{z-3}{2}$ ...(1)

and $\dfrac{x-3}{1}=\dfrac{y-1}{2}=\dfrac{z-2}{3}$ ...(2)

Any point on line (1) is P $(3\lambda+1, \lambda+2, 2\lambda+3)$ and on line (2) is Q $(\mu + 3, 2\mu + 1, 3\mu + 2)$.

On solving $3\lambda + 1 = \mu + 3$ and $\lambda + 2 = 2\mu + 1$

we get $\lambda = 1$, $\mu = 1$

$\therefore$ Point of intersection of two lines is R (4, 3, 5)

So, equation of plane $\perp$ to OR where O is (0, 0, 0) and passing through R is

$4x+3y+5z=50$

110. **(c)** $2x+y+2z-8=0$ ....(Plane 1)

$2x+y+2z+\dfrac{5}{2}=0$ ....(Plane 2)

Distance between Plane 1 and 2

$$=\left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right|=\left|\frac{-21}{6}\right|=\frac{7}{2}$$

111. **(c)** Equation of a plane through the line of intersection of the planes

$x+2y=3, y-2z+1=0$ is

$(x+2y-3)+\lambda(y-2z+1)=0$

$\Rightarrow x+(2+\lambda)y-2\lambda(z)-3+\lambda=0$ (i)

Now, plane (i) is $\perp$ to $x+2y=3$

$\therefore$ Their dot product is zero

i.e. $1+2(2+\lambda)=0 \Rightarrow \lambda=-\dfrac{5}{2}$

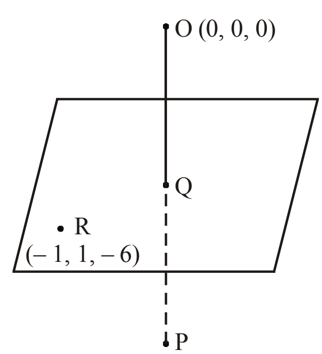
Thus, required plane is

$$x+\left(2-\frac{5}{2}\right)y-2\times\frac{-5}{2}(z)-3-\frac{5}{2}=0$$

$$\Rightarrow x-\frac{y}{2}+5z-\frac{11}{2}=0$$

$$\Rightarrow 2x-y+10z-11=0$$

112. **(c)** Let P be the image of O in the given plane.



Equation of the plane, $4x-3y+z+13=0$

OP is normal to the plane, therefore direction ratio of OP are proportional to 4, – 3, 1

Since OP passes through (0, 0, 0) and has direction ratio proportional to 4, –3, 1. Therefore equation of OP is

$$\frac{x-0}{4}=\frac{y-0}{-3}=\frac{z-0}{1}=r \text{ (let)}$$

$\therefore x=4r, y=-3r, z=r$

Let the coordinate of P be $(4r, -3r, r)$

Since Q be the mid point of OP

$$\therefore Q=\left(2r, -\frac{3}{2}r, \frac{r}{2}\right)$$

Since Q lies in the given plane

$4x-3y+z+13=0$

$$\therefore 8r+\frac{9}{2}r+\frac{r}{2}+13=0$$

$$\Rightarrow r=\frac{-13}{8+\frac{9}{2}+\frac{1}{2}}=\frac{-26}{26}=-1$$

$$\therefore Q=\left(-2, \frac{3}{2}, -\frac{1}{2}\right)$$

$$QR=\sqrt{(-1+2)^2+\left(1-\frac{3}{2}\right)^2+\left(-6+\frac{1}{2}\right)^2}$$

$$=\sqrt{1+\frac{1}{4}+\frac{121}{4}}=3\sqrt{\frac{7}{2}}$$

113. **(b)** Direction cosines of $\vec{n}$ are $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}$.

Equation of the plane,

$$\frac{1}{2}(x-\sqrt{2})+\frac{1}{4}(y+1)+\frac{1}{2}(z-1)=0$$

$$\Rightarrow 2(x-\sqrt{2})+(y+1)+2(z-1)=0$$

$$\Rightarrow 2x+y+2z=2\sqrt{2}-1+2$$

$$\Rightarrow 2x+y+2z=2\sqrt{2}+1$$

114. **(a)** Given that, equation of a plane is

$x-2y+2z-5=0$

So, Equation of parallel plane is

$x-2y+2z+d=0$

Now, it is given that distance from origin to the parallel plane is 1.

$$\therefore \left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right|=1 \Rightarrow d=\pm 3$$

So equation of required plane

$x-2y+2z\pm 3=0$

115. **(a)** The equation of the plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ is a $(x+1)+b(y-3)+c(z+2)=0$

where

$-3a+2b+c=0$ ...(A)

This passes through (0, 7, – 7)

$\therefore a(0+1)+b(7-3)+c(-7+2)=0$

$\Rightarrow a+4b-5c=0$ ...(B)

On solving equation (A) and (B) we get

$a=1, b=1, c=1$

$\therefore$ Required plane is

$x+1+y-3+z+2=0$

$\Rightarrow x+y+z=0$

116. **(d)** Given planes are

$P: x+y-2z+7=0$

$Q: x+y+2z+2=0$

and $R: 3x+3y-6z-11=0$

Consider Plane P and R.

Here $a_1=1, b_1=1, c_1=-2$

and $a_2=3, b_2=3, c_2=-6$

Since, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{3}$

therefore P and R are parallel.

117. **(b)** Let the direction ratios of the common line be l, m and n.

$\therefore l\times 1+m\times 0+n\times 0=0 \Rightarrow l=0$ ....(1)

$2l-5ma+3n=0 \Rightarrow 5ma-3n=0$ ....(2)

$3lb+m-3n=0 \Rightarrow m-3n=0$ ....(3)

Subtracting (3) from (1), we get

$m(5a-1)=0$

Now, value of m can not be zero because if $m=0$ then $n=0$

$\Rightarrow l=m=n=0$ which is not possible.

Hence, $5a-1=0 \Rightarrow a=\frac{1}{5}$

Thus, option (b) is correct.

118. **(c)** Point P is $(2, -1, 2)$

Let this line meet at $Q(h, k, w)$

Direction ratio of this line is

$(h-2, k+1, w-2)$

Since, dc_s are equal & dr_s are also equal,

So, $h-2=k+1+w-2$

$\Rightarrow k=h-3$ and $w=h$

This line meets the plane

$2x+y+z=9$ at Q, so,

$2h+k+w=9$ or $2h+h-3+h=9$

$\Rightarrow 4h-3=9 \Rightarrow h=3$

and $k=0$ and $w=3$

Distance

$$PQ=\sqrt{(3-2)^2+(0-(-1))^2+(3-2)^2}$$

$= \sqrt{1^2+1^2+1^2} = \sqrt{3}$

119. (d) Given equation of plane is

$3x+4y-12z+13=0$

$(1, a, 1)$ and $(-3, 0, a)$ satisfy the equation of plane.

$\therefore$ We have

$3+4(a)-12+13=0$ and $\quad 3(-3)-12(a)+13=0$

$\Rightarrow \quad 4+4a=0$ and $4-12a=0$

$\Rightarrow \quad a=-1$ and $a=\dfrac{1}{3}$

Since, $(1, a, 1)$ and $(-3, 0, a)$ lie on the opposite sides of the plane $\therefore a=0$

120. (a) Equation of line through $P(1,-5,9)$ and parallel to the line $x=y=z$ is

$\dfrac{x-1}{1}=\dfrac{y+5}{1}=\dfrac{z-9}{1}=\lambda(say)$

$Q=(x=1+\lambda, y=-5+\lambda, z=9+\lambda)$

Since Q lies on plane $x-y+z=5$

$\therefore \quad 1+\lambda+5-\lambda+9+\lambda=5$

$\Rightarrow \quad \lambda=-10$

$\therefore \quad Q=(-9,-15,-1)$

$\therefore \quad PQ=\sqrt{(1+9)^2+(15-5)^2+(9+1)^2}$

$=\sqrt{300}=10\sqrt{3}$

121. (d) Let θ be the angle between the given line and plane, then

$$\sin\theta=\frac{1\times1+2\times2+\lambda\times3}{\sqrt{1^2+2^2+\lambda^2}.\sqrt{1^2+2^2+3^2}}=\frac{5+3\lambda}{\sqrt{14}.\sqrt{5+\lambda^2}}$$

$$\Rightarrow \quad \cos\theta=\sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

$$\Rightarrow \quad \sqrt{\frac{5}{14}}=\sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

Squaring both sides, we get

$$\frac{5}{14}=\frac{5\lambda^2-30\lambda+45}{14(5+\lambda^2)}$$

$\Rightarrow \quad \lambda=\dfrac{2}{3}$

122. (a) A(3, 1, 6); B = (1, 3, 4)

Putting coordinate of mid-point of AB = (2, 2, 5) in plane $x-y+z=5$ then $2-2+5=5$, satisfy

So, mid-point of AB = (2, 2, 5) lies on the plane.

d.r's of AB = (2, –2, 2)

d.r's of normal to plane = (1, –1, 1).

Direction ratio of AB and normal to the plane are proportional therefore,

AB is perpendicular to the normal of plane

$\therefore$ A is image of B

Statement-1 is correct.

Statement-2 is also correct but it is not correct explanation.

123. (a) Given that, the line $\dfrac{x-2}{3}=\dfrac{y-1}{-5}=\dfrac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$

$\therefore \quad Pt\,(2, 1, -2)$ lies on the plane

i.e. $\quad 2+3+2\alpha+\beta=0$

or $\quad 2\alpha+\beta+5=0 \quad$ ....(i)

Also normal to plane will be perpendicular to line,

$\therefore \quad 3\times1-5\times3+2\times(-\alpha)=0$

$\Rightarrow \quad \alpha=-6$

From equation (i) then, $\beta=7$

$\therefore (\alpha,\beta)=(-6,7)$

124. (c) Equation of line through $(5, 1, a)$ and

$(3, b, 1)$ is $\dfrac{x-5}{-2}=\dfrac{y-1}{b-1}=\dfrac{z-a}{1-a}=\lambda$

$x=-2\lambda+5$

$y=(b-1)\lambda+1$

$z=(1-a)\lambda+a$

$\therefore \quad$ Any point on this line is a

$[-2\lambda+5, (b-1)\lambda+1, (1-a)\lambda+a]$

Given that it crosses yz plane $\therefore -2\lambda+5=0$

$\lambda=\dfrac{5}{2}$

$$\therefore \left(0,(b-1)\frac{5}{2}+1, (1-a)\frac{5}{2}+a\right)=\left(0,\frac{17}{2},\frac{-13}{2}\right)$$

$\Rightarrow \quad (b-1)\dfrac{5}{2}+1=\dfrac{17}{2}$

and $\quad (1-a)\dfrac{5}{2}+a=-\dfrac{13}{2}$

$\Rightarrow \quad b=4$ and $a=6$

125. (c) Let the direction cosines of line L be l, m, n. Since line L lies on both planes.

$\therefore \quad 2l+3m+n=0 \quad$ ....(i)

and $\quad l+3m+2n=0 \quad$ ....(ii)

on solving equation (i) and (ii), we get

$$\frac{l}{6-3}=\frac{m}{1-4}=\frac{n}{6-3} \quad \Rightarrow \quad \frac{l}{3}=\frac{m}{-3}=\frac{n}{3}$$

Now $\dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}=\dfrac{\sqrt{l^2+m^2+n^2}}{\sqrt{3^2+(-3)^2+3^2}}$

$\because \quad l^2+m^2+n^2=1$

$\therefore \quad \dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}=\dfrac{1}{\sqrt{27}}$

$\Rightarrow \quad l=\dfrac{3}{\sqrt{27}}=\dfrac{1}{\sqrt{3}}, m=-\dfrac{1}{\sqrt{3}}, n=\dfrac{1}{\sqrt{3}}$

Line L, makes an angle α with +ve x-axis

$\therefore \quad l=\cos\alpha \Rightarrow \cos\alpha=\dfrac{1}{\sqrt{3}}$

126. **(d)** Let (α, β, γ) be the image, then mid point of (α, β, γ) and $(-1, 3, 4)$ must lie on $x - 2y = 0$

$$\therefore \frac{\alpha-1}{2} - 2\left(\frac{\beta+3}{2}\right) = 0$$

$$\therefore \alpha - 1 - 2\beta - 6 = 0 \Rightarrow \alpha - 2\beta = 7 \quad \text{....(1)}$$

Also line joining (α, β, γ) and $(-1, 3, 4)$ should be parallel to the normal of the plane $x - 2y = 0$

$$\therefore \frac{\alpha+1}{1} = \frac{\beta-3}{-2} = \frac{\gamma-4}{0} = \lambda$$

$$\Rightarrow \alpha = \lambda - 1, \beta = -2\lambda + 3, \gamma = 4 \quad \text{....(2)}$$

From (1) and (2)

$$\alpha = \frac{9}{5}, \ \beta = -\frac{13}{5}, \gamma = 4$$

None of the option matches.

127. **(b)** The given line is $\vec{r} = 2\vec{i} - 2\vec{j} + 3\vec{k} + \lambda(\vec{i} - \vec{j} + 4\vec{k})$

and the plane is $\vec{r} \cdot (\vec{i} + 5\vec{j} + \vec{k}) = 5$

$$\Rightarrow x + 5y + z = 5$$

$$\text{Required distance} = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$$= \left|\frac{2-10+3-5}{\sqrt{1+25+1}}\right| = \frac{10}{3\sqrt{3}}$$

128. **(a)** Let θ is the angle between line and plane then

$$\sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

$$= \frac{(\hat{i} + 2\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j} + \sqrt{\lambda}\hat{k})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}} = \frac{2-2+2\sqrt{\lambda}}{3 \times \sqrt{5+\lambda}}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \frac{1}{3} \Rightarrow 4\lambda = 5 + \lambda$$

$$\Rightarrow \lambda = \frac{5}{3}.$$

129. **(c)** The planes are $2x + y + 2z - 8 = 0$ ...(1)

and $4x + 2y + 4z + 5 = 0$

or $2x + y + 2z + \frac{5}{2} = 0$ ...(2)

Since, both planes are parallel

$\therefore$ Distance between (1) and (2)

$$= \left|\frac{\frac{5}{2}+8}{\sqrt{2^2+1^2+2^2}}\right| = \left|\frac{21}{2\sqrt{9}}\right| = \frac{7}{2}$$

130. **(a)** Equation of planes in intercept form be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

& $\frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1$

($\perp r$ distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = \left|\frac{-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}\right|$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$$

131. **(b)** Equation of plane through (1, 0, 0) is

$a(x-1) + by + cz = 0$ ...(i)

It is also passes through (0, 1, 0).

$$\therefore -a + b = 0 \Rightarrow b = a;$$

$$\cos 45° = \frac{a+a}{\sqrt{2(2a^2+c^2)}}$$

$$\Rightarrow 2a = \sqrt{2a^2+c^2} \Rightarrow 2a^2 = c^2 \Rightarrow c = \sqrt{2}a.$$

So d.r of normal are a, a $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

132. **(a)** Since the point (3, 2, 0) lies on the given line

$$\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$$

$\therefore$ There can be infinite many planes passing through this line. We observed that only option (a) is satisfied by the coordinates of both the points (3, 2, 0) and (4, 7, 4)

$\therefore$ $x - y + z = 1$ is the required plane.

133. **(c)** We know that centre of sphere

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

is $(-u, -v, -w)$

Given that, $x^2+y^2+z^2-6x-12y-2z+20=0$

$\therefore$ Centre $\equiv (3, 6, 1)$

Coordinates of one end of diameter of the sphere are (2, 3, 5).

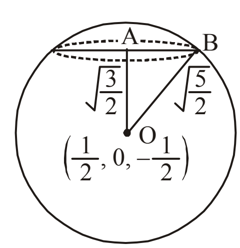
Let the coordinates of the other end of diameter are (α, β, γ)

$$\therefore \frac{\alpha+2}{2} = 3, \ \frac{\beta+3}{2} = 6, \ \frac{\gamma+5}{2} = 1$$

$$\Rightarrow \alpha = 4, \ \beta = 9 \text{ and } \gamma = -3$$

$\therefore$ Coordinate of other end of diameter are (4, 9, –3)

134. **(b)**



Centre of sphere $= \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$ and radius of sphere

$$= \sqrt{\frac{1}{4} + \frac{1}{4} + 2} = \sqrt{\frac{5}{2}}$$

Perpendicular distance OA of centre from $x+2y-z=4$ is given by

$$\frac{\left|\frac{1}{2}+\frac{1}{2}-4\right|}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

$\therefore$ radius of circle

$$AB=\sqrt{OB^2-OA^2}=\sqrt{\frac{5}{2}-\frac{3}{2}}=1.$$

135. (c) Plane $2ax-3ay+4az+6=0$ passes through the mid point of the line joining the centres of spheres

$x^2+y^2+z^2+6x-8y-2z=13$ and

$x^2+y^2+z^2-10x+4y-2z=8$

respectively centre of spheres are $c_1(-3, 4, 1)$ and $c_2(5, -2, 1)$. Mid point of c_1c_2 is $(1, 1, 1)$.

Satisfying this in the equation of plane, we get

$2a-3a+4a+6=0$

$\Rightarrow \quad a=-2.$

136. (a) Given that, the equations of spheres are

$S_1: x^2+y^2+z^2+7x-2y-z-13=0$ and

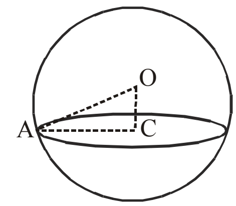
$S_2: x^2+y^2+z^2-3x+3y+4z-8=0$

We know that eqn. of intersection plane be

$S_1-S_2=0 \Rightarrow 10x-5y-5z-5=0$

$\Rightarrow \quad 2x-y-z=1$

137. (d)



Centre of sphere $=(-1, 1, 2)$

Radius of sphere $\sqrt{1+1+4+19}=5$

Perpendicular distance from centre to the plane

$$OC=d=\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right|=\frac{12}{3}=4.$$

In right, ΔAOC

$AC^2=AO^2-OC^2=5^2-4^2=9$

$\Rightarrow \quad AC=3$

138. (d) Centre of sphere be $(-2, 1, 3)$ and radius 13

We know that,

Shortest distance = perpendicular distance between the plane and sphere = distance of plane from centre of sphere – radius

$$=\left|\frac{-2\times 12+4\times 1+3\times 3-327}{\sqrt{144+9+16}}\right|-13$$

$=26-13=13$

27

Probability

TOPIC 1 Multiplication Theorem on Probability, Independent events, Conditional Probability, Baye's Theorem

1. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is : **[Sep. 04, 2020 (II)]**

(a) $\frac{5}{31}$ (b) $\frac{31}{61}$

(c) $\frac{5}{6}$ (d) $\frac{30}{61}$

2. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is : **[Sep. 03, 2020 (I)]**

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{8}$ (d) $\frac{1}{9}$

3. The probability that a randomly chosen 5-digit number is made from exactly two digits is : **[Sep. 03, 2020 (II)]**

(a) $\frac{135}{10^4}$ (b) $\frac{121}{10^4}$

(c) $\frac{150}{10^4}$ (d) $\frac{134}{10^4}$

4. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is : **[Sep. 02, 2020 (I)]**

(a) $\frac{2}{3}$ (b) $\frac{8}{17}$

(c) $\frac{4}{17}$ (d) $\frac{2}{5}$

5. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$.

Then $P(E_2^C \cap E_3^C / E_1)$ is equal to : **[Sep. 02, 2020 (II)]**

(a) $P(E_2^C) + P(E_3)$ (b) $P(E_3^C) - P(E_2^C)$

(c) $P(E_3) - P(E_2^C)$ (d) $P(E_3^C) - P(E_2)$

6. In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is : **[Jan. 9, 2020 (I)]**

(a) $\frac{9}{16}$ (b) $\frac{11}{16}$

(c) $\frac{13}{16}$ (d) $\frac{15}{16}$

7. Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE ? **[Jan. 8, 2020 (I)]**

(a) $P(A/B) = \frac{2}{3}$ (b) $P(A/B') = \frac{1}{3}$

(c) $P(A'/B') = \frac{1}{3}$ (d) $P(A/(A \cup B)) = \frac{1}{4}$

8. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. Then the expected value of X, is: **[Jan. 7, 2020 (I)]**

(a) $\frac{3}{16}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{16}$ (d) $-\frac{1}{8}$

9. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to: **[Jan. 7, 2020 (II)]**

(a) $\frac{17}{8}$ (b) $\frac{17}{4}$

(c) $\frac{17}{2}$ (d) 4

10. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is : **[April 12, 2019 (II)]**

(a) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (b) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

(c) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$ (d) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

11. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is: **[April 10, 2019 (I)]**

(a) $\frac{1}{11}$ (b) $\frac{1}{10}$

(c) $\frac{1}{12}$ (d) $\frac{1}{17}$

12. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is : **[April 10, 2019 (II)]**

(a) 5 (b) 6

(c) 8 (d) 7

13. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is: **[April 09, 2019 (I)]**

(a) $\frac{25}{192}$ (b) $\frac{7}{32}$

(c) $\frac{1}{192}$ (d) $\frac{25}{32}$

14. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct? **[April 08, 2019 (I)]**

(a) $P(A|B) = P(B) - P(A)$ (b) $P(A|B) \geq P(A)$

(c) $P(A|B) \leq P(A)$ (d) $P(A|B) = 1$

15. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is : **[April. 08, 2019 (II)]**

(a) 5 (b) 3

(c) 4 (d) 2

16. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to : **[Jan. 12, 2019 (I)]**

(a) $\frac{200}{6^5}$ (b) $\frac{150}{6^5}$

(c) $\frac{225}{6^5}$ (d) $\frac{175}{6^5}$

17. In a game, a man wins ₹ 100 if he gets 5 or 6 on a throw of a fair die and loses ₹ 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is : **[Jan. 12, 2019 (II)]**

(a) $\frac{400}{9}$ loss (b) 0

(c) $\frac{400}{3}$ gain (d) $\frac{400}{3}$ loss

18. Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is : **[Jan. 11, 2019 (I)]**

(a) $\frac{7}{10}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

19. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ..., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is: **[Jan 10, 2019 (I)]**

(a) $\frac{13}{36}$ (b) $\frac{15}{72}$ (c) $\frac{19}{72}$ (d) $\frac{19}{36}$

20. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is: **[Jan. 10, 2019 (II)]**

(a) 3 (b) 6

(c) 5 (d) 4

21. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X=1)+P(X=2)$ equals: **[Jan 09, 2019 (I)]**

(a) 49/169 (b) 52/169

(c) 24/169 (d) 25/169

22. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red is: **[Jan. 09, 2019 (II)]**

(a) $\frac{21}{49}$ (b) $\frac{27}{49}$

(c) $\frac{26}{49}$ (d) $\frac{32}{49}$

23. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is : **[2018]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{4}$ (d) $\frac{3}{10}$

24. Let A, B and C be three events, which are pair-wise independence and $\overline{E}$ denotes the complement of an event E. If $P(A\cap B\cap C)=0$ and $P(C)>0$, then $P[(\overline{A}\cap\overline{B})|C]$ is equal to. **[Online April 16, 2018]**

(a) $P(A)+P(\overline{B})$ (b) $P(\overline{A})-P(\overline{B})$

(c) $P(\overline{A})-P(B)$ (d) $P(\overline{A})+P(\overline{B})$

25. A player *X* has a biased coin whose probability of showing heads is *p* and a player *Y* has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If *X* starts the game, and the probability of winning the game by both the players is equal, then the value of '*p*' is **[Online April 15, 2018]**

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$

(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

26. If two different numbers are taken from the set (0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is : **[2017]**

(a) $\frac{7}{55}$ (b) $\frac{6}{55}$

(c) $\frac{12}{55}$ (d) $\frac{14}{55}$

27. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is : **[Online April 9, 2017]**

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

28. Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4},\frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is : **[Online April 8, 2017]**

(a) $\frac{21}{64}$ (b) $\frac{9}{64}$

(c) $\frac{15}{64}$ (d) $\frac{39}{64}$

29. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is : **[Online April 8, 2017]**

(a) $\frac{255}{256}$ (b) $\frac{127}{128}$

(c) $\frac{63}{64}$ (d) $\frac{1}{2}$

30. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ? **[2016]**

(a) E_1 and E_3 are independent.

(b) E_1, E_2 and E_3 are independent.

(c) E_1 and E_2 are independent.

(d) E_2 and E_3 are independent.

31. If A and B are any two events such that $P(A)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$, then the conditional probability, $P(A\mid A'\cup B'))$, where A' denotes the complement of A, is equal to : **[Online April 9, 2016]**

(a) $11/20$ (b) $5/17$

(c) $8/17$ (d) $1/4$

32. Let X be a set containing 10 elements and P(X) be its power set. If A and B are picked up at random from P(X), with replacement, then the probability that A and B have equal number elements, is : **[Online April 10, 2015]**

(a) $\frac{\left(2^{10}-1\right)}{2^{10}}$ (b) $\frac{{}^{20}C_{10}}{2^{10}}$

(c) $\frac{\left(2^{10}-1\right)}{2^{20}}$ (d) $\frac{{}^{20}C_{10}}{2^{20}}$

33. Let A and B be two events such that $P\left(\overline{A \cup B}\right)=\frac{1}{6}$, $P\left(\overline{A \cap B}\right)=\frac{1}{4}$ and $P\left(\overline{A}\right)=\frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are **[2014]**

(a) independent but not equally likely.
(b) independent and equally likely.
(c) mutually exclusive and independent.
(d) equally likely but not independent.

34. Let A and E be any two events with positive probabilities:

Statement - 1: $P(E/A) \geq P(A/E)\,P(E)$

Statement - 2: $P(A/E) \geq P(A \cap E)$

[Online April 19, 2014]

(a) Both the statements are true
(b) Both the statements are false
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

35. A, B, C try to hit a target simultaneously but independently. Their respective probabilities of hitting the targets are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is : **[Online April 23, 2013]**

(a) 21/64 (b) 7/8
(c) 7/32 (d) 9/64

36. Given two independent events, if the probability that exactly one of them occurs is $\frac{26}{49}$ and the probability that none of them occurs is $\frac{15}{49}$, then the probability of more probable of the two events is : **[Online April 22, 2013]**

(a) 4/7 (b) 6/7
(c) 3/7 (d) 5/7

37. The probability of a man hitting a target is $\frac{2}{5}$. He fires at the target k times (k, a given number). Then the minimum k, so that the probability of hitting the target at least once is more than $\frac{7}{10}$, is : **[Online April 9, 2013]**

(a) 3 (b) 5
(c) 2 (d) 4

38. Three numbers are chosen at random without replacement from $\{1,2,3,..8\}$. The probability that their minimum is 3, given that their maximum is 6, is : **[2012]**

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$

(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

39. Let A, B, C, be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then $P\left(A^c \cap B^c / C\right)$. **[2011RS]**

(a) $P\left(B^c\right) - P(B)$ (b) $P\left(A^c\right) + P\left(B^c\right)$

(c) $P\left(A^c\right) - P\left(B^c\right)$ (d) $P\left(A^c\right) - P(B)$

40. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is **[2011]**

(a) $P(C \mid D) \geq P(C)$ (b) $P(C \mid D) < P(C)$

(c) $P(C \mid D) = \frac{P(D)}{P(C)}$ (d) $P(C \mid D) = P(C)$

41. One ticket is selected at random from 50 tickets numbered 00,01,02,...,49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals: **[2009]**

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$

(c) $\frac{1}{50}$ (d) $\frac{1}{14}$

42. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A \mid B)=\frac{1}{2}$ and $P(B \mid A)=\frac{2}{3}$. Then $P(B)$ is **[2008]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

43. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is **[2007]**

(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14.

44. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is **[2005]**

(a) $\frac{2}{9}$ (b) $\frac{1}{9}$

(c) $\frac{8}{9}$ (d) $\frac{7}{9}$

45. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are **[2005]**

(a) equally likely and mutually exclusive

(b) equally likely but not independent

(c) independent but not equally likely

(d) mutually exclusive and independent

46. The probability that A speaks truth is $\frac{4}{5}$, while the probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is **[2004]**

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{7}{20}$ (d) $\frac{3}{20}$

47. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is **[2002]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

TOPIC 2 Random Variables, Probability Distribution, Bernoulli Trails, Binomial Distribution, Poisson Distribution

48. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ______. **[NA Sep. 05, 2020 (I)]**

49. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _________. **[NA Sep. 05, 2020 (II)]**

50. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ___________. **[NA Sep. 04, 2020 (I)]**

51. A random variable X has the following probability distribution:

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then, $P(X > 2)$ is equal to: **[Jan. 9, 2020 (II)]**

(a) $\frac{7}{12}$ (b) $\frac{1}{36}$

(c) $\frac{1}{6}$ (d) $\frac{23}{36}$

52. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X\ d''\ 2) = \frac{k}{2^{16}}$, then k is equal to: **[April 12, 2019 (I)]**

(a) 17 (b) 121

(c) 1 (d) 137

53. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is : **[April 12, 2019 (II)]**

(a) $\frac{1}{2}$ gain (b) $\frac{1}{4}$ loss

(c) $\frac{1}{2}$ loss (d) 2 gain

54. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to: **[Jan. 11, 2019 (II)]**

(a) 4 (b) $4\sqrt{3}$ (c) $3\sqrt{2}$ (d) $\frac{4\sqrt{3}}{3}$

55. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is: **[2017]**

(a) $\frac{6}{25}$ (b) $\frac{12}{5}$ (c) 6 (d) 4

56. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is : **[Online April 10, 2016]**

(a) $\frac{496}{729}$ (b) $\frac{192}{729}$

(c) $\frac{240}{729}$ (d) $\frac{256}{729}$

57. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to one is : **[Online April 11, 2015]**

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{1}{16}$ (d) $\frac{15}{16}$

58. If X has a binomial distribution, B(n, p) with parameters n and p such that P(X = 2) = P (X = 3), then E(X), the mean of variable X, is **[Online April 11, 2014]**

(a) $2 - p$ (b) $3 - p$ (c) $\frac{p}{2}$ (d) $\frac{p}{3}$

59. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is: **[2013]**

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$ (c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

60. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval **[2011]**

(a) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (b) $\left[0, \frac{1}{2}\right]$

(c) $\left(\frac{11}{12}, 1\right]$ (d) $\left(\frac{1}{2}, \frac{3}{4}\right]$

61. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than: **[2009]**

(a) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (b) $\frac{9}{\log_{10} 4 - \log_{10} 3}$

(c) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (d) $\frac{1}{\log_{10} 4 - \log_{10} 3}$

62. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is **[2007]**

(a) 8/729 (b) 8/243 (c) 1/729 (d) 8/9.

63. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is **[2006]**

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$

64. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals **[2005]**

(a) $\frac{2}{e^2}$ (b) 0 (c) $1 - \frac{3}{e^2}$ (d) $\frac{3}{e^2}$

65. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is **[2004]**

(a) $\frac{28}{256}$ (b) $\frac{219}{256}$ (c) $\frac{128}{256}$ (d) $\frac{37}{256}$

66. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.2	0.2	0.1	0.1	0.2	0.1	0.1	0.1

For the events $E = \{X$ is a prime number $\}$ and $F = \{X < 4\}$, the $P(E \cup F)$ is **[2004]**

(a) 0.50 (b) 0.77 (c) 0.35 (d) 0.87

67. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then P $(X = 1)$ is **[2003]**

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$

(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

68. A dice is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is **[2002]**

(a) 8/3 (b) 3/8 (c) 4/5 (d) 5/4

Hints & Solutions

1. **(d)** Probability of sum getting 6, $P(A)=\frac{5}{36}$

Probability of sum getting 7, $P(B)=\frac{6}{36}=\frac{1}{6}$

$P(A \text{ wins})= P(A)+P(\bar{A})P(\bar{B})P(A)$

$+P(\bar{A})\cdot P(\bar{B})P(\bar{A})P(\bar{B})P(A)+.....$

$\Rightarrow \frac{5}{36}+\left(\frac{31}{36}\right)\left(\frac{30}{36}\right)\left(\frac{5}{36}\right)+.....\infty$

$\Rightarrow \frac{5}{36}\left(1+\frac{155}{216}+\left(\frac{155}{216}\right)^2+........\infty\right)$

$\Rightarrow \frac{\frac{5}{36}}{\frac{61}{216}}=\frac{30}{61}$ $\qquad \left(\because S_\infty=\frac{a}{1-r}\right)$

2. **(d)** E_1 [the event for getting score a multiple of 4]
$=(1,3),(3,1),(2,2),(2,6),(6,2),(3,5),(5,3),(4,4)\ \&\ (6,6)$
E_2 [4 has appeared atleast once]
$=(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(4,1),(4,2),(4,3),(4,5)\ \&\ (4,6)$

$E_1\cap E_2=(4,4)$

$P\left(\frac{E_2}{E_1}\right)=\frac{1}{9}$

3. **(a)** Total outcomes $=9(10^4)$
Favourable outcomes

$={}^9C_2(2^5-2)+{}^9C_1(2^4-1)=36(30)+9(15)$

Probability $=\frac{36\times30+9\times15}{9\times10^4}=\frac{4\times30+15}{10^4}=\frac{135}{10^4}$

4. **(b)** Let B_1 and B_2 be the boxes and N be the number of non-prime number.

$\because P(B_1)=P(B_2)=\frac{1}{2}$

and P (non-prime number)

$=P(B_1)\times P\left(\frac{N}{B_1}\right)+P(B_2)\times P\left(\frac{N}{B_2}\right)$

$=\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}$

So,

$$P\left(\frac{B_1}{N}\right)=\frac{P(B_1)\times P\left(\frac{N}{B_1}\right)}{P(B_1)\times P\left(\frac{N}{B_1}\right)+P(B_2)\times P\left(\frac{N}{B_2}\right)}$$

$$=\frac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}}=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{15}{40}}=\frac{8}{17}.$$

5. **(d)** $P\left(\frac{E_2^C\cap E_3^C}{E_1}\right)=\frac{P\left[E_1\cap\left(E_2^C\cap E_3^C\right)\right]}{P(E_1)}$

$=\frac{P(E_1)-P[E_1\cap(E_2\cup E_3)]}{P(E_1)}$

$[\because P(A\cap B^C)=P(A)-P(A\cap B)]$

$=\frac{P(E_1)-P[(E_1\cap E_2)\cup(E_1\cap E_3)]}{P(E_1)}$

$=\frac{P(E_1)-[P(E_1\cap E_2)+P(E_1\cap E_3)-P(E_1\cap E_2\cap E_3)]}{P(E_1)}$

$=\frac{P(E_1)-P(E_1\cap E_2)-P(E_1\cap E_3)+0}{P(E_1)}$

$=1-P(E_2)-P(E_3)$ $\qquad [\because P(A\cap B)=P(A)\cdot P(B)]$

$=P(E_2^C)-P(E_3)$ or $P(E_3^C)-P(E_2)$

6. **(b)** P(second A – card appears before the third B– card)
$=P(AA)+P(ABA)+P(BAA)+P(ABBA)+P(BBAA)$
$+P(BABA)$

$=\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{11}{16}$

7. **(b)** A and B are independent events.

So, $P\left(\frac{A}{B'}\right)=\frac{P(A\cap B')}{P(B')}=\frac{\frac{1}{3}-\frac{1}{3}\cdot\frac{1}{6}}{\frac{1}{6}}=\frac{1}{3}$

8. **(b)**

k	0	1	2	3	4	5
$P(k)$	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = No. of times head occur consecutively

Now expectation

$$= \sum xP(k) = (-1)\times\frac{1}{32}+(-1)\times\frac{12}{32}+(-1)\times\frac{11}{32}$$

$$+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32}=\frac{1}{8}$$

9. **(a)** Required probability = when no machine has fault + when only one machine has fault + when only two machines have fault.

$$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$=\frac{243}{1024}+\frac{405}{1024}+\frac{270}{1024}=\frac{918}{1024}=\frac{459}{512}=\frac{27\times17}{64\times8}$$

$$=\left(\frac{3}{4}\right)^3\times k=\left(\frac{3}{4}\right)^3\times\frac{17}{8}$$

$$\therefore \quad k=\frac{17}{8}$$

10. **(c)** Let p is the probability that candidate can solve a problem and q is the probability that candidate can not not solve a problem.

$$p=\frac{4}{5} \text{ and } q=\frac{1}{5} \qquad (\because \ p+q=1)$$

Probability of solving either 50 or 49 problem by the candidate

$$= {}^{50}C_{50}\cdot p^{50}\cdot q^0 + {}^{50}C_{49}\cdot p^{49}\cdot q^1 = p^{49}\,[p+50q]$$

$$=\left(\frac{4}{5}\right)^{49}.\left(\frac{4}{5}+\frac{50}{5}\right)=\frac{54}{5}.\left(\frac{4}{5}\right)^{49}$$

11. **(a)** Let, A = At least two girls

B = All girls

$$P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{P(B)}{P(A)}$$

$$=\frac{\left(\frac{1}{4}\right)^4}{1-{}^4C_0\left(\frac{1}{2}\right)^4-{}^4C_1\left(\frac{1}{2}\right)^4}=\frac{1}{16-1-4}=\frac{1}{11}$$

12. **(d)** According to the question,

$$1-\left(\frac{1}{2}\right)^n>\frac{99}{100}\Rightarrow\left(\frac{1}{2}\right)^n<\frac{1}{100}\Rightarrow n\geq 7$$

Hence, minimum value is 7.

13. **(d)** P (at least one hits the target)

= 1 – P (none of them hits the target)

$$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)$$

$$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{7}{8}=1-\frac{7}{32}=\frac{25}{32}$$

14. **(b)** $\because\ A\subset B$; so $A\cap B=A$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$$\Rightarrow \quad P\left(\frac{A}{B}\right)=\frac{P(A)}{P(B)}$$

$\because \quad P(B)\leq 1$

$$\Rightarrow P\left(\frac{A}{B}\right)\geq P(A)$$

15. **(c)** Let, p is probability for getting head and is probability for getting tail.

$$p=P(H)=\frac{1}{2}, q=1-p=\frac{1}{2}$$

$$P(x\geq1)\geq\frac{9}{10}\Rightarrow 1-P(x=0)\geq\frac{9}{10}$$

$$1-{}^nC_0\left(\frac{1}{2}\right)^n\geq\frac{9}{10}\Rightarrow\frac{1}{2^n}\leq1-\frac{9}{10}\Rightarrow\frac{1}{2^n}\leq\frac{1}{10}$$

$$2^n\geq10 \ \Rightarrow n\geq4 \ \Rightarrow n_{\min}=4$$

16. **(d)** Since, the experiment will end in the fifth throw. Hence, the possibilities are 4 * * 4 4, * 4 * 4 4, * * * 4 4 (where * is any number except 4)

$$\text{Required Probability}=\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$$

$$+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2$$

$$=\frac{25+25+125}{6^5}=\frac{175}{6^5}$$

17. **(b)** Probability of getting 5 or 6 $=P(E)=\frac{2}{6}=\frac{1}{3}$

Probability of not getting 5 or 6 $=P(E)=1-\frac{1}{3}=\frac{2}{3}$

E will consider as success.

Event	Success in Ist attempt	Success in IInd attempt	Success in IIIrd attempt	No success in IIIrd attempt
Probability	$\frac{1}{3}$	$\frac{2}{3}\times\frac{1}{3}$	$\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$	$\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$
Gain/loss	100	50	0	–150

His expected gain/loss

$$=\frac{1}{3}\times100+\frac{2}{9}\times50+\frac{8}{27}\times(-150)$$

$$=\frac{900+300-1200}{27}=0$$

18. **(c)** Probability of getting sum of selected two numbers is even

$$= P(E_1) = \frac{{}^5C_2 + {}^5C_2}{{}^{11}C_2}$$

Probability of getting sum is even and selected numbers are also even $P(E_2) = \frac{{}^5C_2}{{}^{11}C_2}$

Hence, $P\left(\frac{E_2}{E_1}\right) = \frac{{}^5C_2}{{}^6C_2 + {}^5C_2} = \frac{10}{15+10} = \frac{2}{5}$.

19. **(c)** $P(\text{Outcome is head}) = \frac{1}{2}$

$P(\text{Outcome is tail}) = \frac{1}{2}$

$P(\text{7 or 8 is the sum of two dice}) = \frac{6}{36} + \frac{5}{36} = \frac{11}{36}$

$P(\text{7 or 8 is the number of card}) = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$

Required probability $= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9}$

$= \frac{1}{2}\left(\frac{11+8}{36}\right) = \frac{19}{72}$

20. **(c)** Let the number of independent shots required to hit the target at least once be n, then

$1 - \left(\frac{2}{3}\right)^n > \frac{5}{6}\ \left(\frac{2}{3}\right)^n < \frac{1}{6}$

Hence, the above inequality holds when least value of n is 5.

21. **(d)** X = number of aces drawn

$\therefore\ P(X=1) + P(X=2)$

$= \left\{\frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}\right\} + \left\{\frac{4}{52} \times \frac{4}{52}\right\}$

$= \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$

22. **(d)** Let G represents drawing a green ball and R represents drawing a red ball

So, the probability that second drawn ball is red

$= P(G) \cdot P\left(\frac{R}{G}\right) + P(R)P\left(\frac{R}{R}\right)$

$= \frac{2}{7} \times \frac{6}{7} + \frac{5}{7} \times \frac{4}{7}$

$= \frac{12+20}{49}$

$= \frac{32}{49}$

23. **(a)** Let R_t be the even of drawing red ball in t^{th} draw and B_t be the event of drawing black ball in t^{th} draw.

Now, in the given bag there are 4 red and 6 black balls.

$\therefore\ P(R_1) = \frac{4}{10}$ and $P(B_1) = \frac{6}{10}$

And, $P\left(\frac{R_2}{R_1}\right) = \frac{6}{12}$ and $P\left(\frac{R_2}{B_1}\right) = \frac{4}{12}$

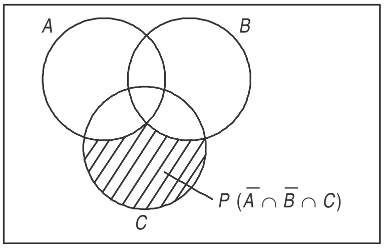
Now, required probability

$= P(R_1) \times P\left(\frac{R_2}{R_1}\right) + P(B_1) \times P\left(\frac{R_2}{B_1}\right)$

$= \left(\frac{4}{10} \times \frac{6}{12}\right) + \left(\frac{6}{10} \times \frac{4}{12}\right) = \frac{2}{5}$

24. **(c)** Here, $P(\bar{A} \cap \bar{B}\ |C) = \frac{P(\bar{A} \cap \bar{B} \cap C)}{P(C)}$.

$= \frac{P(C) - P(A \cap C - P(B \cap C) + P(A \cap B \cap C))}{P(C)}$



$= 1 - \left[\frac{P(A).P(C) + P(B).P(C)}{P(C)}\right]$

$= 1 - P(A) - P(B) = P(\bar{A}) - P(B)(\because P(A \cap B \cap C) = 0)$

25. **(a)** If the outcome is one of the following: $H, TTH, TTTTH, \ldots$, then X wins.

As subsequent tosses are independent, so the probability that X wins is

$p + \frac{p}{4} + \frac{p}{16} + \ldots = \frac{4p}{3}$.

Similarly Y wins if the outcome is one of the following: $TH, TTTH, TTTTTH, \ldots$

Therefore, the probability that Y wins is

$\frac{1-p}{2} + \frac{1-p}{8} + \frac{1-p}{32} = \frac{2(1-p)}{3}$

Since, the probability of winning the game by both the players is equal then, we have

$\frac{4p}{3} = \frac{2(1-p)}{3} \Rightarrow p = \frac{1}{3}$

26. **(b)** Let $A \equiv \{0, 1, 2, 3, 4, \ldots\ldots, 10\}$

$n(S) = {}^{11}C_2 = 55$ where 'S' denotes sample space

Let E be the given event

$\therefore E \equiv \{(0,4), (0,8), (2,6), (2,10), (4,8), (6,10)\}$

$\Rightarrow n(E) = 6$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \dfrac{6}{55}$

27. **(a)** $P(E \cap F) = P(E).P(F) = \dfrac{1}{12}$

$P(\bar{E} \cap \bar{F}) = P(\bar{E}).P(\bar{F}) = \dfrac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F)) = \dfrac{1}{2}$

Let $P(E) = x$
$P(F) = y$

$\Rightarrow 1 - x - y + xy = \dfrac{1}{2}$

$\Rightarrow 1 - x - y = \dfrac{1}{2} - \dfrac{1}{12} = \dfrac{5}{12}$

$\Rightarrow \boxed{x + y = \dfrac{7}{12}}$

$\Rightarrow x + \dfrac{1}{12x} = \dfrac{7}{12}$ $\quad \left[\because x.y = \dfrac{1}{12}\right]$

$\Rightarrow 12x^2 - 7x + 1 = 0$

$\Rightarrow 12x^2 - 4x - 3x + 1 = 0$

$\Rightarrow (4x - 1)(3x - 1) = 0$

$\Rightarrow x = \dfrac{1}{3}, x = \dfrac{1}{4}$

and $y = \dfrac{1}{4}, y = \dfrac{1}{3}$

$\therefore \dfrac{x}{y} = \dfrac{1/3}{1/4} = \dfrac{4}{3}$ *or* $\dfrac{1/4}{1/3} = \dfrac{3}{4}$

28. **(a)** Required probability =

$$\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)$$

$$= \frac{12 + 9}{64} = \frac{21}{64}$$

29. **(b)** Required probability $= 1 - \{P(\text{All Head}) + P(\text{All Tail})\}$

$$= 1 - \left\{\frac{1}{2^8} + \frac{1}{2^8}\right\}$$

$$= 1 - \left\{\frac{1}{2^7}\right\}$$

$$= 1 - \left\{\frac{1}{128}\right\} = \frac{127}{128}$$

30. **(b)** $P(E_1) = \dfrac{1}{6}$; $P(E_2) = \dfrac{1}{6}$; $P(E_3) = \dfrac{1}{2}$

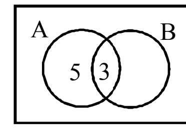
$$P(E_1 \cap E_2) = \frac{1}{36}, P(E_2 \cap E_3) = \frac{1}{12}, P(E_1 \cap E_3) = \frac{1}{12}$$

And $P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1).P(E_2).P(E_3)$

$\Rightarrow E_1, E_2, E_3$ are not independent.

31. **(b)** $P(A) = \dfrac{2}{5} = \dfrac{8}{20}$; $P(A \cap B) = \dfrac{3}{20}$

$P(\overline{A \cap B}) = 1 - \dfrac{3}{20}$



$\Rightarrow P(\bar{A} \cup \bar{B}) = \dfrac{17}{20}$

$A \cap (A' \cup B')$
$= A - (A \cap B)$

$\therefore P(A - (A \cap B)) = \dfrac{5}{20}$

$\therefore P(A/(A' \cap B')) = \dfrac{P(A - (A \cap B))}{P(\bar{A} \cup \bar{B})} = \dfrac{5}{17}$

32. **(d)** Required probability is

$$\frac{({}^{10}C_0)^2 + ({}^{10}C_1)^2 + ({}^{10}C_2)^2 + ... + ({}^{10}C_{10})^2}{2^{10}}$$

$$= \frac{{}^{20}C_{10}}{2^{20}}$$

33. **(a)** Given, $P(\overline{A \cup B}) = \dfrac{1}{6} \Rightarrow P(A \cup B) = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

$P(\bar{A}) = \dfrac{1}{4} \Rightarrow P(A) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \dfrac{5}{6} = \dfrac{3}{4} + P(B) - \dfrac{1}{4}$

$\left(\because P(A \cap B) = \dfrac{1}{4}\right)$

$\Rightarrow P(B) = \dfrac{1}{3}$

$\because P(A) \neq P(B)$ so they are not equally likely.

Also $P(A) \times P(B) = \dfrac{3}{4} \times \dfrac{1}{3} = \dfrac{1}{4} = P(A \cap B)$

So A & B are independent.

34. **(a)** Let A and E be any two events with positive probabilities.

Consider statement-1 :

$P(E/A) \geq P(A/E)P(E)$

LHS : $P(E/A) = \dfrac{P(E \cap A)}{P(A)}$ ...(1)

RHS : $P(A/E).P(E) = \dfrac{P(E \cap A)}{P(E)} \cdot P(E)$

$= P(A \cap E)$ ...(2)

Clearly, from (1) and (2), we have

$P(E/A) \geq P(A \cap E)$

Thus, statement-1 is true.

Similarly, statement-2 is also true.

35. **(a)** $P(\text{A or B but not by C}) = P((A \cup B) \cap \overline{C})$

$= P(A \cup B) \times P(\overline{C})$

$= [P(A) + P(B) - P(A \cap B)] \times P(\overline{C})$

$$= \left[\frac{3}{4} + \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}\right] \times \frac{3}{8} = \left(\frac{6+4-3}{8}\right) \times \frac{3}{8} = \frac{21}{64}$$

36. **(a)** Let the probability of occurrence of first event A, be 'a'

i.e., $P(A) = a$

$\therefore$ $P(\text{not } A) = 1 - a$

And also suppose that probability of occurrence of second event B, $P(B) = b$,

$\therefore$ $P(\text{not } B) = 1 - b$

Now, $P(\text{A and not B}) + P(\text{not A and B}) = \frac{26}{49}$

$\Rightarrow$ $P(A) \times P(\text{not B}) + P(\text{not A}) \times P(B) = \frac{26}{49}$

$\Rightarrow$ $a \times (1 - b) + (1 - a)\, b = \frac{26}{49}$

$\Rightarrow$ $a + b - 2ab = \frac{26}{49}$ ...(i)

And $P(\text{not A and not B}) = \frac{15}{49}$

$\Rightarrow$ $P(\text{not A}) \times P(\text{not B}) = \frac{15}{49}$

$\Rightarrow$ $(1 - a) \times (1 - b) = \frac{15}{49}$

$\Rightarrow$ $1 - b - a + ab = \frac{15}{49}$

$\Rightarrow$ $a + b - ab = \frac{34}{49}$ ...(ii)

From (i) and (ii),

$a + b = \frac{42}{49}$ ...(iii)

and $ab = \frac{8}{49}$

$$(a - b)^2 = (a + b)^2 - 4ab = \frac{42}{49} \times \frac{42}{49} - \frac{4 \times 8}{49} = \frac{196}{2401}$$

$\therefore$ $a - b = \frac{14}{49}$ ...(iv)

From (iii) and (iv),

$a = \frac{4}{7}, b = \frac{2}{7}$

Hence probability of more probable of the two events $= \frac{4}{7}$

37. **(a)** $\frac{2}{5} + \frac{3}{5} \times \frac{2}{5} + \left(\frac{3}{5}\right)^2 \times \frac{2}{5} + \ldots\ldots + \left(\frac{3}{5}\right)^k . \frac{2}{5} > \frac{7}{10}$

$$\Rightarrow \frac{2}{5}\left[1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \ldots\ldots + \left(\frac{3}{5}\right)^k\right] > \frac{7}{10}$$

$$\Rightarrow \frac{2}{5} \times \frac{1 - \left(\frac{3}{5}\right)^k}{1 - \frac{3}{5}} > \frac{7}{10} \Rightarrow 1 - \left(\frac{3}{5}\right)^k > \frac{7}{10}$$

$$\Rightarrow \left(\frac{3}{5}\right)^k < \frac{3}{10} \Rightarrow k \geq 3$$

Hence minimum value of $k = 3$

38. **(b)** Given three numbers are chosen without replacement from $= \{1,2,3,\ldots.,8\}$

Let Event

F : Maximum of three numbers is 6.

E : Minimum of three numbers is 3.

This is the case of conditional probability

We have to find P (minimum) is 3 when it is given that P (maximum) is 6.

$$s\therefore\ P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$$

39. **(d)** $P(A^c \cap B^c / C) =$

$$\frac{P\left(\left(A^c \cap B^c\right) \cap C\right)}{P(C)} = \frac{P\left((A \cup B)^c \cap C\right)}{P(C)}$$

$$= \frac{P\left((S - A \cup B) \cap C\right)}{P(C)}$$

$$= \frac{P\left((S - A - B + A \cap B) \cap C\right)}{P(C)}$$

$$= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$$

$$= \frac{P(C) - P(A).P(C) - P(B)P(C) + 0}{P(C)}$$

$= 1 - P(A) - P(B)$ $[\because P(A^C) = 1 - P(A)]$

$= P(A^c) - P(B)$

40. **(a)** We know,

$$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \qquad [\because C \subset D]$$

Where, $0 \leq P(D) \leq 1$, hence

$$P\left(\frac{C}{D}\right) \geq P(C)$$

41. **(d)** Given that

$$P = \frac{1}{4} \Rightarrow q = 1 - \frac{1}{4} = \frac{3}{4}$$

and $P(x \geq 1) \geq \frac{9}{10}$

$\Rightarrow \quad 1-P(x=0) \geq \frac{9}{10}$

$\Rightarrow \quad 1-{}^nC_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow \quad 1-\frac{9}{10} \geq \left(\frac{3}{4}\right)^n$

$\Rightarrow \quad \left(\frac{3}{4}\right)^n \leq \left(\frac{1}{10}\right)$

Taking log at the base 3/4, on both sides, we get

$n \log_{3/4}\left(\frac{3}{4}\right) \geq \log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow \quad n \geq -\log_{3/4} 10 = \frac{-\log_{10} 10}{\log_{10}\left(\frac{3}{4}\right)} = \frac{-1}{\log_{10} 3 - \log_{10} 4}$

$\Rightarrow \quad n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$

42. **(b)** Given that $P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A)\,P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

43. **(d)** Given that P(I) = 0.3 and P(II) = 0.2

$\therefore P(\overline{I}) = 1-0.3 = 0.7$

$\therefore$ The required probability

$= P(\overline{I} \cap II) = P(\overline{I}).P(II) = 0.7 \times 0.2 = 0.14$

44. **(b)** Probability of particular house being selected $= \frac{1}{3}$

P (all the persons apply for the same house)

$= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right)3 = \frac{1}{9}$.

45. **(c)** $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$

$\Rightarrow P(A \cup B) = \frac{5}{6},\ P(A) = \frac{3}{4}$

Also $\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}$

$\Rightarrow P(A)\ P(B) = \frac{3}{4} - \frac{1}{3} = \frac{1}{4} = P(A \cap B)$

Hence A and B are independent but not equally likely.

46. **(c)** A and B will contradict each other if one speaks truth and other false . So , the required probability

$P(A \cap \overline{B}) + P(\overline{A} \cap B) = \frac{4}{5}\left(1-\frac{3}{4}\right) + \left(1-\frac{4}{5}\right)\frac{3}{4}$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$

47. **(a)** Given that $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and

$P(C) = \frac{1}{4}$; $P(AUBUC) = 1 - P(\bar{A})P(\bar{B})P(\bar{C})$

$= 1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) = 1-\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$

48. **(11)**

Probability of getting at least two 3's or 5's in one trial

$= {}^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {}^4C_4\left(\frac{2}{6}\right)^4$

$= \frac{33}{3^4} = \frac{11}{27}$

$E(x) = np = 27\left(\frac{11}{27}\right) = 11.$

49. **(11.00)**

Let 'n' bombs are required, then

$1-{}^nC_1 \cdot \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} - {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$

$\Rightarrow \frac{1}{100} \geq \frac{n+1}{2^n} \Rightarrow 2^n \geq 100(n+1) \Rightarrow n \geq 11$

50. **(3.00)**

$p = \frac{1}{10}, q = \frac{9}{10}$

P (not hitting target in n trials) $= \left(\frac{9}{10}\right)^n$

P (at least one hit) $= 1-\left(\frac{9}{10}\right)^n$

$\because 1-\left(\frac{9}{10}\right)^n > \frac{1}{4} \Rightarrow (0.9)^n < 0.75$

$\therefore n_{\text{minimum}} = 3.$

51. **(d)** $\sum P(K) = 1 \quad \Rightarrow \quad 6K^2 + 5K = 1$

$6K^2 + 5K - 1 = 0$

$6K^2 + 6K - K - 1 = 0$

$\Rightarrow \quad (6K-1)(K+1) = 0$

$\Rightarrow \quad K = \frac{1}{6}$ ($K = -1$ rejected)

$P(X > 2) = K + 2K + 5K^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6+12+5}{36} = \frac{23}{36}$

52. **(d)** Given mean $\mu = 8$ and variance $\sigma^2 = 4$

$\Rightarrow \mu = np = 8$ and $\sigma^2 = npq = 4$.

$p + q = 1 \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}$ and $n = 16$

$\because P(X \le 2) = \frac{k}{2^{16}}$

$\therefore \quad {}^{16}C_0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^{16} + {}^{16}C_2\left(\frac{1}{2}\right)^{16} = \frac{k}{2^{16}}$

$\Rightarrow k = (1 + 16 + 120) = 137$

53. **(c)** Let X be the random variable which denotes the Rs gained by the person.

Total cases $= 6 \times 6 = 36$.

Favorable cases for the person on winning ₹ 15 are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) i.e., 6 cases.

$\therefore \quad P(X = 15) = \frac{6}{36} = \frac{1}{6}$

Favorable cases for the person on winning ₹ 12 are (6, 3), (5, 4), (4, 5), (3, 6) i.e., 4.

$\therefore \quad P(X = 12) = \frac{4}{36} = \frac{1}{9}$

Remaining cases $= 36 - 6 - 4 = 26$

$\therefore \quad P(X = -6) = \frac{26}{36} = \frac{13}{18}$

X	15	12	-6
$P(X)$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{13}{18}$
$X.P(X)$	$\frac{5}{2}$	$\frac{4}{3}$	$\frac{-13}{3}$

Hence, $E(X) = \sum X.P(X) = \frac{5}{2} + \frac{4}{3} - \frac{13}{3} = -\frac{1}{2}$

54. **(b)** $P(\text{white ball}) = \frac{30}{40} = \frac{3}{4}$, $Q(\text{red ball}) = \frac{10}{40} = \frac{1}{4}$, $n = 16$

$$\frac{\text{Mean of } X}{\text{standard deviation of } X} = \frac{nP}{\sqrt{nPQ}} = \frac{\sqrt{nP}}{\sqrt{Q}}$$

$$= \sqrt{\frac{16 \times \frac{3}{4}}{\frac{1}{4}}} = \sqrt{48} = 4\sqrt{3}$$

55. **(b)** We can apply binomial probability distribution

We have n = 10

p = Probability of drawing a green ball $= \frac{15}{25} = \frac{3}{5}$

Also $q = 1 - \frac{3}{5} = \frac{2}{5}$

Variance $= npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

56. **(d)** Let $p(F) = p \Rightarrow p(S) = 2p$

$\because \quad p + 2p = 1 \Rightarrow p = \frac{1}{3}$

$p(x \ge 5) = p(x = 5) + p(x = 6)$

$= {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + {}^6C_5\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$

$= \left(\frac{2}{3}\right)^5\left(6 \times \frac{1}{3} + \frac{2}{3}\right) = \frac{256}{729}$

57. **(d)** Let mean $= np = 2$ ...(1)

and variance $= npq = 1$ ...(2)

On solving eqn (1) and (2), we get

$q = \frac{1}{2}$ and $p = \frac{1}{2}$

From eqn (1), we have

$n = 4$

$P(x \ge 1) = {}^4C_1p^1q^3 + {}^4C_2p^2q^2 + {}^4C_3p^3q + {}^4C_4p^4$

$= 1 - {}^4C_0p^0q^4 = 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$

58. **(b)** Since X has a binomial distribution, B (n, p)

$\therefore \quad P(X = 2) = {}^nC_2\,(p)^2\,(1-p)^{n-2}$

and $P(X = 3) = {}^nC_3\,(p)^3\,(1-p)^{n-3}$

Given $P(X = 2) = P(X = 3)$

$\Rightarrow {}^nC_2\,p^2\,(1-p)^{n-2} = {}^nC_3\,(p)^3\,(1-p)^{n-3}$

$\Rightarrow \frac{n!}{2!(n-2)!}.\frac{p^2(1-p)^n}{(1-p)^2} = \frac{n!}{3!(n-3)!}.\frac{p^3(1-p)^n}{(1-p)^3}$

$\Rightarrow \frac{1}{n-2} = \frac{1}{3}.\frac{p}{1-p} \Rightarrow 3(1-p) = p(n-2)$

$\Rightarrow 3 - 3p = np - 2p$

$\Rightarrow np = 3 - p$

$\Rightarrow E(X) = \text{mean} = 3 - p$

($\because$ mean of B $(n, p) = np$)

59. **(c)** $p = p$ (correct answer), $q = p$ (wrong answer)

$\Rightarrow P = \frac{1}{3}, q = \frac{2}{3}, n = 5$

By using Binomial distribution

Required probability

$P(x \ge 4) = {}^5C_4\left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + {}^5C_5\left(\frac{1}{3}\right)^5$

$= 5 \cdot \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

60. **(b)** Given that p (at least one failure) $\ge \frac{31}{32}$

$\Rightarrow 1 - p$ (no failure) $\ge \frac{31}{32}$

$\Rightarrow 1 - p^5 \ge \frac{31}{32}$

$\Rightarrow p^5 \leq \frac{1}{32} \Rightarrow p \leq \frac{1}{2}$

But $p \geq 0$

Hence p lies in the interval $\left[0, \frac{1}{2}\right]$.

61. **(d)** Given that

$P = \frac{1}{4} \Rightarrow q = 1 - \frac{1}{4} = \frac{3}{4}$

and $P(x \geq 1) \geq \frac{9}{10}$

$\Rightarrow \quad 1 - P(x = 0) \geq \frac{9}{10}$

$\Rightarrow \quad 1 - {}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow \quad 1 - \frac{9}{10} \geq \left(\frac{3}{4}\right)^n$

$\Rightarrow \quad \left(\frac{3}{4}\right)^n \leq \left(\frac{1}{10}\right)$

Taking log at the base 3/4, on both sides, we get

$n \log_{3/4}\left(\frac{3}{4}\right) \geq \log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow \quad n \geq -\log_{3/4} 10 = \frac{-\log_{10} 10}{\log_{10}\left(\frac{3}{4}\right)} = \frac{-1}{\log_{10} 3 - \log_{10} 4}$

$\Rightarrow \quad n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$

62. **(b)** The sample space of pair of fair dice is thrown,

$S = (1, 1), (1, 2)\, (1, 3) \ldots = 36$

Sum 9 are $(5, 4), (4, 5), (6, 3), (3, 6)$

$\text{P(score 9)} = \frac{4}{36} = \frac{1}{9}$

Number of trial = 3

$\therefore$ Probability of getting score 9 exactly twice

$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 . \left(1 - \frac{1}{9}\right) = \frac{3!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9}$

$= \frac{3.2!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{243}$

63. **(d)** From poission distribution

$P(X = r) = \frac{e^{-m} m^r}{r!}$

Given mean $(m) = 5$

P (at most 1 phone call)

$= P(X \leq 1) = P(X = 0) + P(X = 1)$

$= e^{-5} + 5 \times e^{-5} = \frac{6}{e^5}$

64. **(c)** From poission distribution, probability of getting k successes is

$P(x = k) = e^{-\lambda} \frac{\lambda^k}{k!}$

Given mean $(\lambda) = 2$

$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$

$= 1 - e^{-\lambda} - e^{-\lambda}\left(\frac{\lambda}{1!}\right) = 1 - \frac{3}{e^2}$.

65. **(a)** Given that mean = $np = 4$ and variance = $npq = 2$

$\Rightarrow p = q = \frac{1}{2}$ and $n = 8$

$\therefore \quad P(2 \text{ success}) = {}^8C_2 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2$

$= \frac{28}{2^8} = \frac{28}{256}$

66. **(b)** $P(E) = P(\text{ 2 or 3 or 5 or 7})$

$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

$P(F) = P(1 \text{ or } 2 \text{ or } 3) = 0.15 + 0.23 + 0.12 = 0.50$

$P(E \cap F) = P(2 \text{ or } 3) = 0.23 + 0.12 = 0.35$

We know that

$P(EUF) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.50 - 0.35 = 0.77$

67. **(b)** Given that $np = 4$ and

$npq = 2 \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$P(X = 1) = {}^8C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8.\frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$

68. **(d)** The experiment follows binomial distribution with $n = 5, p = 3/6 = 1/2$.

$q = 1 - p = 1/2.$;

$\therefore$ Variance $= npq = 5/4$.

28

Properties of Triangles

TOPIC 1 Properties of Triangle, Solutions of Triangles, Inscribed & Enscribed Circles, Regular Polygons

1. Let A(3, 0, –1), B(2, 10, 6) and C (1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then cos (∠GOA) (O being the origin) is equal to : **[April 10, 2019 (I)]**

(a) $\frac{1}{2\sqrt{15}}$ (b) $\frac{1}{\sqrt{15}}$

(c) $\frac{1}{6\sqrt{10}}$ (d) $\frac{1}{\sqrt{30}}$

2. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is : **[April 10, 2019 (II)]**

(a) $\frac{2}{\sqrt{3}}$ (b) $4\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\frac{4}{\sqrt{3}}$

3. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is : **[April 08, 2019 (II)]**

(a) 5 : 9 : 13 (b) 4 : 5 : 6

(c) 3 : 4 : 5 (d) 5 : 6 : 7

4. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. if $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is : **[Jan. 11, 2019 (I)]**

(a) $\frac{3}{2}y$ (b) $\frac{c}{\sqrt{3}}$

(c) $\frac{c}{3}$ (d) $\frac{y}{\sqrt{3}}$

5. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value : **[Jan. 11, 2019 (II)]**

(a) (7, 19, 25) (b) (3, 4, 5)

(c) (5, 12, 13) (d) (19, 7, 25)

6. With the usual notation, in ΔABC, if ∠A + ∠B = 120°, $a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$, then the ratio ∠A : ∠B, is: **[Jan. 10, 2019 (II)]**

(a) 7 : 1 (b) 5 : 3

(c) 9 : 7 (d) 3 : 1

7. In a ΔABC, $\frac{a}{b} = 2 + \sqrt{3}$ and ∠C = 60°. Then the ordered pair (∠A, ∠B) is equal to : **[Online April 10, 2015]**

(a) (45°, 75°) (b) (105°, 15°)

(c) (15°, 105°) (d) (75°, 45°)

8. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to : **[2013]**

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

(c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

9. If in a triangle ABC, $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$, then cosA is equal to **[2012]**

(a) 5/7 (b) 1/5

(c) 35/19 (d) 19/35

10. In a ΔPQR, If 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to : **[2012]**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

11. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is **[2010]**

(a) There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$

(b) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$

(c) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$

(d) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$

12. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P, then $\sin A$, $\sin B$, $\sin C$ are in **[2005]**

(a) G.P. (b) A. P.
(c) A.P-G.P. (d) H. P

13. In a triangle ABC, let $\angle C=\frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r+R)$ equals **[2005]**

(a) $b+c$ (b) $a+b$
(c) $a+b+c$ (d) $c+a$

14. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is **[2004]**

(a) 150° (b) 90°
(c) 120° (d) 60°

15. If in a ΔABC $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides a, b and c **[2003]**

(a) satisfy $a+b=c$ (b) are in A.P
(c) are in G.P (d) are in H.P

16. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB=\frac{\pi}{6}$ and $\angle ABE=\frac{\pi}{3}$, then the area of the ΔABC is **[2003]**

(a) $\frac{64}{3}$ (b) $\frac{8}{3}$
(c) $\frac{16}{3}$ (d) $\frac{32}{3\sqrt{3}}$

17. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is **[2003]**

(a) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{n}\right)$

(c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{2n}\right)$.

18. In a triangle with sides a, b, c, $r_1>r_2>r_3$ (which are the ex-radii) then **[2002]**

(a) $a>b>c$ (b) $a<b<c$
(c) $a>b$ and $b<c$ (d) $a<b$ and $b>c$

19. The sides of a triangle are $3x+4y$, $4x+3y$ and $5x+5y$ where $x, y>0$ then the triangle is **[2002]**

(a) right angled (b) obtuse angled
(c) equilateral (d) none of these

TOPIC 2 Heights & Distances

20. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line $x=1$ at the point A. The ray gets reflected on the line $x=1$ and meets x-axis at the point B. Then, the line AB passes through the point: **[Sep. 06, 2020 (I)]**

(a) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (b) $\left(4,-\frac{\sqrt{3}}{2}\right)$

(c) $(3,-\sqrt{3})$ (d) $(4,-\sqrt{3})$

21. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45º. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30º to the horizontal plane, the angle of elevation of the top of the hill becomes 75º. Then the height of the hill (in meters) is ______. **[Sep. 06, 2020 (I)]**

22. The angle of elevation of the summit of a mountain from a point on the ground is 45º. After climbing up on km towards the summit at an inclination of 30º from the ground, the angle of elevation of the summit is found to be 60º. Then the height (in km) of the summit from the ground is: **[Sep. 06, 2020 (II)]**

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(c) $\frac{1}{\sqrt{3}-1}$ (d) $\frac{1}{\sqrt{3}+1}$

23. Two vertical poles $AB = 15$ m and $CD = 10$ m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is : **[Sep. 04, 2020 (I)]**

(a) 20/3 (b) 5
(c) 10/3 (d) 6

24. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to : **[Sep. 04, 2020 (II)]**

(a) 100 (b) $200\sqrt{3}$
(c) 400 (d) $400\sqrt{3}$

25. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is : **[April 10, 2019 (I)]**

(a) $\frac{100}{3\sqrt{3}}$ (b) $10\sqrt{5}$
(c) 20 (d) 25

26. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is: **[Jan. 12, 2019 (II)]**

(a) 60 (b) 50
(c) 45 (d) 42

27. Consider a triangular plot ABC with sides AB = 7 m, BC = 5 m and CA = 6 m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is: **[Jan. 10, 2019 (I)]**

(a) $\frac{3}{2}\sqrt{21}$ (b) $\frac{2}{3}\sqrt{21}$
(c) $2\sqrt{21}$ (d) $7\sqrt{3}$

28. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is : **[2018]**

(a) 50 (b) $100\sqrt{3}$
(c) $50\sqrt{2}$ (d) 100

29. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min, for the angle of depression of the car to change from 30° to 45°, then after this, the time taken (in min) by the car to reach the foot of the tower, is. **[Online April 16, 2018]**

(a) $9(1+\sqrt{3})$ (b) $\frac{9}{2}(\sqrt{3}-1)$
(c) $18(1+\sqrt{3})$ (d) $18(\sqrt{3}-1)$

30. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}km$ above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane is **[Online April 15, 2018]**

(a) 1500 (b) 750
(c) 720 (d) 1440

31. A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is **[Online April 15, 2018]**

(a) $20\sqrt{2}$ (b) $10\sqrt{2}$
(c) $10\sqrt{3}$ (d) $20\sqrt{3}$

32. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to : **[2017]**

(a) $\frac{4}{9}$ (b) $\frac{6}{7}$
(c) $\frac{1}{4}$ (d) $\frac{2}{9}$

33. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is: **[2016]**

(a) 20 (b) 5
(c) 6 (d) 10

34. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is : **[Online April 10, 2016]**

(a) 108 (b) $36\sqrt{3}$
(c) $54\sqrt{3}$ (d) 54

(which are the ex-radii) then **[2002]**

35. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is : **[2015]**

(a) $1:\sqrt{3}$ (b) $2:3$
(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

36. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is 'h' and the distance of the foot of the smallest pole from O is 'a'; then the distance between two consecutive poles, is : **[Online April 11, 2015]**

(a) $\dfrac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$ (b) $\dfrac{h\sin\alpha + a\cos\alpha}{9\sin\alpha}$

(c) $\dfrac{h\cos\alpha - a\sin\alpha}{9\cos\alpha}$ (d) $\dfrac{h\sin\alpha - a\cos\alpha}{9\cos\alpha}$

37. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is $45°$. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to $30°$. Then the speed (in m/s) of the bird is **[2014]**

(a) $20\sqrt{2}$ (b) $20\left(\sqrt{3}-1\right)$

(c) $40\left(\sqrt{2}-1\right)$ (d) $40\left(\sqrt{3}-\sqrt{2}\right)$

38. The angle of elevation of the top of a vertical tower from a point P on the horizontal ground was observed to be α. After moving a distance 2 metres from P towards the foot of the tower, the angle of elevation changes to β. Then the height (in metres) of the tower is: **[Online April 11, 2014]**

(a) $\dfrac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ (b) $\dfrac{\sin\alpha\sin\beta}{\cos(\beta-\alpha)}$

(c) $\dfrac{2\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ (d) $\dfrac{\cos(\beta-\alpha)}{\sin\alpha\sin\beta}$

39. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45°. Then the height of the pole is **[2008]**

(a) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}-1}m$ (b) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

(c) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}-1)m$ (d) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}+1}m$

40. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB\ (= a)$ subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A and B is 30°. The height of the tower is **[2007]**

(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$

(c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$

41. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is **[2004]**

(a) 60m (b) 30 m

(c) 40m (d) 20 m

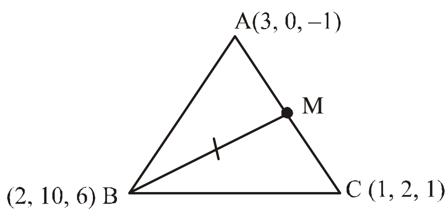
42. The upper $\dfrac{3}{4}$ th portion of a vertical pole subtends an angle $\tan^{-1}\dfrac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is **[2003]**

(a) 80 m (b) 20 m

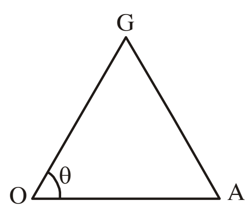
(c) 40 m (d) 60 m.

Hints & Solutions

1. **(b)** G is the centroid of ΔABC.



$\Rightarrow G \equiv \left(\frac{3+2+1}{3}, \frac{0+10+2}{3}, \frac{-1+6+1}{3}\right) \equiv (2, 4, 2)$



$OG = \sqrt{4+16+4}$, $OA = \sqrt{9+1}$, $AG = \sqrt{1+16+9}$

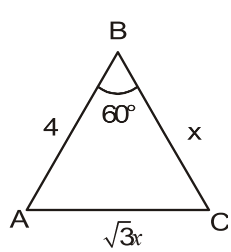
$\therefore \cos\theta = \frac{(OG)^2 + (OA)^2 - (AG)^2}{2(OG)(OA)} = \frac{24+10-26}{2\sqrt{24}\sqrt{10}}$

$= \frac{8}{2\sqrt{8\times3\times2\times5}} = \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$

2. **(c)** Given that A, B, C, are in A.P. $\Rightarrow 2B = A + C$

Now, $A + B + C = \pi \Rightarrow B = \frac{\pi}{3}$

Area $= \frac{1}{2}(4x)\sin 60° = \sqrt{3}x$



Now $\cos 60° = \frac{16 + x^2 - 3x^2}{8x}$

$\Rightarrow 4x = 16 - 2x^2 \Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow x = 2$ [$\because x$ can't be negative]

Hence, area $= 2\sqrt{3}$ sq. cm

3. **(b)** Let the sides of triangle are $a > b > c$ where

Given $A = 2C$

$\because A + B + C = \pi$ and $A = 2C$

$\Rightarrow B = \pi - 3C$ ...(i)

$\because a, b, c$ are in A.P. $\Rightarrow a + c = 2b$

$\Rightarrow \sin A + \sin C = 2\sin B$ ...(ii)

$\Rightarrow \sin A = \sin(2C)$ and $\sin B = \sin 3C$

From (ii),

$\sin 2C + \sin C = 2\sin 3C$

$\Rightarrow (2\cos C + 1)\sin C = 2\sin C\,(3 - 4\sin^2 C)$

$\Rightarrow 2\cos C + 1 = 6 - 8(1 - \cos^2 C)$

$\Rightarrow 8\cos^2 C - 2\cos C - 3 = 0$

$\Rightarrow \cos C = \frac{3}{4}$ or $\cos C = -\frac{1}{2}$

$\because$ C is acute angle

$\Rightarrow \cos C = \frac{3}{4} \Rightarrow \sin C = \frac{\sqrt{7}}{4}$

and $\sin A = 2\sin C\cos C = 2 \times \frac{\sqrt{7}}{4} \times \frac{3}{4} = \frac{3\sqrt{7}}{8}$

$\sin B = \frac{3\sqrt{7}}{4} - \frac{4\sqrt{7}}{4} \times \frac{7}{16} = \frac{5\sqrt{7}}{16}$

$\Rightarrow \sin A : \sin B : \sin C :: a : b : c$ is $6 : 5 : 4$

4. **(b)** Let two sides of triangle are a and b.

$a + b = x$

$ab = y$

$x^2 - c^2 = y \Rightarrow (a+b)^2 - c^2 = ab$

$\Rightarrow (a + b - c)(a + b + c) = ab$

$\Rightarrow 2(s - c)(2s) = ab$

$\Rightarrow 4s(s - c) = ab$

$\Rightarrow \frac{s(s-c)}{ab} = \frac{1}{4}$

$\Rightarrow \cos^2\frac{c}{2} = \frac{1}{4}$

$\Rightarrow \cos c = -\frac{1}{2} \Rightarrow c = 120°$

$\therefore$ Area of triangle is,

$\Delta = \frac{1}{2}ab(\sin 120°) = \frac{\sqrt{3}}{4}ab$

$\because \quad R = \dfrac{abc}{4\Delta}$

$\because \quad R = \dfrac{abc}{\sqrt{3}\,ab} = \dfrac{c}{\sqrt{3}}$

5. **(a)** Let $\dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13} = k$ (Say).

$\therefore \quad b+c = 11k,\ c+a = 12k,\ a+b = 13k$

$\therefore \quad a+b+c = 18k$

$\therefore \quad a = 7k,\ b = 6k$ and $c = 5k$

$\therefore \quad \cos A = \dfrac{36k^2 + 25k^2 - 49k^2}{2.30k^2} = \dfrac{1}{5}$

and $\cos B = \dfrac{49k^2 + 25k^2 - 36k^2}{2.35k^2} = \dfrac{19}{35}$

and $\cos C = \dfrac{49k^2 + 36k^2 - 25k^2}{2.42k^2} = \dfrac{5}{7}$

$\therefore \quad \cos A : \cos B : \cos C = 7 : 19 : 25$

$\therefore \quad \dfrac{\cos A}{7} = \dfrac{\cos B}{19} = \dfrac{\cos C}{25}$

Hence, required ordered triplet is (7, 19, 25).

6. **(a)** $\angle A + \angle B = 120°$ ...(1)

$\Rightarrow \quad \angle C = 180° - 120° = 60°$

$\tan\left(\dfrac{A-B}{2}\right) = \dfrac{a-b}{a+b}\cot\left(\dfrac{C}{2}\right)$

$= \dfrac{2}{2\sqrt{3}}(\cot 30°) = \dfrac{1}{\sqrt{3}} \times \sqrt{3} = 1$

$\Rightarrow \quad \dfrac{\angle A - \angle B}{2} = \dfrac{\pi}{4}$ ($\angle$ is angle)

$\Rightarrow \quad \angle A - \angle B = 90°$ ...(2)

From eqn (1) and (2)

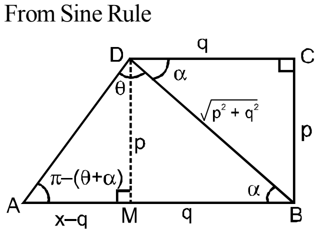
$\angle A = 105°,\ \angle B = 15°$

Then, $\angle A : \angle B = 7 : 1$

7. **(b)** $\dfrac{\sin A}{\sin B} = 2 + \sqrt{3}$

$\dfrac{\sin(105°)}{\sin(15°)} = 2 + \sqrt{3} \quad \dfrac{\cos 15°}{\sin 15°} = 2 + \sqrt{3}$

8. **(a)** From Sine Rule



$\dfrac{AB}{\sin\theta} = \dfrac{\sqrt{p^2+q^2}}{\sin(\pi - (\theta + \alpha))}$

$AB = \dfrac{\sqrt{p^2+q^2}\,\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha}$

$= \dfrac{(p^2+q^2)\sin\theta}{q\sin\theta + p\cos\theta}$

$\left(\because \cos\alpha = \dfrac{q}{\sqrt{p^2+q^2}} \text{ and } \sin\alpha = \dfrac{p}{\sqrt{p^2+q^2}}\right)$

9. **(b)** In a triangle ABC.

Let $\dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13} = K$

$\Rightarrow b+c = 11K,\ c+a = 12K,\ a+b = 13K$

On solving these equations, we get

$a = 7K,\ b = 6K,\ c = 5K$

Now we know,

$\cos A = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{36K^2 + 25K^2 - 49K^2}{2(6K)(5K)} = \dfrac{1}{5}$

10. **(b)** Given that $3\sin P + 4\cos Q = 6$ ...(i)

$4\sin Q + 3\cos P = 1$ ...(ii)

Squaring and adding (i) & (ii) we get

$9\sin^2 P + 16\cos^2 Q + 24\sin P\cos Q$
$+ 16\sin^2 Q + 9\cos^2 P + 24\sin Q\cos P$
$= 36 + 1 = 37$

$\Rightarrow \quad 9(\sin^2 P + \cos^2 P) + 16(\sin^2 Q + \cos^2 Q)$
$+ 24(\sin P\cos Q + \cos P\sin Q) = 37$

$\Rightarrow \quad 9 + 16 + 24\sin(P+Q) = 37$

$[\because \sin^2\theta + \cos^2\theta = 1$ and $\sin A\cos B + \cos A\sin B = \sin(A+B)]$

$\Rightarrow \quad \sin(P+Q) = \dfrac{1}{2}$

$\Rightarrow \quad P + Q = \dfrac{\pi}{6}$ *or* $\dfrac{5\pi}{6}$

$\Rightarrow \quad R = \dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$ $\quad (\because P + Q + R = \pi)$

If $\quad R = \dfrac{5\pi}{6}$ then $0 < Q,\ P < \dfrac{\pi}{6}$

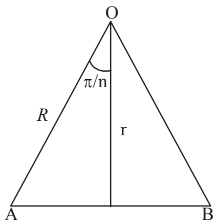
$\Rightarrow \quad \cos Q < 1$ and $\sin P < \dfrac{1}{2}$

$\Rightarrow \quad 3\sin P + 4\cos Q < \dfrac{11}{2}$

But given that $3\sin P + 4\sin Q = 6$

So, $R = \dfrac{\pi}{6}$

11. **(b)** Let O is centre of polygon of n sides and AB is one of the side, then by figure



$$\cos\frac{\pi}{n}=\frac{r}{R}$$

$$\Rightarrow \quad \frac{r}{R}=\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}$$

for $n = 3, 4, 6$ respectively.

12. **(b)** Let altitudes from A, B and C be p_1, p_2 and p_3 resp.

$$\therefore \quad \Delta=\frac{1}{2}p_1 a=\frac{1}{2}p_2 b=\frac{1}{2}p_3 b$$

Given that, p_1, p_2, p_3, are in H.P.

$$\Rightarrow \quad \frac{2\Delta}{a},\frac{2\Delta}{b},\frac{2\Delta}{c} \text{ are in H.P.}$$

$$\Rightarrow \quad \frac{1}{a},\frac{1}{b},\frac{1}{c} \text{ are in H.P.}$$

$\Rightarrow \quad a, b, c$ are in A.P.

By sine formula

$\Rightarrow \quad K \sin A, K \sin B, K \sin C$ are in AP

$\Rightarrow \quad \sin A, \sin B, \sin C$ are in A.P.

13. **(b)** We know that for the circle circumscribing a right triangle, hypotenutse is the diameter

$\because \quad \angle C = 90°$

$$\therefore \quad 2R = c \Rightarrow R=\frac{c}{2}$$

$$\text{also } r=\frac{\Delta}{s}=\frac{\frac{1}{2}\times a\times b}{\frac{a+b+c}{2}}$$

$$\Rightarrow \quad r=\frac{ab}{a+b+c}$$

$$\therefore \quad 2r+2R=\frac{2ab}{a+b+c}+c=\frac{2ab+ac+bc+c^2}{a+b+c}$$

$$=\frac{2ab+ac+bc+a^2+b^2}{a+b+c} \qquad (\because c^2=a^2+b^2)$$

$$=\frac{(a+b)^2+(a+b)c}{a+b+c}=(a+b)$$

14. **(c)** Let $a = \sin\alpha$, $b = \cos\alpha$ and

$c=\sqrt{1+\sin\alpha\cos\alpha}$

Clearly a and $b < 1$ but $c > 1$ as $\sin\alpha > 0$ and $\cos\alpha > 0$

$\therefore$ c is the greatest side and greatest angle is C.

$$\text{We know that, } \cos C=\frac{a^2+b^2-c^2}{2ab}$$

$$=\frac{\sin^2\alpha+\cos^2\alpha-1-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}=-\frac{1}{2}$$

$\therefore C = 120°$

15. **(b)** Given that, $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$

$a[\cos C + 1] + c[\cos A + 1] = 3b$

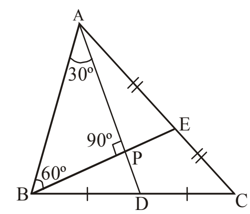
$(a + c) + (a \cos C + c \cos B) = 3b$

We know that, $b = a \cos C + c \cos B$

$a + c + b = 3b$ or $a + c = 2b$

or a, b, c are in A.P.

16. **(d)**



We know that median divides each other in ratio 2 :1

$$AP=\frac{2}{3}AD=\frac{8}{3}; \quad PD=\frac{4}{3}; \quad \text{Let } PB=x$$

$$\tan 60^o=\frac{8/3}{x} \text{ or } x=\frac{8}{3\sqrt{3}}$$

$$\text{Area of } \Delta ABD=\frac{1}{2}\times 4\times\frac{8}{3\sqrt{3}}=\frac{16}{3\sqrt{3}}$$

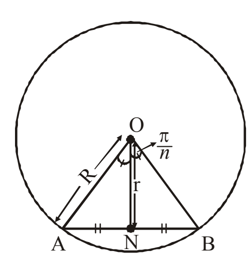
$$\therefore \text{Area of } \Delta ABC=2\times\frac{16}{3\sqrt{3}}=\frac{32}{3\sqrt{3}}$$

[$\because$ Median of a Δ divides it into two Δ's of equal area.]

17. **(c)** We know that, $\tan\left(\frac{\pi}{n}\right)=\frac{a}{2r}; \sin\left(\frac{\pi}{n}\right)=\frac{a}{2R}$

$$\Rightarrow \quad r=\frac{a}{2}\cot\frac{\pi}{n}; \text{R}=\frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

$$r+R=\frac{a}{2}\left[\cot\frac{\pi}{n}+\operatorname{cosec}\frac{\pi}{n}\right]$$



$$=\frac{a}{2}\left[\frac{\cos\frac{\pi}{n}+1}{\sin\frac{\pi}{n}}\right]=\frac{a}{2}\left[\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right]=\frac{a}{2}\cot\frac{\pi}{2\pi}$$

18. (a) We know that, $r_1 = \dfrac{\Delta}{s-a}, r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$

Given that,

$r_1 > r_2 > r_3 \Rightarrow \dfrac{\Delta}{s-a} > \dfrac{\Delta}{s-b} > \dfrac{\Delta}{s-c}$;

$\Rightarrow \quad s-a < s-b < s-c$

$\Rightarrow \quad -a < -b < -c \Rightarrow a > b > c$

19. (b) Let $a = 3x + 4y$, $b = 4x + 3y$ and $c = 5x + 5y$

as $x, y > 0$, $c = 5x + 5y$ is the largest side

$\therefore$ C is the largest angle . Now

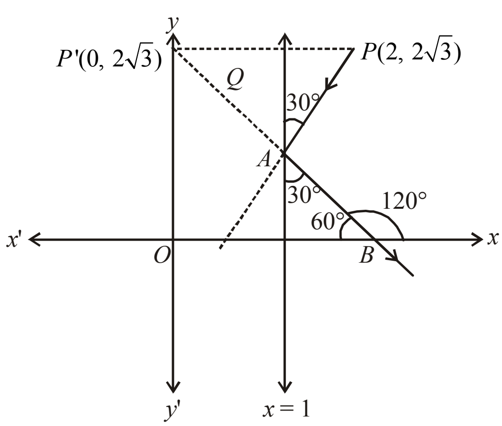
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(3x+4y)^2 + (4x+3y)^3 - (5x+5y)^2}{2(3x+4y)(4x+3y)}$$

$$= \frac{-2xy}{2(3x+4y)(4x+3y)} < 0$$

$\therefore$ C is obtuse angle $\Rightarrow \Delta ABC$ is obtuse angled

20. (c)



Slope of $AB = \tan 120° = -\sqrt{3}$

$\therefore$ Equation of line AB (i.e. BP') :

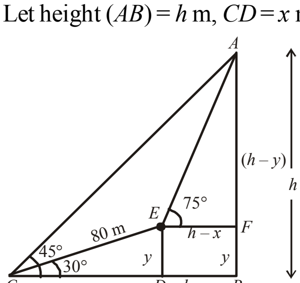
$$y - 2\sqrt{3} = -\sqrt{3}(x - 0)$$

$$\Rightarrow \sqrt{3}x + y = 2\sqrt{3}$$

$\therefore$ Point $(3, -\sqrt{3})$ lies on line AB.

21. (80)

Let height $(AB) = h$ m, $CD = x$ m and $ED = y$ m



In rt. ΔCDE,

$$\sin 30° = \frac{y}{80} \Rightarrow y = 40$$

$$\cos 30° = \frac{x}{80} \Rightarrow x = 40\sqrt{3}$$

Now, in ΔAEF,

$$\tan 75° = \frac{h-y}{h-x}$$

$$\Rightarrow (2+\sqrt{3}) = \frac{h-40}{h-40\sqrt{3}}$$

$$\Rightarrow (2+\sqrt{3})(h - 40\sqrt{3}) = h - 40$$

$$\Rightarrow 2h - 80\sqrt{3} + \sqrt{3}h - 120 = h - 40$$

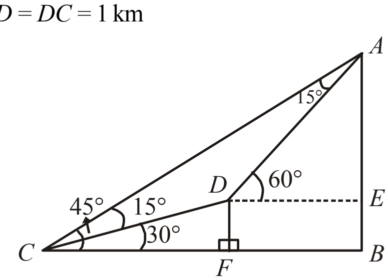
$$\Rightarrow h + \sqrt{3}h = 80 + 80\sqrt{3}$$

$$\Rightarrow (\sqrt{3}+1)h = 80(\sqrt{3}+1)$$

$\therefore h = 80$ m

22. (c) $\because \angle DCA = \angle DAC = 30°$

$\therefore AD = DC = 1$ km



In ΔDEA,

$$\frac{AE}{AD} = \sin 60° \Rightarrow AE = \frac{\sqrt{3}}{2} \text{ km}$$

In ΔCDF, $\sin 30° = \dfrac{DF}{CD} \Rightarrow DF = \dfrac{1}{2}$ km

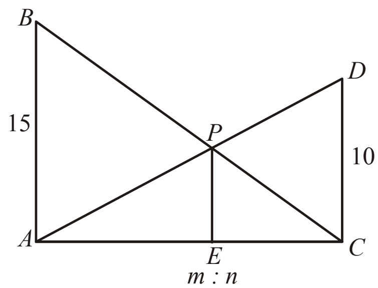
$\therefore EB = DF = \dfrac{1}{2}$ km

$\therefore$ Height of mountain $= AE + EB$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \left(\frac{\sqrt{3}+1}{2}\right) \text{ km}$$

$$= \frac{1}{\sqrt{3}-1} \text{ km}$$

23. (d)



Let $PE \perp AC$ and $\frac{AE}{EC} = \frac{m}{n}$

$\because \Delta AEP \sim \Delta ACD,\ \frac{m}{PE} = \frac{m+n}{10}$

$\Rightarrow PE = \frac{10m}{m+n}$ ...(i)

$\because \Delta CEP \sim \Delta CAB,\ \frac{n}{PE} = \frac{m+n}{15}$

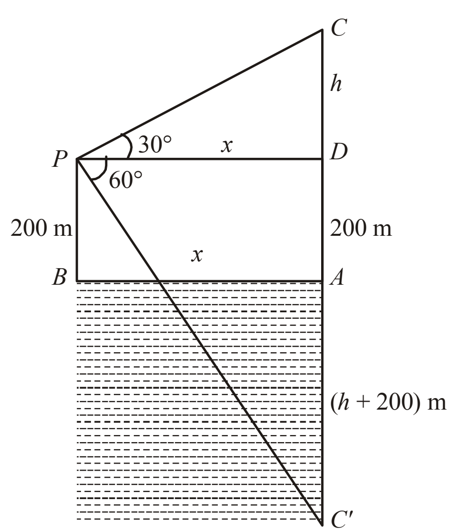
$\Rightarrow PE = \frac{15n}{m+n}$ ...(ii)

From (i) and (ii),

$10m = 15n \Rightarrow m = \frac{3}{2}n$

So, $PE = 6$

24. (c)



Here in ΔPCD,

$\sin 30° = \frac{h}{PC} \Rightarrow PC = 2h$ ...(i)

$\tan 30° = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$ ...(ii)

Now, in right $\Delta PC'D$

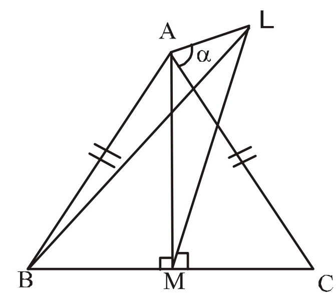
$\tan 60° = \frac{h+400}{x}$

$\Rightarrow \sqrt{3}x = h + 400 \Rightarrow 3h = h + 400$ [From (ii)]

$\Rightarrow h = 200$

So, $PC = 400$ m [From (i)]

25. (3) Let the height of the vertical tower situated at the mid point of BC be h.



In ΔALM,

$\cot A = \frac{AM}{LM}$

$\Rightarrow 3\sqrt{2} = \frac{AM}{h} \Rightarrow AM = 3\sqrt{2}h$

In ΔBLM,

$\cot B = \frac{BM}{LM} \Rightarrow \sqrt{7} = \frac{BM}{h} \Rightarrow BM = \sqrt{7}h$

In ΔABM by Pythagoras theorem

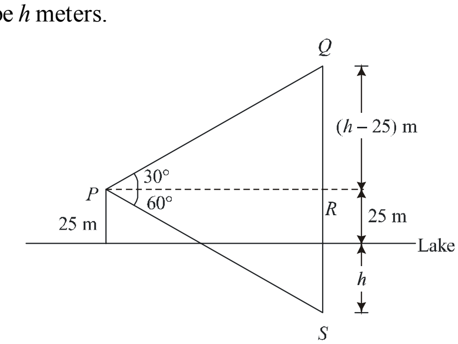
$AM^2 + MB^2 = AB^2$

$\therefore\ AM^2 + MB^2 = (100)^2$

$\Rightarrow 18h^2 + 7h^2 = 100 \times 100$

$\Rightarrow h^2 = 4 \times 100 \Rightarrow h = 20$

26. (2) Let height of the cloud from the surface of the lake be h meters.



$\therefore$ In ΔPRQ:

$\tan 30° = \dfrac{h-25}{PR}$

$\therefore \quad PR = (h-25)\sqrt{3}$ ...(i)

and in ΔPRS : $\tan 60° = \dfrac{h+25}{PR}$

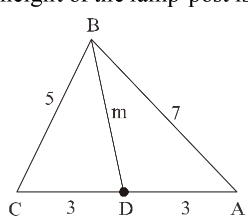
$PR = \dfrac{h+25}{\sqrt{3}}$ ...(ii)

Then, from eq. (i) and (ii),

$(h-25)\sqrt{3} = \dfrac{h+25}{\sqrt{3}}$

$\therefore \quad h = 50\,\text{m}$

27. **(b)** Let the height of the lamp-post is h.



By Appolonius Theorem,

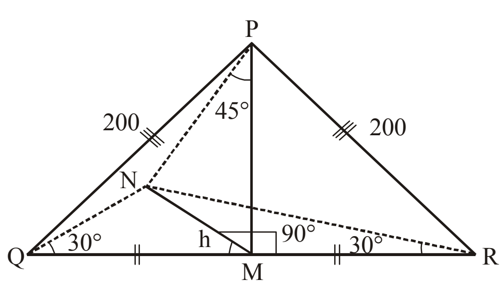
$$2\left(BD^2 + \left(\frac{AC}{2}\right)^2\right) = BC^2 + AB^2$$

$\Rightarrow \quad 2(m^2 + 3^2) = 25 + 49 \Rightarrow m = 2\sqrt{7}$

$\tan 30° = \dfrac{h}{BD}$

$\Rightarrow \quad h = 2\sqrt{7} \times \dfrac{1}{\sqrt{3}} = \dfrac{2\sqrt{21}}{3}$

28. **(d)**



Let height of tower MN = h

In ΔQMN we have

$\tan 30° = \dfrac{\text{MN}}{\text{QM}}$

$\therefore \quad \text{QM} = \sqrt{3}\text{h} = \text{MR}$ ...(1)

Now in ΔMNP

MN = PM ...(2)

In ΔPMQ we have :

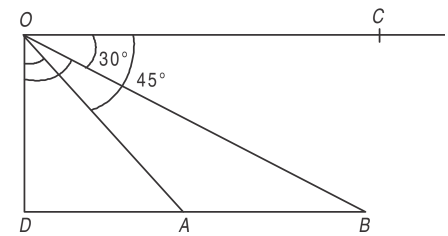
$\text{MP} = \sqrt{(200)^2 - (\sqrt{3}\,\text{h})^2}$

$\therefore$ From (2), we get :

$\sqrt{(200)^2 - (\sqrt{3}\,\text{h})^2} = \text{h} \Rightarrow \text{h} = 100\text{m}$

29. **(a)** Here; $\angle$DOA = 45°; $\angle$DOB = 60°

Now, let height of tower = h.



In ΔDOA, $\tan(\angle \text{DOA}) = \dfrac{DA}{OD}$

$\Rightarrow \quad \tan 45° = \dfrac{DA}{h} \quad \Rightarrow h = \text{DA}$

Now, in Δ DOB

$\tan(\angle \text{DOB}) = \dfrac{BD}{OD}$

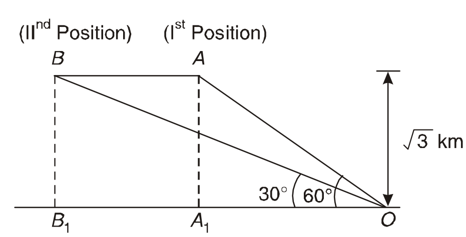
$\Rightarrow \tan 60° = \dfrac{BD}{h} \quad \Rightarrow \text{BD} = \sqrt{3}\ h.$

$\therefore$ speed for the distance BA $= \dfrac{BD - AD}{18} = \dfrac{(\sqrt{3}-1)\,h}{18}$

$\therefore$ required time taken

$= \dfrac{AD}{\text{speed}} = \dfrac{h \times 18}{(\sqrt{3}-1)\,\text{h}} = \dfrac{18}{\sqrt{3}-1} = 9(\sqrt{3}+1)$

30. **(d)** For Δ OA, A, $\text{OA}_1 = \dfrac{\sqrt{3}}{\tan 60°} = 1$ km.

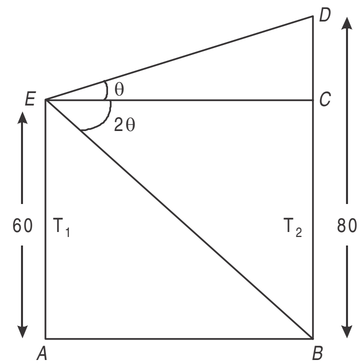


For Δ OB_1, B, $\text{OB}_1 = \dfrac{\sqrt{3}}{\tan 30°} = 3$ *km.*

As, a distance of 3 – 1 = 2 km is covered in 5 seconds. Therefore the speed of the plane is

$\dfrac{2 \times 3600}{5} = 1440$ km / hr

31. **(d)** Let the distance between T_1 and T_2 be x



From the figure

$EA = 60$ m (T_1) and $DB = 80$ m (T_2)

$\angle DEC = \theta$ and $\angle BEC = 2\theta$

Now in ΔDEC,

$$\tan\theta = \frac{DC}{AB} = \frac{20}{x}$$

and in ΔBEC,

$$\tan 2\theta = \frac{BC}{CE} = \frac{60}{x}$$

We know that

$$\tan 2\theta = \frac{2\tan\theta}{1-(\tan\theta)^2}$$

$$\Rightarrow \frac{60}{x} = \frac{2\left(\frac{20}{x}\right)}{1-\left(\frac{20}{x}\right)^2}$$

$$\Rightarrow x^2 = 1200 \quad \Rightarrow x = 20\sqrt{3}$$

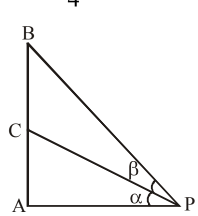
32. **(d)** Since $AP = 2AB \Rightarrow \frac{AB}{AP} = \frac{1}{2}$ ...(i)

Let $\angle APC = \alpha$

$$\therefore \quad \tan\alpha = \frac{AC}{AP} = \frac{1}{2}\frac{AB}{AP} = \frac{1}{4}$$

($\because$ C is the mid point) ($\therefore AC = \frac{1}{2}AB$)

$$\Rightarrow \tan\alpha = \frac{1}{4}$$



As $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$$\Rightarrow \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{1}{2}$$

$$\left[\begin{aligned}&\because \tan(\alpha+\beta) = \frac{AB}{AP}\\ &\tan(\alpha+\beta) = \frac{1}{2}\ [\text{From}(1)]\end{aligned}\right]$$

$$\Rightarrow \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta} = \frac{1}{2} \quad \therefore \ \tan\beta = \frac{2}{9}$$

33. **(b)** $\tan 30° = \frac{h}{x+a}$

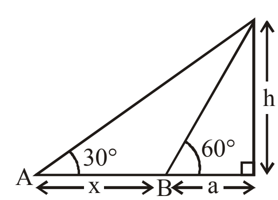
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+a} \Rightarrow \sqrt{3}h = x+a$$

...(1)

$$\tan 60° = \frac{h}{a} \Rightarrow \sqrt{3} = \frac{h}{a}$$

$$\Rightarrow h = \sqrt{3}a$$

...(2)



From (1) and (2)

$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover $a = \frac{10}{2}$ minutes = 5 minutes

34. **(d)** Let $AP = x$

$BP = y$

$$\tan 45° = \frac{H}{x} \Rightarrow H = x$$

$$\tan 30° = \frac{H}{y} \Rightarrow y = \sqrt{3}H$$

$$x^2 + (54\sqrt{2})^2 = y^2$$

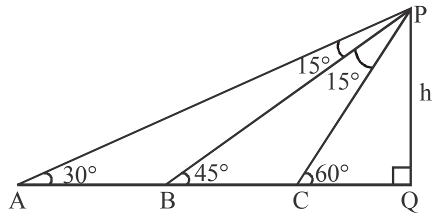
$$H^2 + (54\sqrt{2})^2 = 3H^2$$

$$(54\sqrt{2})^2 = 2H^2$$

$$54\sqrt{2} = \sqrt{2}H$$

$$54 = H$$

35. (c)



$\because$ PB bisects $\angle APC$, therefore

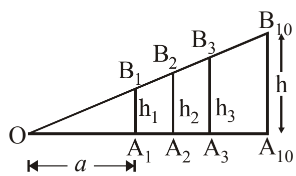
$AB : BC = PA : PC$

Also in ΔAPQ, $\sin 30° = \dfrac{h}{PA} \Rightarrow PA = 2h$

and in ΔCPQ, $\sin 60° = \dfrac{h}{PC} \Rightarrow PC = \dfrac{2h}{\sqrt{3}}$

$\therefore \quad AB : BC = 2h : \dfrac{2h}{\sqrt{3}} = \sqrt{3} : 1$

36. (a)



$\Delta OA_1B_1, \Delta OA_2B_2, \Delta OA_3B_3, ..., \Delta OA_{10}B_{10}$ all are similar triangles.

$\Rightarrow \quad \dfrac{h_1}{a_1} = \dfrac{h_2}{a_2} = \dfrac{h_3}{a_3} = ... = \dfrac{h_{10}}{a_{10}} = \tan\alpha$.

Since, $h_{10} = h = a_{10} \tan\alpha$ ...(1)

and $a_1 = a \Rightarrow h_1 = a \tan\alpha$ ...(2)

$\Rightarrow$ h = (a + 9d) tan α where d is distance between poles

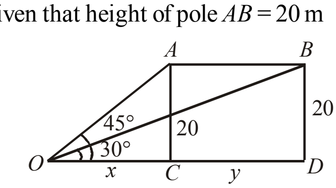
$(\because a_{10} = a + 9d)$

$\Rightarrow \quad h = a\tan\alpha + 9d\tan\alpha$

$$\Rightarrow \quad \frac{h - a\tan\alpha}{9\tan\alpha} = d \Rightarrow \frac{h - \dfrac{a\sin\alpha}{\cos\alpha}}{9\dfrac{\sin\alpha}{\cos\alpha}} = d$$

$$\Rightarrow \quad d = \frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$$

37. (b) Given that height of pole $AB = 20$ m



Let O be the point on the ground such that $\angle AOC = 45°$

Let $OC = x$ and $CD = y$

In right ΔAOC, $\tan 45° = \dfrac{20}{x}$ ...(i)

In right ΔBOD, $\tan 30° = \dfrac{20}{x+y}$ ...(ii)

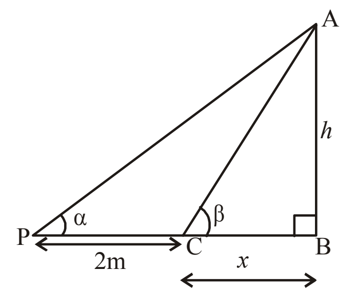
From (i) and (ii), we have $x = 20$ and $\dfrac{1}{\sqrt{3}} = \dfrac{20}{x+y}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{20}{20+y} \Rightarrow 20 + y = 20\sqrt{3}$$

So, $y = 20(\sqrt{3} - 1)$ m and time = 1s (Given)

Hence, speed $= 20(\sqrt{3} - 1)$ m/s

38. (a) Let AB be the tower of height 'h'.



Given : In ΔABP

$\tan\alpha = \dfrac{AB}{PB}$

or $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{h}{x+2}$

$\Rightarrow \quad (x + 2)\sin\alpha = h\cos\alpha$

$\Rightarrow \quad h = \dfrac{x\sin\alpha + 2\sin\alpha}{\cos\alpha}$ ...(1)

Now, In ΔABC, $\tan\beta = \dfrac{AB}{BC}$

$\Rightarrow \quad \dfrac{\sin\beta}{\cos\beta} = \dfrac{h}{x} \Rightarrow x = \dfrac{h\cos\beta}{\sin\beta}$ ...(2)

Putting the value of x in eq. (2) to eq. (1), we get

$$h = \frac{\dfrac{h\cos\beta\sin\alpha}{\sin\beta} + \dfrac{2\sin\alpha}{1}}{\cos\alpha}$$

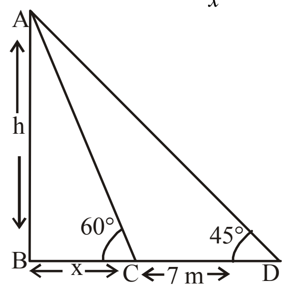
$$\Rightarrow \quad h = \frac{h\cos\beta.\sin\alpha + 2\sin\alpha\sin\beta}{\sin\beta.\cos\alpha}$$

$\Rightarrow \quad h(\sin\beta.\cos\alpha - \cos\beta.\sin\alpha) = 2\sin\alpha.\sin\beta$

$\Rightarrow \quad h[\sin(\beta - \alpha)] = 2\sin\alpha.\sin\beta$

$$\Rightarrow \quad h = \frac{2\sin\alpha.\sin\beta}{\sin(\beta - \alpha)}$$

39. **(b)** In right, ΔABC $\tan 60° = \frac{h}{x} = \sqrt{3}$



$\Rightarrow \quad x = \frac{h}{\sqrt{3}}$ ....(i)

In right, $\Delta ABD = \tan 45° = \frac{h}{x+7} = 1$

$\Rightarrow \quad h = x + 7 \Rightarrow h - \frac{h}{\sqrt{3}} = 7$ [From (i)]

$\Rightarrow \quad h = \frac{7\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow \quad h = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

40. **(a)** In the ΔAOB given that $\angle$AOB = 60°

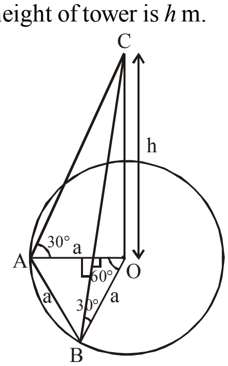
and OA = OB = radius

$\therefore \quad \angle OBA = \angle OAB = 60°$

$\therefore \quad \Delta$ AOB is a equilateral triangle.

$\Rightarrow \quad OA = OB = AB = a$

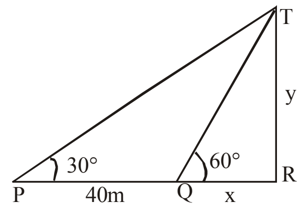
Let the height of tower is h m.



In ΔOAC, $\tan 30° = \frac{h}{a} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$

$\Rightarrow h = \frac{a}{\sqrt{3}}$

41. **(d)**



In right ΔQTR

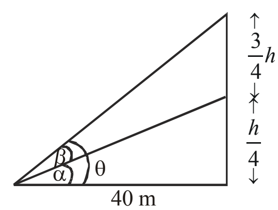
$\tan 60° = \frac{y}{x} \Rightarrow y = \sqrt{3}x$ ....(1)

In right ΔPTR

$\tan 30° = \frac{y}{x+40} \Rightarrow y = \frac{x+40}{\sqrt{3}}$ ....(2)

From (1) and (2), $\sqrt{3}x = \frac{x+40}{\sqrt{3}} \Rightarrow x = 20m$

42. **(c)**



$\theta = \alpha + \beta, \beta = \tan^{-1}\left(\frac{3}{5}\right)$

or $\quad \beta = \theta - \alpha$

$\Rightarrow \quad \tan\beta = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta . \tan\alpha}$

or $\quad \frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40} . \frac{h}{160}}$

$h^2 - 200h + 6400 = 0$

$\Rightarrow \quad h = 40$ or 160 metre

$\therefore \quad$ possible height = 40 metre

MOCK TEST 1

PART-I (Multiple Choice Questions)

1. In a box containing 100 bulbs, 10 are faulty. The probability that from a sample of 5 bulbs none are defective.

(a) $\left(\frac{1}{10}\right)^5$ (b) $\left(\frac{9}{10}\right)^5$ (c) $\frac{9}{10^5}$ (d) $\frac{1}{5}$

2. If $2\sec 2\alpha = \tan\beta + \cot\beta$ then one of the values of $(\alpha+\beta)$ =

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) None

3. The value of $\sum_{r=1}^{5} r\frac{{}^nC_r}{{}^nC_{r-1}} =$

(a) $5(n-3)$ (b) $5(n-2)$ (c) $5n$ (d) $5(2n-9)$

4. ${}^{14}C_7 + \sum_{i=1}^{3} {}^{17-i}C_6 =$

(a) ${}^{16}C_7$ (b) ${}^{17}C_7$ (c) ${}^{17}C_8$ (d) ${}^{16}C_8$

5. The angle between the two lines $\frac{x+1}{2}=\frac{y+3}{2}=\frac{z-4}{-1}$ & $\frac{x-4}{1}=\frac{y+4}{2}=\frac{z+1}{2}$ is

(a) $\cos^{-1}\left(\frac{4}{9}\right)$ (b) $\cos^{-1}\left(\frac{3}{9}\right)$

(c) $\cos^{-1}\left(\frac{2}{9}\right)$ (d) $\cos^{-1}\left(\frac{1}{9}\right)$

6. The contrapositive of $(p \vee q) \Rightarrow r$ is

(a) $r \Rightarrow (p \vee q)$ (b) $\sim r \Rightarrow (p \vee q)$

(c) $\sim r \Rightarrow \sim p \wedge \sim q$ (d) $p \Rightarrow (q \vee r)$

7. If (2, 3, 5) are ends of the diameter of a sphere $x^2+y^2+z^2-6x-12y-2z+20=0$. Then coordinates of the other end are

(a) (4, 9, –3) (b) (4, 3, 5)

(c) (4, 3, –3) (d) (4, –3, 9)

8. If $f(x) = |x-2|$ and $g(x) = f(f(x))$ then for $x > 10$, $g'(x)$ equal

(a) –1 (b) 0 (c) 1 (d) $2x-4$

9. If a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. then a, c, e are in

(a) A.P. (b) G.P. (c) H.P. (d) None

Space for Rough Work

10. The coefficient of x^{10} in the expansion of $\left(3x^2 - \frac{1}{x^2}\right)^{15}$ is

(a) $\frac{15!}{10!\,5!}3^{10}$ (b) $-\frac{15!}{10!\,5!}3^{10}$ (c) $-\frac{15!\,3^5}{10!\,5!}$ (d) $\frac{15!}{7!\,8!}3^8$

11. The mean of discrete observations $y_1, y_2, y_3, \ldots, y_n$ is given by

(a) $\frac{\sum_{i=1}^{n} y_i}{n}$ (b) $\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$ (c) $\frac{\sum_{i=1}^{n} y_i f_i}{n}$ (d) $\frac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

12. $\int \frac{dx}{(x-\beta)\sqrt{(x-\alpha)(\beta-x)}}$ is

(a) $\frac{2}{\alpha-\beta}\sqrt{\frac{x-\alpha}{\beta-x}}+c$

(b) $\frac{2}{\alpha-\beta}\sqrt{(x-\alpha)(\beta-x)}+c$

(c) $\frac{\alpha-\beta}{2}(x-\alpha)\sqrt{\beta-x}$ (d) None of these.

13. The spheres $x^2+y^2+z^2+x+y+z-1=0$ and $x^2+y^2+z^2+x+y+z-5=0$

(a) intersect in a plane
(b) intersect in five points
(c) do not intersect
(d) None of these

14. The domain of the function $f(x)=\exp(\sqrt{5x-3-2x^2})$ is

(a) $[3/2, \infty)$ (b) $[1, 3/2]$ (c) $(-\infty, 1]$ (d) $(1, 3/2)$

15. The value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix}$$ is zero, if

(a) $\sin x = 0$ (b) $\cos x = 0$

(c) $a = 0$ (d) $\cos x = \frac{1+a^2}{2a}$

16. If AB = 0, then for the matrices

$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and

$B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$, $\theta-\phi$ is

(a) an odd number of $\frac{\pi}{2}$
(b) an odd multiple of π
(c) an even multiple of $\frac{\pi}{2}$
(d) 0

17. If $f'(x) = (x-1)(x-2)(x-3)$ then $f(x)$ in monotonically increasing in

(a) $x<1$ (b) $x>3, x<1$
(c) $x>3, 1<x<2$ (d) $x<1, 2<x<3$

18. An inverted cone is 10 cm in diameter and 10 cm deep. Water is poured into it at the rate of 4cm^3/min. When the depth of water level is 6 cm, then it is rising at the rate

(a) $\frac{9}{4\pi}\text{cm}^3/\text{min}$. (b) $\frac{1}{4\pi}\text{cm}^3/\text{min}$.

(c) $\frac{1}{9\pi}\text{cm}^3/\text{min}$. (d) $\frac{4}{9\pi}\text{cm}^3/\text{min}$.

19. The equation of tangent to $4x^2 - 9y^2 = 36$ which are perpendicular to straight line $5x+2y-10=0$ are

(a) $5(y-3) = 2\left(x - \frac{\sqrt{117}}{2}\right)$

(b) $2y-5x+10-2\sqrt{18}=0$

Space for Rough Work

(c) $2y - 5x - 10 - 2\sqrt{18} = 0$

(d) None of these

20. $\int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x} \sec^2\left(\frac{1}{3}e^{2x}\right)dx$ is equal to :

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{3\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$

PART-II (Numerical Answer Questions)

21. The probability of getting the sum more than 7 when a pair of dice is tossed is

22. Three persons A, B, C throw a die in succession. The one getting 'six' wins. If A starts then the probability of B winning is

23. The eccentricity of the ellipse represented by $25x^2 + 16y^2 - 150x - 175 = 0$ is

24. If $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + ... + \infty = \frac{\pi^4}{90}$, then the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...\infty$ is $\frac{\pi^y}{k}$, then k is:

25. The area enclosed by the curve $y = x^5$, the x-axis and the ordinates $x = -1$, $x = 1$ in sq. units is

RESPONSE SHEET

1.	ⓐⓑⓒⓓ	**2.**	ⓐⓑⓒⓓ	**3.**	ⓐⓑⓒⓓ	**4.**	ⓐⓑⓒⓓ	**5.**	ⓐⓑⓒⓓ
6.	ⓐⓑⓒⓓ	**7.**	ⓐⓑⓒⓓ	**8.**	ⓐⓑⓒⓓ	**9.**	ⓐⓑⓒⓓ	**10.**	ⓐⓑⓒⓓ
11.	ⓐⓑⓒⓓ	**12.**	ⓐⓑⓒⓓ	**13.**	ⓐⓑⓒⓓ	**14.**	ⓐⓑⓒⓓ	**15.**	ⓐⓑⓒⓓ
16.	ⓐⓑⓒⓓ	**17.**	ⓐⓑⓒⓓ	**18.**	ⓐⓑⓒⓓ	**19.**	ⓐⓑⓒⓓ	**20.**	ⓐⓑⓒⓓ
21.		**22.**		**23.**		**24.**		**25.**	

Space for Rough Work

HINTS & SOLUTIONS

MOCK TEST-1

1. (b) $p = \frac{1}{10} \quad q = \frac{9}{10}$

$\therefore$ Probability that none are defective $= \left(\frac{9}{10}\right)^5$

2. (c) $\frac{2}{\cos 2\alpha} = \frac{\tan^2\beta + 1}{\tan\beta} = \frac{1}{\sin\beta\cos\beta}$

$\Rightarrow \sin 2\beta = \cos 2\alpha = \sin(90\text{-}2\alpha) \Rightarrow \alpha + \beta = \frac{\pi}{4}$

3. (b) $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{r!\,n!}{r!\,(n-r)!} \cdot \frac{(r-1)!\,(n-r+1)!}{n!} = \frac{(n-r+1)!}{(n-r)!}$

$= \frac{(n-r+1)!}{(n-r)!} = n - r + 1$

$\therefore \sum_{r=1}^{5} = n + (n-1) + (n-2) + (n-3) + (n-4)$
$= 5n - 10 = 5(n-2)$

4. (b) ${}^{14}C_7 + \sum_{i=1}^{3} {}^{17-i}C_6 = {}^{14}C_7 + {}^{14}C_6 + {}^{15}C_6 + {}^{16}C_6$

$= {}^{15}C_7 + {}^{15}C_6 + {}^{16}C_6 = {}^{16}C_7 + {}^{16}C_6 = {}^{17}C_7$

5. (a) $a_1 = 2, b_1 = 2, c_1 = -1$ and $a_2 = 1, b_2 = 2, c_2 = 2$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2 + 4 - 2}{\sqrt{4+4+1}\sqrt{1+4+4}} = \pm\frac{4}{9}.$$

6. (c) Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

$\therefore$ contrapositive of $(p \vee q) \Rightarrow r$ is

$\sim r \Rightarrow \sim (p \vee q)$ i.e. $\sim r \Rightarrow (\sim p \wedge \sim q)$

7. (a) Let the co-ordiante of other ends are (x, y, z).
The centre of sphere is C(3, 6, 1)

Therefore, $\frac{x+2}{2} = 3 \Rightarrow x = 4$

$\frac{y+3}{2} = 6 \Rightarrow y = 9$ and $\frac{z+5}{2} = 1 \Rightarrow z = -3$

8. (c) For $x > 10$, $f(x) = x - 2$.
Therefore, $g(x) = x - 2 - 2 = x - 4$
$\therefore g'(x) = 1$.

9. (b) a, b, c in A.P. $\Rightarrow a + c = 2b$; b, c, d in G.P.

$\Rightarrow bd = c^2$; c, d, e in H.P. $\Rightarrow d = \frac{2ce}{c+e}$

$\therefore \frac{a+c}{2} \times \frac{2ce}{c+e} = c^2 \Rightarrow (a+c)e = (c+e)c \Rightarrow c^2 = ae.$

Therefore, in G.P.

10. (b) $\left(3x^2 - \frac{1}{x^2}\right)^{15}$

$T_{r+1} = {}^{15}C_r (3x^2)^{15-r}\left(-\frac{1}{x^2}\right)^r$
$= {}^{15}C_r 3^{15-r}(-1)^r x^{30-2r-2r}$

Therefore, $30 - 4r = 10 \Rightarrow r = 5$.

Therefore, $T_6 = -{}^{15}C_5 3^{10} = \frac{-15!}{10!\,5!}3^{10}$.

11. (a)

12. (a) $I = \int \frac{dx}{(x-\beta)\sqrt{(x-\alpha)(\beta-x)}}$

Put $x = \alpha\sin^2\theta + \beta\cos^2\theta$
[see the standard substitutions]
$dx = 2(\alpha - \beta)\sin\theta\cos\theta\, d\theta$

Also, $(x-\alpha) = (\beta-\alpha)\cos^2\theta$

$(x-\beta) = (\alpha-\beta)\sin^2\theta$

$$\therefore I = \int \frac{2(\alpha-\beta)\sin\theta\cos\theta\, d\theta}{(\alpha-\beta)\sin^2\theta(\beta-\alpha)\sin\theta\cos\theta}$$

$$= \frac{2}{\beta-\alpha}\int\frac{d\theta}{\sin^2\theta} = \frac{2}{\beta-\alpha}\int \operatorname{cosec}^2\theta\, d\theta$$

$= \frac{2}{\beta-\alpha}(-\cot\theta)+C = \frac{2}{\alpha-\beta}\cot\theta + C$

Now, $x = \alpha\sin^2\theta + \beta\cos^2\theta$

$\Rightarrow x\,\text{cosec}^2\theta = \alpha + \beta\cot^2\theta$

$\Rightarrow x(1+\cot^2\theta) = \alpha + \beta\cot^2\theta$

$\therefore \cot\theta = \sqrt{\frac{x-\alpha}{\beta-x}}$; $\therefore\ I = \frac{2}{\alpha-\beta}\sqrt{\frac{x-\alpha}{\beta-x}} + C$

13. (c) As the given spheres both have same cenre and different radii therefore they are concentric and they do not have any point in common. Hence they do not intersect.

14. (b) We have, $f(x) = \exp\left(\sqrt{5x-3-2x^2}\right)$

i.e., $f(x) = e^{\sqrt{5x-3-2x^2}}$

For Domain of $f(x)$, $\sqrt{5x-3-2x^2}$ should be +ve.

i.e., $\sqrt{5x-3-2x^2} \geq 0$

$\Rightarrow 2x^2 - 5x + 3 \leq 0$ (By taking –ve sign common)

$\Rightarrow 2x(x-1) - 3(x-1) \leq 0$

$\Rightarrow (2x-3)(x-1) \leq 0$

$\Rightarrow 2x - 3 \leq 0$ or $x - 1 \geq 0$

$\Rightarrow x \leq \frac{3}{2}$ or $x \geq 1$

$\therefore\ 1 \leq x \leq \frac{3}{2}$ i.e., $x \in \left[1, \frac{3}{2}\right]$

Hence, domain of the given function is $[1, \frac{3}{2}]$.

15. (a) Given determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1+a^2-2a\cos x & a & a^2 \\ 0 & \cos nx & \cos(n+1)x \\ 0 & \sin nx & \sin(n+1)x \end{vmatrix} = 0$$

By applying $C_1 \to C_1 + C_3 - 2\cos x\, C_2$

By expanding

$(1 + a^2 - 2a\cos x)\,[\cos nx \sin(n+1)x - \sin nx \cos(n+1)x] = 0$

Now, $(1 + a^2 - 2a\cos x)\sin(n+1-n)x = 0$

$\Rightarrow (1 + a^2 - 2a\cos x)\sin x = 0$

$\sin x = 0$ or $\cos x = \frac{1+a^2}{2a}$

As $a \neq 1\ \therefore \left(\frac{1+a^2}{2a}\right) > 1$

$\Rightarrow \cos x > 1$ It is not possible.

$\therefore\ \sin x = 0$

16. (a) We have,

$$AB = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\cos\phi\sin\theta\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\cos\phi\sin\theta\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \cos(\theta-\phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\theta\sin\phi \end{bmatrix}$$

Since, $AB = 0$, $\therefore\ \cos(\theta-\phi) = 0$

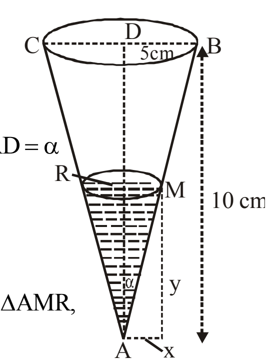
$\therefore\ \theta - \phi$ is an odd multiple of $\frac{\pi}{2}$

17. (c) When $x > 3$, $f'(x) > 0$; when $2 < x < 3$, $f'(x) < 0$; when $1 < x < 2$, $f'(x) > 0$; when $x < 1$, $f'(x) < 0$.

18. (d) Let y be the level of water at time t and x the radius of the surface and V, the volume of water.

We know that the volume of cone

$= \frac{1}{3}\pi\ (\text{radius})^2 \times \text{height}$



$\therefore\ V = \frac{1}{3}\pi x^2 y$. Let $\angle BAD = \alpha$

$\Rightarrow \tan\alpha = \frac{BD}{AD} = \frac{5}{10} = \frac{1}{2}$.

Again, from right angled ΔAMR, we have

$$\tan\alpha = \frac{MR}{AR} = \frac{x}{y}; \Rightarrow \frac{1}{2} = \frac{x}{y}; \quad \therefore x = \frac{y}{2}.$$

$$\therefore V = \frac{1}{3}\pi x^2 y = \frac{1}{3}\pi\left(\frac{y}{2}\right)^2 . y = \frac{\pi}{12}y^2 \qquad ......(1).$$

By question, the rate of change of volume

$= \frac{dV}{dt}. = 4$ cub.cm./min.

We have to find out the rate of increase of water-level

i.e. $\frac{dy}{dt}$.

Differentiating (1) with respect to t, we get

$$\frac{dV}{dt} = \frac{\pi}{12}.3y^2.\frac{dy}{dt}; \quad \therefore 4 = \frac{\pi}{4}y^2.\frac{dy}{dt}; \quad \therefore \frac{dy}{dt} = \frac{16}{\pi y^2}.$$

When $y = 6$ cm, $\frac{dy}{dt} = \frac{16}{\pi 6^2} = \frac{4}{9\pi}$ cub.cm./min.

19. (d) Slope of the equations $4x^2 - 9y^2 = 36$

$$8x - 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{4x}{9y} \text{ or } m_1 = \frac{4x}{9y}$$

Slope of the straight line, $5x + 2y - 10 = 0$ is $m_2 = -\frac{5}{2}$

Therefore, for the perpendicularity, $m_1 m_2 = -1$

Now, $\frac{4x}{9y} \times \frac{-5}{2} = -1 \Rightarrow y = \frac{10x}{9}$.

Putting $y = \frac{10x}{9}$ in $4x^2 - 9y^2 = 36$ gives imaginary roots resulting in no tangents.

20. (a) $I = \int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$

Put $e^{2x} = t \Rightarrow 2e^{2x}dx = dt$

When $x = \log\sqrt{\pi/2}$, $t = e^{2\log\sqrt{\pi/2}} = e^{\log\pi/2} = \frac{\pi}{2}$

When $x = \log\sqrt{\pi}$, $t = e^{2\log\sqrt{\pi}} = \pi$

$$\therefore I = \int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}\sec^2\left(\frac{1}{3}t\right)dt = \frac{1}{2}.\frac{1}{\frac{1}{3}}\left[\tan\frac{t}{3}.\right]_{\pi/2}^{\pi}$$

$$= \frac{3}{2}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right] = \frac{3}{2}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = \sqrt{3}$$

21. (0.42) Sum of 7 can be obtained when (2,6), (3,5) (3, 6), (4, 4), (4,5), (4, 6)(5, 3)(5, 4) (5, 5)(5, 6)(6, 2)(6, 3)(6, 4)(6,5)(6, 6)

$$\therefore \text{Probability of sum} > 7 = \frac{15}{36} = \frac{5}{12} = 0.4167 = 0.42$$

22. (0.33) $P(\bar{E}E) + P(\bar{E}\bar{E}\bar{E}\bar{E}\bar{E})$

$$= \frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \left(\frac{5}{6}\right)^8 \frac{1}{6}...\infty$$

$$= \frac{5}{36}\left[1 + \left(\frac{5}{6}\right)^3 ......\right] = \frac{30}{91} = 0.329 = 0.33$$

23. (0.6) The ellipse

$$\Rightarrow \frac{(x-3)^2}{16} + \frac{y^2}{25} = 1 \qquad \therefore \frac{16}{25} = 1 - e^2 \Rightarrow e = \frac{3}{5}.$$

24. (96) $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + ...\infty = \frac{\pi^4}{90}$

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...+\infty + \frac{1}{2^4}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + ...\infty\right)$$

$$= \frac{\pi^4}{90}$$

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + ... + \infty + \frac{1}{16}\times\frac{\pi^4}{90} = \frac{\pi^4}{90}$$

$$\therefore \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + ... + \infty = \frac{\pi^4}{90} - \frac{1}{16}\left(\frac{\pi^4}{90}\right)$$

$$= \frac{15}{16}\left(\frac{\pi^4}{90}\right) = \frac{\pi^4}{96}.$$

25. (0.33) Required area

$$= \int_{-1}^{1}|y|\,dx = \int_{-1}^{1}|x^5|\,dx = 2\int_{0}^{1}|x^5|\,dx$$

$$= 2\int_{0}^{1}x^5 dx = 2\left[\frac{x^6}{6}\right]_0^1 = \frac{2}{6} = \frac{1}{3}$$

MOCK TEST 2

PART-I (Multiple Choice Questions)

1. Equation of straight line $ax + by + c = 0$ where $3a+4b+ c = 0$, which is at maximum distance from $(1, -2)$, is
(a) $3x + y - 17 = 0$ (b) $4x + 3y - 24 = 0$
(c) $3x + 4y - 25 = 0$ (d) $x + 3y - 15 = 0$

2. Given $f(x) = \begin{cases} \sqrt{10-x^2} & \text{if } -3 < x < 3 \\ 2 - e^{x-3} & \text{if } x \geq 3 \end{cases}$

The graph of $f(x)$ is –
(a) continuous and differentiable at $x = 3$
(b) continuous but not differentiable at $x = 3$
(c) differentiable but not continuous at $x = 3$
(d) neither differentiable nor continuous at $x = 3$

3. The solution of the equation $2z = |z| + 2i$, where z is a complex number, is –
(a) $z = \frac{\sqrt{3}}{3} - i$ (b) $z = \frac{\sqrt{3}}{3} + i$
(c) $z = \frac{\sqrt{3}}{3} \pm i$ (d) None of these

4. The equation $(5x - 1)^2 + (5y - 2)^2 = (\lambda^2 - 4\lambda + 4)(3x + 4y - 1)^2$ represents an ellipse if $\lambda \in$
(a) $(0, 1]$ (b) $(-1, 2)$ (c) $(2, 3)$ (d) $(-1, 0)$

5. The straight line $y = m(x - a)$ meets the parabola $y^2 = 4ax$ in two distinct points for –
(a) all $m \in R$ (b) all $m \in [-1, 1]$
(c) all $m \in R - \{0\}$ (d) None of these

6. $p \vee (p \wedge q)$ is equivalent to –
(a) q (b) p (c) $\sim p$ (d) $\sim q$

7. Find $\int e^{\sin x} \left(\frac{x \cos^3 x - \sin x}{\cos^2 x} \right) dx$
(a) $x\, e^{\sin x} - e^{\sin x} \sec x + C$
(b) $x\, e^{\cos x} - e^{\sin x} \sec x + C$
(c) $x^2 e^{\sin x} + e^{\sin x} \sec x + C$
(d) $2x\, e^{\sin x} - e^{\sin x} \tan x + C$

8. The function $f : [2, \infty) \to (0, \infty)$ defined by $f(x) = x^2 - 4x + a$, then the set of values of 'a' for which f(x) becomes onto is
(a) $(4, \infty)$ (b) $[4, \infty)$ (c) $\{4\}$ (d) ϕ

9. If α and β are the real roots of the equation $x^2 - (k - 2)x + (k^2 + 3k + 5) = 0$ $(k \in R)$.
Find the maximum and minimum values of $(\alpha^2 + \beta^2)$.
(a) 18, 50/9 (b) 18, 25/9
(c) 27, 50/9 (d) None of these

10. The sum of the coefficient of all the terms in the expansion of $(2x - y + z)^{20}$ in which y do not appear at all while x appears in even powers and z appears in odd powers is –
(a) 0 (b) $\frac{2^{20}-1}{2}$ (c) 2^{19} (d) $\frac{3^{20}-1}{2}$

Space for Rough Work

11. All the five digit numbers in which each successive digit exceeds is predecessor are arranged in the increasing order. The $(105)^{th}$ number does not contain the digit
(a) 1 (b) 2 (c) 6 (d) All of these

12. Three people each flip two fair coins. The probability that exactly two of the people flipped one head and one tail, is–
(a) 1/2 (b) 3/8 (c) 5/8 (d) 3/4

13. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vector such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{\sqrt{2}}(\vec{b} + \vec{c})$ then the angle between the vectors $\vec{a}, \vec{b}$ is
(a) $3\pi/4$ (b) $\pi/4$ (c) $\pi/8$ (d) $\pi/2$

14. The greatest and the least value of $|z_1 + z_2|$ if $z_1 = 24 + 7i$ and $|z_2| = 6$ respectively are
(a) 25, 19 (b) 19, 25 (c) –19, –25 (d) –25, –19

15. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is
(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3y} = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$.

16. If $\int \sqrt{2}\sqrt{1 + \sin x}\, dx = -4\cos(ax + b) + C$ then the value of (a, b) is :
(a) $\frac{1}{2}, \frac{\pi}{4}$ (b) $1, \frac{\pi}{2}$
(c) 1, 1 (d) None of these

17. Let $f : R \to R$ and $f_n(x) = f(f_{n-1}(x))\ \forall\ n \geq 2, n \in N$, the roots of equation $f_3(x)\, f_2(x)\, f(x) - 25 f_2(x)\, f(x) + 175\, f(x) = 375$. Which also satisfy equation $f(x) = x$ will be
(a) 5 (b) 15
(c) 10 (d) Both (a) and (b)

18. A triangle ABC satisfies the relation $2\sec 4C + \sin^2 2A + \sqrt{\sin B} = 0$ and a point P is taken on the longest side of the triangle such that it divides the side in the ratio 1 : 3. Let Q and R be the circumcentre and orthocentre of ΔABC. If PQ : QR : RP = 1 : α : β, then the value of $\alpha^2 + \beta^2$.
(a) 9 (b) 8 (c) 6 (d) 7

19. The value of $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt$ is
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) 1

20. If a is real and $\sqrt{2}ax + \sin By + \cos Bz = 0$, $x + \cos By + \sin Bz = 0$, $-x + \sin By - \cos Bz = 0$, then the set of all values of a for which the system of linear equations has a non-trivial solution, is –
(a) [1,2] (b) [–1,1] (c) $[1,\infty]$ (d) $[2^{-1/2}, 2^{1/2}]$

PART-II (Numerical Answer Questions)

21. Box contains 2 one rupee, 2 five rupee, 2 ten rupee and 2 twenty rupee coin. Two coins are drawn at random simultaneously. The probability that their sum is Rs. 20 or more, is

22. The value of the definite integral, $\int_{\theta_1}^{\theta_2} \frac{d\theta}{1 + \tan\theta} = \frac{501\pi}{K}$ where $\theta_2 = \frac{1003\pi}{2008}$ and $\theta_1 = \frac{\pi}{2008}$. The value of K equals

23. The expansion of $(1 + x)^n$ has 3 consecutive terms with coefficients in the ratio 1 : 2 : 3 and can be written in the form ${}^nC_k : {}^nC_{k+1} : {}^nC_{k+2}$. The sum of all possible values of (n + k) is –

24. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new standard deviation of the resulting observations.

25. $\lim_{x \to 0} \frac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3.\sqrt{x}}$ equals

Space for Rough Work

RESPONSE SHEET

1.	ⓐⓑⓒⓓ	**2.**	ⓐⓑⓒⓓ	**3.**	ⓐⓑⓒⓓ	**4.**	ⓐⓑⓒⓓ	**5.**	ⓐⓑⓒⓓ
6.	ⓐⓑⓒⓓ	**7.**	ⓐⓑⓒⓓ	**8.**	ⓐⓑⓒⓓ	**9.**	ⓐⓑⓒⓓ	**10.**	ⓐⓑⓒⓓ
11.	ⓐⓑⓒⓓ	**12.**	ⓐⓑⓒⓓ	**13.**	ⓐⓑⓒⓓ	**14.**	ⓐⓑⓒⓓ	**15.**	ⓐⓑⓒⓓ
16.	ⓐⓑⓒⓓ	**17.**	ⓐⓑⓒⓓ	**18.**	ⓐⓑⓒⓓ	**19.**	ⓐⓑⓒⓓ	**20.**	ⓐⓑⓒⓓ
21.		**22.**		**23.**		**24.**		**25.**	

HINTS & SOLUTIONS

MOCK TEST-2

1. (d) It passes through a fixed point (3, 4)
Slope of line joining (3, 4) and (1, –2) is –6/–2 = 3
$\therefore$ Slope of required line = –1/3

Equation is $y-4=-\frac{1}{3}(x-3)$

$x+3y-15=0$

2. (b) $f'(3^+)=\lim_{h\to0}\frac{f(3+h)-f(3)}{h}$

$=\lim_{h\to0}\frac{(2-e^h)-1}{h}=-\lim_{h\to0}\left(\frac{e^h-1}{h}\right)=-1$

$f'(3^-)=\lim_{h\to0}\frac{f(3-h)-f(3)}{-h}$

$=\lim_{h\to0}\frac{\sqrt{10-(3-h)^2}-1}{-h}=-\lim_{h\to0}\frac{\sqrt{1+(6h-h^2)}-1}{-h}$

$=\lim_{h\to0}\frac{6h-h^2}{-h(\sqrt{1+6h-h^2}+1)}=\frac{-6}{2}=-3$

Hence, $f'(3^+)\neq f'(3^-)$

3. (b) $2(x+iy)=\sqrt{x^2+y^2}+2i$

$2x=\sqrt{x^2+y^2}$ and $2y=2$ i.e., $y=1$

$4x^2=x^2+1$ i.e., $3x^2=1$ i.e., $x=\pm\frac{1}{\sqrt3}$

$x=\frac{1}{\sqrt3}$ $(\because x\geq0)$ $\therefore$ $z=\frac{1}{\sqrt3}+i=\frac{\sqrt3}{3}+i$

4. (c) $\left(x-\frac{1}{5}\right)^2+\left(y-\frac{2}{5}\right)^2=(\lambda^2-4\lambda+4)\left(\frac{3x+4y-1}{5}\right)^2$

i.e., $\sqrt{\left(x-\frac{1}{5}\right)^2+\left(y-\frac{2}{5}\right)^2}=|\lambda-2|\left|\frac{3x+4y-1}{\sqrt5}\right|$

is an ellipse.
If $0<|\lambda-2|<1$ i.e., $\lambda\in(1,2)\cup(2,3)$

5. (c) $y^2=4a\left(\frac{y+am}{m}\right)$ i.e., $my^2-4ay-4a^2m=0$

$m\neq0$; $16a^2+16a^2m^2>0$ which is true $\forall$ m.
$\therefore$ $m\in R-\{0\}$

6. (b) $p\vee(p\wedge q)$ is equivalent to p.

7. (a) $\int e^{\sin x}\left(\frac{x\cos^3x-\sin x}{\cos^2x}\right)dx$

$=\int e^{\sin x}\,x\cos x\,dx-\int e^{\sin x}\tan x\sec x\,dx$

$=\int x\,d\left(e^{\sin x}\right)-\int e^{\sin x}\,d(\sec x)$

$=\left\{x\,e^{\sin x}-\int e^{\sin x}dx\right\}$

$-\left\{e^{\sin x}\sec x-\int e^{\sin x}\sec x\cos x\,dx\right\}$

$=x\,e^{\sin x}-e^{\sin x}\sec x+C$

8. (d) $f(x)=x^2-4x+a$ always attains its minimum value.
So its range must be closed.
So, $a=\{\phi\}$

9. (a) For real roots $D\geq0$
$(k-2)^2-4(k^2+3k+5)\geq0$
$(k^2+4-4k)-4k^2-12k-20\geq0$
$-3k^2-16k-16\geq0$; $3k^2+16k+16\leq0$

$\left(k+\frac{4}{3}\right)(k+4)\leq0$

Now $E=\alpha^2+\beta^2$; $E=(\alpha+\beta)^2-2\alpha\beta$
$E=(k-2)^2-2(k^2+3k+5)=-k^2-10k-6$
$E=-(k^2+10k+6)=-[(k+5)^2-19]=19-(k+5)^2$
$\therefore$ E_{min} occurs when $k=-4/3$

$\therefore E_{min}=19-\frac{121}{9}=\frac{171-121}{9}=\frac{50}{9}$

E_{max} occurs when $k=-4$
$E_{max}=19-1=18$

10. (a) $\frac{20!}{p!q!r!}(2x)^p(-y)^q(z)^r=\frac{20!}{p!q!r!}2^p(-1)^q\,x^py^qz^r$

$p+q+r=20$, $q=0$
$p+r=20$ (p is even and r is odd).
even + odd = even (never possible)
Coefficient of such power never occur
$\therefore$ coefficient is zero

11. (a) Starting with 1 | 1 | | | | | 2 3 4 5 6 7 8 9
$={}^8C_4=70$

Starting with 2 | 2 | | | | | 3 4 5 6 7 8 9
$={}^7C_4=35$
Total = 105
$(105)^{th}$ number 26789

12. **(b)** $n = 3$, P (success) = P (HT or TH) = 1/2

$\Rightarrow p = q = \frac{1}{2}$ and $r = 2$

$P(r = 2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} = \frac{3}{8}$

13. **(a)** $(\hat{a}.\hat{c})\hat{b} - (\hat{a}.\hat{b})\hat{c} = \frac{1}{\sqrt{2}}\hat{b} + \frac{1}{\sqrt{2}}\hat{c}$

$\therefore \hat{a}.\hat{c} = \frac{1}{\sqrt{2}}$ and $\hat{a}.\hat{b} = -\frac{1}{\sqrt{2}}$

$\Rightarrow$ angle between $\hat{a}$ and $\hat{c} = \frac{\pi}{4}$ and angle between $\hat{a}$ and $\hat{b} = \frac{3\pi}{4}$

14. **(a)** $|z_1 + z_2| \le |z_1| + |z_2| = |24 + 7i| + 6 = 25 + 6 = 31$

Also, $|z_1 + z_2| = |z_1 - (-z_2)| \ge ||z_1| - |z_2||$

$\Rightarrow |z_1 + z_2| \ge |25 - 6| = 19$

Hence the least value of $|z_1 + z_2|$ is 19 and the greatest value is 25.

15. **(c)** The coordinates of points P, Q, R are (–1, 0), (0, 0), $(3, 3\sqrt{3})$ respectively.

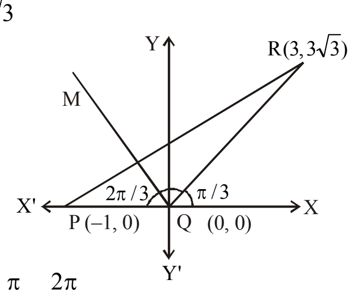
Slope of QR = $\sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{3}$

$\Rightarrow \angle RQX = \frac{\pi}{3}$

$\therefore \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$



Let QM bisects the $\angle PQR$,

$\therefore$ Slope of the line QM = $\tan\frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y - 0) = -\sqrt{3}(x - 0)$

$\Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

16. **(a)** Given $\int \sqrt{2}\sqrt{1 + \sin x}\, dx = -4\cos(ax + b) + C$

$\Rightarrow \int \sqrt{2}\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)dx = -4\cos(ax + b) + C$

$\Rightarrow \int \sqrt{2}\,.\,\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\frac{x}{2} + \frac{1}{\sqrt{2}}\cos\frac{x}{2}\right)dx$

$= -4\cos(ax + b) + C$

$\Rightarrow \int 2\left(\cos\frac{\pi}{4}\sin\frac{x}{2} + \sin\frac{\pi}{4}\cos\frac{x}{2}\right)dx$

$= -4\cos(ax + b) + C$

$\Rightarrow \int 2\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)dx = -4\cos(ax + b) + C$

$\Rightarrow -4\cos\left(\frac{x}{2} + \frac{\pi}{4}\right) = -4\cos(ax + b) + C$

$\Rightarrow a = \frac{1}{2},\ b = \frac{\pi}{4}$

17. **(d)** $f_2(x) = f(f(x)) = f(x) = x$

$f_3(x) = f(f_2(x)) = f(x) = x$

$\Rightarrow x^3 - 25x^2 + 175x - 375 = 0$

$(x - 5)(x^2 - 20x + 75) = 0$

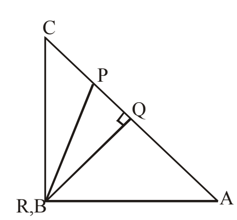
$(x - 5)^2(x - 15) = 0 \Rightarrow x = 5, 15$

18. **(a)** $2\sec 4C + \sin^2 2A + \sqrt{\sin B} = 0$

$A = 45°$, $B = 90°$ and $C = 45°$

Let AQ = a, then BP = $\frac{a}{2}$,

PQ = $\frac{a}{2}$ and QR = a



$\therefore PR = \sqrt{a^2 + \frac{a^2}{4}} = \frac{\sqrt{5}a}{2}$

$\therefore 1 : \alpha : \beta = \frac{a}{2} : a : \frac{\sqrt{5}a}{2} = 1 : 2 : \sqrt{5}$

$\therefore \alpha = 2$ and $\beta = \sqrt{5}$ $\quad\therefore \alpha^2 + \beta^2 = 9$

19. **(c)** Let $I_1 = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt$

Put $t = \sin^2 u \Rightarrow dt = 2\sin u\cos u\, du$

$\Rightarrow dt = \sin 2u\, du$

$\therefore I_1 = \int_0^x u\sin 2u\, du$

Let $I_2 = \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\ dt$

Put $t = \cos^2 v \Rightarrow dt = -2\cos v \sin v dv$

$\Rightarrow dt = -\sin 2v\, dv$

$\therefore I_2 = \int_{\frac{\pi}{2}}^{x} v(-\sin 2v)dv = -\int_{\frac{\pi}{2}}^{x} v\sin 2v dv$

$= -\int_{\frac{\pi}{2}}^{x} u\sin 2u du$ [change of variable]

$\therefore I = I_1 + I_2 = \int_0^x u\sin 2u du - \int_{\frac{\pi}{2}}^{x} u\sin 2u du$

$$= \int_0^{\frac{\pi}{2}} u\sin 2u du + \int_{\frac{\pi}{2}}^{x} u\sin 2u du - \int_{\frac{\pi}{2}}^{x} u\sin 2u du$$

$$= \int_0^{\frac{\pi}{2}} u\sin 2u du = \frac{\pi}{4} \quad \text{[Integrate by parts]}$$

20. **(b)** For non-trivial solution,

$$\Delta = \begin{vmatrix} \sqrt{2}a & \sin B & \cos B \\ 1 & \cos B & \sin B \\ -1 & \sin B & -\cos B \end{vmatrix} = 0$$

$\Rightarrow a\sqrt{2}[-\cos^2 B - \sin^2 B] - \sin B[-\cos B + \sin B]$

$+ \cos B[\sin B + \cos B] = 0$

$\Rightarrow -a\sqrt{2} + \sin 2B + \cos 2B = 0 \Rightarrow a \in [-1, 1]$

21. **(0.5)** Let A be the event such that sum is Rs. 20 or more

$\therefore P(1) = 1 - P(\text{Total value is} < 20)$

$$= 1 - \frac{{}^6C_2 - {}^2C_2}{{}^8C_2} = 1 - \frac{14}{28} = 1 - \frac{1}{2} = \frac{1}{2}$$

8 → 1,1 ; 5,5 ; 10,10 ; 20,20

22. **(2008)** $\theta_1 + \theta_2 = \frac{\pi}{2}$

$$\therefore I = \int_{\theta_1}^{\theta_2} \frac{d\theta}{1+\tan\left(\frac{\pi}{2}-\theta\right)} = \int_{\theta_1}^{\theta_2} \frac{\tan\theta\, d\theta}{1+\tan\theta}$$

and also $I = \int_{\theta 1}^{\theta 2} \frac{d\theta}{1+\tan\theta}$

$$\therefore 2I = \int_{\theta_1}^{\theta_2} d\theta = \theta_2 - \theta_1 = \frac{1002\pi}{2008} \Rightarrow I = \frac{501\pi}{2008}$$

Hence, $K = 2008$.

23. **(18)** $\frac{{}^nC_k}{{}^nC_{k+1}} = \frac{1}{2} \Rightarrow \frac{n!}{k!(n-k)!}\frac{(k+1)!(n-k-1)!}{n!} = \frac{1}{2}$

or $\frac{k+1}{n-k} = \frac{1}{2}$

$2k + 2 = n - k$

$n - 3k = 2$ .... (1)

Similarly, $\frac{{}^nC_{k+1}}{{}^nC_{k+2}} = \frac{2}{3}$

$$\frac{n!}{(k+1)!(n-k-1)!}\cdot\frac{(k+2)!(n-k-2)!}{n!} = \frac{2}{3}$$

$\frac{k+2}{n-k-1} = \frac{2}{3}$

$3k + 6 = 2n - 2k - 2$

$2n - 5k = 8$ .... (2)

From (1) and (2)

$n = 14$ and $k = 4$

$\therefore n + k = 18$

24. **(12)** Let the observations be x_1, x_2, x_3, x_4, x_5 and x_6, so

their mean $\bar{x} = \frac{\sum_{i=1}^{6} x_i}{6} = 8 \Rightarrow \sum_{i=1}^{6} x_i = 48$

On multiplying each observation by 3, we get the new observations as $3x_1, 3x_2, 3x_3, 3x_4, 3x_5$ and $3x_6$.

Now, their mean $= \bar{x} = \frac{\sum_{i=1}^{6} 3x_i}{6} = \frac{3\times 48}{6} = 24$

Variance of new observations

$$= \frac{\sum_{i=1}^{6}(3x_i - 24)^2}{6} = \frac{3^2\sum_{i=1}^{6}(x_i - 8)^2}{6}$$

$= \frac{9}{1} \times$ Variance of old observations $= 9 \times 4^2 = 144$

Thus, standard deviation of new observations

$= \sqrt{\text{Variance}} = \sqrt{144} = 12$

25. **(0.75)** $\lim_{x\to 0} \frac{(\tan x)^{3/2}[1-(\cos x)^{3/2}]}{x^{3/2}.x^2}$.

$$= 1 \times \lim_{x\to 0} \frac{1-\cos^3 x}{x^2}\cdot\frac{1}{1+(\cos x)^{3/2}}$$

$$= \frac{1}{2}\cdot\frac{1}{2}(1+\cos x+\cos^2 x) = \frac{3}{4}$$